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Localization and the interface between quantum mechanics, quantum field theory and quantum gravity I (The two antagonistic localizations and their asymptotic compatibility)

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Localization and the interface between quantum mechanics, quantum field theory and quantum gravity I (The two antagonistic localizations and their asymptotic compatibility)

dedicated to the memory of Rob Clifton

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Abstract

It is shown that there are significant conceptual differences between QM and QFT which make it difficult to view the latter as just a relativistic extension of the principles of QM. At the root of this is a fundamental distiction between Born-localization in QM (which in the relativistic context changes its name to Newton-Wigner localization) and *modular localization* which is the localization underlying QFT, after one separates it from its standard presentation in terms of field coordinates. The first comes with a probability notion and projection operators, whereas the latter describes causal propagation in QFT and leads to thermal aspects of locally reduced finite energy states. The Born-Newton-Wigner localization in QFT is only applicable asymptotically and the covariant correlation between asymptotic in and out localization projectors is the basis of the existence of an invariant scattering matrix.

In this first part of a two part essay the modular localization (the intrinsic content of field localization) and its philosophical consequences take the center stage. Important physical consequences of vacuum polarization will be the main topic of part II. The present division into two semi-autonomous essays is the result of a partition and extension of an originally one-part manuscript.

1 Introductory remarks

Ever since the discovery of quantum mechanics (QM), the conceptual differences between classical theory and QM have been the subject of fundamental investigations with profound physical and philosophical consequences. But the conceptual relation between quantum field theory (QFT) and QM, which is at least as challenging and rich of surprises, has not received the same amount of attention and scrutiny, and often the subsuming of QFT under "relativistic QM" nourished prejudices and prevented a critical foundational debate. Apart from some admirable work on the significant changes which the theory of measurements must undergo in order to be consistent with the structure of QFT, which emanated from people who are or have been affiliated with the Philosophy of Science Department of the University of Pittsburgh [1][2][3], as well as some related deep mathematical and conceptual work from quantum field theorists [4][91][92], this subject has remained in the mind of a few individuals working on the foundations of QFT and is still far from being part of the collective knowledge of the foundation of QT community.

Often results of this kind which involve advanced knowledge of QFT do not attract much attention even when they have bearings on the foundations of QT as e.g. the issue of *Bell states* in *local quantum physics* (LQP[1]) [6] or the important relations between causal disjointedness with the existence of uncorrelated states as well as the issue to what extent causal independence is a consequence of statistical independence [7]. The

[1]We use this terminology instead of QFT if we want to direct the reader's attention away from the textbook Lagrangian quantization towards the underlying principles [5]. QFT (the content of QFT textbooks) and LQP deal with the same physical principles but LQP is less comitted to a particular formalism (Lagrangian quantization, functional integrals) and rather procures always the most adaequate mathematical concepts for their implementation. It includes of course all the results of the standard perturbative Lagrangian quantization but presents them in a conceptually and mathematically more satisfactory way. Most of the subjects in this article are outside of textbook QFT.

reason is not so much a lack of interest but rather that QFT is often thought to be just a kind of relativistic quantum mechanics and that possible differences are of a technical nature. This may explain why there has been an amazing lack of balance between the very detailed and sophisticated literature about interpretational aspects of QM and its relation with quantum information theory (aiming sometimes at some very fine, if not to say academic/metaphoric points e.g. the multiworld interpretation), and the almost complete lack of profound interpretive activities about our most fundamental quantum field theory of matter. Although the name QT usually appears in the title of foundational papers, this mostly hides the fact that they deal exclusively with concepts from QM leaving out QFT.

If on the other hand some foundational motivated quantum theorist become aware of the deep conceptual differences between particles and fields, they tend to look at them as antagonistic and create a battleground; the fact that they are fully compatible where for physical reasons they must agree, namely in the asymptotic region of scattering theory, remains often uncommented.

The aim of this essay is to show that at the root of these differences there are two localization concepts: the quantum mechanical Born-Newton-Wigner localization and the modular localization of LQP. The BNW localization is not Poincaré covariant but attains this property in a certain asymptotic limit namely the one on which scattering theory is founded. Modular localization on the other hand is causal and covariant at all distances but provides no projectors on subspaces as they arise from spectral decompositions of selfadjoint or unitary operators, instead the linear spaces of localized states are usually dense in the Hilbert space of all states. One of the aims of this article is to collect some facts which, somewhat oversimplified, show that besides sharing the notion of Hilbert space, operators, states and Planck's constant $\hbar$, QM and QFT are conceptually worlds apart and yet they harmonize perfectly in the asymptotic region of scattering theory.

For a long time the subtle distinction between the non-covariant BNW localization based on the existence of a position operator and the autonomous covariant localization concept of QFT was insufficiently understood. It has been claimed (private communication by Rudolf Haag) that the reason for Wigner, who together with Jordan significantly enriched QFT, to become later disenchanted with this theory was that he failed to obtain a covariant localization concept which was able to directly connect his representation theory of the Poincaré group with QFT [2]. The application of the non-covariant and hence frame-dependent BNW localization to finite distances leads to incorrect results in particular to superluminal phenomena. But only after the publication of an article [8] in which it was claimed that Fermi's result about his two-atom Gedankenexperiment contradicts understandings about spacetime localization and signal propagation came the issue of BNW- and modular- localization to a climax. The editor of *Nature* at that time wrote an article in which the main (sensational, but not incorrect) conclusion was that the claimed superluminal propagation makes time-machines theoretically possible where upon the world press especially in the US and Europe took up the sensational subject.

Fortunately this was not the end of this episode. The same journal which published the article based on the use of BNW localization accepted a second article [9] in which

[2] This is precisely what modular localization achieves (section 6).

Fermi's conclusions about finite propagation speed were reinforced on the basis of modular localization[3]. This episode underlines the subtlety of localization in QFT and most of the content of both parts of this essay will consist in explaining why this is such a delicate problem which led to many misunderstandings.

It is not our intention to present a new axiomatic setting (for an older presentation of the existing one see [5]). Such a goal would be too ambitious in view of the fact that we are confronting a theory where, in contradistinction to QM, no conceptual closure is yet in sight. Although there has been some remarkable nonperturbative progress concerning constructive control (i.e. solving the existence problem) of models, the main knowledge about models of QFT is still limited to numerically successful, but nevertheless diverging perturbative series.

Here the more modest aim is to collect some either unknown or little known facts which could present some food for thoughts about a more inclusive measurement theory, including all of quantum theory (QT) end not just QM. On the other hand one would like to improve the understanding about the interface between QFT in CST (curved spacetime) and the still elusive QG.

Since both expressions QFT and LQP are used do denote the same theory, let me emphasize again that there is no difference in the physical aims since LQP originated from QFT and incorporated all concepts and computational results of QFT including renormalized perturbation theory; LQP is used instead of QFT whenever the conceptual level of the presentations gets beyond what the reader is able to find in standard textbooks of QFT, more specifically whenever one is interested in nonperturbative mathematically controlled constructions of models in terms of intrinsic ("field-coordinatization independent") structures. There is one recommendable exception, namely Rudolf Haag's book "Local Quantum Physics" [5]; but in a fast developing area of particle physics two decades (referring to the time it was written) are a long time.

The paper consists of two parts, the first is entirely dedicated to the exposition of the differences between (relativistic[4]) QM and LQP and their coexistence at large time separations within the setting of scattering theory. The second part which will appear as a separate contribution deals with thermal and entropic consequences of vacuum polarization caused by causal localization as well as some consequences for QFT in curved spacetime (CST). A quantum gravity theory (QG) theory does not yet exist, but a profound understanding of those foundational aspects are expected to be important to arrive at one.

The sections of the paper at hand are as follows. The first sections presents the little known theory of *direct particle interactions* (DPI), a framework which incorporates all those properties of a relativistic theory which one is able to formulate solely in terms of relativistic particles; some of them already appeared in the pre Feynman S-matrix work of E.C.G. Stückelberg. In contradistinction to nonrelativistic QM where the cluster factorization follows from the additivity of two-particle interactions, its enforcement in DPI requires more refined arguments. As a closely related result, DPI does not allow

[3]In the publications during the 90s, this terminology was not yet available. It was sufficient to simply think in terms of the kind of localization which is intrinsic to pointlike covariant fields.

[4]In order to show, that by making QM relativistic, one does not remove the fundamental differences with QFT, the next section will be on the relativistic setting of "direct particle interactions".

a second quantization presentation, even though it is a perfect legitimate multiparticle theory in which n-particles are linked to n+1 particles by cluster factorization. Most particle physicists tend to believe that a relativistic particle theory, consistent with macro-causality and a Poincaré-invariant S-matrix, must be equivalent to QFT[5], therefore it may be helpful to show that this is not correct. DPI theories fulfill all the physical requirements which one is able to formulate solely in terms relativistic particles without recourse to fields, as Poincaré covariance, unitary and macro-causality of the resulting S-matrix (which includes cluster factorization).

In this way one learns to appreciate the fundamental difference between quantum theories which have no algebraically built-in maximal velocity and those which have. As a quantum mechanical theory DPI only leads to statistical "effective" finite velocity propagation for asymptotically large time-like separations between localized events as they occur in scattering theory. With other words the causal propagation between Born-localized events is a macroscopic phenomenon for which in analogy to the acoustic velocity in QM, the large time behavior of dissipating wave packets is important, whereas in QFT the maximal velocity is imprinted on the algebra. DPI does not possess covariant local operators, the only covariant object is the Poincaré invariant S-matrix; from this viewpoint DPI is an S-matrix theory.

At the root of the QM-QFT (particle-field) antagonism is the existence of two very different concepts of localization namely the *Born localization*[6] (which is the only localization for QM), and the *modular localization* which underlies the causal locality in QFT. The Born localization and the related position operator has been adapted to the covariant normalization of relativistic wave functions in a paper by Newton and Wigner [11] and will henceforth be referred to as the BNW localization. Whereas relativistic QM permits only the BNW localization, QFT needs both, the modular localization[7] in order to implement causal propagation and the BNW localization to get to the indispensable scattering probabilities (cross sections). Without the BNW localization QFT would remain a beautiful mathematical construct with no accessible physical content; on the other hand without modular localization QFT would not have interaction-induced vacuum polarization and its description of reality at finite distances would contain acausal poltergeist-daemons.

Particles are objects with a well-defined ontological status, whereas (basic and composite) fields form an infinite set of coordinatizations which generate the local algebras. By this we mean that particles are the truly real and unique objects which are subject to direct observations and independent of any "field-coordinatization", a property which is not derogated by the fact that their existence is only an asymptotic. What is referred to as

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[5]The related folklore one finds in the literature amounts to the dictum: relativistic quantum theory of particles + cluster factorization property = QFT. Apparently this conjecture goes back to S. Weinberg.

[6]It is interesting to note that Born introduced the probability concept in QM in the context of the Born approximation of what we call nowadays the cross section and not of the Schroedinger wave function [10]. With other words he introduced it in the asymptotic region where it is indidpensible and where the BNW localization becomes independent of the reference frame.

[7]Modular localization is the same as the causal localization inherent in QFT after one liberates the letter from the contingencies of particular selected fields. It is a property of the local equivalence class of relatively loca; fields (the Borchers class) or of the associated sysrem of local algebras. If one considers, as it is done in algebraic QFT, the local fields as coordinatizations of the local algebras, modular localization is independent of the "field coordinatization".

an (asymptotic) n-particle state is a state in which n well separated coincidence counters (in a world cobbled with counters) click simultaneously and apart from the localization of the counters, the number n does not change at later times.

Quantum fields on the other hand have a more fleeting existence and there are always infinitely many fields which are associated with one particle. This finds its expression in the terminology "interpolating fields" used in the LSZ scattering theory of the 50s. But as epistemic entities fields or local algebras are indispensable since all our intuition about local interactions and their causal localization properties is injected on the level of fields or directly into the local observable algebras which they generate. Without Born localization and the associated projectors, there would be no scattering theory leading to cross sections and hence QFT would be reduced to just a mathematical playground.

In contradistinction to DPI, in interacting QFT there is no way in which in the presence of interactions the notion of *particles at finite times* can be saved. The statement that an isolated relativistic particle cannot be localized below its Compton wave length refers to the (Newton-Wigner adaptation of the) Born localization and, as all statements involving Born localization, it is meant in an *effective* probabilistic sense. Only in the timelike asymptotic limit between two events the BNW localization becomes a sharp geometric relations in terms of momenta with c being the maximal velocity which is independent of the reference frame; fortunately this is precisely what one needs to obtain a Poincaré invariant macrocausal S-matrix.

The maximal velocity in the sense of asymptotic expectations in suitable states of relativistic particle theories plays a similar role as acoustic velocity in nonrelativistic QM leads to (material-dependent) acoustic velocities. Placing our interpretation in the context of prior work on this subject [2][12], Reeh-Schlieder neither "defeats" Newton-Wigner, nor does Newton-Wigner "meet" Reeh-Schlieder, rather *both indispensable localization schemes* approximate each other asymptotically at $t \rightarrow \pm\infty$ where the Newton-Wigner localization becomes covariant, macro-causality coalesces with micro-causality, and last not least the modular localization shares the asymptotic probability notion with BNW, i.e. no defeat of either one but harmony at the only places where both are valid.

The next section contains some remarks about the history of the growing awareness about properties which separate QM from QFT. This is followed in section 3 with the presentation of a little known consistent setting of interacting relativistic particles without fields: the direct particle interaction theory (DPI) by Coester and Polyzou. Sections 4 and 5 focus on the radical difference between the Newton-Wigner (NW) localization (the name for the Born localization after the adaptation to the relativistic particle setting) and the localization which is inherent in QFT, which in its intrinsic form, i.e. liberated from singular pointlike "field coordinatizations", is referred to as *modular localization* [13][14][15]. The terminology has its origin in the fact that it is backed up by a mathematical theory within the setting of operator algebras which bears the name Tomita-Takesaki[8] *modular theory.* Within the setting of thermal QFT, physicists independently discovered various aspects of this theory [5]. Its relevance for causal localization was only spotted a decade later [17] and the appreciation of its role in problems of thermal behavior at causal- and event- horizons and black hole physics had to wait another decade [18].

[8]Tomita was a Japanese mathematician who discovered the main properties of the theory in the first half of the 60s, but it needed a lot of polishing by Takesaki in order to be accepted.

Sections 6-10 are all centered around an in-depth exposition of various aspects of modular localization starting from the modular localization of states and passing to its more restrictive algebraic counterpart. Among its very recent application is the notion of semiinfinite spacelike string localization which on the one hand settled the age old problem of the appropriate localization for the Wigner infinite spin representation but also shows that the object of string theory is really an infinite component field (section 7).

The penultimate section presents LQP as the result of *relative positioning* of a finite (and rather small) number of *monads* within a Hilbert space; here we are using a terminology which Leibniz introduced in a philosophical context. This shows the enormous conceptual distance between QM and LQP. Whereas a single monad also appears in different contexts e.g. KMS states on open quantum systems and the information theoretical interpretation of bipartite spin algebras in suitable singular states [4][19], the modular positioning of several copies is totally characteristic for LQP. Although its physical and mathematical content is quite different from Mermin's [20] new look (the "Ithaca-interpretation" of QM) at quantum mechanical reality exclusively in terms of correlations between subsystems, the two concepts share the aspect of viewing reality in relational terms. Mathematically a monad in the sense of this article is the unique hyperfinite type III_1 factor algebra to which all local algebras in LQP are isomorphic, so all concrete monads are copies of the abstract monad. Naturally a monade in isolation is an abstract entity without structure, the reality emerges from relations between monads within the same Hilbert space.

Whereas for Newton physical reality consisted of matter moving in a fixed space according to a universal time, reality for Leibniz emerges from interrelations between monads with spacetime serving as ordering device. The modular positioning of monads goes one step further in that even the Minkowski spacetime together with its invariance group the Poincaré group appears as a consequence of positioning in a more abstract sense namely of a finite number of monads in a joint Hilbert space (subsection 7). For actual constructions of interacting LQP models it is however advantageous to start with one monad and the action of the Poincaré group on it.

The algebraic structure of QM on the other hand, relativistic or not, has no such monad structure; the global algebra as well as all Born-localized subalgebras in ground states are always of type I i.e. either the algebra of all bounded operators $B(H)$ in an appropriate Hilbert space or multiples thereof. Correlations are characteristic features of quantum mechanical states, whereas for the characterization of a QM system global operators as the Hamiltonian are indispensable.

Part I of this essay closes with a section on the split inclusion which shows how in the ubiquitous presence of vacuum polarization some of the notion known from QM (tensor factorization of disjoint subsystem, entanglement) can be recuperated. The second part will present many more applications of modular localization and the split property notably those related to thermal and entropic properties which are of potential astrophysical and cosmological relevance.

2 Historical remarks on the interface between QM and QFT

Shortly after the discovery of field quantization in the second half of the 1920s, there were two opposed viewpoints about its content and purpose represented by Dirac and Jordan [21]. Dirac's maintained that quantum theory should stand for *quantizing a real classical reality* which meant field quantization for electromagnetism and quantization of classical mechanics for particles. Jordan, on the other hand, proposed an uncompromising field quantization point of view; his guiding theme was that all what can be quantized should be quantized, independent of whether there is a classical reality or not. The more radical field quantization including particles finally won the argument, but ironically it was Dirac's particle setting (the hole theory) and not Jordan's "version of Murphy's law" ("everything which can be quantized must be quantized") to all field objects which contributed the richest structural property to QFT, namely charge-anticharge symmetry leading to the necessary presence of antiparticles.

It was also Dirac's hole theory setting in which the first perturbative QED computations (which entered the textbooks of Heitler and Wenzel) were done, before it was recognized that this setting was not really consistent. This inconsistency showed up in problems involving renormalization in which *vacuum polarization* plays the essential role. The successful perturbative renormalization of QED in the charge symmetric description was also the end of hole theory as well as the start of Dirac's late conversion to QFT as the general setting for relativistic particle physics at the beginning of the 50s.

Vacuum polarization is a very peculiar phenomenon which in the special context of currents and the associated local charges of a complex free Bose field was noticed already in the 30s by Heisenberg [22]. But only when Furry and Oppenheimer [23] studied perturbative interactions of Lagrangian fields and became aware to their amazement that the Lagrangian field applied to the vacuum created inevitably some additional particle-antiparticle pairs in addition to the expected one-particle state, the subtlety of the particle-field relation within interacting QFT begun to be noticed. The number of these pairs increase with the perturbative order, pointing towards the fact that in case of sharp localization ("banging" with sharply localized operators onto the vacuum) one has to deal with infinite polarization clouds containing arbitrary high energy components.

Whenever one tries in an interacting theory to create particles via local disturbances of the vacuum, the vacuum polarization clouds corrupt the observation of those particles which one intends to create, but after a sufficient amount of time the particle content separates from the polarization cloud. In the presence of interactions the notion of particles in local regions at a fixed time is, strictly speaking, meaningless because even the field with the "mildest" vacuum polarization taken from the class of all possible relative local fields which all interpolate the same particle still generates an infinite vacuum polarization cloud which sticks inseparably to the particle of interest[9]. It is somewhat ironic

[9]Only if one allows noncompact localization regions one is able to find "PFGs" i.e. operators which applied to the vacuum generate one-particle states without polarization admixture. Wedge regions in Minkowski space lead to the best compromise between particles and fields and play a fundamental role in recent model constructions [24][25][26] and is at the root of the crossing property [27]. For a philosophical viewpoint see [28].

that particles, which are the main bridge between QFT and its laboratory reality (and which are the basic objects of QM), have only an asymptotic existence as incoming and outgoing asymptotic particle configurations.

In the next subsection it will be shown that relativistic QM in the form of DPI, in contradistinction of what most particle theorists believe, can be consistently formulated [29] and this setting can even be extended to incorporate creation and annihilation channels [30]. This goes along way to vindicate Dirac's relativistic particle viewpoint. But it does not vindicate it completely, since theories which start as particle theories but then lead to vacuum polarization as Dirac's hole theory are at the end inconsistent unless one converts their content into a charge symmetric field theoretic setting (in which case the connection with Dirac's whole theory is lost).

By contrasting QFT with DPI, one obtains a better appreciation of the conceptual depth of QFT, in particular one becomes aware of its still unexplored regions. DPI is basically a relativistic *particle* setting i.e. it deals only with properties which can be formulated in terms of particles; this limits causality properties to *macro-causality* i.e. spacelike cluster factorization and timelike causal rescattering. Apart from the fact that the multi-particle representation theory of the Poincaré group is incompatible with the additivity of interaction terms which complicates the implementation of the cluster factorization property and prevents an elegant second quantization description in Wigner-Fock space, the DPI setting is as well understood as nonrelativistic QM. In contrast nobody who has studied QFT beyond a textbook level would claim that QFT is anywhere near its closure. The last section illustrates this point by an unexpected new abstract characterization of QFT which is different from any previous axiomatic attempt.

3 Direct particle interactions, relativistic QM

The Coester-Polyzou theory of *direct particle interactions* (DPI) (where "direct" means "not field-mediated") is a relativistic theory in the sense of representation theory of the Poincaré group which, among other things, leads to a Poincaré invariant S-matrix. Every property which can be formulated in terms of particles, as the cluster factorization into systems with a lesser number of particles and other timelike aspects of macrocausality, can be implemented in this setting. The S-matrix does however not fulfill such analyticity properties as the crossing [27] property whose derivation relies on the existence of local interpolating fields.

In contradistinction to the more fundamental locally covariant QFT, DPI is primarily a phenomenological setting, but one which is consistent with every property which can be expressed in terms of relativistic particles only. So instead of approximating nonperturbative QFT in a metaphoric way outside conceptional-mathematical control, the idea of DPI is to arrange phenomenological calculations in such a way that at least the principles of relativistic mechanics and macro-causality are maintained [29].

For the interaction of two relativistic particles the introduction of relativistic interactions amounted to add to the free mass operator (the Hamiltonian in the c.m. system) an interact which depends on the relative position and momentum. The exigencies of representation theory of the Poincaré group are then fulfilled and the cluster property stating that $S \to \mathbf{1}$ for large spatial separation is a consequence of the short ranged interaction. Assuming for simplicity identical scalar Bosons, the c.m. invariant energy operator is $2\sqrt{p^2+m^2}$ and the interaction is introduced by adding an interaction term v

$$M = 2\sqrt{\vec{p}^2+m^2}+v, \quad H=\sqrt{\vec{P}^2+M^2} \tag{1}$$

where the invariant potential v depends on the relative c.m. variables p, q in an invariant manner i.e. such that M commutes with the Poincaré generators of the 2-particle system which is a tensor product of two one-particle systems.

One may follow Bakamjian and Thomas (BT) [31] and choose the Poincaré generators in their way so that the interaction only appears in the Hamiltonian. Denoting the interaction-free generators by a subscript 0, one arrives at the following system of two-particle generators

$$\begin{aligned} \vec{K} &= \frac{1}{2}(\vec{X}_0 H + H\vec{X}_0) - \vec{J}\times\vec{P}_0(M+H)^{-1} \\ \vec{J} &= \vec{J}_0 - \vec{X}_0\times\vec{P}_0 \end{aligned} \tag{2}$$

The interaction v may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum p in the c.m. system; but since the scheme anyhow does not lead to local differential equations, there is not much to be gained from such a choice. The Wigner canonical spin $\vec{J}_0$ commutes with $\vec{P}=\vec{P}_0$ and $\vec{X}=\vec{X}_{.0}$ and is related to the Pauli-Lubanski vector $W_\mu = \varepsilon_{\mu\nu\kappa\lambda}P^\nu M^{\kappa\lambda}$.

As in the nonrelativistic setting, short ranged interactions v lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian

$$\Omega_\pm(H,H_0) = \lim_{t\to\pm\infty} e^{iHt}e^{-H_0 t} \tag{3}$$

$$\begin{aligned} \Omega_\pm(M,M_0) &= \Omega_\pm(H,H_0) \\ S &= \Omega_+^*\Omega_- \end{aligned} \tag{4}$$

The identity in the second line is the consequence of a theorem which say that the limit is not affected if instead of M one takes take a positive function of M (4) as $H(M)$, as long as H_0 is the same function of M_0. This insures the the asymptotic *frame-independence of objects as the Møller operators and the S-matrix* but not necessarily that of semi asymptotic operators as formfactors of local operators between ket in and bra out particle states. Apart from this *identity for operators and their positive functions* (4) which is not needed in the nonrelativistic scattering, the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property is the statement that $\Omega_\pm^{(2)} \to \mathbf{1}$, $S^{(2)} \to \mathbf{1}$, i.e. the scattering formalism is identical. In particular the two particle cluster property, which says that for short range interactions the S-matrix approaches the identity if one separates the center of the wave packets of the two incoming particles, holds also for the relativistic case.

The implementation of clustering is more delicate for three particles as can be seen from the fact that the first attempts were started in 1965 by Coester [32] and considerably later generalized (in collaboration with Polyzou [29]) to an arbitrary high particle number.

To anticipate the result below, DPI leads to a consistent scheme which fulfills cluster factorization but it has no useful second quantized formulation so it may stand accused of lack of elegance; one is inclined to view less elegant theories also as less fundamental. It is also more nonlocal and nonlinear than QM, This had to be expected since adding interacting particles does not mean adding up interactions as in Schroedinger QM.

The BT form for the generators can be achieved inductively for an arbitrary number of particles. As will be seen, the advantage of this form is that in passing from n-1 to n-particles the interactions add after appropriate Poincaré transformations to the joint c.m. system and in this way one ends up with Poincaré group generators for an interacting n-particle system. But for $n > 2$ the aforementioned subtle problem with the cluster property arises; whereas this iterative construction in the nonrelativistic setting complies with cluster separability, this is not the case in the relativistic context.

This problem shows up for the first time in the presence of 3 particles [32]. The BT iteration from 2 to 3 particles gives the 3-particle mass operator

$$\begin{aligned} M &= M_0 + V_{12} + V_{13} + V_{23} + V_{123} \\ V_{12} &= M(12,3) - M_0(12;3), \;\; M(12,3) = \sqrt{\vec{p}_{12,3}^2 + M_{12}^2} + \sqrt{\vec{p}_{12,3}^2 + m^2} \end{aligned} \tag{5}$$

and the $M(ij,k)$ result from cyclic permutations. Here $M(12,3)$ denotes the 3-particle invariant mass in case the third particle is a "spectator", which by definition does not interact with 1 and 2. The momentum in the last line is the relative momentum between the (12)-cluster and particle 3 in the joint c.m. system and M_{12} is the associated two-particle mass i.e. the invariant energy in the (12) c.m system. Written in terms of the original two-particle interaction v, the 3-particle mass term appears very nonlinear.

As in the nonrelativistic case, one can always add a totally connected contribution. Setting this contribution to zero, the 3-particle mass operator only depends on the two-particle interaction v. But contrary to the nonrelativistic case, the BT generators constructed with M as it stands does not fulfill the cluster separability requirement. The latter demands that if the interaction between two clusters is removed, the unitary representation factorizes into that of the product of the two clusters.

One expects that shifting the third particle to infinity will render it a spectator and result in a factorization $U_{12,3} \rightarrow U_{12} \otimes U_3$. Unfortunately what really happens is that the (12) interaction also gets switched off i.e. $U_{123} \rightarrow U_1 \otimes U_2 \otimes U_3$. The reason for this violation of the cluster separability property, as a simple calculation (using the transformation formula from c.m. variables to the original p_i, i = 1, 2, 3 shows [29]), is that although the spatial translation in the original system (instead of the $12,3$ c.m. system) does remove the third particle to infinity as it should, unfortunately it also drives the two-particle mass operator (with which it does not commute) towards its free value which violates clustering.

In other words the BT produces a Poincaré covariant 3-particle interaction which is additive in the respective c.m. interaction terms (5), but the Poincaré representation U of the resulting system will not be cluster-separable. However this is the time for intervention of a saving grace: *scattering equivalence.*

As shown first in [32], even though the 3-particle representation of the Poincaré group arrived at by the above arguments violates clustering, the 3-particle S-matrix computed in the additive BT scheme turns out to have the cluster factorization property. But without implementing the correct cluster factorization also for the 3-particle Poincaré generators there is no chance to proceed to a clustering 4-particle S-matrix.

Fortunately there always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix*. Such transformations are called *scattering equivalences.* They were first introduced into QM by Sokolov [33] and their intuitive content is related to a certain insensitivity of the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is reminiscent of the insensitivity of the S-matrix against local changes in the interpolating field-coordinatizations[10] in QFT by e.g. using composites instead of the Lagrangian field.

The notion of scattering equivalences is conveniently described in terms of a subalgebra of *asymptotically constant operators* C defined by

$$\lim_{t\to\pm\infty} C^{\#} e^{iH_0 t}\psi = 0 \qquad (6)$$
$$\lim_{t\to\pm\infty} \left(V^{\#}-1\right) e^{iH_0 t}\psi = 0$$

where $C^{\#}$ stands for both C and C^{*}. These operators, which vanish on dissipating free wave packets in configuration space, form a *-subalgebra which extends naturally to a C^{*}-algebra $\mathcal{C}$. A scattering equivalence is a unitary member $V\in\mathcal{C}$ which is asymptotically equal to the identity (the content of the second line). Applying this asymptotic equivalence relation to the Møller operator one obtains

$$\Omega_{\pm}(VHV^{*},VH_0V^{*}) = V\Omega_{\pm}(H,H_0) \qquad (7)$$

so that the V cancels out in the S-matrix. Scattering equivalences do however change the interacting representations of the Poincaré group according to $U(\Lambda,a)\to VU(\Lambda,a)V^{*}$.

The upshot is that there exists a clustering Hamiltonian H_{clu} which is unitarily related to the BT Hamiltonian H_{BT} i.e. $H_{clu}=BH_{BT}B^{*}$ such that $B\in\mathcal{C}$ is uniquely determined in terms of the scattering data computed from H_{BT}. It is precisely this clustering of H_{clu} which is needed for obtaining a clustering 4-particle S-matrix which is cluster-associated with the $S^{(3)}$. With the help of M_{clu} one defines a 4-particle interaction following the additive BT prescription; the subsequent scattering formalism leads to a clustering 4-particle S-matrix and again one would not be able to go to n=5 without passing from the BT to the cluster-factorizing 4-particle Poincaré group representation. Coester and Polyzou showed [29] that this procedure can be iterated and doing this one arrives at the following statement

Statement: *The freedom of choosing scattering equivalences can be used to convert the Bakamijan-Thomas presentation of multi-particle Poincaré generators into a cluster-factorizing representation. In this way a cluster-factorizing S-matrix* $S^{(n)}$ *associated to a BT representation* H_{BT} *(in which clustering mass operator* $M_{clu}^{(n-1)}$ *was used) leads via*

[10]In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class) or in the setting of AQFT by picking another operator from a local operator algebra.

the construction of $M_{clu}^{(n)}$ to a S-matrix $S^{(n+1)}$ which clusters in terms of all the previously determined $S^{(k)}, k < n$. The use of scattering equivalences prevents the existence of a 2^{nd} quantized formalism.

For a proof we refer to the original papers [29][30]. In passing we mention that the minimal extension, i.e. the one determined uniquely in terms of the two-particle interaction v) from n to n+1 for $n > 3$, contains *connected 3-and higher particle interactions* which are nonlinear expressions (involving nested roots) in terms of the original two-particle v. This is another unexpected phenomenon as compared to the nonrelativistic case.

This theorem shows that it is possible to construct a relativistic theory which only uses particle concepts only, thus correcting an old folklore which says relativity + clustering = QFT. Whether one should call this DPI theory "relativistic QM" or just a relativistic S-matrix theory in a QM setting is a matter of taste; it depends on what significance one attributes to those unusual scattering equivalences. In any case it defines a *relativistic S-matrix setting* with the correct particle behavior i.e. all properties which one is able to formulate in terms of particles (without the use of fields) as unitarity, Poincaré invariance and macrocausality are fulfilled. In this context one should also mention that the S-matrix bootstrap approach never addressed these macro-causality problems of the DPI approach; it was a grand self-deluding design for a unique theory of all non-gravitational interactions in which important physical details were arrogantly ignored.

As mentioned above Coester and Polyzou also showed that this relativistic setting can be extended to processes which maintain cluster factorization in the presence of a finite number of creation/annihilation channels, thus demonstrating, as mentioned before, that *the mere presence of particle creation is not characteristic for QFT* (but rather the presence of infinite vacuum polarization clouds from "banging" with localized operators onto the vacuum, see section 7). Different from the nonrelativistic Schroedinger QM, the superselection rule for masses of particles which results from Galilei invariance for nonrelativistic QM does not carry over to the relativistic setting; in this respect DPI is less restrictive than its Galilei-invariant QM counterpart where such creation processes are forbidden.

One may consider the DPI setting of Coester and Polyzou as that scheme which results from implementing the mentioned particle properties within a n-particle Wigner representation setting in the presence of interaction [29]. Apparently the work of these mathematical nuclear physicists has not been noted by particle physicists since the authors have published most of their results in nuclear physics journals. What makes it worthwhile to mention this work is that even physicists of great renown as Steven Weinberg did not believe that such a theory exists because otherwise they would not have conjectured that the implementation of cluster factorization properties in a relativistic setting leads to QFT [34].

Certain properties which are consequences of locality in QFT and can be formulated but not derived in a particle setting as the TCP symmetry, the spin-statistics connection and the existence of anti-particles, can be added "by hand" to the DPI setting. Other properties which are on-shell relics of locality which QFT imprints on the S-matrix and which require the notion of analytic continuation in particle momenta (as e.g. the crossing property for formfactors) cannot be implemented in the QM setting of DPI.

4 First brush with the intricacies of the particles-field problems in QFT

In contrast to QM (Schrödinger-QM or relativistic DPI), interacting QFT does not admit a particle interpretation at finite times[11]. If it would not be for the asymptotic scattering interpretation in terms of incoming/outgoing particles associated with the free in/out fields, there would be hardly anything of a non-fleeting measurable nature. In QFT in CST and thermal QFT where even this asymptotically valid particle concept is missing, the set of conceivable measurements is essentially reduced to energy- and entropy- densities in thermal states and in black hole states with event horizons as well as of cosmological states describing the microwave background radiation.

Since the notion of particle is often used in a more general sense than in this paper, it may be helpful to have a brief interlude on this issue. By particle I mean an asymptotically stable object which forms the Fock space tensor product basis for an asymptotically complete description. It is precisely this particle concept which furnishes QFT with a (LSZ, Haag-Ruelle) complete asymptotic particle interpretation[12], so that the Hilbert space of such an interacting theory has a Fock space tensor structure. The physics behind is the idea [35] that if we were to cobble the asymptotic spacetime region with counters which monitor coincidences/anticoincidences of localization events (local deviations from the vacuum) after a collision of two incoming particles has taken place, then the defining property of an outgoing n-particle state is the stable n-fold coincidence/anticoincidence (the latter in order to insure that we registered all particles) between n counters. The intuitive idea is that after some time the n would not change and the n-fold local excitations from the vacuum would move along trajectories of free relativistic particles. would eventually remain stable because the far removed localization centers would have ceased to interact and from there on move freely. The occurrence rate of these coincidences as well as their correlation with that of the incoming coincidences is independent of the frame of reference even though BNW localization at finite spacetime regions is frame dependent. In popular textbooks this is expressed as. the BNW localization becomes "effectively" covariant for distances beyond a Compton wavelength (exactly covariant only in the large time limit.) The Newton-Wigner adaptation of the Born position operator would lead to genuinely Poincaré invariant frame independent transition probabilities between incoming and outgoing Newton-Wigner localization events.

The particle concept in QFT is therefore precisely applicable where it is needed, namely for asymptotically separated BNW-localized events for which the probability interpretation and covariance become compatible. In fact the use of the BNW localization for finite distances is known to lead to trouble in form of unphysical superluminal effects; in that case one should formulate the problem in the setting of the modular localization which has instead of probabilities and projectors dense subsets of states (the Reeh-Schlieder

[11] Although the one-particle states and their multiparticle counterparts are global states in the Hilbert space, they are not accessible by acting locally on the vacuum. Scattering theory is the only known nonlocal intervention.

[12] The asymptotic completeness property was for the first time established (together with a recent existence proof) in a family of factorizing two-dimensional models (see the section on modular localization) with nontrivial scattering.

property [5]).

Tying the particle concept in QFT to asymptotically stable coincidences of counters can be traced back to a seminal paper by Haag and Swieca [35]. These authors noticed for the first time that the phase space degree of freedom density in QFT, unlike that in QM, is not finite, rather its cardinality is mildly infinite (the phase space is *nuclear*). The larger number of degrees of freedom in form of an enhanced phase space density is yet another line of unexpected different consequences [36][37] resulting from the different localization concepts in QM and QFT, but this interesting topic will not be pursued here.

Not all particles comply with this definition; in fact all electrically charged particles are *infraparticles* i.e. objects which are asymptotically stable but in contrast to Wigner particles they are inexorably attached to an unobserved cloud of infinitely many infrared photons which are persistent even in the asymptotic large time limit. The existence of an electron as a Wigner particle associated with a sharp mass hyperboloid on top of a photon background is a fiction which is incompatible with QED. Rather electrically charged particles have instead of a mass shell delta function in their Kallen-Lehmann two´point function a cut which starts at $p^2 = m^2$ which makes a precise description of such *infraparticles* [38] and their scattering theory more involved. The application of LSZ scattering theory leads to infrared divergencies which cannot be cured by renormalizing parameters. The use of the conceptual "sledge hammer" of an infrared cutoff and compensating the divergence in the scattering amplitude against an infinite phase space factor obtained from summing over inclusive photons gives an observational satisfactory recipe for calculating an inclusive cross section with photon resolution Δ. Such successful recipes hide the fact that the root of the problem is a radical change of the particle concept which entails a fundamental adjustment [39].

In contrast to Wigner particles which are representation theoretical objects of the Poincaré group, infraparticles exist only in QED-like interacting theories in which the quantum adaptation of the Gauss law holds. The most dramatic differences between infraparticles and Wigner particles show up in localization aspects. Whereas Wigner particles "are pointlike" i.e. have pointlike generating wave functions, the sharpest localized generators for infraparticles are semiinfinite stringlike. On a formal level this has been known for a long time as expressed in the Dirac-Jordan-Mandelstam formulas in which a Dirac spinor is multiplied by an exponential semiinfinite line integral over the vector-potential (31). Their modern exposition would be an important part of an essay about various localization concepts. However the description of string-localized infraparticles is too subtle and would require a presentation which goes much beyond the content of this essay. We hope to return to issue in a separate paper.

It is the *asymptotic particle structure* which leads to the observational richness of QFT. Once we leave this setting by going to curved spacetime or to QFT in KMS thermal representations, or if we restrict a Minkowski spacetime theory to a Rindler wedge with the Hamiltonian being now the boost operator with its two-sided spectrum, in all these cases we are loosing not only the setting of scattering theory but also the very notion of particles as elementary systems with respect to the Poincaré group. With it also most of the observational wealth related to scattering theory is lost. Any deviation from Poincaré covariance also endangers the existence of a vacuum. The restriction to the Rindler world preserves the Fock space particle structure of the free field Minkowski QFT, but it looses its intrinsic physical significance with respect to the Rindler situation[13]. Since the Minkowski vacuum restricted to the Rindler world is now a thermal KMS state, there is no particle scattering theory in the "boost time" in such a thermal situation. The remaining observable phenomena are Hawking-like [43] radiation densities and their fluctuations i.e. observables such as they are presently studied in the cosmic background radiation. Some of the conceptual problems related to the Unruh effect [42] have been addressed in the philosophically oriented literature [40][41]. Quantum fields are not directly accessible to measurements[14] and therefore the problem what happens to the wealth of particle physics in such QFT requires more research.

Formally the local covariance principle forces the construction of a QFT on all causally complete manifolds and their submanifolds at once. So the QFT in Minkowski spacetime with its particle interpretation is always part of the solution. What one would like to have is a more direct physical connection e.g. a particle concept in the tangent space or something in this direction.

The conceptual differences between a DPI relativistic QM and QFT are enormous, but in order to appreciate this, one has to become acquainted with structural properties of QFT which are somewhat removed from the standard properties of the Lagrangian setting and therefore have not entered textbooks; it is the main purpose of the following sections to highlight these contrasts by going more deeply into QFT.

There are certain folkloric statements about the relation QM–QFT whose refutation does not require much conceptual sophistication. For example in trying to make QFT more susceptive to newcomers it is sometimes said that a free field is nothing more than a collection of infinitely many coupled oscillators. Although not outright wrong, this characterization misses the most important property of how spacetime enters as an ordering principle into QFT. It would not help any newcomer who knows the quantum oscillator, but has not met a free field before, to construct a free field from such a verbal description. Even if he manages to write down the formula of the free field he would still have to appreciate that the most important aspect is the causal localization and not that what oscillates. This is somewhat reminiscent of the alleged virtue from equating QM via Schrödinger's formulation with classical wave theory. What may be gained for a newcomer by appealing to his computational abilities acquired in classical electrodynamics, is more than lost in the conceptual problems which he confronts later when facing the subtleties of entanglement in quantum physics.

5 More on Born versus covariant localization

In this section it will be shown that the difference between QM and LQP in terms of their localization result in a surprising distinction in their notion of entanglement. We will continue to use the word *Born localization* for the probability density of the x-space Schroedinger wave function $p(x) = |\psi(x)|^2$; whereas its adaptation to the invariant inner

[13]There is of course the mathematical possibility of choosing a groundstate representation for a Rindler world instead of restricting the Minkowski vacuum and to have a finite number of "quanta" (excitations). But there is no reason for believing that these objects fall into the range of validity of the Haag-Ruelle scattering theory which is the hallmark of particle physics as we know it.

[14]An opposing opinion to this "interpolating field point of view" can be found in [44].

product of relativistic wave functions which was done by Newton and Wigner [11] and will referred to as BNW localization. Being a bona fide probability density, one may characterize the BNW localization in a spatial region $R \in \mathbb{R}^3$ at a given time in terms of a localization projector $P(R)$ which appears in the spectral decomposition of the selfadjoint position operator. The standard version of QM and the various settings of measurement theory rely heavily on these projectors; without BNW localization and the ensuing projectors it would be impossible to formulate the conceptual basis for the time-dependent scattering theory of QM and QFT.

The BNW position operator and its family of spatial region-dependent projectors $P(R)$ is not covariant under Lorentz boosts. For Wigner, to whom modular localization was not available, this frame dependence raised doubts about the conceptual soundness of QFT. Apparently the existence of completely covariant correlation functions in renormalized perturbation theory did not satisfy him, he wanted an understanding from first principles and not as an outgrowth of some formalism.

The lack of covariance of BNW localization in finite time propagation leads to frame-dependence and superluminal effects, which is why the terminology "relativistic QM" has to be taken with a grain of salt. However, as already emphasized, in the asymptotic limit of large timelike separation as required in scattering theory, the covariance, frame-independence and causal relations are recovered. As shown in section 3 one obtains a Poincaré-invariant unitary Møller operator and S-matrix whose DPI construction within an interacting n-particle Wigner representation of the Poincaré group which also guaranties the validity of all the macro-causality requirements (spacelike clustering, absence of timelike precursors, causal rescattering) which can be formulated in a particle setting i.e. without taking recourse to interpolating local fields. Even though the localizations of the individual particles are frame-dependent, the asymptotic relation between BNW-localized events is given in terms of the geometrically associated *covariant on-shell momenta* or 4-velocities which describe the asymptotic movement of the c.m. of wave packets. In fact *all observations on particles always involve BNW localization* measurements.

The situation of propagation of DPI is similar to that of propagation of acoustic waves in an elastic medium; although in neither case there is a limiting velocity, there exists a maximal "effective" velocity, for DPI this is c and in the acoustic case this is the velocity of sound in the particular medium.

In comparing QM with QFT it is often convenient in discussions about conceptual issues to rephrase the content of (nonrelativistic) QM in terms of operator algebras and states (in the sense of positive expectation functional on operator algebras); in this way one also achieves more similarity with the formalism of QFT and develops a greater awareness for genuine conceptual antinomies. In this Fock space setting the basic quantum mechanical operators are the creation/annihilation operators $a^{\#}(\mathbf{x})$ with

$$[a(\mathbf{x}), a^*(\mathbf{y})]_{grad} = \delta(\mathbf{x} - \mathbf{y}) \tag{8}$$

where for Fermions the graded commutator stands for the anticommutator. In the QFT setting it is not forbidden to work with such operators (the Fourier transforms of the Wigner creation/annihilation operators), except that it becomes nearly impossible to keep track of covariance and express local observables in terms of them[15].

[15] In fact local observables would appear nonlocal. The incorrect use of these operators led Irving Segal

The ground state for T=0 zero matter density states is annihilated by $a(x)$, whereas for finite density one encounters a state in which the levels are occupied up to the Fermi surface in case of Fermions, and contains a Bose-Einstein condensate groundstate in case of Bosons.

In QFT the identification of pure states with state-vectors of a Hilbert space has no intrinsic meaning and often cannot be maintained in concrete situations. For the reason of facilitating the comparison with QM we use the unified Fock space setting instead of the Schroedinger formulation. Although DPI is formulated in Fock space, there is no useful second quantized formalism (8).

The global algebra which contains all observables independent of their localization is the algebra $B(H)$ of all bounded operators in Hilbert space. Physically important unbounded operators are not members but rather have the mathematical status of being affiliated with $B(H)$ and its subalgebras; this bookkeeping makes it possible to apply powerful theorems from the theory of operator algebras (whereas unbounded operators are treated on a case to case basis). $B(H)$ is the correct global description whenever the physical system under discussion arises as the weak closure of a ground state representation of an irreducible system of operators[16] be it QM or LQP. According to the classification of operator algebras, $B(H)$ and all its multiples are of Murray von Neumann type I_∞ whose characteristic property is the existence of minimal projectors; in the irreducible case these are the one-dimensional projectors belonging to measurements which cannot be refined. There are prominent physical states which lead to different global situations as e.g. thermal KMS states, but for the time being our interest is in ground states.

The structural differences between QM and LQP emerge as soon as one defines a physical substructure on the basis of localization. It is well known that a dissection of space into nonoverlapping spatial regions i.e. $\mathbb{R}^3 = \cup_i R_i$ implies via Born localization a tensor factorization of $B(H)$ and H

$$
\begin{aligned}
B(H) &= \bigotimes_i B(H(R_i)) && (9) \\
H &= \bigotimes_i H(R_i),\ \ P(R_i)H = H(R_i) \\
\tilde{\mathbf{X}}_{op} &= \int a^*(\vec{x})\vec{x}a(\vec{x})d^3x = \int \vec{x}dP(\vec{x}) && (10)
\end{aligned}
$$

where the third line contains the definition of the position operator and its spectral decomposition in the bosonic Fock space. Hence there is orthogonality between subspaces belonging to localizations in nonoverlapping regions (orthogonal Born projectors) and one may talk about states which are pure in $H(R_i)$. As well known from the discussion of entanglement, a pure state in the global algebra $B(H)$ may not be of the special tensor product form but rather be a superposition of factorizing states; the Schmidt decompo-

to the conclusion that local observable subalgebras in QFT are quantum mechanical type I factors a claim which he withdrew after becoming aware of the results by Araki [45] who showed that they are of type III (later refined to the unique "hyperfinite type III_1").

[16] The closure in a thermal equilibrium state associated with a continuous spectrum Hamiltonian leads to a unitarily inequivalent (type III) operator algebra without minimal projectors.

sition is a method to achieve this with an intrinsically determined basis in the case of a bipartite tensor factorization.

States which are not tensor products, but rather superpositions of such, are called entangled; their reduced density matrix obtained by averaging over the environment of R_i describes a mixed state on $B(H(R_i))$. This is the standard formulation of QM in which pure states are vectors and mixed states are density matrices.

Although this quantum mechanical entanglement can be related to the notion of entropy, it is an entropy in the sense of *information theory* and not in the *thermal sense.* One cannot create a physical temperature as a quantitative measure of the degree of quantum mechanical entanglement in this way. which results from BNW-restricting pure global states to a finite region and its outside environment. In particular the ground state always factorizes, a spatial tensor factorization never causes vacuum polarization and entanglement in QM setting. The net structure of $B(H)$ in terms the subalgebras $B(H(R_i))$ is of a kinematical kind; although the reduced state may be impure, there is no $B(H(R)$ reduced Hamiltonian relative to which an impure state in QM becomes a KMS state. Here QM stands for any QT without a maximal propagation speed i.e. one which lacks causal propagation and vacuum polarization.

The LQP counterpart of the Born-localized subalgebras at a fixed time are the observable algebras $\mathcal{A}(\mathcal{O})$ for spacetime double cone regions $\mathcal{O}$ obtained from spatial regions R by causal completion $\mathcal{O} = R''$ (causal complement taken twice); they form what is called in the terminology of LQP a *local net* $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset M}$ of operator algebras indexed by regions in Minkowski spacetime $\cup\mathcal{O} = \mathcal{M}$ which is subject to the natural and obvious requirements of isotony ($\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ if $O_1 \subset O_2$) and causal locality, i.e. the algebras commute for spacelike separated regions.

The connection with the standard formulation of QFT in terms of pointlike fields is that smeared fields $\Phi(f) = \int \Phi(x)f(x)d^4x$ with *suppf* $\subset \mathcal{O}$ under reasonable general conditions generate local algebras. Pointlike fields, which by themselves are too singular to be operators (even if admitting unboundedness), have a well-defined mathematical meaning as operator-valued distributions briefly referred to as generators of algebras. The singular nature of generating fields is therefore not a pathological aspect leading to inescapable ultraviolet catastrophes, but rather a natural attribute of passing from classical to quantum fields.

The real cumbersome aspect is not their singular behavior but their multitude; there are myriads of fields which generate the same net of local operator algebras and interpolate the same particles whereas in classical field theory they could be distinguished by classical field measurements.

In this sense generating fields play a similar role in LQP as coordinates in modern differential geometry i.e. they coordinatize the net of spacetime indexed operator algebras and only the latter has an intrinsic meaning; in particular the particles and their collision theory can be obtained from the local net without being forced to distinguish individual operators within a local algebra. But as the use of particular coordinates often facilitates geometrical calculations, the use of particular fields, with e.g. the one with the lowest short-distance dimension within the infinite charge equivalence class of fields, can greatly simplify[17] calculations in QFT. Therefore it is a problem of practical importance to construct a covariant basis of locally covariant pointlike fields of an equivalence class.

For massive free fields and for massless free fields of finite helicity such a basis is especially simple; the "Wick-basis" of composite fields still follows in part the logic of classical composites (apart from the definition of the double dot : :). This remains so even in the presence of interactions in which case the Wick-ordering gets replaced by the technically more demanding "normal ordering" [46]. For free fields in curved spacetime (CST) and the definition of their composites it is important to require the *local covariant transformation behavior* under local isometries [47]. The conceptual framework of QFT in CST in the presence of *interactions* has also been largely understood [48].

We now return to the main question namely: what changes if we pass from the BNW localization of QM/DPI to the causal localization of LQP? The crucial property is that a localized algebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ together with its commutant $\mathcal{A}(\mathcal{O})'$ (which under very general conditions[18] is equal to the algebra of the causal disjoint of $\mathcal{O}$ i.e. $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}'))$ are two von Neumann factor algebras i.e.

$$B(H) = \mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O})',\ \mathcal{A}(\mathcal{O}) \cap \mathcal{A}(\mathcal{O})' = \mathbb{C}\mathbf{1} \tag{11}$$

In contrast to the QM algebras the local factor algebras are not of type I and $B(H)$ does *not tensor-factorize* in terms of them, in fact they cannot even be embedded into a $B(H_1) \otimes B(H_2)$ tensor product. The prize to pay for ignoring this important fact and imposing wrong structures is the appearance of spurious *ultraviolet divergences,* the typical way of a QFT model to resist enforcing an incompatible structure on it.

On the positive side, as will be explained in the second part of this essay, without this significant change in the nature of algebras there would be no holography onto causal horizons and the resulting huge symmetry enhancement to infinite-dimensional (BMS) groups, and of course there would be no thermal behavior caused by localization and a fortiori no area-proportional localization entropy.

The situation in LQP is radically different from that of entanglement and pure versus mixed states in QM since local algebras as $\mathcal{A}(\mathcal{O})$ have *no pure states at all*; so the dichotomy between pure and mixed states breaks down and the kind of entanglement caused by field theoretic localization is much more violent then that coming from BNW-localization[19], in the terminology of Ruetsche [3] these states are *instrinsically mixed.* This implies that the standard pure-mixed dichotomy does not extend beyond QM i.e. such intrinsically mixed states do not exist in any natural way on $B(H)$. At the moment in which they come into being as e.g. thermodynamic limit states in the infinite volume limit, the algebra has ceased to be of the quantum mechanical B(H) type and become a type III operator algebra [90]. The thermodynamic limit construction at finite temperature

[17] The field which is "basic" in the sense of a Lagrangian field in a Lagrangian approach is generally simpler to deal with than composites of that fields (the Massive Thirring field is simpler than the Sine-Gordon field which maybe derived from it).

[18] In fact this duality relation can always be achieved by a process of maximalization (Haag dualization) which increases the degrees of freedom inside $\mathcal{O}$. A pedagogical illustration based on the "generalized free field" can be found in [49].

[19] By introducing in addtion to free fields $A(x)$ which are covariant Fourier transforms also noncovariant Fourier transforms $a(\vec{x},t), a^*(\vec{x},t)$ one can explicitly that the latter are relatively nonlocal.

gives also the correct hint to the nature of intrinsically mixed states; they are typically "singular" KMS states i.e. KMS states which although being the thermodynamic limits of Gibbs state cannot themselves be represented in the Gibbs form because the KMS Hamiltonian has continuous spectrum.

Unlike Born localization, causal localization is not related to position operators and projectors $P(R)$; rather the operator algebras $\mathcal{A}(\mathcal{O})$ are of an entirely different kind than those met in ground state (zero temperature) QM ; they are all isomorphic to one abstract object, the hyperfinite type III_1 von Neumann factor also referred to as *the monad* the unique factor behind Araki's 1963 discovery [45]. As will be seen later LQP creates its wealthy mansion from just this one kind of brick; all its structural richness comes from positioning the bricks, there is nothing hidden in the structure of one bricks. In a later section it will be explained how this emerges from modular localization and a related operator formalism.

The situation does not change if one takes for $\mathcal{O}$ a region R at a fixed time; as stated before, in a theory with finite propagation speed one has$\mathcal{A}(R) = \mathcal{A}(D(R))$, where $D(R)$ is the diamond shaped double cone subtended by R (the causal shadow of R). Even if there are no pointlike generators and if the theory (as the result of the existence of an elementary length) only admits a macroscopically localized net of algebras (e.g. a net of non-trivial wedge-localized factor algebras $\mathcal{A}(W)$ with trivial double cone intersection algebras $\mathcal{A}(\mathcal{O}) = \{c\mathbf{1}\}$), the algebras would still not tensor factorize $B(H) \neq \mathcal{A}(W) \otimes \mathcal{A}(W')$. Hence the properties under discussion are not directly related to the presence of singular generating pointlike/stringlike fields but are connected to the existence of well-defined (sharp) causal shadows. There is a hidden singular aspect in the sharpness of the $\mathcal{O}$-localization which generates infinitely large vacuum polarization clouds on the causal horizon of the localization. In the last section a method (splitting) will be presented which permits to define a split-distance dependent, but otherwise intrinsically defined finite thermal entropy.

Most divergencies (but not all, since the divergence of localization entropy for vanishing splitting distance is an unavoidable consequence of the principles) in QFT are the result of conceptual errors in the formulation resulting from tacitly identifying QFT with some sort of relativistic QM[20] and in this way ignoring the intrinsically singular nature of pointlike localized fields.

Often it is thought that the avoidance of locality in favor of *nonlocal* covariant operators eliminates the singular short distance behavior. But this is not quite true as evidenced by the Kallen-Lehmann representation of a covariant scalar object

$$\langle A(x)A(y)\rangle = \int \Delta_+(x-y,\kappa^2)\rho(\kappa^2)d\kappa^2 \qquad (12)$$

which was proposed precisely to show that even without demanding locality, but retaining only covariance and the Hilbert space structure (positivity), a certain singular behavior of covariant objects is unavoidable. In the DPI scheme this was avoided, because even though there are particles at all times, there are no covariant (tensors, spinors) objects

[20]The correct treatment of perturbation theory which takes into account the singular nature of pointlike quantum fields may yield more free parameters than in the classical setting, but one is never required to confront infinities or cut-offs.

at finite times, the only covariant quantity arises in the form of the invariant S-matrix in the $t \to \infty$ limit. The next section shows that a separation between covariance and localization in the pursuit of a less singular more nonlocal theory is a futile endeavour, at least as long as one does not subject spacetime itself to a radical revision.

In the algebraic formulation the covariance requirement refers to the geometry of the localization region $\mathcal{A}(\mathcal{O})$ i.e.

$$U(a,\Lambda)\mathcal{A}(\mathcal{O})U(a,\Lambda)^* = \mathcal{A}(\mathcal{O}_{a,\Lambda}) \tag{13}$$

whereas no additional requirement about the transformation behavior under finite dimensional (tensor, spinor) Lorentz representations (which would bring back the unboundedness and thus prevent the use of powerful theorems in operator algebras) is imposed for the individual operators. The singular nature of pointlike generators (if they exist) is then a purely mathematical consequence. Using such singular objects in pointlike interactions in the same way as one uses operators in QM leads to self-inflicted divergence problems.

We have seen that although QM and QFT can be described under a common mathematical roof of C^*-algebras with a state functional, as soon as one introduces the physically important localization structure, significant conceptual differences appear. These differences show up in the presence of vacuum polarization in QFT as a result of causal localization and they tend to have dramatic consequences; the most prominent ones will be presented in this and the subsequent sections, more will be contained in the second essay.

The net structure of the observables allows a *local comparison of states*: two states are locally equal in a region $\mathcal{O}$ if and only if the expectation values of all operators in are the same in both states. Local deviations from any state, in particular from the vacuum state, can be measured in this manner; states which are equal on the causal complement $\mathcal{A}(\mathcal{O}')$ that are indistinguishable from the vacuum are called localizable in $\mathcal{A}(\mathcal{O})$ ("strictly localized states" in the sense of Licht [50]) can be defined. Due to the unavoidable correlations in the vacuum state in relativistic quantum theory (the Reeh-Schlieder property [5]), the space $H(\mathcal{O})$ obtained by applying the operators in $\mathcal{A}(\mathcal{O})$ to the vacuum is, for any open region $\mathcal{O}$, dense in the Hilbert space and thus far from being orthogonal to $H(\mathcal{O}')$. This somewhat counter-intuitive fact is inseparably linked with a structural difference between the local algebras and the algebras encountered in non-relativistic quantum mechanics (or the global algebra of a quantum field associated with the entire Minkowski space-time) as mentioned in connection with the breakdown of tensor-factorization (11).

The result is a particular benevolent form of "Murphy's law" for interacting QFT: *everything which is not forbidden (by superselection rules) to couple, really does couple.* On the level of interacting particles this has been termed *nuclear democracy*: any particle whose superselected charge is contained in the spectrum which results from fusing the charges in a cluster of particles can be viewed as a bound state of that cluster of particles. Nuclear democracy even strips a particle with a fundamental charge of its individuality since such an object can be considered as bound of itself + an arbitrary number of particles with non-fundamental charges. This renders interacting QFT conceptually much more attractive and fundamental than QM, but it also contributes to its computational complexity i.e. the benevolent character of Murphy's LQP law unfortunately does not necessarily extend to the computational side, at least if one limits oneself to the standard tools of QT.

The Reeh-Schlieder property [5] (in more popular but less precise terminology: the "state-field relation") is perhaps the strongest realization of Murphy's law since it secures the existence of a localization region dependent dense subspace $H(\mathcal{O}) = \mathcal{A}(\mathcal{O})\Omega \subset H$ which cannot be associated with a nontrivial projector. It also implies that the expectation value of a projection operator localized in a bounded region *cannot* be interpreted as the probability of detecting a particle-like object in that region, since it is necessarily nonzero if acting on the vacuum state. The $\mathcal{A}(\mathcal{O})$-reduced ground state is a KMS thermal state at a appropriately normalized (Hawking) temperature (more in part II). The intrinsically defined *modular "Hamiltonian"* associated via modular operator theory[21] to a "standard pair" $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ is always available in the mathematical sense but allows a physical interpretation only in those rare cases when there exists an invariance group of $\mathcal{O}$ which is a subgroup of the spacetime group leaving Ω_{vac} invariant. Well known cases are the Lorentz boost for the wedge region in Minkowski spacetime (the Unruh effect) and the generator of a double-cone preserving conformal transformation in a conformal theory and certain Killing symmetries in black hole physics. Its general purpose is to give an intrinsic description of the $\mathcal{A}(\mathcal{O})$- reduced vacuum state in terms of an KMS state of an Hamiltoian "movement" where we used brackets in order to highlight the fact that this is generally not a geometric movement but only an algebraic automorphism of $\mathcal{A}(\mathcal{O})$ (and simultaneously of $\mathcal{A}(\mathcal{O}'))$ which respects the geometric boundaries (the causal horizon) of $\mathcal{O}$ [22]. It is never the Hamiltonian associated with a globally inertial reference frame as in case of heat bath thermal systems.

There exists in fact a whole family *of modular Hamiltonians* since the operators in $\mathcal{A}(\mathcal{O})$ naturally fulfill the KMS condition for any standard pair $(\mathcal{A}(\check{\mathcal{O}}), \Omega_{vac})$ for $\check{\mathcal{O}} \supset \mathcal{O}$: i.e. the different modular Hamiltonians and the KMS states change with the causally closed world $\check{\mathcal{O}}$ of the observer. The surprising aspect is that the causal localization structure of one QFT leads to an infinite supply of different Hamiltonians without any change of interactions. The change of the modular Hamiltonian $K_{\mathcal{O}}$ via a change of the localization region will lead to a new Hamiltonian whose automorphic movement maintains the new region but leaves (after some "modular time") the old region i.e. this is not a family of Hamiltonians on a quantum mechanical algebra. Of particular interest is the restriction of a modular automorphism to the horizon of a causally closed region $Hor(\mathcal{O})$; there are good indications that this defines a diffeomorphism which belongs to the infinite dimensional Bondi-Metzner-Sachs subgroup of a gigantic symmetry group of holographic projection onto horizons (see part II on holography).

The situation just described is one of extreme "virtuality", i.e. there is generally not even the possibility to view it in terms of an Gedankenexperiment of a non-inertial (accelerated) $\mathcal{O}$-confined observer for whom the modular movement is an $\mathcal{O}$ - preserving diffeomorphism; such pure algebraic movements without individual orbits are often called "fuzzy". Whenever the modular movement passes to a diffeomorphism one can at least envisage a Gedankenexperiment which keeps the observer on an $\mathcal{O}$-preserving track by

[21]The modular Hamitonian is the infinitesimal generator $K_{\rm mod}$ of the modular group $\Delta^{it} \equiv e^{-itK_{\rm mod}}$.(see next two sections).

[22]In fact it induces a geometric movement *on* the horizon.

appropriate accelerations. The only geometric case in Minkowski spacetime is the situation proposed first by Unruh [42], when $\mathcal{O}$ is a wedge i.e. a region W which is bounded by two intersecting lightfronts which only share the 2-dim. edge of their intersection.. Conformal theories for which the observables live in the Dirac-Weyl compactification $\widetilde{M}$ of the Minkowski spacetime lead to modular diffeomorphisms even for compact double cones $\mathcal{D}$[23].

The most interesting and prominent case comes about when spacetime curvature is creating a black hole. In case there are time-like Killing orbits and an extension of the spacetime such that the black hole horizon is a event horizon in the sense of dividing the extended manifold into a causally inside/outside with separate Killing movements, one is in the classical Hawking-like situation. What one in additions needs for the quantum setting is the existence of a quantum state which is invariant under the Killing group action.

In the case of the Schwarzschild black hole all these requirements are fulfilled, the extension is the Schwarzschild-Kruskal extension and the invariant state is the Hartle-Hawking state Ω_{H-H}. In this case $(\mathcal{A}(\mathcal{O}_{S-K}), \Omega_{H-H})$ is a standard pair and the modular movement is the Killing orbit which respects the black hole event horizon. Whereas the causal horizons in the previous Minkowski spacetime examples was an extremely "fleeting" object, a black hole event horizon has an intrinsic metric-imprinted position. Besides their astrophysical interests, black holes are therefore of considerable philosophical interest. The only future development which could still enforce a significant modification of the present concepts is the still unknown *quantum gravity* (more remarks on QG in part II).

For computations of thermal properties, including thermal entropy, it does not matter whether the horizon is a "fleeting" observer-dependent causal localization[24] horizon or a fixed curvature generated black hole *event horizon*; only its direct observable significance depends on the black hole event horizon. This leads to a picture about the LQP-QG (quantum gravity) interface which is somewhat different from that in most of the literature; we will return to these issues in connection with the presentation of the split property in part II of the essay.

Causality in relativistic quantum field theory is mathematically expressed through local commutativity, i.e., mutual commutativity of the algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}')$. There is an intimate connection of this property with the possibility of preparing states that exhibit no mutual correlations for a given pair of causally disjoint regions. In fact, in a recent paper Buchholz and Summers [7] show that local commutativity is a necessary condition for the existence of such uncorrelated states.

Conversely, in combination with some further properties related to degrees of freedom densities (split property [53], existence of scaling limits [54]), local commutativity leads to a very satisfactory picture of *statistical independence* and *local preparabilty of states* in relativistic quantum field theory. We refer to [55][56] for thorough discussions of these matters and [51][15] for a brief review of some physical consequences. The last two pa-

[23]The region obtained by intersecting a forward lightcone with arbitrary apex with an backward lightcone;

[24]The localization entropy which depends on the "split" size (see below) is however an important property of the model, even if it not directy experimentally accessible.

pers also explain how the above mentioned concepts avoids spurious problems rooted in assumptions that are in conflict with basic principles of relativistic quantum physics. In particular it can be shown how an alleged difficulty [8][9] with Fermi's famous Gedanken-experiment [52], which Fermi proposed in order to show that the velocity of light is also the limiting propagation velocity in quantum electrodynamics, can be resolved by taking [51] into account the progress on the conceptual issues of causal localization and the gain in mathematical rigor since the times of Fermi.

After having discussed some significant conceptual differences between QM and LQP, one naturally asks for an argument why and in which way QM appears as a nonrelativistic limit of LQP. The standard kinematical reasoning of the textbooks is acceptable for fermionic/bosonic systems in the sense of "FAPP", but has not much strength on the conceptual level. To see its weakness, imagine for a moment that we would live in a 3-dim. world of anyons (abelian plektons, where plektons are Wigner particles with braid group statistics). Such relativistic objects are by their very statistics so tightly interwoven that there simply are no compactly localized free fields which only create a localized anyon without a vacuum polarization cloud admixture. In such a world no nonrelativistic limit which maintains the spin-statistic connection could lead to QM, the limiting theory would rather *remain a nonrelativistic QFT*. In order to avoid misunderstandings, its is not claimed here that the issue of nonrelativistic limits of any interacting relativistic QFT is mathematically understood[25], rather the statement is that plektonic (braid-group) commutation relations, relativistic or nonrelativistic, interacting or not, are incompatible with the structure of (Schroedinger) QM. In 4-dimensional spacetime there is no such obstacle against QM, simply because it is not the Fermi/Bose statistics which causes vacuum polarization; to formulate it more provocatively: there would be no Schroedinger QM without the existence of free relativistic fermions/bosons.

6 Modular localization

Previously it was mentioned on several occasions that the localization underlying QFT can be freed from the contingencies of field coordinatizations. This is achieved by a physically as well mathematically impressive, but for historic and sociological reasons little known theory. Its name "modular theory" is of mathematical origin and refers to a vast generalization of the (uni)modularity encountered in the relation between left/right Haar measure in group representation theory. In the middle 60s the mathematician Tomita presented a significant generalization of this theory to operator algebras and in the subsequent years this theory received essential improvements from Takesaki and later from Connes.

At the same time Haag, Hugenholtz and Winnink published their work on statistical mechanics of open systems [5]. When the physicists and mathematicians met at a conference in Baton Rouge in 1966, there was surprise about the similarity of concepts, followed by deep appreciation about the perfection with which these independent developments

[25]The arguments about the nonrelativistic limit og QFT have remained metaphoric; however the existence of exactly solved interacting 2-dim. QFTs raises now hopes that age old problem will be better understood.

supported each other [57]. Physicists not only adapted mathematical terminology, but mathematicians also took some of their terminology from physicists as e.g. *KMS states* which refer to Kubo, Martin and Schwinger who introduced an analytic property of Gibbs states merely as a computational tool (in order to avoid computing traces), Haag, Hugenholtz and Winnink realized that this property (which they termed the KMS property) is the only aspect which survives in the thermodynamic limit when the trace formulas loses its meaning and must be replaced by the analytic KMS boundary condition.

This turned out to be the right concept for formulating and solving problems directly in the setting of open systems. In the present work the terminology is mainly used for thermal states of open systems which are not Gibbs states. They are typical for LQP, for example every multiparticle state $\Omega_{particle}$ of finite energy, including the vacuum, (i.e. every physical particle state) upon restriction to a local algebra $\mathcal{A}(\mathcal{O})$ becomes a KMS state with respect to a "modular Hamiltonian" which is canonically determined by $(\mathcal{A}(\mathcal{O}), \Omega_{particle})$.

Connes, in his path-breaking work on the classification of von Neumann factors [59], made full use of this hybrid math.-phys. terminology which developed after Baton Rouge. Nowadays one can meet mathematicians who use the KMS property but do not know that this was a mere computational tool by 3 physicists (Kubo. Martin and Schwinger) to avoid calculating traces and that the conceptual aspect was only realized later by Haag Hugenholtz and Winnink who gave it its final name. One can hardly think of any other confluence of mathematical and physical ideas on such a profound and at the same time equal and natural level as in modular theory; even the Hilbert space formalism of QM already existed for many years before quantum theorists became aware of its use.

About 10 years after Baton Rouge, Bisognano and Wichmann [17] discovered that a vacuum state restricted to a wedge-localized operator algebra $\mathcal{A}(W)$ in QFT defines a modular setting in which the restricted vacuum becomes a thermal KMS state with respect to the wedge-affiliated L-boost "Hamiltonian". This step marks the beginning of a very natural yet unexpected relation between thermal and geometric properties, one which is totally characteristic for QFT i.e. which is not shared by classical theory nor by QM. Thermal aspects of black holes were however discovered independent of this work, and the first physicist who saw the connection with modular theory was Geoffrey Sewell [18].

The theory becomes more accessible for physicists if one introduces it first in its more limited spatial- instead of its full algebraic- context. Since as a foundational structure of LQP it merits more attention than it hitherto received from the particle physics community, some of its methods and achievements will be presented in the sequel.

It has been realized by Brunetti, Guido and Longo[26] [13] that there exists a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. The starting point is an irreducible representation $U_1$of the Poincaré´group on a Hilbert space H_1 that after "second quantization" becomes the single-particle subspace of the Hilbert space (Wigner-Fock-space) H_{WF} of the quantum fields act[27]. In the bosonic case the construction then proceeds according to the following steps [13][58][15].

[26]In a more limited context and with less mathematical rigor this was independently proposed in [14].
[27]The construction works for arbitrary positive energy representations, not only irreducible ones.

One first fixes a reference wedge region, e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$ and considers the one-parametric L-boost group (the hyperbolic rotation by χ in the $x^{d-1} - x^0$ plane) which leaves W_0 invariant; one also needs the reflection j_{W_0} across the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The $\mathfrak{j}_{W_0}$ extended Wigner representation is then used to define two commuting wedge-affiliated operators

$$\delta_{W_0}^{it} = \mathfrak{u}(0, \Lambda_{W_0}(\chi = -2\pi t)),\ \mathfrak{j}_{W_0} = \mathfrak{u}(0, j_{W_0}) \tag{14}$$

where attention should be paid to the fact that in a positive energy representation any operator which inverts time is necessarily antilinear[28]. A unitary one- parametric strongly continuous subgroup as $\delta_{W_0}^{it}$ can be written in terms of a selfadjoint generator K as $\delta_{W_0}^{it} = e^{-itK_{W_0}}$ and therefore permits an "analytic continuation" in t to an unbounded densely defined positive operators $\delta_{W_0}^{s}$. With the help of this operator one defines the unbounded antilinear operator which has the same dense domain as its "radial" part

$$\mathfrak{s}_{W_0} = \mathfrak{j}_{W_0}\delta_{W_0}^{\frac{1}{2}},\ \mathfrak{j}\delta^{\frac{1}{2}}\mathfrak{j} = \delta^{-\frac{1}{2}} \tag{15}$$

Whereas the unitary operator $\delta_{W_0}^{it}$ commutes with the reflection, the antiunitarity of the reflection changes the sign in the analytic continuation which leads the commutation relation between δ and $\mathfrak{j}$ in (15). This causes the involutivity of the s-operator on its domain, as well as the identity of its range with its domain

$$\mathfrak{s}_{W_0}^2 \subset \mathbf{1}$$
$$dom\ \mathfrak{s} = ran\ \mathfrak{s}$$

Such operators which *are unbounded and yet involutive* on their domain are very unusual; according to my best knowledge they only appear in modular theory and it is precisely these unusual properties which are capable to encode geometric localization properties into domain properties of abstract quantum operators, a fantastic achievement completely unknown in QM. The more general algebraic context in which Tomita discovered modular theory will be mentioned later.

The idempotency means that the s-operator has ± 1 eigenspaces; since it is antilinear, the +space multiplied with i changes the sign and becomes the - space; hence it suffices to introduce a notation for just one eigenspace

$$\begin{aligned} \mathfrak{K}(W_0) &= \{domain\ of\ \Delta_{W_0}^{\frac{1}{2}},\ \mathfrak{s}_{W_0}\psi = \psi\} \\ \mathfrak{j}_{W_0}\mathfrak{K}(W_0) &= \mathfrak{K}(W_0') = \mathfrak{K}(W_0)',\ duality \\ \overline{\mathfrak{K}(W_0) + i\mathfrak{K}(W_0)} &= H_1,\ \mathfrak{K}(W_0) \cap i\mathfrak{K}(W_0) = 0 \end{aligned} \tag{16}$$

It is important to be aware that, unlike QM, we are here dealing with real (closed) subspaces $\mathfrak{K}$ of the complex one-particle Wigner representation space H_1. An alternative which avoids the use of real subspaces is to directly deal with complex dense subspaces as in the third line. Introducing the graph norm of the dense space the complex subspace in the third line becomes a Hilbert space in its own right. The second and third line require

[28]The wedge reflection $\mathfrak{j}_{W_0}$ differs from the TCP operator only by a π-rotation around the W_0 axis.

some explanation. The upper dash on regions denotes the causal disjoint (which is the opposite wedge) whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on H_1.

The two properties in the third line are the defining property of what is called the *standardness property* of a real subspace[29]; any standard K space permits to define an abstract s-operator

$$\mathfrak{s}(\psi + i\varphi) = \psi - i\varphi \tag{17}$$
$$\mathfrak{s} = \mathfrak{j}\delta^{\frac{1}{2}}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group δ^{it} and a antiunitary reflection which generally have however no geometric significance. The domain of the Tomita $\mathfrak{s}$-operator is the same as the domain of $\delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that this domain is determined solely in terms of Wigner group representation theory.

It is easy to obtain a net of K-spaces by $U(a,\Lambda)$-transforming the K-space for the distinguished W_0. A bit more tricky is the construction of sharper localized subspaces via intersections

$$\mathfrak{K}(\mathcal{O}) = \bigcap_{W \supset \mathcal{O}} \mathfrak{K}(W) \tag{18}$$

where $\mathcal{O}$ denotes a causally complete smaller region (noncompact spacelike cone, compact double cone). Intersection may not be standard, in fact they may be zero in which case the theory allows localization in W (it always does) but not in $\mathcal{O}$. Such a theory is still causal but not local in the sense that its associated free fields are pointlike. One can show that the intersection for spacelike cones $\mathcal{O} = \mathcal{C}$ for all positive energy is always standard.

Note that the relativistic DPI setting also starts from Wigner particles but it completely ignores the presence of this modular localization structure which, as will be seen in brief, is the royal path into QFT which would have pleased Wigner and reconciled him with the conceptual structure of QFT.

There are three classes of irreducible positive energy representation, the family of massive representations $(m > 0, s)$ with half-integer spin s and the family of massless representation which consists really of two subfamilies with quite different properties namely the $(0, h =$ half-integer$)$ class, often called the neutrino-photon class, and the rather large class of $(0, \kappa > 0)$ infinite helicity representations parametrized by a continuous-valued Casimir invariant κ [15].

For the first two classes the $\mathfrak{K}$-space the standardness property also holds for double cone intersections $\mathcal{O} = \mathcal{D}$ for arbitrarily small $\mathcal{D}$, but this is definitely not the case for the infinite helicity family for which the localization spaces for compact spacetime regions turn out to be trivial[30]. Passing from localized subspaces $\mathcal{K}$ in the representation theoretical

[29]According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

[30]It is quite easy to prove the standardness for spacelike cone localization (leading to singular stringlike generating fields) just from the positive energy property which is shared by all three families [13].

setting to singular covariant generating wave functions (the first quantized analogs of generating fields) one can show that the $\mathcal{D}$ localization leads to pointlike singular generators (state-valued distributions) whereas the spacelike cone localization $\mathcal{C}$ is associated with semiinfinite spacelike stringlike singular generators [15]. Their second quantized counterparts are pointlike or stringlike covariant fields. It is remarkable that one does not need to introduce generators which are localized on hypersurfaces (branes).

Although the observation that the third Wigner representation class is not pointlike generated was made many decades ago, the statement that it is semiinfinite string-generated and that this is the worst possible case of state localization (which needs the knowledge of modular theory) is of a more recent vintage [13][15].

There is a very subtle aspect of modular localization which one encounters in the second Wigner representation class of massless finite helicity representations (the photon, graviton..class). Whereas in the massive case all spinorial fields $\Psi^{(A,\dot{B})}$ the relation of the physical spin s with the two spinorial indices follows the naive angular momentum composition rules [16]

$$\left|A-\dot{B}\right| \leq s \leq \left|A+\dot{B}\right|,\ m>0 \tag{19}$$
$$s=\left|A-\dot{B}\right|,\ m=0$$

the second line contains the significantly reduced number of spinorial descriptions for zero mass and finite helicity representations. What is going on here, why is there, in contradistinction to classical field theory no covariant s=1 vector-potential A_μ or no $g_{\mu\nu}$ in case of s=2 ? Why are the admissible covariant generators of the Wigner representation in this case limited to field strengths (for s=2 the linearized Riemann tensor)?

The short answer is that all these missing generators exist as stringlike covariant objects, the above restriction in the massless case only results from the covariantization to pointlike generators. The full range of spinorial possibilities (19) returns in terms of string localized fields $\Psi^{(A,\dot{B})}(x,e)$ if $s \neq \left|A-\dot{B}\right|$. These generating free fields are covariant and "string-local"

$$U(\Lambda)\Psi^{(A,\dot{B})}(x,e)U^*(\Lambda) = D^{(A,\dot{B})}(\Lambda^{-1})\Psi^{(A,\dot{B})}(\Lambda x,\Lambda e) \tag{20}$$
$$\left[\Psi^{(A,\dot{B})}(x,e),\Psi^{(A',\dot{B}')}(x',e'\right]_\pm = 0,\ x+\mathbb{R}_+e >< x'+\mathbb{R}_+e'$$

Here the unit vector e is the spacelike direction of the semiinfinite string and the last line expresses the spacelike fermionic/bosonic spacelike commutation. The best known illustration is the $(m=0,s=1)$ vectorpotential representation; in this case it is well-known that although a generating pointlike field strength exists, there is no *pointlike* vectorpotential acting in a Hilbert space.

According to (20) the modular localization approach offers as a substitute a stringlike covariant vector potential $A_\mu(x,e)$. In the case $(m=0,s=2)$ the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor (it is often referred to as the *linearized* Riemann tensor). In this case the string-localized potential is of the form $g_{\mu\nu}(x,e)$ i.e. resembles the metric tensor of general relativity. Some consequences of this localization for a reformulation of gauge theory will be mentioned in section 8.

Even in case of massive free theories where the representation theoretical approach of Wigner does not require to go beyond pointlike localization, covariant stringlike localized fields exist. Their attractive property is that they improve the short distance behavior e.g. a massive pointlike vector-potential of *sdd=2* passes to a string localized vector potential of *sdd=1*. In this way the increase of the sdd of pointlike fields with spin s can be traded against string localized fields of spin independent dimension with sdd=1. This observation would suggest the possibility of an enormous potential enlargement of perturbatively accessible higher spin interaction in the sense of power counting.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [60]. The amazing power of the modular localization approach is that it preempts the spin-statistics connection already in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the symplectic complement with the causal complement the generalized duality relation

$$\mathfrak{K}(\mathcal{O}') = Z\mathfrak{K}(\mathcal{O})' \tag{21}$$

where the square of the twist operator $Z = e^{\pi i s}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase $= Z^2$ [60].

The fact that one never has to go beyond string localization (and fact, apart from $s \geq 1$, never beyond point localization) in order to obtain generating fields for a QFT is remarkable in view of the many attempts to introduce extended objects into QFT.

It is helpful to be again reminded that modular localization which goes with real subspaces (or dense complex subspaces), unlike BNW localization, cannot be connected with probabilities and projectors. It is rather related to causal localization aspects; the standardness of the K-space for a compact region is nothing else then the one-particle version of the Reeh-Schlieder property. As will be seen in the next section modular localization is also an important tool in the non-perturbative construction of interacting models.

7 Algebraic aspects of modular theory

A net of real subspaces $\mathfrak{K}(\mathcal{O}) \subset H_1$ for an finite spin (helicity) Wigner representation can be "second quantized"[31] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the bosonic or fermionic Fock space $H = Fock(H_1)$ built over the one-particle Wigner space H_1. For integer spin/helicity values the modular localization in Wigner space implies the identification of the symplectic complement with the geometric complement in the sense of relativistic causality, i.e. $\mathfrak{K}(\mathcal{O})' = \mathfrak{K}(\mathcal{O}')$ (spatial Haag duality in H_1). The Weyl

[31] The terminology 2^{nd} quantization is a misdemeanor since one is dealing with a rigorously defined functor within QT which has little in common with the artful use of that parallellism to classical theory called "quantization". In Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

functor takes this spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: for each Wigner wave function $\varphi \in H_1$ the associated (unitary) Weyl operator is defined as

$$\begin{aligned} Weyl(\varphi) &:= expi\{a^*(\varphi) + a(\varphi)\} \in B(H) \\ \mathcal{A}(\mathcal{O}) &:= alg\{Weyl(\varphi)|\varphi \in \mathfrak{K}(\mathcal{O})\}'', \;\; \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned} \tag{22}$$

where $a^*(\varphi)$ and $a(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function φ. We then define the von Neumann algebra corresponding to the localization region $\mathcal{O}$ in terms of the operator algebra generated by the functorial image of the modular constructed localized subspace $\mathfrak{K}(\mathcal{O})$ as in the second line. By the von Neumann double commutant theorem, our generated operator algebra is weakly closed by definition.

The functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization structure and hence the spatial duality passes to its algebraic counterpart. The functor also commutes with the improvement of localization through intersections $\cap$ according to $\mathfrak{K}(\mathcal{O}) = \cap_{W\supset O}\mathfrak{K}(W)$, $\mathcal{A}(\mathcal{O}) = \cap_{W\supset O}\mathcal{A}(W)$ as expressed in the commuting diagram

$$\begin{array}{ccc} \{\mathfrak{K}(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow\cap & & \downarrow\cap \\ \mathfrak{K}(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{23}$$

Here the vertical arrows denote the tightening of localization by intersection whereas the horizontal ones denote the action of the Weyl functor.

The case of half-integer spin representations is analogous [58], apart from the fact that there is a mismatch between the causal and symplectic complements which must be taken care of by a *twist operator* $\mathcal{Z}$ and as a result one has to use the CAR functor instead of the Weyl functor.

In case of the large family of irreducible zero mass infinite spin representations in which the lightlike little group is faithfully represented, the finitely localized K-spaces are trivial $\mathfrak{K}(\mathcal{O}) = \{0\}$ and the *most tightly localized nontrivial spaces are of the form* $\mathfrak{K}(\mathcal{C})$ for $\mathcal{C}$ an arbitrarily narrow *spacelike cone*. As a double cone contracts to its core which is a point, the core of a spacelike cone is a *covariant spacelike semiinfinite string*. The above functorial construction works the same way for the Wigner infinite spin representation, except that in that case there are no nontrivial algebras which have a smaller localization than $\mathcal{A}(\mathcal{C})$ and there is no field which is sharper localized than a semiinfinite string. As stated before, stringlike generators, which are also available in the pointlike case, turn out to have an improved short distance behavior which makes them preferable from the point of view of formulating interactions within the power counting limit. They can be constructed from the unique Wigner representation by so called intertwiners between the unique canonical and the many possible covariant (dotted-undotted spinorial) representations. The Euler-Lagrange aspects plays no direct role in these construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization and also because most of the spinorial higher spin representations

(19) anyhow cannot be characterized in terms of Euler-Lagrange equations. The modular localization is the more general method of implementating causal propagation than that from hyperbolic equations of motions.

A basis of local covariant field coordinatizations is then defined by Wick composites of the free fields. The case which deviates furthest from classical behavior is the pure string-like infinite spin case which relates a *continuous* family of free fields with one irreducible infinite spin representation. Its non-classical aspects, in particular the absence of a Lagrangian, is the reason why the spacetime description in terms of semiinfinite string fields has been discovered only recently rather than at the time of Jordan's field quantization or Wigner's representation theoretical approach.

Using the standard notation Γ for the second quantization functor which maps real localized (one-particle) subspaces into localized von Neumann algebras and extending this functor in a natural way to include the images of the $\mathfrak{K}(\mathcal{O})$-associated s,δ,j which are denoted by S,Δ,J, one arrives at the Tomita Takesaki theory of the interaction-free local algebra $(\mathcal{A}(\mathcal{O}),\Omega)$ in standard position[32]

$$\begin{aligned} &H_{Fock}=\Gamma(H_1)=e^{H_1},\ \ \left(e^h,e^k\right)=e^{(h,k)} \\ &\Delta=\Gamma(\delta),\ J=\Gamma(j),\ S=\Gamma(s) \\ &SA\Omega=A^*\Omega,\ A\in\mathcal{A}(O),\ S=J\Delta^{\frac{1}{2}} \end{aligned} \tag{24}$$

With this we arrive at the core statement of the Tomita-Takesaki theorem which is a statement about the two modular objects Δ^{it} and J on the algebra

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O}))\equiv\Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it}=\mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J=\mathcal{A}(\mathcal{O})'=\mathcal{A}(\mathcal{O}') \end{aligned} \tag{25}$$

in words: the reflection J maps an algebra (in standard position) into its von Neumann commutant and the unitary group Δ^{it} defines an one-parametric automorphism-group σ_t of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem hold in complete mathematical generality for standard pairs $(\mathcal{A},\Omega)$. The free fields and their Wick composites are "coordinatizing" singular generators of this $\mathcal{O}$-indexed net of operator algebras in the sense that the smeared fields $A(f)$ with $supp f\subset\mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$ and in a certain sense generate $\mathcal{A}(\mathcal{O})$.

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic disjoint passes via the functorial operation to the operator algebra commutant (19) and the spatial one-particle automorphism goes into its algebraic counterpart. The definition of the Tomita involution S through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega=A^*\Omega$ and the action of the two modular objects Δ,J (24) is part of the general setting of the modular Tomita-Takesaki theory of abstract operator algebras in "standard position"; standardness is the mathematical terminology for the physicists Reeh-Schlieder property i.e. the existence[33] of a

[32] The functor Γ preserves the standardness i.e. maps the spatial one-particle standardness into its algebraic counterpart.

[33] In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in H.

vector $\Omega \in H$ with respect to which the algebra acts cyclic and has no "annihilators" of Ω. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras is more involved[34].

The domain of the unbounded Tomita involution S turns out to be "kinematical" in the sense that the dense set which features in the Reeh-Schlieder theorem is determined in terms of the representation of the connected part of the Poincaré group i.e. the particle/spin spectrum[35]. In other words the Reeh-Schlieder domains in an interacting theory with asymptotic completeness are identical to those of the incoming or outgoing free field theory.

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for $(\mathcal{A}(W), \Omega)$ the antiunitary involution J depends on the interaction, whereas Δ^{it} continues to be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [14] time that J is related with its free counterpart J_0 through the scattering matrix

$$J = J_0 S_{scat} \tag{26}$$

This modular role of the scattering matrix as a relative modular invariant between an interacting theory and its free counterpart comes as a surprise. It is precisely this property which opens the way for an inverse scattering construction. If one only looks at the dense localization of states which features in the Reeh-Schlieder theorem, one misses the dynamics. There is presently no other way to inject dynamics than generating these states by applying operators from operator algebras. The properties of J are essentially determined by the relation of localized operators A to their Hermitian adjoints A^*[36].

The physically relevant facts emerging from modular theory can be condensed into the following statements:

- *The domain of the unbounded operators $S(\mathcal{O})$ is fixed in terms of intersections of the wedge domains associated to $S(W)$; in other words it is determined by the particle content alone and therefore of a kinematical nature. These dense domains change with $\mathcal{O}$ i.e. the dense set of localized states has a bundle structure.*

- *The complex domains $DomS(\mathcal{O}) = K(\mathcal{O}) + iK(\mathcal{O})$ decompose into real subspaces $K(\mathcal{O}) = \overline{\mathcal{A}(\mathcal{O})^{sa}\Omega}$. This decomposition contains dynamical information which in case $\mathcal{O} = W$ reduces to the S-matrix (26). Assuming the validity of the crossing properties for formfactors, the S-matrix fixes $\mathcal{A}(W)$ uniquely [24].*

[34]The local algebras of QFT are (as a consequence of the split property) hyperfinite; for such operator algebras Longo has given an elegant proof [62].

[35]For a wedge W the domain of S_W is determined in terms of the domain of the "analytic continuation" $\Delta_W^{\frac{1}{2}}$ of the wedge-associated Lorentz-boost subgroup $\Lambda_W(\chi)$, and for subwedge localization regions $\mathcal{O}$ the dense domain is obtained in terms of intersections of wedge domains.

[36]According to a theorem of Alain Connes [59] the existence of operator algebras in standard position can be inferred if the real subspace K permit a decompositions into a natural positive cone and its opposite with certain facial properties of positive subcones. Although this construction has been highly useful in Connes classification of von Neumann factors, it has not yet been possible to relate this to physical concepts.

The remainder of this subsection contains some comments about a remarkable constructive success of these modular methods with respect to a particular family of interacting theories. For this one needs some additional terminology. Let us enlarge the algebraic setting by admitting unbounded operators with Wightman domains which are affiliated to $\mathcal{A}(\mathcal{O})$ and let us agree to just talk about "$\mathcal{O}$-localized operators" when we do not want to distinguish between bounded and affiliated unbounded operators. We call an $\mathcal{O}$-localized operators a vacuum **p**olarization **f**ree **g**enerator (PFG) if applied to the vacuum it generated a one-particle state without admixture of a vacuum-polarization cloud. The following three theorems have turned out to be useful in a constructive approach based on modular theory.

Theorem ([25]): *The existence of an $\mathcal{O}$-localized PFG for a causally complete subwedge region $\mathcal{O} \subset W$ implies the absence of interactions i.e. the generating fields are (* a slight generalization [25] of the Jost-Schroer theorem (referred to in [61][63]) which still used the existence of pointlike covariant fields).

Theorem ([25]): *Modular theory for wedge algebras insures the existence of wedge-localized PFGs. Hence the wedge region permits the best compromise between interacting fields and one-particle states*[37]*.*

Theorem ([25]): *Wedge localized PFGs with good (Wightman-like) domain properties ("temperate" PFGs) lead to the absence of particle creation (pure elasic S_{scat}) which in turn is only possible in d=1+1 and leads to the factorizing models (which hitherto were studied in the setting of the bootstrap-formfactor program [64]). The compact localized interacting subalgebras $\mathcal{A}(\mathcal{O})$ have no PFGs and possess the full interaction-induced vacuum polarization clouds.*

Some additional comments will be helpful. The first theorem gives an intrinsic (not dependent on any Lagrangian or other extraneous properties) local definition of the presence of interaction, even though it is not capable to differentiate between different kind of interactions (which would be reflected in the shapes of interaction-induced polarization clouds). The other two theorems suggest that the knowledge of the wedge algebra $\mathcal{A}(W) \subset B(H)$ may serve as a useful starting point for classifying and constructing models of LQP in a completely intrinsic fashion. Knowing generating operators of $\mathcal{A}(W)$ including their transformation properties under the Poincaré group is certainly sufficient and constitutes the most practical way for getting the construction started (for additional informations see later section).

All wedge algebras possess affiliated PFGs but only in case they come with reasonable domain properties ("temperate") they can presently be used in computations. This requirement only leaves models in d=1+1 which in addition must be factorizing (integrable); in fact the modular theory used in establishing these connections shows that there is a deep connection between integrability in QFT and vacuum polarization properties [25].

Temperate PFGs which generate wedge algebra for factorizing models have a rather simple algebraic structure. They are of the form (in the absence of boundstates)

$$Z(x) = \int \left(\tilde{Z}(\theta) e^{-ipx} + h.c. \right) \frac{dp}{2p_0} \tag{27}$$

[37] It is the smallest causally closed region (its localization representing a field aspect) which contains one-particle creators.

where in the simplest case $\tilde{Z}(\theta), \tilde{Z}^*(\theta)$ are one-component objects[38] which obey the Zamolodchikov-Faddeev commutation relations [24]. In this way the formal Z-F device which encoded the two-particle S-matrix into the commutation structure of the Z-F algebra receives a profound spacetime interpretation. Like free fields these wedge fields are on mass shell, but their Z-F commutation relations renders them non-local, more precisely wedge-local [24].

The simplicity of the wedge generators in factorizing models is in stark contrast to the richness of compactly localized operators e.g. of operators affiliated to a spacetime double cone $\mathcal{D}$ which arises as a relative commutant $\mathcal{A}(\mathcal{D}) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$. The wedge algebra $\mathcal{A}(W)$ has simple generators and the full space of formal operators affiliated with $\mathcal{A}(W)$ has the form of an infinite series in the Z-F operators with coefficient functions $a(\theta_1, ...\theta_n)$ with analyticity properties in a θ-strip

$$A(x) = \sum \frac{1}{n!} \int_{\partial S(0,\pi)} d\theta_1 ... \int_{\partial S(0,\pi)} d\theta_n e^{-ix\sum p(\theta_i)} a(\theta_1, ...\theta_n) : \tilde{Z}(\theta_1)...\tilde{Z}(\theta_1) : \qquad (28)$$

where for the purpose of a compact notation we view the creation part $\tilde{Z}^*(\theta)$ as $\tilde{Z}(\theta + i\pi)$ i.e. as coming from the upper part of the strip $S(0, \pi)$[39]. The requirement that the series (28) commutes with the translated generator $A(f_a) \equiv U(a)A(f)U^*(a)$ affiliated with $\mathcal{A}(W_a)$ defines formally a subspace of operators affiliated with $\mathcal{A}(\mathcal{D}) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$.

As a result of the simplicity of the $\tilde{Z}$ generators one can characterize these subspaces in terms of analytic properties of the coefficient functions $a(\theta_1, ...\theta_n)$. The latter are related to the formfactors of A which are the matrix elements of A between "ket" in and "bra" out particle states. The coefficient functions in (28) obey the crossing property. In this way the computational rules of the bootstrap-formfactor program [64] are explained in terms of an algebraic construction [14].

This is similar to the old Glaser-Lehmann-Zimmermann representation for the interacting Heisenberg field [65] in terms of incoming free field. Their use has the disadvantage that the coefficient functions are not related by the crossing property to one analytic master function. The convergence of both series has remaind an open problem. So unlike the perturbative series resulting from renormalized perturbation theory which have been shown to diverge even in models with optimal short distance behavior (even Borel resummability does not help), the status of the GLZ and formfactor series remains unresolved.

The main property one has to establish, if one's aim is to secure the existence of a QFT with local observables, is the standardness of the double cone intersection $\mathcal{A}(\mathcal{D}) = \cap_{W \supset \mathcal{D}} \mathcal{A}(W)$. Based on nuclearity properties of degrees of freedom in phase space (discovered by Buchholz and Wichmann [69]), Lechner has established the standardness of these intersections and in this way demonstrated the nontriviality of the model as a localized QFT [26][70]. For the first time in the history of QFT one now has a construction method which goes beyond the Hamiltonian- and measure-theoretical approach of the 60s [71].

[38] This case leads to the Sinh-Gordon theory and related models.

[39] The notation is suggested by the the strip analyticity coming from wedge localization. Of course only certain matrix elements and expectation values, but not field operators or their Fourier transforms, can be analytic; therefore the notation is symbolic.

The old approach could only deal with superrenormalizable models i.e. models whose basic fields did not have a short distance dimension beyond that of a free field.

The factorizing models form an interesting theoretical laboratory where problems, which accompanied QFT almost since its birth, resurface in a completely new light. The very existence of these theories, whose fields have anomalous trans-canonical short distance dimensions with interaction-dependent strengths, shows that there is nothing intrinsically threatening about singular short distance behavior. Whereas in renormalized perturbation theory the power counting rule only permits logarithmic corrections to the canonical (free field) dimensions, the construction of factorizing models starting from wedge algebras and their Z generators allow arbitrary high powers. That many problems of QFT are not intrinsic but rather caused by a particular method of quantization had already been suspected by the protagonist of QFT Pascual Jordan who, as far back as 1929, pleaded for a formulation "without (classic) crutches" [72]. The above construction of factorizing models which does not use any of the quantization schemes and in which the model does not even come with a Lagrangian name may be considered at the first realization of Jordan's plea at which he arrived on purely philosophically grounds.

The significant conceptual distance between QM and LQP begs the question in what sense the statement that QM is a nonrelativistic limit of LQP should be understood. By this we do not mean a formal manipulation in a Lagrangian or functional integral representation, but an argument which starts from the correlation functions or operator algebras of an interacting LQP and explains in what way an interacting QFT looses its modular localization + vacuum polarization and moves into the conceptual setting of QM. This is far from evident since in certain cases as that of 3-dimensional plektonic statistics the nonrelativistic limit retains the vacuum polarization, which is necessary to sustain the braid group statistics and thus becomes a nonrelativistic QFT instead of QM.

Apparently such arguments do not yet exist. One attempt in this direction could consist in starting from the known formfactors of a factorizing model (as e.g. the Sinh-Gordon model) and study the simplifications for small rapidity θ. An insight of this kind would constitute an essential improvement of our understanding of the QM-QFT interface.

Since modular theory continues to play an important role in the remaining section as well as part II, some care is required in avoiding potential misunderstandings. It is crucial to be aware of the fact that by restricting the global vacuum state to, a say double cone algebra $\mathcal{A}(\mathcal{D})$ whereupon it becomes a thermal KMS state, there is no change in the values of the global vacuum expectation values

$$(\Omega_{vac}, A\Omega_{vac}) = (\Omega_{\mathrm{mod},\beta}, A\Omega_{\mathrm{mod},\beta}), \quad A \in \mathcal{A}(\mathcal{D}) \tag{29}$$

where for the standard normalization of the modular Hamiltonian[40] $\beta = -1$. This notation on the right hand side means that the vacuum expectation values, if restricted to $A \in \mathcal{A}(\mathcal{D})$, fulfill an additional property (which without the restriction to the local algebra would not hold), namely the KMS relation

$$(\Omega_{\mathrm{mod},\beta}, AB\Omega_{\mathrm{mod},\beta}) = \left(\Omega_{\mathrm{mod},\beta}, B\Delta_{\mathcal{A}(\mathcal{O})}A\Omega_{\mathrm{mod},\beta}\right) \tag{30}$$

[40]The modular Hamiltonian lead to fuzzy motions within $\mathcal{A}(\mathcal{O})$ except in case of $\mathcal{O} = W$ when the modular Hamiltonian is identical to the boost generator.

At this point one may wonder how a global vacuum state can turn into a thermal state on a smaller algebra without any thermal exchange taking place. The answer is that the in terms of $(\mathcal{A}(\mathcal{D}), \Omega_{vac})$ canonically defined modular Hamiltonian K_{mod} with $\Delta = e^{-K_{\text{mod}}}$ is very different from the original translative Hamiltonian H_{tr} whose lowest energy eigenstate defines the vacuum, whereas K_{mod} is the generator of a modular automorphism of $\mathcal{A}(\mathcal{D})$ which in the geometric terminology preferred by physicists (even when it becomes inappropriate) describes a "fuzzy" motion inside $\mathcal{D}$.

The modular automorphism is actually defined on the global algebra $B(H)$ where it acts in such a way that $\mathcal{A}(\mathcal{D})$ and $\mathcal{A}(\mathcal{D})' = \mathcal{A}(\mathcal{D}')$ are automorphically mapped into themselves. The state vector $\Omega_{vac} \in H$ is a zero eigenvalue of K_{mod} which sits in the middle of a symmetric two-sided spectrum. What has changed through the process of restriction is not the state but rather the way of looking at it: H_{mod} describes the dynamics of an "observer" confined to $\mathcal{D}$ whereas H_{tr} has obviously no intrinsic meaning in a world restricted to $\mathcal{D}$. In fact it turns out that the fuzzy automorphism becomes geometric near the causal horizon of the region $\mathcal{O}$ (see second part)

The thermal aspect of modular theory refers to the modular Hamiltonian; it does not mean that one is creating heat with respect to the usual inertial frame Hamiltonian; its energy conservation is always maintained and observer-relevant heat is never generated as long as the observer's system remains inertial. Already in this context of inertial observer in the ground state and a modular observer for whom this state becomes thermal, the attentive reader may correctly presume an anticipation of the thermal manifestations of black holes as localized restrictions of a larger system (the Kruskal extension of the Schwartzschild black hole).

Going back to the Unruh Gedankenexperiment featuring a non-inertial observer which in order to follow the path of the modular Hamiltonian of a Rindler wedge W must be uniformely accelerated in some spatial direction, the standard question is the thermal aspect of the W-reduced vacuum real or is it a mathematical aspect carried too far ? The Unruh effect claims that this is really what the non-inertial observer measures in his taken along counter. Although the effect is so tiny that it will probably never be observed, the existence of the thermal radiation is a inescapable consequence of our most successful theories. One is accustomed to all kind of forces in noninertial systems but where does the nonzero thermal radiation density come from?

In order to create a causal horizon the observer must be uniformely accelerated which requires feeding energy into the system. In other words the realization of the innocent looking restriction in localization requires an enormous energy expenditure thus revealing in one example what is behind the physics of the harmless sounding word "restriction". Only when the modular Hamiltonian describes a movement which corresponds to a diffeomorphism of spacetime is there a chance to think in terms of an Unruh kind of Gedankenexperiment. As was explained before the modular situation is more physical in black hole situations where the position of event horizons is fixed by the metric independent of what an observer does. This is underlined by the earlier mentioned existence of a pure state on the Kruskal extension of the Schwarzschild solution (the Hartle-Hawking state); this state has the position of the event horizon worked in and does not need any observer for its definition. Restricted to the region outside of the black hole the modular automorphism describes the timelike Killing movement which is as close as one can come to an inertial path. The correponding Killing Hamiltonian is the closest analog of the inertial Hamiltonian in Minkowski spacetime.

There remains the question to what extent quantum physics in an Unruh frame is different from that in an inertial frame. There are no particles (in the sense of Wigner) since the vacuum behaves like a thermal densitiy in which counter experiments only permit the measurement of radiation densities as in standard thermal radiation or cosmic microwave background radiation. In fact it is quite straightforward to show the *LSZ scattering limit does not exist in the Unruh boost time*, a fact which is related to the two-sided spectrum of the modular Hamiltonian with respect to the W-reduced reduced ground state of the original inertial system. To wit, the global zero temperature Wigner-Fock space can be used also after the wedge restriction, but the global n-particle states loose their intrinsic physical meaning. Apart from the modular aspects the problems of the Unruh effect have been treated by many authors including authors from the foundational community [40][41].

In fact there is a *continuous family of modular "Hamiltonians"* which are the generators the modular unitaries for sequences of included regions. The modular Hamiltonian of the larger region will spread the smaller localized algebra into the larger region.

Besides the thermal description of restricted states there is one other macroscopic manifestation of vacuum polarization which has caused unbelieving amazement in philosophical circles namely the cyclicity of the vacuum (the Reeh-Schlieder property) with respect to algebras localized in arbitrarily small spacetime region or in its more metaphoric presentation the idea that by doing something in a small earthly laboratory for an arbitrary small fraction of time one can approximate any state "behind the moon" with arbitrary precision by (however with ever increasing expenditure in energy [5]).

Both consequences of vacuum polarization and as such interconnected, they [41] are manifestations of an *holistic* behavior which in this extreme form is absent in QM. Instead of the division between an object to be measured, the measuring apparatus and the environment, without which the modern quantum mechanical measurement theory cannot be formulated, in LQP such a separation is called into question. By restricting the vacuum to the inside, one already specifies the vacuum polarization driven dynamic on the causal disjoint. In the "state behind the moon argument" the difficulty in a system-environment dichotomy is even more palpable.

This is indeed an extremely surprising feature which goes considerably beyond the kinematical change caused by entanglement as the result of the quantum mechanical division into measured system and environment. It is this dependence of the reduced vacuum state on the localization region inside which it is tested with localized algebras which raises doubts about what are really non-fleeting persistent properties of a material substance. The monad description in the next section strengthens this little holistic aspect of LQP.

As we have seen the thermal aspects of modular localization are very rich from an epistemic viewpoint. The ontological content of these observations on the other hand is quite weak; it is only when the (imagined) causal localization horizons passes from a Gedanken objects to a (real) event horizons through the curvature of spacetime, that the

[41] Sometimes used as a metaphor for the Reeh-Schlieder property.

fleeting aspect of causal horizons of observers pass to an intrinsic ontological property of spacetime in the case of black holes. But even if one's main interest is to do black hole physics, it is wise to avoid a presently popular "shut up and compute" attitude and to understand the conceptual basis in LQP of the thermal aspect of localization and the peculiar thermal entanglement which contrasts the information-theoretical quantum mechanical entanglement. Ignoring these conceptual aspects one may easily be drawn into a fruitless and protractive arguments as it happened (and still happens) with the entropy/information loss issue.

Up to now the terminology "localization" was used both for states and for subalgebras. In the absence of interactions they are synonymous; this is because free fields are uniquely determined by positive energy representations of the Poincaré, in fact the generators of covariant wave functions pass directly to generating fields. A representation which has no infinite spin components is always pointlike generated. This applies in particular to string theory which is a misnomer for infinite component field theory [27]. Such a close relation between algebraic and state localization breaks down in the presence of interactions. It is perfectly conceivable to have a theory with "topological charges" [5] which by definition cannot be described by compactly localizable operators but need spacelike string localizable generating fields. In that case the neutral observable algebra has the usual compact localizability whereas the charge-carrying part of the total algebra may need semiinfinite string generators for its description [66]. The fact that this possibility could even occur in massive QCD like theories makes it very interesting, but unfortunately there is no illustrative example.

The problem of localization is of pivotal relevance for QFT. But nowhere is glory and failure so interlinked as in this issue. The misunderstandings range from the comparatively harmless confusion between the BNW localization of states and the modular localization of observables to the very serious misunderstanding of string theory.

In particular the 10 dimensional covariant infinite component unitary superstring representation of the Poincaré group coming from the quantization of the bilinearized Nambu-Goto Lagrangian is according to the before mentioned theorem (for representations which do not contain Wigner's infinite spin representation) a pointlike localized object, and this also applies to its predecessor, the dual resonance model. For a more detailed presentation of these points see [27]. Every explicit computation of the (graded) commutator of two string fields carried out by string theorists has confirmed the infinite component pointlike nature [67][68], but there is a strange ideological spirit which pervades the string community which prevents them from saying clearly what they really compute. Reading the two cited papers is a strange experience because it shows that correct computations in times of a dominating metaphorical idea are no guaranty for a correct interpretation. The authors come up with all kinds of metaphoric ideas (including that of a string of which one only sees a point) in order to avoid having to say "infinite component pointlike field".

Any philosophically motivated historian who wants to understand the Zeitgeist which led to string theory and its various revolutions in the service of a theory of everything, should find these (computationally correct but conceptually strange) papers a rich source of information. Less than 7 decades after Bohr and Heisenberg removed the metaphoric arguments of the old quantum theory by introducing the concept of observables, the discourse within the string theory community is trying to re-introduce metaphoric arguments into the relativistic particle discourse. Surely one does not want to miss the kind of fruitful transient metaphors which at the end led to valuable insights, but what is a reasonable attitude with respect to an obviously incorrect metaphor which hovers over particle theory ever since its beginnings in the 70s?

8 String-localization and gauge theory

Zero mass fields of finite helicity play a crucial role in gauge theory. Whereas in classical gauge theory a pointlike massless vectorpotential is a perfectly acceptable concept, the situation changes in QT as a consequence of the Hilbert space positivity which for massless unitary representations leads to the loss of many spinorial realizations as expressed in the second line of (19), in particular to that of the vector-potential without which it is hardly possible to formulate perturbative QED. The traditional way to deal with this situation has been to allow vector-potentials in an indefinite metric space and to add ghost degrees of freedom in intermediate calculations in such a way that the physical objects in form of the local observables in a Hilbert space coalesce with the gauge invariant objects under a suitably defined gauge group action.

Despite the undeniable success of this kind of quantum adaptation of the perturbative gauge setting, there are two arguments against considering the present formulation as the end of the story. One is of a more philosophical kind and the other points towards a serious limitation of the gauge formalism. ¿From a philosophical point of view this setting violates the maxim of Bohr and Heisenberg that one should always look for a formulation in which the computational steps (and not only the final result) can be formulated in terms of observables. More tangible is the objection that the existing gauge formalism aims only at *local* observables. There are interacting generators of physical objects which do not admit pointlike generators but whose sharpest possible localization is semiinfinite stringlike; the most prominent ones are electric charge-carrying operators [69]. Their construction is *not* part of the standard perturbative formalism but they have to be defined "by hand".

The best localization for a charged generating field is that of a semiinfinite Dirac-Jordan-Mandelstam string characterized *formally* by the well-known expression

$$\Psi(x,e) = \text{"}\psi(x)e^{\int_0^\infty ie_{el}A^\mu(x+\lambda e)d\lambda}\text{"} \tag{31}$$

Using a version of perturbation theory which was especially designed to incorporate this formal DJM expression into the n^{th} order renormalization setting, Steinmann [73] succeeded to attribute a renormalized perturbative meaning to this formal expression. Connected with this nonlocality aspect is the subtle relation of electrically charged fields to charged particles which shows up in infrared divergencies of on mass shell objects. In addition a charged particle, even after a long time of having left the scattering region, will never be without an infinite cloud of infrared real (not virtual!) photons whose energy is below the (arbitrarily small but nonvanishing) registering resolution and which therefore remain "invisible". This makes charge particles "infraparticles" i.e. objects whose scattering theory does not lead to scattering amplitudes but only to inclusive cross sections.

The infrared divergence problems in QED, first studied in a simpler model by Bloch and Nordsiek, whose phenomenological remedy required to trade scattering amplitudes with inclusive cross section [74], turned out to have a very profound conceptual explanation: the Hilbert space of QED does not contain an irreducible representation with a sharp mass, rather the electron two-point function starts with a cut at m_e which depends on e. For this to occur the presence of zero mass particles is necessary but not sufficient. Their coupling for low energies must also be sufficiently strong, a requirement which is fulfilled for the minimal coupling of photons in QED but e.g. not for the π-N coupling with massless pions. Also the converse holds, if the theory allows for one particle states in the sense that the theory has a mass-shell than even if this mass shell is not separated from the continuum by a gap) the theory possess a standard (LSZ) scattering theory [76].

For global gauge symmetries, the idea that the local observables in their vacuum representation determines all charged representation and, by suitably combining them, lead to the physical charged fields, was one of the most seminal conceptual conquests in local quantum physics [5]. The superselected charge-carrying fields are in this way (up to some conventions) uniquely determined in terms of the vacuum representation of the local observables. In $d \geq 4$ these fields are Bose/Fermi fields which act irreducibly in a Hilbert space which contains all superselected sectors associated to the system of local observables. They transform according to a compact internal symmetry group whose existence is preempted by the discrete presence of a copy of the dual of a group within the net of local observables; the latter in turn is the is the fixpoint subalgebra of the field algebra under the action of the internal symmetry. Each compact group with the exception of supersymmetry can appear as the internal symmetry of a QFT. With this structural insight a long path of the somewhat mysterious[42] concept of internal symmetries, which begun with Heisenberg's SU(2) isospin in nuclear physics, came to a beautiful conclusion. Global symmetry groups are a tool by which the quantum locality principle arranges the various inequivalent local representations of a given observable algebra belonging to different superselection charges.

In d=3,2, the commutation relations may be *plektonic* or *solitonic*, meaning that the fields obey braid group or soliton commutation relations which require semiinfinite string-like localization and lead to a generalized spin&statistics theorem [60] and to a situation in which the internal and spacetime symmetries allow no clearcut separation among them. But as before, the net of local observables determines modulo some conventions its full field algebra which incorporates the superselected charge-carrying fields. In both cases, the low- and higher- dimensional case, there is no better characterization for the inverse problem *neutral observables* $\rightarrow$ *charge-carrying fields* than the metaphor which Mark Kac used in connection with an acoustic inverse problem: "how to hear the shape of a drum".

It is a natural question to ask whether this reconstruction permits a more concrete formulation in the form of reconstructing bilocals by breaking up local expressions as the electric current $\bar{\psi}(x)\gamma_\mu\psi(x)$ in an analogous manner as was done in the 60s in order to reconstruct bilocals $A(x)A(y)$ from Wick-ordered locals $:A^2(x):$ via a lightlike limiting process [77]. In the case that the local operators are associated with a local gauge theory as QED, one expects bilocals with "gauge-bridges" between the two points. The partial

[42]This concept, which is central to local quantum physics, does not exist in classical physics.

results on this problem are scarce but encouraging [78]. It would be a major progress in gauge theory if electrically charged bilocals including gauge bridges could be obtained from local currents by such a lightlike splitting, so that formally the stringlike DJM charge generating field (31) appears in the limit of dumping one charge at infinity.

The problem of possible presence of interacting nonlocal generating fields in the physical Hilbert space becomes more serious in theories involving vectorfields coupled among themselves. Whereas one believes to have a physical understanding of the local (= gauge invariant) composites (whose perturbation expansion in terms of invariant correlation functions has incurable infrared divergencies[43]), there is no convincing idea about the conceptual status of the degrees of freedom which are the analogs of the charged fields in QED. For many decades we have been exposed to such evocating metaphoric words as quark- and gluon- confinement. Whereas such ideas are quite natural in QM where they point to enclosing quantum matter in a potential vault, QFT has no mechanism of hiding degrees of freedom by localizing them. The only mechanism through which degrees of freedom may escape observations in a theory in which localization is the dominating physical principle is a *weakening of localization* i.e. the opposite of a quantum mechanical vault[44]. The delocalization of electrically charged particles due to surrounding photon clouds in QED is obviously not sufficient. What one needs is the understanding of a situation in which the gluon plays a double role at the same time: that of a charge carrier and that of the photons hovering around it. Contrary to the formal DJM expression for charged fields, there is little chance that the formal spacetime structure of such a "hermaphroditic" object can be guessed "by hand".

A potential alternative to the present gauge method in which the pointlike localization of covariant vectorpotentials is paid for by the unphysical ghost formalism and the subsequent restriction to local observables, is to use string-localized potentials $A_\mu(x, e)$ from the start. As was explained in the section on modular localization (19) one can find intertwiners from the Wigner representation to all spinor representations if one admits string-localized potentials. In particular the one with the string-localized field of lowest Lorentz spin ($A_\mu(x, e)$ for helicity h=1, $g_{\mu\nu}(x, e)$ for h=2,..) has the lowest short distance dimension namely sdd=1 and hence the optimal behavior from the viewpoint of renormalization theory. This poses completely new and largely unsolved problems. But before commenting on this new task, it is helpful to delineate what one expects of such an alternative approach.

Superficially the use of such string-localized fields seem to be indistinguishable from the axial gauge; in both cases the conditions $\partial^\mu A_\mu(x, e) = 0 = e^\mu A_\mu(x, e)$ are obeyed. In the axial gauge interpretation the e is a gauge parameter and does not participate in Lorentz transformations, whereas in case of string-localized field the spacelike unit vector transforms as a string direction or, what is the same, as a point in a 3-dimensional de Sitter spacetime. The axial gauge failed as a perturbative computational tool as a result of its incurable infrared divergence problems. In a way the string-localized approach explains this as a consequence of quantum fluctuations *both* in x and e which makes it

[43]Only if perturbation theory is formulated in a pure algebraic setting and the problem of states is treated as a second step there is a chance to control the infrared divergencies.

[44]The use of lattice theory has also its limitations; for example there is no lattice description of infraparticles.

necessary to use testfunction smearing in x and e. The guiding idea is that the use of string dependent potentials delocalizes the charged field automatically so that there is no necessity to use ad hoc formulas as (31) and to engage in the difficult task to construct their renormalized counterpart apart from the standard formalism.

But this string-localized approach in the presence of interactions poses one hurdle which at the time of writing this essay had not yet been overcome: the adaptation of the perturbative Epstein-Glaser iteration [79]. In the pointlike case the knowledge of the n^{th} order determines the $n+1$ order up to a term on the total diagonal which limits the freedom to the addition of pointlike composites. The presence of string-like fields invalidates this argument. What one hopes for is that the freedom can be described in terms of some suitably defined string composites so that the interaction does not lead out of the family of string-localized fields inasmuch as it did not lead out of the pointlike setting. The aspect which makes the idea of formulating interactions in terms of stringlike localized fields attractive is the fact that their short distance dimension is sdd=1, independent of the spin.

This leads to many more interactions which for which the sdd of the interaction polynomial does not surpass the power-counting limit sdd=4 than with point-localized fields. Among them are all interactions which became renormalizable in the Gupta-Bleuler or BRST ghost setting of gauge theory but now nonlocal objects as electrically charged fields would be part of the formalism. Finally, the use of ghosts and other intermediate technical devices, as useful as they may appear, do not remove the desire to become philosophically clean and look for arguments which in the spirit of Bohr and Heisenberg eliminate non-observable structures from calculations altogether. In the present context this means that, although gauge theory is the QFT with the largest reality content already in its present form, it is also that area of QFT where, even after more than 3 decades of stagnation, one still can expect fundamental conceptual and mathematical changes.

It is my conviction that this will not be possible without restarting the discussion of what four decades ago was appropriately called the *Schwinger-Higgs screening mechanism* [36]. It was Schwinger who proposed for the first time the idea that a gauge theory as QED may not follow the logic of an electrically charged (infra)particles coupled to photons, but there may exist another phase in which the charge is screened (leading to a loss of the charge conservation rule, hence a loss of symmetry) and the photon becomes massive. Since Schwinger was unable to find a convincing argument in 4-dimensional spinor QED, he invented the soluable Schwinger model (2-dim. massless QED) as the simplest non perturbative model which exemplifies his idea of screening[45]). Higgs took instead of the model of scalar QED i.e. a charged complex field coupled minimally to a photon. This model has one more coupling parameter (the quadrilinear selfcoupling) than spinor QED. Its screened (or broken) phase may be reached perturbatively by appropriate choice of its 3 parameters. In this phase the complex scalar field becomes real and half of its degrees of freedom get hooked onto the photon which thus becomes a massive vector meson. The screened model lost its nonlocal charge-carrying sectors and describes a fully local system of a massive vectormeson interacting with a copiously produced neutral particle.

The important question which remained unanswered in the 70s is this screening mech-

[45]There is a mathematical theorem (Swieca's screening theorem) which says that the only way a photon in a Maxwellian setting can become massive is through charge screening [36].

anism a peculiar illustration for how an interacting massive vectormeson can be part of a pointlike local QFT. Perhaps this is a special case of a general more general intrinsic mechanism which states that in order to maintain locality interacting massive higher spin particles must always be accompanied by lower spin objects? Different spins have been linked together by the invention of supersymmetry, but it would be more natural to understand this as a consequence of the locality principle. i.e. smaller spin companions of a high spin interacting particle are necessary in order to maintain locality. An supporting argument was given within the BRST setting [80]: if one starts with a massive vectormeson, the Higgs meson (but now with vanishing vacuum expectation) has to be introduced for maintaining consistency of the BRST formalism. Only by removing the non-intrinsic BRST formalism by the use of stringlike sdd=1 vector-potentials one hopes to begin see the crucial role of locality in a conjectured *lower spin companion* mechanism behind the Higgs issue.

In view of the impending LHC data it would a great leap forward to get the Schwinger-Higgs screening mechanism away from its present "the God particle" metaphor back to to its origins.

9 Building LQP via *positioning of monads* in a Hilbert space

We have seen that modular localization of states and algebras is an intrinsic i.e. field-coordinatization-independent way to formulate the kind of localization which is characteristic for QFT. It is deeply satisfying that it also leads to a new constructive view of QFT.

Definition (Wiesbrock [84])**:** *An inclusion of standard operator algebras* $(\mathcal{A}\subset\mathcal{B},\Omega)$ *is "modular" if* $(\mathcal{A},\Omega)$ *and* $(\mathcal{B},\Omega)$ *are standard and* $\Delta_{\mathcal{B}}^{it}$ *acts like a compression on* $\mathcal{A}$ *i.e.* $Ad\Delta_{\mathcal{B}}^{it}\mathcal{A}\subset\mathcal{A}$*. A modular inclusion is said to be standard if in addition the relative commutant* $(\mathcal{A}'\cap\mathcal{B},\Omega)$ *is standard. If this holds for* $t<0$ *one speaks about a -modular inclusion.*

The study of inclusions of operator algebras has been an area of considerable mathematical interest. Particle physics uses three different kind of inclusions; besides the modular inclusions, which play the principal role in this section, there are *split inclusions* and inclusions with conditional expectations (or using the name of their protagonist, Vaughn *Jones inclusions).* Split inclusions play an important role in structural investigation and are indispensable in the study of thermal aspects of localization, notably localization entropy (see second part). Inclusions with conditional expectations result from reformulating the DHR theory of superselection sectors which in its original formulation uses the setting of localized endomorphisms of observable algebras [5].

Inclusions $\mathcal{A}\subset\mathcal{B}$ with conditional expectation $E(\mathcal{B})$ cannot be modular and the precise understanding of the reason discloses interesting insights. According to a theorem of Takesaki [81] the existence of a conditional expectation is tantamount to the modular group of the smaller algebra being equal to the restriction of that of the bigger. Hence the natural generalization of this situation is that the group $Ad\Delta_{\mathcal{B}}^{it}$ of the larger algebra acts on $\mathcal{A}$ for either $t<0$ or for $t>0$ as a bona fide compression (endomorphism) which precludes the existence of a conditional expectation. Intuitively speaking modular inclusions are "too deep" to allow conditional expectations. Continuing this line of speculative reasoning one would expects that inasmuch as "flat" inclusions with conditional expectations are related to inner symmetries, "deep" inclusions of the modular kind should lead to spacetime symmetries.

This rough guess turns out to be correct. The main aim of modular inclusions is really twofold, on the one hand to *generate spacetime symmetry* which than acts on the original algebras and creates a *net of spacetime indexed algebras* which are covariant under these symmetries. For the above modular inclusion of two algebras this is done as follows: from the two modular groups $\Delta_{\mathcal{B}}^{it}, \Delta_{\mathcal{A}}^{it}$ one can form a unitary group $U(a)$ which together with the modular unitary group of the smaller algebra $\Delta_{\mathcal{B}}^{it}$ leads to the commutation relation $\Delta_{\mathcal{B}}^{it}U(a) = U(e^{-2\pi t}a)\Delta_{\mathcal{B}}^{it}$ which characterizes the 2-parametric translation-dilation (Anosov) group. One also obtains a system of local algebras by applying these symmetries to the relative commutant $\mathcal{A}' \cap \mathcal{B}$. ¿From these relative commutants one may form a new algebra $\mathcal{C}$

$$\mathcal{C} \equiv \overline{\bigcup_t Ad\Delta_{\mathcal{B}}^{it}(\mathcal{A}' \cap \mathcal{B})} \tag{32}$$

In general $\mathcal{C} \subset \mathcal{B}$ and we are in a situation of a nontrivial inclusion to which the Takesaki theorem is applicable (the modular group of $\mathcal{C}$ is the restriction of that of $\mathcal{B}$) which leads to a conditional expectation $E : \mathcal{B} \to \mathcal{C}$; $\mathcal{C}$ may also be trivial. The most interesting situation arises if the modular inclusion is *standard* i.e. all three algebras $\mathcal{A}, \mathcal{B}, \mathcal{A}' \cap \mathcal{B}$ are standard with respect to Ω; in that case we arrive at a chiral QFT.

Theorem: (Guido,Longo and Wiesbrock [82]) *Standard modular inclusions are in one-to-one correspondence with strongly additive chiral LQP.*

Here chiral LQP is a net of local algebras indexed by the intervals on a line with a Moebius-invariant vacuum vector and *strongly additive* refers to the fact that the removal of a point from an interval does not "damage" the algebra i.e. the von Neumann algebra generated by the two pieces is still the original algebra. One can show via a dualization process that there is a unique association of a chiral net on $S^1 = \dot{\mathbb{R}}$ to a strongly additive net on $\mathbb{R}$. Although in our definition of modular inclusion we have not said anything about the nature of the von Neumann algebras, it turns out that the very requirement of the inclusion being modular forces both algebras to be hyperfinite type III_1 algebras.

The closeness to Leibniz's idea about (physical) reality of originating from relations between monads (with each monad in isolation being void of individual attributes) more than justifies our choice of name; besides that "monad" is much shorter than the somewhat long winded mathematical terminology "hyperfinite type III_1 Murray-von Neumann factor algebra". The nice aspect of chiral models is that one can pass between the operator algebra formulation and the construction with pointlike fields without having to make additional technical assumptions[46]. Another interesting constructive aspect is that the operator-algebraic setting permits to establish the existence of algebraic nets in the sense of LQP for all $c < 1$ representations of the energy-momentum tensor algebra. This is much more than the vertex algebra approach is able to do since that formal power series

[46]The group theoretic arguments which go into that theorem [83] seem to be available also for higher dimensional conformal QFT.

approach is blind against the dense domains which change with the localization regions.

The idea of placing the monad into modular positions within a common Hilbert space may be generalized to more than two copies. For this purpose it is convenient to define the concept of a *modular intersection* in terms of modular inclusion.

Definition (Wiesbrock [84]): *Consider two monads A and B positioned in such a way that their intersection $A \cap B$ together with A and B are in standard position with respect to the vector $\Omega \in H$. Assume furthermore*

$$(\mathcal{A} \cap \mathcal{B} \subset \mathcal{A}) \text{ and } (\mathcal{A} \cap \mathcal{B} \subset \mathcal{B}) \text{ are } \pm mi \qquad (33)$$
$$J_{\mathcal{A}} \lim_{t \to \mp} \Delta_{\mathcal{A}}^{it} \Delta_{\mathcal{B}}^{-it} J_{\mathcal{A}} = \lim_{t \to \mp} \Delta_{\mathcal{B}}^{it} \Delta_{\mathcal{A}}^{-it}$$

then (A, B, Ω) is said to have the $\pm$ modular intersection property ($\pm$ mi).

It can be shown that this property is stable under taking commutants i.e. if $(\mathcal{A}, \mathcal{B}, \Omega) \pm mi$ then $(\mathcal{A}', \mathcal{B}', \Omega)$ is $\mp mi$.

The minimal number of monads needed to characterize a 2+1 dimensional QFT through their modular positioning in a joint Hilbert space is three. The relevant theorem is as follows

Theorem: (Wiesbrock [85]) *Let A_{12},A_{13} and A_{23} be three monads*[47] *which have the standardness property with respect to $\Omega \in H$. Assume furthermore that*

$$\begin{aligned} &(\mathcal{A}_{12}, \mathcal{A}_{13}, \Omega) \text{ is } -mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{13}, \Omega) \text{ is } +mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{12}', \Omega) \text{ is } -mi \end{aligned} \qquad (34)$$

then the modular groups Δ_{12}^{it}, Δ_{13}^{it} and Δ_{23}^{it} generate the Lorentz group $SO(2,1)$.

Extending this setting by placing an additional monad $\mathcal{B}$ into a suitable position with respect to the $\mathcal{A}_{ik}$ of the theorem, one arrives at the Poincaré group $\mathcal{P}(2,1)$ [86]. The action of this Poincaré group on the four monads generates a spacetime indexed net i.e. a LQP model and all LQP have a monad presentation.

To arrive at d=3+1 LQP one needs 6 monads [87]. The number of monads increases with the spacetime dimensions. Whereas in low spacetime dimensions the algebraic positioning is natural within the logic of modular inclusions, in higher dimensions it is presently necessary to take some additional guidance from geometry, since the number of possible modular arrangements for more than 3 monads increases. There is an approach with similar aims of characterizing a QFT by its modular data by Buchholz and Summers [88]. Instead of the modular groups these authors use the modular reflections J. For our purpose of characterizing local quantum physics in terms of positioning of monads the approach proposed by Wiesbrock based on modular inclusions and intersections is more convenient. Its orgin dates back to the observation that the Moebius group can be extracted from the modular groups of the quarter circle algebras [89].

We have presented these mathematical results and used a terminology in such a way that the relation to Leibniz philosophical view is visible.

[47] As in the case of a modular inclusion, the monad property is a consequence of the modular setting. But for the presentation it is more convenient and elegant to talk about monads from the start.

Since this is not the place to give a comprehensive account, but only to direct the attention of the reader to this (in my view) startling conceptual development in the heart of QFT.

Besides the radically different conceptual-philosophical outlook on what constitutes QFT, the modular setting offers new methods of construction. For that purpose it is however more convenient to start from one monad $\mathcal{A} \subset B(H)$ and assume that one knows the action of the Poincaré group via unitaries $U(a,\Lambda)$ on $\mathcal{A}$. If one interprets the monad $\mathcal{A}$ as a wedge algebra $\mathcal{A} = \mathcal{A}(W)$ than the Poincaré action generates a net of wedge algebras $\{\mathcal{A}(W)\}_{W\in\mathcal{W}}$. A QFT is supposed to have local observables and hence if the double cone intersections[48] $\mathcal{A}(\mathcal{D})$ turn out to be trivial (multiples of the identity algebra), the net of wedge algebras does not leads to a QFT. This is expected to be the algebraic counterpart of a Lagrangian which does not have a have a corresponding QFT. If however these intersections are nontrivial, we would have a rigorous existence proof; the existence of a generating field for those double cone algebras is then merely a technical problem. There are of course two obvious sticking points: (1) to find the action of the Poincaré on $\mathcal{A}(W_0)$ and (2) a method which establishes the non-triviality of intersections of wedge algebras and leads to formulas for their generating elements.

As was explained in the previous section, both problems have been solved within a class of factorizing models [26]. Nothing is known about how to address these two points in the more general setting i.e. when the tempered PFG are not available.

The monad setting has only been formulated for Poincaré-covariant QFT. A extension to locally covariant QFT in CST is expected to present a new path into the still elusive Quantum Gravity. It is tempting to think of the diffeomorphisms of AQFT in CST to be of modular origin. A particularly simple illustration is $Diff(S^1)$, the diffeomorphism group of chiral theories on a circle. It is well known that the vacuum is only invariant under the Moebius subgroup and there are no states which are invariant under higher diffeomorphisms. The candidates for the higher modular groups are the diffeomorphisms which fix more than two points which can be obtained from a covering construction involving roots. The resulting multi-interval construction suggests to look for the modular group of a multi-interval; the problem is to find the appropriate states which lead to a geometric modular group. This problem was solved very recently by Longo, Kawahigashi and Rehren [93]. The interesting aspect of their solution (in agreement with the absence of eigenstates of higher diffeomorphisms) is that the resulting modular groups are only partially geometric i.e. geometric only inside the multi-interval. This is of course what one expects in the case of isometries in CST.

Another interesting problem which is on the verge of being solved is the problem of the higher Aharonov-Bohm effect. The A-B effect in the setting of AQFT is the statement that the zero mass spin=1 electromagnetic free field shows a violation of Haag duality for a non simply connected toroidal spacetime region $\mathcal{T}$

$$\mathcal{A}(\mathcal{T}) \subset \mathcal{A}(\mathcal{T}')' \tag{35}$$

whereas for simply connected regions the equality (Haag duality) holds. For higher spin massive fields Haag duality holds for any region. The A-B interpretation is that that

[48]Double cones are the typical causally complete compact regions which can be obtained by intersecting wedges.

the right hand side contains observables which cannot be constructed from field strengths in the torus. This violation of Haag duality has been shown in an old unpublished work before the modular methods became available. A modular approach to this problem yields more than just the violation of Haag duality, one also can compute a modular group and there is a close relation to the previous 4-fix point problem. What makes this problem so fascinating is the fact that it has a nontrivial extension to zero mass s>1 in which case higher genus A-B fluxes result. So it places s=1 gauge theory and the higher spin extensions on the same A-B footing.

Finally we should mention one unsolved long-lasting issue of modular theory: the modular group of the free massive double cone algebra (with respect to the vacuum) which is known to act "fuzzy" (non-geometric) and has been conjectured to have a Hamiltonian which acts as a pseudo-differential operator [94]. There are rather convincing arguments that the holographic projection of such a situation leads to a geometric modular movement on the horizon. This suggested the idea that if one knew a formula for the propagation of characteristic massive data on the horizon into the inside of the double cone, the fuzzy action may simply come about by applying this formula to the geometric group on the horizon. Such a formula has recently appeared in [95] and it is trivial to check that it reproduces the modular group of the wedge from that of its horizon but its application to the double-cone horizon is more involved. So it looks that there is some new movement on this long-lasting issue. A solution of this problem in the interaction-free case would also shed light onto the interacting case since the modular groups are determined by the representation theory of the Poincaré group i.e. they do not require the knowledge about the interaction dependent modular reflections J.

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