NOTAS DE FÍSICA

VOLUME VI

Nº 5

# PHOTON DECAY OF HYPERONS

by

Prem Prakash and A. H. Zimerman

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1960

Notas de Física - Volume VI - Nº 5

# PHOTON DECAY OF HYPERONS

Prem Prakash*

Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil

and

A. H. Zimerman

Instituto de Física Teórica, São Paulo, Brazil

(Received June 25th, 1960)

SUMMARY: The weak photon decay of hyperon in which the initial hyperon decays into a baryon and a photon is examined. The lowest order perturbation theory estimates on the three parameters required to describe the angular distribution of the decay products is made assuming weak derivative coupling (vector minus axial vector) for hyperon nucleon pion interaction together with the $|\Delta\vec{I}| = \frac{1}{2}$ rule in the isotopic spin space. The theoretical rate of decay seems to be higher than the experimentally observed rate.

---

* Supported in part by the Conselho Nacional de Pesquisas of Brazil.

## 1. INTRODUCTION

The two-body pionic (and three-body leptonic) decays of hyperons are experimentally known. We study here another less frequent two body mode in which a hyperon decays into a photon and another baryon with a change of strangeness[1], first studied by Kawaguchi and Nishijima[2],

$$y \longrightarrow y' + n$$

Such decays should compete favourably with the leptonic decays as the recent experiment of Quareni and Vignudelli[3] shows. They observe three cases of $\Sigma^+$ into a proton and a photon with a branching ratio, compared to the neutral pion decay of the order $\sim 1\%$.[3]

The above mentioned photon decay proceeds, assuming the pion mode of decay to be primary, through a combination of weak and strong interactions, with an intermediate pion-baryon system. A perturbation theory estimate was done recently by us using intermediate vector meson[4] and by Behrends[5] assuming a weak pion-baryon interaction of scalar and pseudoscalar type $(\bar{\psi}_2 (a + b\gamma_5) \psi_1 \phi)$ and the result of the vector and axial vector coupling case was derived assuming an equivalence theorem (footnote 9 in ref. (4)). However, Umezawa et. al.[6] have shown that the (S + P) interaction does not give the correct asymmetry parameters in the pionic decays. Moreover the equivalence theorem does not seem to be justified when the pion-baryon pair is in the intermediate virtual state.

The recent experiments on hyperon decays seem to support fairly well the hypothesis of direct weak pion hyperon interaction of the vector and axial type[7] (derivative coupling)[8]:

$$\bar{\psi}_1 \gamma_\mu \ (1 + \lambda \gamma_5) \psi_2 \ \frac{\partial \varphi}{\partial x_\mu}$$

with the additional assumption that the $|\Delta I| = \frac{1}{2}$ rule[9] is valid in the isotopic spin space. The lowest order perturbation calculation with these assumptions is presented in section (3) and the resulting branching ratios under various constraints on the coupling constants are given in section (4). Before proceeding to perturbation calculation we discuss in section (2) the general form of the matrix element for the process considering the Lorentz invariance and the electromagnetic gauge invariance and the angular distribution in the case of polarized particles is discussed.

## 2. PHENOMENOLOGICAL MATRIX ELEMENT: ANGULAR DISTRIBUTION OF DECAY PRODUCTS:

The most general form of the matrix element in the momentum space is easily written down, requiring the invariance under the Proper Lorentz Group and under the electromagnetic gauge transformation. The only gauge invariant combination possible using the four maomentum $k_\mu$ and the four polarization $\epsilon_\mu$ of one free photon is $f_{\mu\nu} = (\epsilon_\mu k_\nu - \epsilon_\nu k_\mu)$. The antisymmetric tensors available from the baryon current with the initial four momentum $n_1$ and the final four momentum $n_2$ are $(\bar{u}_2 u_1)(n_{1\mu} n_{2\nu} - n_{1\nu} n_{2\mu})$

$\bar{u}_2 \sigma_{\mu\nu} u_1$ and $\bar{u}_2 \gamma_5 \sigma_{\mu\nu} u_1$. Hence the matrix element has the general form

$$T = \bar{u}^{(2)}(A + B\gamma_5)\,(k.\gamma)\,(\epsilon.\gamma)\,u^{(1)} \qquad (1)$$

where A and B are unknown invariant functions of the three four-momenta of which only two are independent ($n_1 = n_2 + k$ ; $k^2 = 0$).

If the final state interactions are ignored in the photon decay the parity conservation requires (AB) = 0, the charge conservation invariance requires Rl $(AB^*) = 0$ and time reversal invariance (or CP invariance) requires Im $(AB^*) = 0$.

However, taking into account the final state interactions (and regarding the photon decay as a secondary process - the viewpoint we will adopt in this paper (section 3) e.g. the pion mode of the hyperon decay be regarded primary instead) these statements are no more true. In fact the condition of CP (or T) invariance is now given by (Appendix I):

$$\bar{u}_2[(A-A^*) + (B-B^*)\gamma_5]\,(k.\gamma)(\epsilon.\gamma)u^{(1)} = -S\langle\pi N|\gamma p\rangle^*_{\substack{out\\ in}}\langle\pi N|H_W|\Sigma\rangle_{\substack{out\\ in}}$$

where $\langle \pi N|\gamma p\rangle$ and $\langle \pi N|H_W|\Sigma\rangle$ are the amplitudes of photoproduction of pion and pion-decay of $\Sigma$ respectively.

The angular distribution in the case of circularly polarized photons derived from (1) has the following form in the rest frame of the initial baryon and with a choice of the gauge such that $\epsilon.n_1 = 0$:

$$|T|^2 \sim \left[ |A|^2 + |B|^2 + (-1)^\rho \, 2 \, \mathrm{R}\ell \, (BA^*) \right] \cdot$$

$$\cdot \left[ \left( \frac{M}{E} (\underline{k}_1 \cdot \underline{S}_1) (\underline{k}_1 \cdot \underline{S}_2) - 1 \right) + (-1)^\rho \left( \underline{k}_1 \cdot \underline{S}_1 - \frac{M}{E} \underline{k}_1 \cdot \underline{S}_2 \right) \right]$$

where M denotes the mass of the emitted baryon and E its energy and $\underline{k}_1$, $\underline{S}_1$ and $\underline{S}_2$ denote the unit 3-vectors along the directions of the photon momentum, of the spin of the initial baryon and the spin of the final baryon respectively. $\rho = +1$ for the right circular polarization while $\rho = -1$ for the left circular polarization of the photon[10].

In the case of linearly polarized photons the distri bution is given by:

$$|T|^2 \sim \left( |A|^2 + |B|^2 \right) \left[ \frac{M}{E} \underline{k}_1 \cdot \underline{S}_1 \;\; \underline{k}_1 \cdot \underline{S}_2 - 1 \right]$$

$$+ \; 2 \, \mathrm{R}\ell \, (BA^*) \left[ \underline{k}_1 \cdot \underline{S}_1 - \frac{M}{E} \underline{k}_1 \cdot \underline{S}_2 \right]$$

$$+ \left( |A|^2 - |B|^2 \right) \left[ \underline{k}_1 \cdot \underline{S}_1 \;\; \underline{k}_1 \cdot \underline{S}_2 - \underline{S}_1 \cdot \underline{S}_2 + 2 \, \underline{\epsilon} \cdot \underline{S}_1 \, \underline{\epsilon} \cdot \underline{S}_2 \right]$$

$$+ \; 2 \, \mathrm{Im} \, (BA^*) \left[ \underline{k}_1 \cdot \underline{S}_1 \times \underline{S}_2 - 2 \, \underline{\epsilon} \cdot \underline{S}_2 \, \underline{\epsilon} \cdot \underline{k}_1 \times S_1 \right]$$

where $\underline{\epsilon}$ is the direction of the linear polarization $(\underline{\epsilon} \cdot k) = 0$).

If the photon polarization and the spin of the final baryon are not observed we obtain the usual forward backward asymmetry for the emission of the photon relative to the direction of the spin of the initial baryon due to the nonconservation of parity[11]. The angular distribution for the photon is

$$N \propto (1 - \alpha \underline{k}_1 \cdot \underline{S}_1) \; d\Omega_k$$

where $d\Omega_k$ is the solid angle of emission for the photon momentum and the asymmetry parameter $\alpha$ is given by

$$\alpha = \frac{2 \, R\ell \, (BA^*)}{(|A|^2 + |B|^2)}$$

It is easily shown that we also have

$$\alpha = (N_L - N_R)/(N_L + N_R)$$

where $N_L$ and $N_R$ are the number of left and right circularly polarized photons respectively when the initial baryon is unpolarized and the final baryon may be emitted with arbitrary spin orientation.

The parameter ( $|A|^2 + |B|^2$ ) is related to the total transition probability of the decay (section (4)). The only other independent parameter is ( $|A|^2 - |B|^2/ (|A|^2 + |B|^2$ ) or $- 2 \, Im \, (BA^*)/ (|A|^2 + |B|^2)$ since the sum of the squares of $\alpha$ and these two is unity. The linear polarization of the photon together with the spins of both the baryons must be detected to observe this term.

3. PERTURBATION THEORY ESTIMATES: INTERACTION HAMILTONION AND MATRIX ELEMENT:

Assuming charge independence for the strong parity conserving pion-baryon interaction, the interaction Hamiltonian (we assume $\Sigma$ and $\Lambda$ have the same parity) is ($\hbar = c = 1$)

$$H_{\pi N} = G_N \left\{ \sqrt{2}(\bar{p}\gamma_5 n \varphi^* + \bar{n}\gamma_5 p\varphi) + (\bar{p}\gamma_5 p - \bar{n}\gamma_5 n)\varphi_0 \right\} + h.c$$

$$H_{\pi\Sigma} = G_\Sigma \left\{ (\bar{\Sigma}^0 \gamma_5 \Sigma^- - \bar{\Sigma}^+ \gamma_5 \Sigma^0)\varphi^* + (\bar{\Sigma}^- \gamma_5 \Sigma^0 - \bar{\Sigma}^0 \gamma_5 \Sigma^+)\varphi \right.$$

$$\left. + (\bar{\Sigma}^+ \gamma_5 \Sigma^+ - \bar{\Sigma}^- \gamma_5 \Sigma^-)\varphi_0 \right\} + h.c$$

$$H_{\pi\Lambda} = G_\Lambda \left\{ \bar{\Sigma}^+ \gamma_5 \Lambda \varphi^* + \bar{\Sigma}^- \gamma_5 \Lambda \varphi + \bar{\Sigma}^0 \gamma_5 \Lambda \varphi_0 \right\} + h.c$$

where $\varphi$ is charged pion $\pi^-$ field operator, $\varphi_0$ the neutral pion field and the baryon field operators are represented by their own symbols.

The strangeness and parity non-conserving weak hyperon-nucleon-pion interaction will be assumed to be primary and the coupling to be derivative. The interaction, assuming the $|\Delta\vec{I}| = \frac{1}{2}$ rule in the isotopic spin space, can be written as follows[6] ($\hbar = c = 1$).

$$H_\Lambda = \sqrt{\frac{1}{3}}\ \frac{ig_\Lambda}{m_\pi} \left\{ \sqrt{2}\ \bar{p}\gamma_\mu(1+\epsilon\gamma_5)\Lambda \frac{\partial\varphi}{\partial x_\mu} - \bar{n}\gamma_\mu(1+\epsilon\gamma_5)\Lambda\frac{\partial\varphi_0}{\partial x_\mu} \right\} + h.c$$

$$H_{\Sigma} = \sqrt{\frac{1}{6}}\ \frac{ig_{\Sigma}}{m_{\pi}} \left\{ \sqrt{2}\,\bar{p}\,\gamma_{\mu}(1+\epsilon\gamma_5)\,\Sigma^{+}\frac{\partial\varphi_0}{\partial x_{\mu}} + \bar{n}\,\gamma_{\mu}(1+\epsilon\gamma_5)\,\Sigma^{+}\frac{\partial\varphi}{\partial x_{\mu}} \right.$$

$$\left. - \sqrt{2}\,\bar{p}\,\gamma_{\mu}(1+\epsilon\gamma_5)\,\Sigma^{0}\frac{\partial\varphi^{*}}{\partial x_{\mu}} - \bar{n}\,\gamma_{\mu}(1+\epsilon\gamma_5)\,\Sigma^{-}\frac{\partial\varphi^{*}}{\partial x_{\mu}} \right\} + h.c$$

$$+ \sqrt{\frac{1}{3}}\ \frac{ig_{\Sigma}'}{m_{\pi}} \left\{ \bar{n}\,\gamma_{\mu}(1+\epsilon'\gamma_5)\,\Sigma^{0}\frac{\partial\varphi_0}{\partial x_{\mu}} + \bar{n}\,\gamma_{\mu}(1+\epsilon'\gamma_5)\,\Sigma^{+}\frac{\partial\varphi}{\partial x_{\mu}} \right.$$

$$\left. + \bar{n}\,\gamma_{\mu}(1+\epsilon'\gamma_5)\,\Sigma^{-}\frac{\partial\varphi^{*}}{\partial x_{\mu}} \right\} + h.c$$

where $\epsilon = -1.2$ and we take $\epsilon' = -\epsilon$

The interaction with the electromagnetic field is:

$$H_{em} = ieA_{\mu}\left(\varphi^{*}\frac{\partial\varphi}{\partial x_{\mu}} - \frac{\partial\varphi^{*}}{\partial x_{\mu}}\varphi\right) - ie'\bar{\psi}\gamma_{\mu}\psi A_{\mu} + H'$$

where $H'$ are the electromagnetic interaction terms arising from the weak interaction Hamiltonian on making the replacements

$$\partial_{\mu}\varphi \longrightarrow (\partial_{\mu} - ieA_{\mu})\varphi \qquad \text{and } \partial_{\mu}\varphi^{+} \longrightarrow (\partial_{\mu} + ieA_{\mu})\ \varphi^{*}.$$

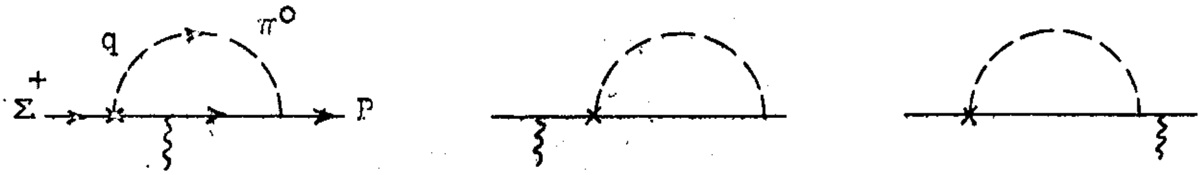
Here e represents the charge of the pion $\pi^{-}$ (field $\varphi$) while e' that of the particle represented by field $\psi$ ( $e'(\Sigma^{+}) = -e$).

In the lowest order perturbation theory two types of graphs for the photon decay of baryon are possible: one in which the photon can be emitted from the intermediate charged pion current and the other in which the photon can be emitted from the intermediate baryon current.

The matrix elements for the decay $\Sigma^+ \longrightarrow p + \gamma$ are as follows:

(a) Decay via Baryon current:

The appropriate graphs to be considered are




(a) (b) (c)

where X indicates the weak interaction vertex and the other pion baryon vertex is the strong interaction vertex. The corresponding matrix elements (apart from common numerical factors) in momentum space are:

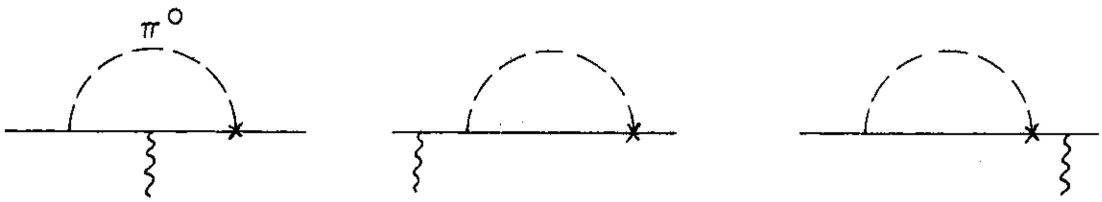
$$M_a = -i\bar{u}_2(n_2) \int d^4q \, \gamma_5 \frac{1}{m+i(n_2-q).\gamma} (\epsilon.\gamma) \frac{1}{m+i(n_1-q).\gamma} (q.\gamma) \frac{(1+\lambda\gamma_5)u_1(n_1)}{(q^2+\mu^2)}$$

$$M_b = -i\bar{u}_2 \int d^4q \; \gamma_5 \frac{1}{m+i(n_2-q).\gamma} (q.\gamma)(1+\lambda\gamma_5) \frac{1}{M_\Sigma + in_2.\gamma} (\epsilon.\gamma)u_1 \frac{1}{(q^2+\mu^2)}$$

$$M_c = -i\bar{u}_2 \int d^4q \;\; \epsilon\cdot\gamma \frac{1}{(m_p+in_1\gamma)} \gamma_5 \frac{1}{m+i(n_1-q)\cdot\gamma} \;\; q\cdot\gamma(1+\lambda\gamma_5)\; u_1 \frac{1}{q^2+\mu^2}$$

where $M_\Sigma$, $M_p$, m and $\mu$ are the masses of $\Sigma$, proton, the intermediate baryon and the pion respectively. k, $n_1$ and $n_2$ are the four momenta of the photon, $\Sigma$ and proton respectively and $\epsilon_\mu$ represents the four polarization of the photon emitted. The sum $M = (M_a + M_b + M_c)$ vanishes identically when $\epsilon \longrightarrow k$ showing that it is gauge invariant.

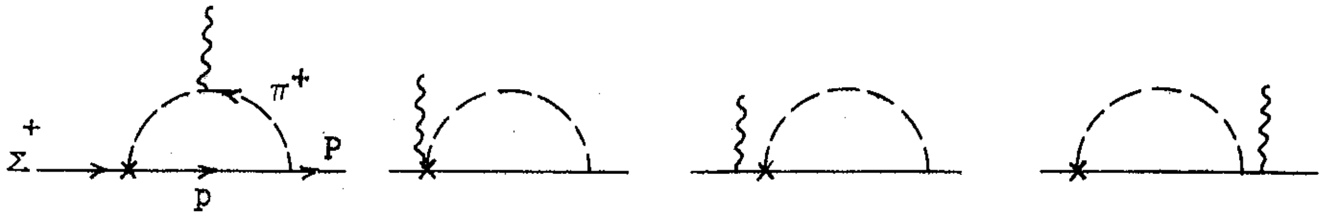
There is also possible another set of graphs in which the weak interaction vertex falls after the strong interaction vertex, e.g:




The contribution M' of the corresponding matrix element can be easily derived from those of the previous set by a change of variables of integration and the parameters involved (Appendix II).

b) Decay via Meson current

The graphs in this case are:




(a) (b) (c) (d)

and the matrix elements in the momentum space are:

$$N_a = \bar{u}_2 \int d^4p\, \gamma_5 \frac{1}{m+i\gamma.p} \frac{[(n_1-p).\gamma](1+\lambda\gamma_5)\, 2\epsilon.(p-n_1)}{[(p-n_1)^2+\mu^2][(p-n_2)^2+\mu^2]} u_1$$

$$N_b = \bar{u}_2 \int d^4p\, \gamma_5 \frac{1}{m+i\gamma.p} \epsilon.\gamma\, (1+\lambda\gamma_5)\, u_1 \frac{1}{[(p-n_2)^2+\mu^2]}$$

$$N_c = -i \int d^4p\, \bar{u}_2 \gamma_5 \frac{1}{m+i\gamma.p} [(n_2-p).\gamma](1+\lambda\gamma_5) \frac{1}{(M_\Sigma+in_2.\gamma)} \epsilon.\gamma u_1 \frac{1}{[(p-n_2)^2+\mu^2]}$$

$$N_d = -i \int d^4p\, \bar{u}_2\, \epsilon.\gamma \frac{1}{m_p+in_1.\gamma} \gamma_5 \frac{1}{(m+i\gamma.p)} [(n_1-p).\gamma]\, (1+\lambda\gamma_5)\, u_1 \frac{1}{[(p-n_1)^2+\mu^2]}$$

The sum $N = [N_a + N_b + N_c + N_d]$ is again gauge invariant. The contribution N' of the other corresponding set of matrix elements in which the weak interaction takes place after the strong interaction is easily derived from these as before.

The contributions from M (and M') as well as from N (and N') are finite and gauge invariant. Their explicit forms are given in the Appendix II.

## 3. TRANSITION PROBABILITY:

The complete S matrix element for the process is given by:

$$S = \frac{1}{(2\pi)^3} \sqrt{\frac{M_\Sigma M_p}{2k_o n_{10} n_{20}}} \, (G_N \, g_\Sigma) \;\; \delta^4(n_1 - n_2 - k) \left(\frac{T}{m_\pi}\right)$$

where

$$T = \bar{u}_2 \, (A + B\gamma_5) \, \not{k} \, \not{a} \, u_1 \qquad ; \; (\not{a} = a \cdot \gamma)$$

$$= \frac{1}{\sqrt{3}} \Big[ \rho_\Sigma \, (M'_\Sigma)_{\lambda=\epsilon} + (M_N)_{\lambda=\epsilon} - (1+\delta)(N_N)_{\lambda=\frac{\epsilon+\delta\epsilon'}{1+\delta}} + \rho_\Sigma \, (N'_\Sigma)_{\lambda=\epsilon} +$$

$$+ \sqrt{2}\rho' \rho_\Lambda \, (N'_\Lambda)_{\lambda=\epsilon} \Big]$$

Here $M'_\Sigma$, $M_N$, $N_N$, $N'_\Sigma$ and $N'_\Lambda$ are the matrix elements corresponding to the decay with the following intermediate states respectively:

$$\Sigma^{+} \Longrightarrow \Sigma^{+} + \pi^{0} \longrightarrow p$$

$$\longrightarrow p + \pi^{0} \Longrightarrow p$$

$$\Longrightarrow \Sigma^{0} + \pi^{+} \longrightarrow p$$

$$\Longrightarrow \Lambda^{0} + \pi^{+} \longrightarrow p$$

$$\longrightarrow n + \pi^{+} \Longrightarrow p$$

the photon may be emitted from any of the charged particle currents.

The parameters $\rho_{\Sigma}$ etc. are defined as follows:

$$\rho_{\Sigma} = \left(\frac{G_{\Sigma}}{G_N}\right) \qquad \rho_{\Lambda} = \left(\frac{G_{\Lambda}}{G_N}\right) \qquad \rho' = \left(\frac{g_{\Lambda}}{g_{\Sigma}}\right) \qquad \delta = \sqrt{2}\left(\frac{g_{\Sigma}'}{g_{\Sigma}}\right)$$

The transition probability per unit time $\omega$ for the decay is

$$\omega = K \quad S' \left| \frac{T}{m_{\pi}} \right|^{2}$$

where

$$K = \frac{1}{2^{10}\pi^{9}} \; e^{2} \, G_N^{\,2} \, g_{\Sigma}^{\,2} \left(1 - \frac{M_p^{\,2}}{M_{\Sigma}^{\,2}}\right) M_p$$

and

$$S' \, |T|^{2} = \frac{2(k \cdot n_1)^{2}}{M_p M_{\Sigma}} \left(|A|^{2} + |B|^{2}\right)$$

where S' indicates the summations over the spin states of the proton, the average over the spin states of $\Sigma$ and the sum over the two states of the photon polarizations.

The matrix elements in which the intermediate state is a nucleon (so that the mass difference between the initial and the final baryon is greater than a real pion mass) contribute also an imaginary part arising from the simple poles in the matrix element (Appendix II). This corresponds to the successive real process of baryon decay into a pion followed by the inverse pion photoproduction as discussed earlier.

In the following table are given the calculated estimates of the branching ratio R for $\Sigma^+ \longrightarrow p + \gamma$ compared to the neutral pion mode.

<u>Table:</u>[12] Branching ratios for $\Sigma^+ \rightarrow p + \gamma$ decay compared to the neutral pion mode of decay ($\Sigma^+ \rightarrow p + \pi^o$).

($m_\pi \simeq$ 138 Mev $\quad M_{\Lambda o}$ = 1115.45 Mev $\quad M_\Sigma$ = 1191.94 Mev

$M_N$ = 938 Mev $\quad G^2/4\pi \simeq 15$)

| $(\delta/\sqrt{2})$ | $\rho_\Sigma$ | $\rho_\Lambda \rho'$ | $R \times 10^2$ |
|---|---|---|---|
| 1 | 0 | 0 | 13 |
| 1 | 0 | -1 | 13 |
| 1 | 0 | 1 | 38 |
| 1 | -1 | 0 | 13 |

| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 38 |
| 1 | 1 | -1 | 13 |
| 1 | -1 | 1 | 12 |
| 1 | -1 | -1 | 7 |
| 1 | 1 | 1 | 90 |
| -1 | 0 | 0 | 20 |
| -1 | 0 | -1 | 26 |
| -1 | 0 | 1 | 38 |
| -1 | -1 | 0 | 27 |
| -1 | 1 | 0 | 37 |
| -1 | 1 | -1 | 15 |
| -1 | -1 | 1 | 20 |
| -1 | -1 | -1 | 50 |
| -1 | 1 | 1 | 100 |

The theoretical estimate is thus seen to be at least one order of magnitude higher than the yet observed experimental ratio. Definite conclusions can only be derived after an improved statistics of data is available.

:.:.:.:.:.:

The authors are grateful to professors J.Leite Lopes and J. Tiomno for many stimulating discussions and encouragement in the problem.

## APPENDIX I*: Time Reversal Invariance and the Effective Matrix Element.

We derive here the conditions on the invariant functions A and B imposed by the requirement of Time Reversal invariance on the interaction Lagrangian when the photon decay is regarded a secondary process taking place via the weak hyperon-nucleon-pion interaction; the strong pion-baryon interaction and the electromagnetic interaction. The matrix element for the photon decay can be written as

$$\langle p \gamma |_{out} \, H_W \, | \Sigma \rangle_{in}$$

where $|\Sigma\rangle_{in}$ and $|p\gamma\rangle_{out}$ are the incoming state of $\Sigma$ and the outgoing state of the photon-proton system respectively. The incoming state (outgoing state) coincides with the Heisenberg state corresponding to the strong and electromagnetic interactions at $t = -\infty$ $(+\infty)$. $H_W$ represents the weak strangeness nonconserving hyperon-nucleon-pion interaction. In the lowest order perturbation theory

$$\langle p |_{out} \, H_W \, |\Sigma\rangle_{in} = \bar{u}_r^{(2)}(p_2)\,(A + B\gamma_5)\, \not k \not \epsilon \; u_s^{(1)}(p_1)$$

where $u_r^{(i)}(p_i)$ is the Dirac spinor for the i-th particle of momentum $p_i$ and longitudinal spin $r = (\pm)$.

---

* The authors are indebted to Dr. S. W. Mac Dowell for clarification of the discussion in this Appendix.

Time reversal invariance requires

$$\langle \gamma p^T |_{out} H_W | \Sigma^T \rangle_{in} = \langle \Sigma |_{out} H_W | \gamma p \rangle_{in}$$

where $\langle \psi^T | = T | \psi \rangle$, T being the (Wigner's) time reversal operator[13]. The right hand side can be expressed as

$$\langle \Sigma |_{out} H_W | \gamma p \rangle_{in} \qquad \text{(since } H_W^\dagger = H_W\text{)}$$

$$= \langle \gamma p |_{in} H_W | \Sigma \rangle_{in}^*$$

Here we have used $|\Sigma\rangle_{out} = |\Sigma\rangle_{in}$ since insofaras strong and electromagnetic interactions are concerned $\Sigma$ is a stable one particle state.

For the $|\gamma p\rangle_{in}$ state, however, we have

$$|\gamma p\rangle_{in} = |\gamma p\rangle_{out} + S |\pi N\rangle_{out} \langle \pi N |_{out} \gamma p \rangle_{in}$$

keeping only the terms to first order in electric charge. S indicates the sum over all intermediate ($\pi$-N) states possible.

Whence

$$\langle \Sigma |_{out} H_W | \gamma p \rangle_{in} = \left[ \langle \gamma p |_{out} H_W | \Sigma \rangle_{in} + S \langle \pi N |_{out} \gamma p \rangle_{in}^* \langle \pi N |_{out} H_W | \Sigma \rangle_{in} \right]^*$$

or

$$\langle \gamma p |_{out} H_W | \Sigma \rangle_{in} = \langle p \gamma^T |_{out} H_W | \Sigma^T \rangle_{in}^* - S \langle \pi N |_{out} \gamma p \rangle_{in}^* \langle \pi N |_{out} H_W | \Sigma \rangle_{in}$$

In the lowest order perturbation theory the first term can be written as:

$$\langle \gamma p^T_{out} \mid H_w \mid \Sigma^T \rangle^*_{in} = \left[ \bar{u}^{(2)}_{-\gamma} (-\underline{p}_2) (A+B\gamma_5)(-\underline{k}\cdot\gamma + k_4\gamma_4)(-\underline{\epsilon}\cdot\underline{\gamma} + \epsilon_4\gamma_4) \; u^{(1)}_{-s} (-\underline{p}_1) \right]^*$$

$$= \bar{u}^{(2)}_r (\underline{p}_2) \gamma_4\gamma_5 \, C^{-1} \left\{ (A+B\gamma_5)(-\underline{k}\cdot\underline{\gamma} + k_4\gamma_4)(-\underline{\epsilon}\cdot\underline{\gamma} + \epsilon_4\gamma_4) \right\}^* C\gamma_5\gamma_4 u^{(1)}_s (\underline{p}_1)$$

$$= \bar{u}^{(2)}_r (\underline{p}_2) (A^* + B^*\gamma_5) \not{k} \not{\epsilon} \, u^{(1)}_s (\underline{p}_1)$$

$$; \; (\not{a} = a\cdot\gamma)$$

Thus

$$\bar{u}_r^{(2)} (\underline{p}_2) \left[ (A-A^*) + (B-B^*)\gamma_5 \right] \not{k} \not{\epsilon} \, u_s^{(1)} (\underline{p}_1) = -\textstyle\sum \langle \pi N_{out} | \gamma p_{in} \rangle^* \langle \pi N_{out} | H_w | \Sigma_{in} \rangle$$

It is then clear that Im $(AB^*)$ need not be zero in this case for time reversal invariance to hold. In fact the imaginary part of the matrix element can be related to the product of the amplitude of the pionic decay of $\Sigma$ and that for the photoproduction of pion.

## APPENDIX II: Explicit Form of the Matrix Elements.

The integration over the momentum variable in M,N etc. is done by the standard method (see for example the theory of Photons and Electrons by J. M. Jauch and F. Rohrlich; Addison Wesley 1955; Appendix A-5).

The contribution of M can be explicitly written as follows:

$$M = (M_1 + M_2 + M_3)$$

where $M_1$ represent the contribution from the actual finite terms in M and $M_2$ and $M_3$ are the finite contributions arising from the linear and the logarithmically divergent terms respectively. The total contribution M is finite and gauge invariant. The explicit expressions for $M_1$ etc. are:

$$M_1 = \pi^2\, \bar{u}_2\, (\epsilon.\gamma)(k.\gamma)\, [-M_\Sigma(-\lambda+\gamma_5)+M_p(-\lambda-\gamma_5)]u_1 . \frac{(m-M_p)(m^2-M_\Sigma^2)}{2k.n_1} .Y_{20}$$

$$M_2 = \pi^2\, \bar{u}_2\, (\epsilon.\gamma)(k.\gamma)\, \Big[M_\Sigma\{M_p(-\lambda+\gamma_5)-M_\Sigma(-\lambda-\gamma_5)\}$$

$$+ m\left\{M_p(-\lambda-\gamma_5) - M_\Sigma(-\lambda+\gamma_5)\right\}\Big]\, u_1 .$$

$$. \frac{1}{2k.n_1} . \left(t^2 Y_{30} + M_p{}^2\, Y_{40} + 2k.n_1\left\{Y_{31} - Y_{41}\right\} - \frac{5}{6}\right).$$

$$M_3 = \pi^2\, \bar{u}_2\, (\epsilon.\gamma)(k.\gamma)\Big\{\Big(- m\, [-M_\Sigma(-\lambda+\gamma_5) + M_p(-\lambda-\gamma_5)]$$

$$- M_\Sigma\, [-M_\Sigma(-\lambda-\gamma_5) + M_p(-\lambda+\gamma_5)]\Big)\, \frac{1}{2k.n_1} .$$

$$. \left(-\tfrac{1}{2} + t^2 Y_{20} + 2M_p^2\, Y_{30} + 2k.n_1 Y_{21} - 4k.n_1 Y_{31}\right)$$

$$+ \frac{mM_\Sigma}{2k.n_1}\left[2M_p M_\Sigma(-\lambda-\gamma_5)-(M_p^2 + M_\Sigma^2)(-\lambda+\gamma_5)\right]\, Y_{30}$$

$$+ \; m \left[ -M_{\Sigma}(-\lambda+\gamma_5) + M_p(-\lambda-\gamma_5) \right] (Y_{30} - Y_{31})$$

$$+ \frac{M_{\Sigma}^2}{2k \cdot n_1} \left[ 2M_p M_{\Sigma}(-\lambda+\gamma_5) - (M_p^2 + M_{\Sigma}^2)(-\lambda-\gamma_5) \right] Y_{30}$$

$$+ M_{\Sigma} \left[ M_p(-\lambda+\gamma_5) - M_{\Sigma}(-\lambda-\gamma_5) \right] (Y_{30} - Y_{31}) \Big\}$$

where

$$k = n_1 - n_2 \qquad 2k \cdot n_1 = M_p^2 - M_{\Sigma}^2 \qquad t^2 = (m^2 - \mu^2 - M_p{}^2)$$

$$Y_{nm} = \int_0^1 dx \int_0^1 dy \; \frac{x^n y^m}{b^2}$$

where

$$b^2 = \mu^2 + (m^2 - \mu^2)\, x + M_p^2\, x\,(x-1) + (M_{\Sigma}{}^2 - M_p{}^2) x (x-1) y$$

Similarly, for $N = N_1 + N_2 + N_3$ we have

$$N_1 = (+\pi^2)\bar{u}_2 \quad M_2 \left[ M_{\Sigma}(\mu^2 - M_p^2)(\lambda+\gamma_5) - M_p(\mu^2 - M_{\Sigma}^2)(\lambda-\gamma_5) \right] Y'_{20}$$

$$+ m\,(\mu^2 - M_{\Sigma}^2) \left[ -M_p(\lambda+\gamma_5) + M_{\Sigma}(\lambda-\gamma_5) \right] Y'_{20}$$

$$+ m\, M_{\Sigma} \cdot 2k \cdot n_1 \cdot (\lambda-\gamma_5)\, Y'_{10} \quad (\epsilon \cdot \gamma)(k \cdot \gamma)\, u_1$$

$$N_2 = \pi^2 \, \bar{u}_2 \, \frac{(m - M_p)}{2k \cdot n_1} \left[ M_{\Sigma}(\lambda-\gamma_5) - M_p(\lambda+\gamma_5) \right] (\epsilon \cdot \gamma)(k \cdot \gamma)\, u_1 .$$

$$\Big( -\tfrac{5}{6} - M_p^2\, Y'_{40} + 2k \cdot n_1\, Y'_{41} + t'^2\, Y'_{30} + 2M_p^2\, Y'_{40} + 2k \cdot n_1 (Y'_{31} - 2Y'_{41}) \Big)$$

$$N_3 = \pi^2 \bar{u}_2 \Big\{ -(\lambda+\gamma_5) \left( -\tfrac{1}{2} - M_p^2\, Y'_{30} + 2k\cdot n_1 Y'_{31} + t'^2 Y'_{20} + 2M_p^2 Y'_{30} + 2k\cdot n_1 [Y'_{21} - 2Y'_{31}] \right)$$

$$- \frac{1}{2k\cdot n_1} \cdot \left( -\tfrac{1}{2} + t'^2\, Y'_{20} + 2M_p^2\, Y'_{30} + 2k\cdot n_1\, [Y'_{21} - 2Y'_{31}] \right) \cdot$$

$$\cdot \left( M_\Sigma [-M_p(\lambda-\gamma_5) + M_\Sigma(\lambda+\gamma_5)] + m\, [-M_p(\lambda+\gamma_5) + M_\Sigma(\lambda-\gamma_5)] \right)$$

$$+ \frac{M_p\, M_\Sigma}{2k\cdot n_1} \left[ -(M_p{}^2 + M_\Sigma{}^2)(\lambda-\gamma_5) + 2M_p M_\Sigma\, (\lambda+\gamma_5) \right] Y'_{30}$$

$$+ M_\Sigma \left[ M_p\, (\lambda-\gamma_5) - M_\Sigma\, (\lambda+\gamma_5) \right] Y'_{31}$$

$$+ \frac{m\, M_p}{2k\cdot n_1} \left[ -(M_p^2 + M_\Sigma^2)(\lambda+\gamma_5) + 2M_p M_\Sigma(\lambda-\gamma_5) \right] Y'_{30} + Y'_{31}\, m \big[ M_p(\lambda+\gamma_5)$$

$$- M_\Sigma(\lambda-\gamma_5) \big] \Big\} (\epsilon\cdot\gamma)(k\cdot\gamma) u_1$$

where $t' = t(\mu^2 \leftrightarrow m^2)$ $\quad Y' = Y\,(\mu^2 \leftrightarrow m^2)$

The $M'$ and $N'$ can be derived from M and N respectively by the following substitutions:

$$M' = M^\dagger \begin{pmatrix} \lambda \rightarrow \lambda \\ n_1 \leftrightarrow n_2 \\ y \rightarrow 1-y \end{pmatrix} \qquad\qquad N' = N^\dagger \begin{pmatrix} \lambda \rightarrow \lambda \\ n_1 \leftrightarrow n_2 \\ y \rightarrow 1-y \end{pmatrix}$$

where $\dagger$ denotes the Hermitian conjugate.

References:


1. The decays (e.g. $\Sigma^0 \rightarrow \Lambda^0 + \gamma$) in which strangeness is conserved proceed solely through strong interactions and are much faster and are not considered here.

2. M.Kawaguchi and K.Nishijima, Progr. Theoret. Phys. (Japan) 15, 182 (1956); see also C. Iso and M. Kawaguchi, Progr. Theoret. Phys. (Japan) 16, 177 (1956).

3. G.Quareni and A.Quareni Vignudelli, Nuovo Cimento 14, 1179 (1959).

4. Prem Prakash and A.H.Zimerman, Nuovo Cimento 11, 869 (1959).

5. R.E.Behrends, Phys. Rev. 111, 1691 (1958).

6. H.Umezawa, M.Konuma and K.Nakagawa, Nucl. Phys. 7, 1699 (1958).

7. In our notation Dirac $\gamma$ - matrices $\gamma_\mu$ and $\gamma_5$ are hermitian and $a.b = a_\mu b_\mu = ( a.b - a_o b_o)$ ; $(\mu = 1, 2, 3, 4)$.

8. K.Nakagawa and H.Umegawa, Nuovo Cimento 8, 945 (1959);
K.Nakagawa, Nucl. Phys. 10, 20 (1959);
S.Bludman, Phys. Rev. 115, 468 (1959);
O.Hara, Nuovo Cimento 14, 114 (1959).

9. R.Dalitz, Rev. Mod. Phys. 31, 823 (1959); See also Crawford et. al. Phys. Rev. Letters 2, 266 (1959); Brown et al. Phys. Rev. Letters 3, 563 (1959).

10. In deriving this result we make use of the following projection operator (L.Michel and A.S.Wightman Phys. Rev. 98, 1190,

(1955), H.A.Tolhoeck, Rev. Mod. Phys. 28, 277, (1956));

$$\frac{u(p,s)\bar{u}(p,s)}{|\bar{u}\,u|} = \frac{1}{2}\,(1-i\gamma_5(\gamma\cdot s))\frac{(-i\gamma\cdot p+M)}{2M} \quad ; \; M \neq 0$$

where s is a four spin vector satisfying $s^2 = 1$, $s.p = 0$ and u (p,s) is Dirac four-spinor representing the fermion with four momentum p and four-spin vector s.

The two circular polarization 3-vectors are defined by

$$\epsilon_\rho = \frac{1}{\sqrt{2}}\left[\,\epsilon_o - i\,(-1)^\rho\,\frac{\underline{k} \times \epsilon_o}{k_o}\right]$$

where $\epsilon_o$ is a unit 3-vector such that $\epsilon_o.\underline{k} = 0, \epsilon_\rho^*\cdot\epsilon_\rho = 1$; $\epsilon_\rho \times \epsilon_\rho^* = i\,\frac{\underline{k}}{k_o}\,(-1)^\rho$ and $\rho = 1$ for right circular polarization and $\rho = -1$ for the left circular polarization of the photon.

11. T.D.Lee and C.N.Yang, Phys. Rev., 105, 1671 (1957); A.Salam, Nuovo Cimento 5, 299 (1957): L.Landau, Nucl.Phys. 3, 127 (1957).

12. $$A = \frac{\pi^2}{\sqrt{3}}\left[\rho_\Sigma\,(4.253) + 2.651 - 5.501i - (1-\delta)(16.221 + 25.072i) + 5.727\,\rho_\Sigma + \rho_\Lambda\rho'\,6.664\right].\,(\epsilon)$$

$$B = \left[\rho_\Sigma\,(2.366) + 24.040 - 5.083i + (1+\delta)(16.467 + 25.456i) + 60.667\,\rho_\Sigma + \rho_\Lambda\rho'\,(63.450)\right].\,\frac{\pi^2}{\sqrt{3}}\,.$$

13. Under T we have the following transformation of the field operators:

$$\psi_T(\underline{x},t) = i\eta_T\;j_5 C^*\bar{\psi}^T(\underline{x},-t) \quad \varphi_T(\underline{x},-t) = \eta_T\varphi^+(\underline{x},-t) \quad \vec{A}_T(\underline{x},t) = -\vec{A}(\underline{x},-t)$$

$$\bar{\psi}_T(\underline{x},t) = i\eta_T^*\;\psi^T(\underline{x},-t)\gamma_4^T C\gamma_4\gamma_5 \quad \varphi_T^+(\underline{x},t) = \eta_T^*\varphi(\underline{x},-t) \quad A_o(\underline{x},t) = +A_o(\underline{x},-t)$$

where $\gamma_\mu$ and $\gamma_5$ are hermitian $\gamma$ matrices and C is the charge conjugation matrix. $(C^* = C^{-1} = C^+)$.