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SOFT-PHOTON EMISSION APPROACH TO THE DECAYS $\omega, \varphi, \rho \rightarrow 2\pi\gamma$

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SUMMARY

The hypothesis of PCTC for the electromagnetic current is used to calculate the rates of the decays $\omega, \varphi \rightarrow 2\pi\gamma$ and $\rho \rightarrow 2\pi\gamma$.

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In this paper we apply the partial conservation of tensor current (PCTC) [1,2] hypothesis for the electromagnetic current to discuss the decays ω, φ, $\rho \rightarrow 2\pi\gamma$. Such assumptions have already led to a successful explanation of the decays $\eta - \pi^0\gamma\gamma$ and $\eta \rightarrow \gamma\gamma$[3] whose branching ratio is otherwise difficult to explain, say, in a simple pole-model calculation [4].

We assume the PCTC for the e.m. current in the form

$$i\partial^\lambda T^{e.m.}_{\lambda\mu}(x) = a j^{e.m.}_\mu(x) ,$$

where $T^{e.m.}_{\lambda\mu} = T^3_{\lambda\mu} + (1/\sqrt{3})T^8_{\lambda\mu}$ and $j^{e.m.}_\mu = j^3_\mu + (1/\sqrt{3})j^8_\mu$; $T^i_{\lambda\mu}$ and j^i_μ are the tensor and vector currents, respectively, contained in the U_{12} algebra and $i = 0, 1, \ldots, 8$ are the SU_3 indices. The constant "a" is given by $a = m_\rho/\sqrt{2}$ [2, 3, 5], where m_ρ is the ρ-meson mass.

We will, in what follows, apply the above form of PCTC only for non-diagonal matrix elements. In this case the difficulties, (for example the vanishing of the charge operator), associated with using it for diagonal matrix elements, as discussed by Ademollo et al.[2] do not appear.

The matrix element for the decay $\omega \rightarrow 2\pi\gamma$ can be written as

$$\langle\pi(q_1)\,\pi(q_2)\,\gamma(k)|\omega(P)\rangle = \frac{i^2\, e}{(2\pi)^3}\cdot\frac{\varepsilon^\mu(k)\,\omega^\nu(P)}{\sqrt{4k_0\, P_0}}\cdot\frac{i}{a}\cdot$$

$$\iint d^4x d^4y \exp\left[i(Py - kx)\right] \langle\pi(q_1)\pi(q_2)|T(\partial^\mu T^{e.m.}_\mu(x) j^{(\omega)}_\nu(y))|0 \quad ,$$

where ε^μ and ω^μ represent the polarization four-vectors of the photon and ω respectively [6]. Using the formal identity

$$\frac{\partial}{\partial x_\lambda} T(T_{\lambda\mu}(x) j_\nu(y)) = T(\partial^\lambda T_{\lambda\mu}(x) j(y)) + \delta(x_o - y_o)\left[T_{o\mu}(x),\ j_\nu(y)\right],$$

we rewrite it, in the soft-photon limit

$$\langle \pi_1 \pi_2 \gamma | \omega \rangle = - \frac{i^2\ e}{(2\pi)^3} \cdot \frac{\varepsilon^\mu\, \omega^\nu}{\sqrt{4 k_o\ P_o}} \cdot$$

$$\cdot\ \frac{i}{a} \iint d^4x d^4y\ \exp\left[i(Py - kx)\right] \delta(x_o - y_o) \langle \pi_1 \pi_2 \left| \left[T_{0\mu}^{e.m.}(x), j_\nu^{(\omega)}(y)\right] \right| 0 \rangle.$$

The equal-time commutator may be evaluated, say, in the quark model [7] to obtain [8, 9] the invariant amplitude in the form [10]

$$M(\omega \longrightarrow 2\pi\gamma) = - i^2 e(\varepsilon . \omega)\ \frac{1}{a} \langle \pi_1 \pi_2 \left| \frac{1}{3} \left(\sqrt{\frac{2}{3}}\ s^o(0) + \sqrt{\frac{1}{3}}\ s^8(0) \right) \right| 0 \rangle ,$$

where S^i are the U_{12} scalar densities.

Consider next the decay $\sigma \longrightarrow \gamma\gamma$. The matrix element is written as

$$\langle \gamma(k)\, \gamma(k')|\sigma(P)\rangle = \frac{i^2\ e^2}{(2\pi)^3} \cdot \frac{\varepsilon^\mu(k)\ \varepsilon^\nu(k')}{\sqrt{4\ k_o\ k_o'}}$$

$$\cdot\ \frac{i}{a} \iint d^4x\ d^4y\ \exp\left[ikx + ik'y\right] \langle 0 \left| T(\partial^\lambda T_{\lambda\mu}^{e.m.}(x)\ j_\nu^{e.m.}(y)) \right| \sigma \rangle,$$

where again only nondiagonal matrix elements of $\partial^\lambda T_{\lambda\mu}^{e.m.}$ contribute. Using the above identity and taking the soft-photon limit we obtain for the invariant amplitude

$$M(\sigma \longrightarrow \gamma\gamma) = i^2 e^2 (\varepsilon . \varepsilon')\ \frac{2}{3a} \langle 0 | \left(2\sqrt{\frac{2}{3}}\ s^o(0) + \sqrt{\frac{1}{3}}\ s^8(0) + s^3(0) \right) | \sigma \rangle.$$

Now we will make the ansatz so that the scalar densities can be identified with the corresponding current of the nonet of scalar σ-particles [11]

$$S^{1,8} = dJ^{\sigma_{1,8}} ,$$

where "d" is a dimensionless constant. The invariant amplitudes can then be written as

$$M(\omega \rightarrow 2\pi\gamma) = - i^2 e(\varepsilon \cdot \omega) \frac{d}{3a}\left[\sqrt{\frac{2}{3}} M(\sigma^1 \rightarrow 2\pi) + \sqrt{\frac{1}{3}} M(\sigma^8 \rightarrow 2\pi)\right],$$

$$M(\sigma \rightarrow 2\gamma) = i^2 e^2 (\varepsilon \cdot \varepsilon') \frac{d}{a}\cdot\frac{2}{3}\left[2\sqrt{\frac{2}{3}} M(\sigma \rightarrow \sigma^1) + \right.$$

$$\left. + \sqrt{\frac{1}{3}} M(\sigma \rightarrow \sigma^8) + M(\sigma \rightarrow \sigma^3)\right],$$

where σ^1, σ^3, σ^8 denote the scalar singlet and the I = 1 and I = 0 members of the octet and where we introduce the following invariant amplitudes:

$$M(\sigma^i \rightarrow 2\pi) = (2\pi)^3 \sqrt{4q_{10}q_{20}} \langle \pi(q_1)\pi(q_2)|J^{\sigma_i}(0)|0\rangle,$$

$$M(\sigma \rightarrow \sigma^i) = (2\pi)^{3/2} \sqrt{2P_0} \langle 0|J^{\sigma_i}(0)|\sigma(P)\rangle .$$

The amplitudes for φ, $\rho \rightarrow 2\pi\gamma$ are similarly derived:

$$M(\varphi \rightarrow 2\pi\gamma) = -i^2 e(\varepsilon \cdot \varphi) \frac{d}{a}\cdot\frac{2}{3}\left[\sqrt{\frac{1}{3}} M(\sigma^1 \rightarrow 2\pi) - \sqrt{\frac{2}{3}} M(\sigma^8 \rightarrow 2\pi)\right],$$

$$M(\rho^0 \rightarrow 2\pi\gamma) = -i^2 e(\varepsilon \cdot \rho) \frac{d}{a}\left[\sqrt{\frac{2}{3}} M(\sigma^1 \rightarrow 2\pi) + \sqrt{\frac{1}{3}} M(\sigma^8 \rightarrow 2\pi)\right],$$

$$M(\rho^\pm \rightarrow 2\pi\gamma) = - i^2 e(\varepsilon \cdot \rho) \frac{d}{a}\cdot\frac{1}{3} M(\sigma^\pm \rightarrow 2\pi) = O(e^3)$$ [4, 10].

We note the following relations for the invariant amplitudes in the soft-photon limit:

$$M(\sigma^3 \to \gamma\gamma) = \frac{1}{\sqrt{3}} M(\sigma^8 \to \gamma\gamma) = 2\sqrt{\frac{2}{3}} M(\sigma^1 \to \gamma\gamma) ,$$

$$M(\omega \to 2\pi\gamma) = \frac{1}{3} M(\rho^0 \to 2\pi\gamma) .$$

Defining $M(\sigma^8 \to \pi^+ + \pi^-) = g_s$ and $M(\sigma^1 \to \pi^+ + \pi^-) = g_1$ and assuming the nonet model $g_1/g_8 = \sqrt{2}$ we find $M(\varphi \to 2\pi\gamma) = 0$. For (physical) isoscalar σ-meson with the projection sin on the SU octet we have [12]

$$M(\sigma \to \pi^+ + \pi^-) = g_1 \cos\varphi + g_8 \sin\varphi = g_s(\sqrt{2}\cos\varphi + \sin\varphi) = g_{\sigma\pi^+\pi^-}$$

where $g^2_{\sigma\pi^+\pi^-} = 1.31\ GeV^2$ for $m_\sigma = 375$ MeV, $\Gamma_\sigma = 70$ MeV and $\varphi \simeq -$

Using the above invariant amplitudes we can calculate the widths of the decays. However we must first recover the gauge invariance [3], obtaining

$$M(\sigma \to \gamma\gamma) = -\frac{d}{a} \cdot \frac{e^2}{3i} \left[2\sqrt{\frac{2}{3}} \cos\varphi + \frac{1}{\sqrt{3}} \sin\varphi \right] F^{\mu\nu} F'_{\mu\nu} ,$$

$$M(\omega(q) \to \pi^+\pi^-\gamma(k)) = \frac{dg_8}{a} \cdot \frac{e\sqrt{3}}{6i} \cdot \frac{F^{\mu\nu} q_\mu \omega_\nu}{q.k} ,$$

with similar relations for the other cases.

The ratio $\Gamma(\omega \to \pi^+\pi^-\gamma)/\Gamma(\sigma \to \gamma\gamma)$ can be calculated independent of the a and d values, and is found to be $0.41 \cdot 10^{+2}$.

Fil'kov, using dispersion theory methods [13], has related the product $\Gamma(\sigma \to \pi\pi)\ \Gamma(\sigma \to \gamma\gamma)$ to the widths of $\omega \to \pi\gamma$ and $\rho \to \pi\gamma$. For our mass for σ we find $\Gamma(\sigma \to \pi\pi)\ \Gamma(\sigma \to \gamma\gamma) =$ $= 0.81 \cdot 10^{-2}\ \mathrm{MeV}^2$. Assuming that σ decays predominantly into two pions, say $\Gamma(\sigma \to 2\pi) \simeq 0.9\ \Gamma_\sigma$ we get $\Gamma(\sigma \to \gamma\gamma) = 1.28 \cdot 10^{-4}$ MeV (which leads to the value for $d \simeq 3.8$).

With this value we can calculate the $\omega \to 2\pi\gamma$ and $\rho \to 2\pi\gamma$ widths ($\Gamma(\varphi \to 2\pi\gamma) = 0$ with nonet hypothesis), which are found to be

$$\Gamma(\omega \to \pi_+\pi_-\gamma) = 5.2 \cdot 10^{-3}\ \mathrm{MeV}\ ,$$
$$\Gamma(\rho^0 \to \pi_+\pi_-\gamma) = 4.7 \cdot 10^{-2}\ \mathrm{MeV}\ ,$$

not inconsistent with the meagre experimental data available at present [14, 15].

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2. M. Ademollo, R. Gatto, S. Longhi and G. Veneziano: Phys. Lett. 22, 521 (1966); Phys. Rev. 153, 1623 (1967).
3. L. Maiani and G. Preparata: Nuovo Cimento, 48A, 550 (1967).
4. G. Oppo and S. Oneda: University of Maryland Preprint, 1967.
5. G. Costa, C. A. Savoy and A. H. Zimerman: Nuovo Cimento, 47, 319 (1967).
6. It is clear that the intermediate states involved here introduce only non diagonal matrix elements of the divergence of the tensor current.
7. M. Gell-Mann: Phys. Rev., 125, 1067 (1962); Physics, 1, 63 (1964).
8. In the quark model

$$T^{e.m.}_{\mu\nu} = \frac{i}{2}\bar{q}\left(\lambda^3 + \frac{\lambda^8}{\sqrt{3}}\right)\sigma_{\mu\nu}\, q, \qquad j^{\omega}_{\nu} = \sqrt{\frac{2}{3}}\, j^{\omega_0}_{\nu} +$$

$$+ \sqrt{\frac{1}{3}}\, j^{\omega_8}_{\nu} = \frac{i}{2}\bar{q}\left(\sqrt{\frac{2}{3}}\,\lambda^0 + \sqrt{\frac{1}{3}}\,\lambda^8\right)\gamma_{\nu}\, q ,$$

giving

$$\left[T^{e.m.}_{o\mu}(x),\, j^{\omega}_{\nu}(0)\right]\delta(x_o) = \frac{1}{3}\left(g_{\mu\nu} - g_{o\nu}\, g_{o\mu}\right)\left[\sqrt{\frac{2}{3}}\, S^o(0) + \right.$$

$$\left. + \sqrt{\frac{1}{3}}\, S^8(0) + 3S^3(0)\right]\delta^4(x) ,$$

where

$$S^i(x) = \frac{i}{2}\left(\bar{q}(x)\,\lambda^i\, q(x)\right)$$

represent the scalar densities. We drop the term $g_{o\nu}\, g_{o\mu}$, which corresponds to neglecting the infinite-mass contributions. See, for example, ref. [9]. We neglect Schwinger terms.

9. C. G. Bollini, J. J. Giambiagi and J. Tiomno: ICTP, Trieste, preprint IC/67/25, to be published in Nuovo Cimento.

10. $\langle \pi_1 \pi_2 | S^3 | 0 \rangle$ vanishes due to Bose statistics for pions.

11. Such an ansatz for the pseudoscalar densities being proportional to the current densities of pseudoscalar octet has been made, for example, by J. W. Moffat: Phys. Lett., 23, 148 (1966) and Asymptotic Theorems Based on Current Algebra and Quark Model University of Toronto Preprint, 1966. See also M. Lévy: Current and Symmetry Breaking, University of Paris Preprint, 1967.

12. V. Silvestrini, L. Maiana and G. Preparata: Nuovo Cimento, <u>48A</u>, 555 (196

13. L. V. Fil'kov: Sov. Phys. JETP Lett., <u>5</u>, 153 (1967).

14. The pole-model calculation using the o intermediate state gives $\Gamma(\omega \to \pi^+ \pi^- \gamma) = 2.10^{-5}$ MeV. See P. Singer: Phys. Rev. <u>128</u>, 2789. The pole model with intermediate o-meson

$$M = \frac{g_{\sigma\pi^+\pi^-} g_{\omega\sigma\gamma}}{\left[(P-k)^2 - m_\sigma^2\right]} \omega^\mu (P) P^\lambda \, (\varepsilon_\mu k_\lambda - \varepsilon_\lambda k_\mu)$$

leads to

$$\Gamma(\omega \to \pi^+ \pi^- \gamma) = \frac{g^2_{\sigma\pi^+\pi^-} g^2_{\omega\sigma\gamma}}{12(2\pi)^2} M_\omega (0.22)$$

giving for the $\omega\sigma\gamma$ coupling constant the value $g_{\omega\sigma\gamma} \sim 2.1 \cdot 10^{-4}$ MeV^{-1}.

15. Recently some calculations seem to require $\Gamma(\sigma \to \gamma\gamma)$ to be comparable to $\Gamma(\sigma \to \pi\pi)$. In this case the value of "d" comes out to be larger and consequently the width $\Gamma(\omega - 2\pi\gamma)$ becomes too large to be consistent with experiment in our model. See, for example, ref. [4] and I. R. Lapidus: Nuovo Cimento, 46A, 668 (1966).