NOTAS DE FÍSICA

VOLUME IV

Nº 5

π-e DECAY AND THE UNIVERSAL FERMI INTERACTION

COLBER G. OLIVEIRA and J. TIOMNO

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1958

Notas de Física, Vol. 4, Nº 5

π-e DECAY AND THE UNIVERSAL INTERACTION*

Colber G. Oliveira and J. Tiomno
Centro Brasileiro de Pesquisas Físicas and
Faculdade Nacional de Filosofia
Rio de Janeiro, D. F.

(Received 19th May, 1958)

It is well known[1] that the experimentally observed process:

$$N \longrightarrow P + e^- + \nu \quad (\beta - \text{decay}) \tag{1}$$

$$P + \mu^- \longrightarrow N + \nu \quad (\mu - \text{capture}) \tag{2}$$

together with the strong interaction of π-mesons and nucleons, lead to the possibility of the decay processes:

$$\pi \longrightarrow e + \nu \tag{3}$$

$$\pi \longrightarrow \mu + \nu \tag{4}$$

Of these, only (4) has been observed, the best upper limit of the relation between (3) and (4) being[2]:

* Work made under the auspices of the Conselho Nacional de Pesquisas.

$$k = \frac{\pi \rightarrow e + \nu}{\pi \rightarrow \mu + \nu} < 10^{-5} \qquad (5)$$

the probability of process (4), as obtained from the lifetime of π_+ mesons is

$$\omega_{\pi \rightarrow \mu} = 3{,}9 \times 10^{7} \text{ sec}^{-1} \qquad (6)$$

Although a direct interaction leading to (4) (and even to (3)) is not to be excluded, there has been a growing tendency to exclude such interactions, since it would be very interesting to explain π-decay only as a consequence of the Fermi interactions.

The idea of the existence of a Universal Fermi interaction responsible for processes (1) and (2) is also very appealing and brings strong restrictions to the relation (5), for instance. The present results on this connection can be summarized as[1]:

A. The value of k for the several forms of Fermi coupling, are:

a) $k = 5{,}2$ (P) (7)

b) $k = 1{,}3 \times 10^{-4}$ (A) (8)

where P and A indicate, respectively the pseudoscalar and axial vector Fermi interaction in the charge exchange ordering.

B. The values of $\omega_{\pi \rightarrow \mu}$ are of the right order of magnitude for P and A, and vanish for S, V, T interactions (in second order perturbation calculation).

So we see that both results given by (7) or (8) are in contradiction with (5).

In this paper, we examine the possibility that the contribution of hyperons to the intermediate states may cancel significantly that of nucleons, to an extent that both $\omega_{\pi \rightarrow \mu}$ and $\omega_{\pi \rightarrow e}$ resulting from the intermediate Baryon-pair, becomes smaller than the experimental values, thus allowing for a direct $\pi \to \mu$ interaction, without a direct $\pi \to e$ interaction, which would be consistent with the experimental results.

I. π-BARYONS INTERACTION AND UNIVERSAL FERMI INTERACTION

We take for the π-Baryon strong interaction the hamiltonian density[3,4]:

$$\mathcal{H}_{in}^{(\pi)} = iG[\bar{N}\gamma_5\vec{\tau}N + \bar{\Xi}\gamma_5\vec{\tau}\Xi \pm (\bar{Y}\gamma_5\vec{\tau}Y + \bar{Z}\gamma_5\vec{\tau}Z)] \cdot \vec{\pi} \qquad (9)$$

where

$$N = \begin{pmatrix} P \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi_o \\ \Xi_- \end{pmatrix}, \quad Y = \begin{pmatrix} \Sigma_+ \\ \frac{\Lambda_o - i\Sigma_o}{\sqrt{2}} \end{pmatrix}, \quad Z = \begin{pmatrix} \frac{\Lambda_o + i\Sigma_o}{\sqrt{2}} \\ \Sigma_- \end{pmatrix} \qquad (10)$$

The interaction (9) is invariant, not only under ordinary isotopic spin space rotations, but also under transformations which permute N, Ξ, Y and Z (global invariance)

$$N \rightleftarrows \Xi \;; \quad Y \rightleftarrows Z \;; \quad \vec{\pi} \rightleftarrows \vec{\pi} \qquad (11)$$

or

$$N \rightleftarrows Y \;; \quad \Xi \rightleftarrows Z \;; \quad \vec{\pi} \rightleftarrows \pm\vec{\pi} \qquad (12)$$

For the U.F.I. we also impose the global invariance, although not the invariance under rotations in isotopic spin space; the non-conservation of parity is not taken into account, as it is unimportant for the present concern. So we take (writing only the terms of interest here):

$$\mathcal{H}_{in}^{(F)} = \sum_k (f_k A_k^{(B)} A_k^{(L)} + h.c.) \qquad (13)$$

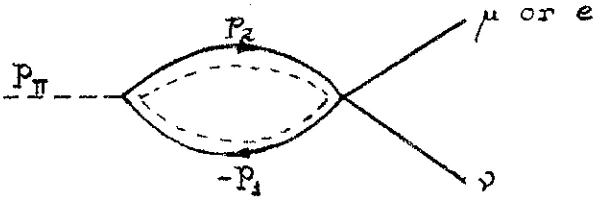
where

$$A_k^{(B)} = \bar{N}\,\Theta_k\,\tau_+\,N + \bar{\Xi}\,\Theta_k\,\tau\,\Xi \pm \ \Theta\,\tau \qquad \bar{\ }\,\Theta\,\tau\ \ ) \tag{14}$$

$$A_k^{(L)} = \bar{e}\,\Theta_k\,\nu \pm \bar{\mu}\,\Theta_k\,\nu \tag{15}$$

One should observe that the ± signs in (9) and (14) are not necessarily in correspondence. Thus depending on the transformation of $A_k^{(B)}$ corresponding to the global transformation (12) (that is, if $A_k^{(B)}$ should remain invariant or if it is allowed to change sign), we can have all the possible correspondences: (+ +), (+-), (-+), (- -).

Now, it is an immediate consequence that if the signs of (Y,Z) terms are taken equal in (9) and (14), then there is a constructive interference in the contributions of the four kinds of baryons to the process:



(16)

The net result being only a multiplication by a factor $\sim 4^2$ of $\omega_{\pi\to\mu}$, or $\omega_{\pi\to e}$, which makes the situation worse than before. If we take those signs as opposite, then there is a destructive interference between the contributions of (Ξ , N) and thos of (Y, Z) terms. If all the baryons masses were equal, they would cancel exactly; as they are nearly equal, we hope that the cancellation will be enough for our purposes. So, we will be mainly interested in the case when those signs are opposite. This situation is similar to that which occurs in the γ-decay of π_o-mesons [4].

II. π-DECAY PROBABILITIES FOR THE AXIAL VECTOR FERMI INTERACTION

Although the contributions of the nucleons to the closed loop (16) has been already computed[5], the reported results are however of no use to us, since we need to know the exact dependence in the mass of intermediate particles.

Thus, we find for the value of the probability of π-decay into a light particle (μ-meson or electron) of mass m, and a neutrino:

$$\omega = \frac{g_A^2}{8\pi} \left(\frac{m}{\mu} \right)^2 \frac{\mu c^2}{\hbar} \left\{ 1 - \left(\frac{m}{\mu} \right)^2 \right\}^2 \tag{17}$$

where:

$$\mu = M_\pi \tag{18}$$

$$g_A = \sqrt{2}\, \frac{f_A \mu^2 c}{16\pi^2 \hbar^3} \frac{G}{\sqrt{\hbar c}} \left(I_N + I_\Xi \pm (I_Y + I_Z) \right) \tag{19}$$

The sign + in (Y,Z) terms corresponds to equal signs in the last terms of (9),(14) and the - sign corresponds to opposite sign in those terms; in the first case we have a constructive interference of all the baryons, and in the second, a destructive one.

The I's are given by divergent integrals involving the intermediates particles;

$$I\, p_\pi^\lambda = \frac{-i}{\mu c^2 \pi^2} \int_\infty^\infty d_4 p_2\, T_r \left[\frac{1}{\not p_1 - M_1} \gamma_5 \gamma^\lambda \frac{1}{\not p_2 - M_2} \gamma_5 \right] \tag{20}$$

Using Feynman's cut-off method of introducing a factor $C(\Lambda, p_2)$, we obtain finite values for the several I's.

Here we neglect the difference of masses of the two intermediate particles:

$$M_1 = M_2 \tag{21}$$

This is not bad for nucleons, but not so good for Λ_o and Σ's; in order to take into account the difference of masses of Λ_o and Σ hyperons, we take:

$$I_Y + I_Z = \frac{1}{2}\left(I_{\Lambda_o} + 3\, I_{\Sigma}\right) \tag{22}$$

We also see the influence of chosen cut-off, considering the two cases:

a) $$C_1(\Lambda,p^2) = \Lambda^2(\Lambda^2 + M^2 - p^2)^{-1} \tag{23}$$

Here, we find:

$$\frac{\mu}{4M}\, I(\Lambda,M) = \frac{2}{\mu}\sqrt{4M^2-\mu^2}\ \mathrm{tg}^{-1}\frac{\mu}{\sqrt{4M^2-\mu^2}} -$$

$$- \frac{\Lambda^2+\mu^2}{2\mu^2}\log\frac{M^2}{\Lambda^2+M^2} - \frac{1}{2\mu^2}\sqrt{\Delta}\log\frac{\Lambda^2+2M^2-\mu^2+\sqrt{\Delta}}{\Lambda^2+2M^2-\mu^2-\sqrt{\Delta}} \tag{24}$$

where:

$$\Delta = \Lambda^4 - 2\mu^2(\Lambda^2+2M^2) + \mu^4 \tag{25}$$

This is correct only if [6]:

$$\Lambda^2 > \mu\,(2M + \mu) \tag{26}$$

Which is the region of interest to us, because we shall take[7]:

$$\Lambda \sim 7\mu \tag{27}$$

b) $$C_2(\Lambda,p^2) = \Lambda^4(\Lambda^2 + M^2 - p^2)^{-2} \tag{28}$$

We find

$$\frac{\mu}{M}\, I = 2\log\frac{\Lambda^2+M^2}{M^2} - \frac{8}{\mu}\sqrt{4M^2-\mu^2}\ \mathrm{tg}^{-1}\frac{\mu}{\sqrt{4M^2-\mu^2}} +$$

$$+ \frac{2\Lambda^2+8M^2-2\mu^2}{\sqrt{\Delta}} \log \frac{\Lambda^2+2M^2-\mu^2+\sqrt{\Delta}}{\Lambda^2+2M^2-\mu^2-\sqrt{\Delta}} \tag{29}$$

if[8]:

$$\Lambda^2 > \mu(2M + \mu)$$

The results of numerical computations, are summarized in Table I.

TABLE I

$\frac{G^2}{4\pi\hbar c} \simeq 16$, $f_A = 10^{-49}$ erg cm^3, $\Lambda = 7\mu$	Probabilities per second	
	Weak Cut-Off: $C_1(\Lambda)$	Strong Cut-Off: $C_2(\Lambda)$
Only nucleons in intermediate state	μ: $3,2 \times 10^7$	$1,7 \times 10^6$
	e: $4,1 \times 10^3$	$2,2 \times 10^2$
Constructive interference of all baryons	μ: $5,1 \times 10^8$	$1,1 \times 10^7$
	e: $6,6 \times 10^4$	$1,4 \times 10^4$
Destructive interference of all baryons	μ: $1,6 \times 10^6$	$8,9 \times 10^4$
	e: $2,1 \times 10^2$	$1,1 \times 10$

The case of contributions only from nucleons is also given, for comparison with the previous papers.

III. π-DECAY PROBABILITIES FOR THE PSEUDO-SCALAR FERMI INTERACTION

In this case, we find for the probability of π decay into a light particle of mass m and a neutrino:

$$\omega = \frac{g_p^2}{8\pi} \frac{\mu c^2}{\hbar} \left\{ 1 - \left(\frac{m}{\mu}\right)^2 \right\}^2 \tag{30}$$

Here:

$$g_p = \sqrt{2} \frac{f_p \mu^2 c}{16\pi^2 \hbar^3} \frac{G}{\sqrt{\hbar c}} \left(J_N + J_\Xi \pm (J_Y + J_Z)\right) \tag{31}$$

where:

$$J = \frac{1}{\mu^2 c^4 \pi^2} \int_{-\infty}^{\infty} d_4 p_2 \, \mathrm{Tr} \left[\frac{1}{\not{p}_1 - M_1} \gamma_5 \frac{1}{\not{p}_2 - M_2} \gamma_5 \right] \tag{32}$$

Using the same approximation as given by (21), and the J_Y, J_Z being given by a relation analogous to (22); we find by the cut-off procedure:

a) $$C_2(\Lambda, p^2) = \Lambda^4 (\Lambda^2 + M^2 - p^2)^{-2} \tag{33}$$

We must observe that in this case, the cut-off $C_1(\Lambda, p^2)$ is not enough to eliminate the divergence of (32).

Here, we obtain:

$$J = -\frac{4}{\mu} \sqrt{4M^2-\mu^2} \, tg^{-1} \frac{\mu}{\sqrt{4M^2-\mu^2}} + \frac{\Lambda^4 - 2M^2\mu^2 + \mu^4}{\mu^4} \log \frac{\Lambda^2+M^2}{M^2} + \frac{2\Lambda^2}{\mu^2} +$$
$$+ \frac{-\Lambda^6 + \mu^2\Lambda^4 + \mu^4\Lambda^2 + 4M^2\mu^4 - \mu^6}{\mu^4\sqrt{\Delta}} \log \frac{\Lambda^2+2M^2-\mu^2+\sqrt{\Delta}}{\Lambda^2+2M^2-\mu^2-\sqrt{\Delta}} \tag{34}$$

if Δ as given by (25), is positive[9]

b) $$C_3(\Lambda, p^2) = \Lambda^6 (\Lambda^2 + M^2 - p^2)^{-3} \tag{35}$$

In this case we obtain the value of J only by derivatives in the former one:

$$J_{C_3} = J - \frac{\Lambda^2}{2} \frac{\partial J}{\partial \Lambda^2}$$

where J is given by (35).

The results of numerical computations for $\Lambda \sim 7\mu$, are given in Ta[illegible] II.

TABLE II

$\frac{G^2}{4\pi\hbar c} \simeq 16$, $f_p = 10^{-49}$ erg cm^3, $\Lambda = 7\mu$	Probabilities per second	
	Weak Cut-Off: $C_2(\Lambda)$	Strong Cut-Off: C[illegible]
Only nucleons in intermediate state	μ: $2,7 \times 10^9$	$5,7 \times 10^7$
	e: $1,4 \times 10^{10}$	$2,9 \times 10^8$
All baryons (Constructive)	μ: $2,5 \times 10^{10}$	$9,1 \times 10^8$
	e: $1,3 \times 10^{11}$	$4,7 \times 10^9$
All baryons (Destructive)	μ: $6,4 \times 10^7$	$5,7 \times 10^5$
	e: $3,3 \times 10^8$	$2,9 \times 10^6$

The influence of the value of Λ in case of the weak cut-off C_2 (Λ) is given in Table III, where the contributions to the decay proba[illegible]lity corresponding to each kind of intermediate baryon are give[illegible] comparison, the same contributions are also given for the Axial [illegible]tor theory.

TABLE III-a: PROBABILITIES PER SECOND

$\frac{\Lambda}{\mu}$	PSEUDOSCALAR THEORY			
	$M = M_N$	$M = M_o$	$M = M$	$M = M_-$
3	μ: $6{,}5 \times 10^{6}$	$3{,}3 \times 10^{6}$	$2{,}6 \times 10^{6}$	$1{,}9 \times 10^{6}$
	e: $3{,}4 \times 10^{7}$	$1{,}7 \times 10^{7}$	$1{,}4 \times 10^{7}$	$9{,}9 \times 10^{6}$
5	μ: $2{,}8 \times 10^{8}$	$1{,}6 \times 10^{8}$	$1{,}2 \times 10^{8}$	$9{,}3 \times 10^{7}$
	e: $1{,}5 \times 10^{9}$	$8{,}3 \times 10^{8}$	$6{,}2 \times 10^{8}$	$4{,}8 \times 10^{8}$
6	μ: $9{,}8 \times 10^{8}$	$5{,}8 \times 10^{8}$	$4{,}6 \times 10^{8}$	$3{,}0 \times 10^{8}$
	e: $5{,}1 \times 10^{9}$	$3{,}0 \times 10^{9}$	$2{,}4 \times 10^{9}$	$1{,}6 \times 10^{9}$
7	μ: $2{,}7 \times 10^{9}$	$1{,}4 \times 10^{9}$	$1{,}3 \times 10^{9}$	$8{,}8 \times 10^{8}$
	e: $1{,}4 \times 10^{10}$	$7{,}3 \times 10^{9}$	$6{,}8 \times 10^{9}$	$4{,}6 \times 10^{9}$
8	μ: $5{,}9 \times 10^{9}$	$3{,}8 \times 10^{9}$	$3{,}0 \times 10^{9}$	$2{,}2 \times 10^{9}$
	e: $3{,}1 \times 10^{10}$	$2{,}0 \times 10^{10}$	$1{,}6 \times 10^{10}$	$1{,}1 \times 10^{10}$
10	μ: $2{,}4 \times 10^{10}$	$1{,}6 \times 10^{10}$	$1{,}2 \times 10^{10}$	$1{,}1 \times 10^{10}$
	e: $1{,}2 \times 10^{11}$	$8{,}3 \times 10^{10}$	$6{,}2 \times 10^{10}$	$5{,}7 \times 10^{10}$
20	μ: $9{,}0 \times 10^{11}$	$6{,}5 \times 10^{11}$	$5{,}8 \times 10^{11}$	$6{,}7 \times 10^{11}$
	e: $4{,}7 \times 10^{12}$	$3{,}4 \times 10^{12}$	$3{,}0 \times 10^{12}$	$3{,}5 \times 10^{12}$

TABLE III-b: PROBABILITIES PER SECOND

$\frac{\Lambda}{\mu}$	AXIAL VECTOR THEORY			
	$M = M_N$	$M = M_o$	$M = M$	$M = M_-$
3	μ: $1,3 \times 10^4$	$1,1 \times 10^4$	$1,1 \times 10^4$	$8,1 \times 10^3$
	e: 1,7	1,4	1,4	1,1
5	μ: $1,7 \times 10^5$	$6,5 \times 10^4$	$5,7 \times 10^4$	$9,8 \times 10^3$
	e: $2,2 \times 10$	8,5	7,4	1,3
6	μ: $6,5 \times 10^5$	$3,2 \times 10^5$	$1,7 \times 10^5$	$1,3 \times 10^5$
	e: $8,5 \times 10$	$4,2 \times 10$	$2,2 \times 10^{10}$	$1,7 \times 10$
7	μ: $1,7 \times 10^6$	$1,1 \times 10^6$	$6,5 \times 10^5$	$4,3 \times 10^5$
	e: $2,2 \times 10^2$	$1,4 \times 10^2$	$8,5 \times 10$	$5,6 \times 10$
8	μ: $3,9 \times 10^6$	$2,1 \times 10^6$	$1,7 \times 10^6$	$1,1 \times 10^6$
	e: $5,1 \times 10^2$	$2,7 \times 10^2$	$2,2 \times 10^2$	$1,4 \times 10^2$
10	μ: $1,2 \times 10^7$	$7,3 \times 10^6$	$6,0 \times 10^6$	$4,1 \times 10^6$
	e: $1,6 \times 10^3$	$9,5 \times 10^2$	$7,8 \times 10^2$	$5,3 \times 10^2$
20	μ: $1,6 \times 10^8$	$1,4 \times 10^8$	$1,2 \times 10^8$	$1,1 \times 10^8$
	e: $2,1 \times 10^4$	$1,8 \times 10^4$	$1,6 \times 10^4$	$1,4 \times 10^4$

IV. ANALYSIS OF THE RESULTS

The analysis of Tables I - III, lead us to the following conclusions:

I) If the pseudo-scalar Fermi coupling is of order of 10^{-49} erg cm^3, no agreement can be obtained with the upper limit of $\pi \to e$ probability, except in one of the following possibilities;

a) The value of the cut-off parameter Λ is much smaller than usually assumed - for instance if $\Lambda \sim \mu$.

In this case the $\pi \to \mu$ probability would be very small, and we must assume a direct $\pi \to \mu$ interaction, without a direct $\pi \to e$ one.

b) The value of Λ is of order of magnitude of M_N, the baryon interference being destructive; in this case we should still take a direct pseudoscalar $\pi \to \mu$ interaction which interferes constructively with the indirect one, and a direct pesudoscalar $\pi \to e$ interaction which interferes destructively with the indirect one.

c) The renormalized couplings of π mesons with the baryons are not identical, but depend on their mass, in such a way that the cancellation would be very strong, even for $\Lambda \sim M_N$. This situation is analogous to case a).

II) If the Axial vector Fermi coupling is of order of 10^{-49} erg cm^3, we have: (The pseudoscalar Fermi coupling vanishes).

a) The value of the cut-off parameter is the usually taken: $\Lambda \sim M_N$. In this case we must take a direct $\pi \to \mu$ interaction, without a direct $\pi \to e$ one. (Argument similar to case (a) before.) In the same manner is valid here an argument similar to case (b) before.

b) The pseudoscalar Fermi coupling does not vanish and is of the order of magnitude of 10^{-53} erg cm^3:

$$f_p \sim 10^{-4} f_A \tag{37}$$

and has a convenient sign in order to produce a strong cancellation of the pseudovector contribution to $\pi \to e$ decay.

It is an unsatisfactory situation, that in any case we must postulate additional interactions in order to justify the experimental results.

1. M. Ruderman and R. Finkelstein, Phys. Rev. 76, 1458, (1949).
2. H. L. Anderson and C. M. G. Lattes, Nuovo Cim. 6, (1957).
3. M. Gell-Mann, Phys. Rev. 106, 1296, (1957).
4. J. Tiomno, Nuovo Cim. 6, 255, (1957).
5. S. B. Treiman and H. W. Wyld, Phys. Rev. 101, 1552, (1956).
6. If $\Delta^2 < \mu(2M + \mu)$, we should only replace the last term in (24) by:

$$- \frac{1}{\mu^2} (-\Delta)^{1/2} \left\{ tg^{-1} \frac{\Delta^2 + \mu^2}{\sqrt{-\Delta}} - tg^{-1} \frac{\Delta^2 + \mu^2}{\sqrt{-\Delta}} \right\}$$

7. R. P. Feynman and G. Speisman, Phys. Rev. 94, 500, (1954).
8. If $\Delta < 0$, the same substitution in the last term of (29) should be done, as given in note (6).
9. If $\Delta < 0$, the same substitution in the last term of (34) may be done, in the same manner that in notes (6) and (8).