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# ANALYSIS OF THE $K^+$-DEUTERIUM EXPERIMENTS

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# ANALYSIS OF THE $K^+$-DEUTERIUM EXPERIMENTS *

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ABSTRACT. Application is made of the results of the analysis of the $K^+$ meson-deuteron scattering in terms of elementary two-particle processes. It is assumed that only S-waves contribute to the $K^+$-neutron interaction. The data recently obtained with a deuterium bubble chamber on the charge exchange process $K^+d \longrightarrow K^0pp$ are used and seem to confirm this assumption. Using these data two kinds of solutions are obtained for the scattering parameters in the isotopic spin zero state of the K-nucleon system. Measurements of the cross section for the inelastic (non-exchange) process $K^+d \longrightarrow K^+np$ will decide for one of the two groups of solutions.

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## I - INTRODUCTION.

Bubble chamber experiments of $K^+$-meson collision on protons and deuterons have been performed by the group of the University of California[1]. It was observed that the $K^+p$ cross-section remains isotropic up to $K^+$ momenta of 812 MeV/c (kinetic energy in lab 456 MeV), thus indicating that with only S-waves one can describe the K-proton interaction at these rather high energies. The angular distributions of the $K^0$ mesons coming out from the charge-exchange scattering $K^+d \longrightarrow K^0pp$ have been experimentally obtained for incident meson momenta of 350 MeV/c, 530 MeV/c and 642 MeV/c. The measured values of the total charge-exchange cross-section increase with the energy, as indicated in Table I. The values of the cross-sections for the inelastic (without charge exchange) scattering $K^+d \longrightarrow K^+np$ have not yet been published.

TABLE I

The total charge exchange $K^+d \longrightarrow K^0pp$ cross section obtained experimentally for several values of the incident meson momentum.

| Lab. mom. | Lab. Energy | $\sigma_{ex}$(mb) |
|---|---|---|
| 230 MeV/c | 52 MeV | $0.9 \begin{smallmatrix} +0.7 \\ -0.3 \end{smallmatrix}$ |
| 350 MeV/c | 111 MeV | $3.0 \pm 0.3$ |
| 530 MeV/c | 230 MeV | $5.8 \pm 0.6$ |
| 642 MeV/c | 315 MeV | $6.7 \pm 0.6$ |

The purpose of this paper is to analyse these data. Assuming charge independence, two scattering amplitudes (those in I = 1 and I = 0 isotopic spin states) describe the K meson-nucleon processes. The $K^+p$ interaction gives information only on the scattering amplitude for the I = 1 state. Since experiments with beams of $K^o$ mesons incident on free protons are not easy to perform, access to the I = 0 state is to be obtained through $K^+$-neutron processes. Experiments with beams of $K^+$- mesons incident on deuterons can be analysed[2] in terms of two-particle scattering amplitudes, so that the value of the amplitude in the I = 0 state can be extracted.

The method of analysis of the meson-deuteron scattering consists in expanding the corresponding scattering amplitude so that the main terms of the expansion contain only two-particle scattering operators and free-particle propagators. Neglecting the corrections that would be due to three-body processes is based on the assumptions that the interaction of the meson with one of the nucleons is restricted to a small region and takes only a short time. In these conditions the presence of the second nucleon does not affect the states of motion of the two colliding particles during the relevant interval of time in which their interaction lasts. Emulsion[3] work have indicated that the $K^+$-n total cross section is not much bigger than the value of 16 mb observed in the $K^+$-p scattering. Judged by the value of this cross section, the K-nucleon interaction range is short compared to the average internucleonic distance in the deuteron, so that the first of the above mentioned conditions seems to be well satisfied. Now, the meson-nucleon interaction will take a short time only if the meson passes fast through the region of interaction. This means that the model here considered is better for higher incident meson energies. This point will be important in the interpretation of the results we have obtained.

Due to the smallness of the K meson-nucleon scattering parameters in both isotopic spin states, double scattering of the incident meson can be neglected as giving small contribution compared to single scattering processes[2]. The rescattering of the two nucleons, however, cannot be neglected. On the other hand, the meson-proton Coulomb interaction is negligible if the incident meson momentum is not small.

We take from the results of experiments in hydrogen that the $K^+p$ interaction is purely S-wave. In all the calculations presented in this paper we assume that the $K^+$-n interaction is also purely S-wave. We claim that with this assumption we can explain the experimental results so far obtained.

## II - INELASTIC (WITHOUT CHARGE-EXCHANGE) SCATTERING.

The differential cross-section for the process $K^+d \longrightarrow K^+pn$ is given in reference 2. If the meson-nucleon interaction is purely S-wave there will be no spin-flip and the two nucleons in the final state will be in a triplet state, like the two nucleons in the deuteron. We assume the nucleon-nucleon interaction in the final state to occur in S and P waves only. For the S-wave phase shift we take the shape independent approximation $\cot \delta_o^1 = -(a_t\, \ell_f)^{-1} + \frac{1}{2} r_{ot}\, \ell_f$ where $\ell_f$ is the momentum of the proton relative to the center of mass of the proton-neutron system. The P-wave phase shifts for proton-neutron scattering are small and not very well determined at low energies. The terms involving the P-wave interaction in the expression for the cross-section which give stronger contributions are those depending only on the average phase-shift $\bar{\delta}_p = \frac{1}{9}(\delta_1^0 + 3\delta_1^1 + 5\,\delta_1^2)$ which experimentally has a value of about $1^o$ at $\ell_f = 100$ MeV/c. We adopted a linear dependence of $\bar{\delta}_p$ on $\ell_f$, i.e. $\bar{\delta}_p = C\,\ell_f$, with C properly chosen so as to fit the above mentioned value. The numerical computations showed that the effect of the P-wave interaction of the two nucleons is negligible in the $K^+d$ inelastic scattering at the incident energies investigated. We then write

$$d\sigma_{inel} = \frac{(2\pi)^4}{v} 4M\ell_f\, d_3\,\vec{q}_f \Big\{ \sum_{\ell=0}^{\infty} (2\ell+1)|a_p + (-1)^\ell\, a_n|^2\, \Gamma_\ell^2(\ell_f,\Delta) - -|a_p+a_n|^2\Gamma_o^2(\ell_f,\Delta)\sin^2\delta_o^1 + |a_p+a_n|^2\Big[-2\sin\delta_o^1\cos\delta_o^1\,\Gamma_o(\ell_f,\Delta)\Lambda_o(\ell_f,\Delta) + \sin^2\delta_o^1\Lambda_o^2(\ell_f,\Delta)\Big]\Big\} \tag{1}$$

where

$$\Lambda_o(\ell_f,\Delta) = \int_0^\infty n_o(\ell_f r) j_o(\tfrac{\Delta}{2}r)\psi_D(r) r^2 dr(1-e^{-Zr}) = \frac{N}{\ell_f\Delta}\Big\{\tan^{-1}\Big(\frac{\beta\Delta}{\ell_f^2-\frac{1}{4}\Delta^2+\beta^2}\Big) - \tan^{-1}\Big(\frac{\alpha\Delta}{\ell_f^2-\frac{1}{4}\Delta^2+\alpha^2}\Big) - \tan^{-1}\Big(\frac{(\beta+Z)\Delta}{\ell_f^2-\frac{1}{4}\Delta^2+(\beta+Z)^2}\Big) + \tan^{-1}\Big(\frac{(\alpha+Z)\Delta}{\ell_f^2-\frac{1}{4}\Delta^2+(\alpha+Z)^2}\Big)\Big\}$$

and

$$\Gamma_\ell(\ell_f,\Delta) = \frac{N}{\ell_f\Delta}\Big\{ Q_\ell\Big(\frac{\alpha^2+\ell_f^2+\frac{1}{4}\Delta^2}{\ell_f\Delta}\Big) - Q_\ell\Big(\frac{\beta^2+\ell_f^2+\frac{1}{4}\Delta^2}{\ell_f\Delta}\Big)\Big\}$$

In Eq. (1), M is the nucleon mass, $\vec{q}_f$ is the final meson momentum,

$\vec{\Delta} = \vec{q}_f - \vec{q}_o$ is the meson momentum transfer, and v is the incident meson velocity. The deuteron wave function is $\psi_D = N(e^{-\alpha r} - e^{-\beta r})/r$ with $\alpha = 45.7$ MeV/c and $\beta = 7\alpha$. In the cut-off factor $(1-e^{-Zr})$ we choose Z = 200 MeV/c.

The sum indicated in the first term of Eq. (1) converges rapidly. This sum comes from the expansion of the final state plane waves in angular-momentum waves. The rapid convergence indicates that the two nucleons come out mainly in low angular momentum states (relative to their center of mass). This is due to the limited size of the deuteron. For higher energies of the incident meson, higher values of $\ell$ will contribute more strongly to the sum. At the energies we have considered the $\ell = 2$ term gives smaller contributions than the $\ell = 0$ terms (even including the S-wave interaction effects). Since the nucleon-nucleon phase-shifts for $\ell = 2$ are much smaller than those in S-waves for all the nucleon-nucleon energies that may occur, the neglect of the nucleon-nucleon interaction in D-waves is perfectly justified.

In terms of the scattering amplitudes $a_o$ and $a_1$ in the I = 0 and I = 1 isotopic spin states we have $a_p = a_1$, and $a_n = \frac{1}{2}(a_1 + a_o)$. Unitarity implies that we can write

$$(2\pi)^2 \left(\frac{k^2\ E_o}{M\ q_o^2}\right)^{\frac{1}{2}} (m^2 + k^2)^{1/4}\ (M^2 + k^2)^{1/4}\ a_1 = R_1(1 - ik\ R_1)^{-1} \quad (2)$$

and an anologous expression for $a_o$. Here $R_1$ and $R_o$ are real quantities, k is the meson momentum in the system of the center of mass of the meson-nucleon system, $E_o$ is the meson incident energy in lab system and m the meson mass. In terms of $R_1$ the $K^+$-proton scattering cross-section is given by

$$d\sigma_1/_{d\Omega} = R_1^2 (1 + k^2 R_1^2)^{-1} \tag{3}$$

In an effective range approximation we would have

$$1/_{R_1} = - \frac{1}{A_1} + \frac{1}{2} r_1 k^2$$

with $A_1$ (the scattering length) and $r_1$ independent of energy.

The momentum spectra of the mesons emergent from the process $K^+d \longrightarrow K^+np$ at angles $45^\circ$ and $90^\circ$ with respect to the direction of the incident meson are plotted in Figures 1 and 2 respectively. The incident meson momentum is 200 MeV/c. In these curves we adopted for $R_1$ the value $-0.3338 \times 10^{-13}$cm which gives the cross section of 14 mb experimentally observed for the $K^+p$ scattering (and the observed constructive interference with the Coulomb potential). The curves are plotted for two different values of $R_o$, namely zero and $-0.1 \times 10^{-13}$cm, so as to show how the value of the I = 0 parameter can affect the form of the spectrum. From the figures we see that the effect of the value of $R_o$ is small, and it appears that, unless the statistics becomes very good, the meson momentum spectrum is not a convenient way of determining the value of $R_o$.

Integrating over the meson momentum spectrum, we have for the differential cross section in lab system

$$\frac{d\sigma_{inel}}{d\Omega} = N^2 M^2 q_o \left[ k^2 (m^2 + k^2)^{\frac{1}{2}} (M^2 + k^2)^{\frac{1}{2}} (1 + k^2 R_o^2)(1 + k^2 R_1^2) \right]^{-1} \left\{ \left[ (3R_1 + R_o)^2 + 16 k^2 R_1^2 R_o^2 \right] F(q_o, \theta) + (R_1 - R_o)^2 \, G(q_o, \theta) \right\} \tag{4}$$

where $\theta$ is the scattering angle of the meson in the lab system, $d\Omega$ is the element of solid angle about the direction of the momentum in the meson-nucleon system, i.e. the center of mass momentum obtained by considering the nucleon at rest inside the deuteron, $k^2 = M q_o (M^2 + m^2 + 2M\, E_o)^{\frac{1}{2}}$.

In Fig. 3 we plot the differential cross section for $q_o$ = 200 MeV/c, considering two different values for $R_o$. The peculiar structure of the cross-section at small angles is due to the interaction of the two nucleons in the final state[4]. This structure disappears at higher incident energies.

The (dimensionless) functions $F(q_o, \theta)$ and $G(q_o, \theta)$ were evaluated numerically for the values of $q_o$ at which the Berkeley experiments were done. They are plotted in Figures 4 and 5. For all the energies investigated the functions $F(q_o, \theta)$ and $G(q_o, \theta)$ have very similar shapes. This implies that the form of the differential cross section cannot be modified to fit experimental curves only by conveniently choosing the values of the parameters $R_1$ and $R_o$. Modifying the parameters pratically changes only the value of the total cross section, not the angular distribution. If the experimental differential cross-section cannot be fitted by curves of the type of those in Figures 4 and 5, then either the Impulse Approximation (plus final state interaction) is not good or we must include P-waves in the meson-neutron interaction.

Integrating (4) over the solid angle we obtain the total inelastic (without charge exchange) cross sections. Let us write

$$\sigma_{inel}(q_o) = (1 + k^2 R_o^2)^{-1} (1 + k^2 R_1^2)^{-1} \left\{ \left[ (3R_1 + R_o)^2 + 16 k^2 R_o^2 R_1^2 \right] f(q_o) + \right.$$

$$+ (R_1 - R_o)^2 g(q_o) \Big\} \quad (5)$$

The values of f and g are given in Table II.

TABLE II

The functions $f(q_o)$ and $g(q_o)$ that enter in $\sigma_{inel}(q_o)$ and the function $h(q_o)$ that enters in $\sigma_{ex}(q_o)$, for several values of the incident momentum $q_o$. k is the momentum in the system of the center of mass of the meson-nucleon system.

| $q_o$ (MeV/c) | k (MeV/c) | $f(q_o)$ | $g(q_o)$ | $h(q_o)$ |
|---|---|---|---|---|
| 200 | 128.7 | 0.46 | 0.62 | 1.59 |
| 350 | 218.4 | 0.95 | 1.04 | 2.31 |
| 530 | 315.5 | 1.16 | 1.19 | 2.58 |
| 642 | 370.4 | 1.20 | 1.20 | 2.53 |
| 810 | 445.9 | 1.23 | 1.22 | 2.54 |

III - CHARGE-EXCHANGE SCATTERING.

If the meson-nucleon interaction is purely S-wave, the two protons in the final state in the process $K^+d \longrightarrow K^opp$ will be in the triplet state. Due to the Pauli Principle, the final system will contain only states in which the angular momentum of the two protons relative to their center of mass is odd. As the phase-

-shifts of the proton-proton interaction in P and higher odd waves are very small for the energies involved in our calculations, the Coulomb interaction of the two protons will play the prominent role.

In reference 2 it is shown how the effect of the Coulomb interaction in the final state can be evaluated. The importance of the contribution of the Coulomb interaction decreases rapidly with the value of the angular momentum of the p-p system (relative to its center of mass). This has been actually shown in the calculations. Taking the Coulomb interaction in states with $\ell = 1$ and $\ell = 3$ only, we write for the charge-exchange scattering cross-section

$$d\sigma_{ex} = \frac{(2\pi)^4}{v} (M/2)\ell_f \, a_{ex}^2 \, d_3 \vec{q}_f \left\{ K_1 + K_3 + \sum_{\ell=5,7\ldots} 16(2\ell+1)\Gamma_\ell^2(\ell_f,\Delta) \right\} \tag{6}$$

where $a_{ex} = \frac{1}{2}(a_1 - a_o)$ is the amplitude for the charge-exchange process $K^+n \longrightarrow K^o p$. Let us call $n = M/(274\,\ell_f)$. For $K_1$ and $K_3$ we have

$$K_1 = 16 \times 6\pi \, N^2 \left[1-\exp(-2\pi n)\right]^{-1} \left[n(1+n^2)\Delta^2 \ell_f^2\right]^{-1} \left\{ \exp|n\varphi(\alpha)| \left[ n \cos\left(\frac{n}{2}\log\frac{v+1}{v-1}\right) + \right.\right.$$

$$\left.\left. + v \sin\left(\frac{n}{2}\log\frac{v+1}{v-1}\right)\right] - \exp\left[n\varphi(\beta)\right]\left[n \cos\left(\frac{n}{2}\log\frac{u+1}{u-1}\right) + u \sin\left(\frac{n}{2}\log\frac{u+1}{u-1}\right)\right]\right\}^2 \tag{7}$$

and

$$K_3 = \frac{16\times7\times36\pi\times N^2\left[1-\exp(-2\pi n)\right]^{-1}}{n(1+n^2)(4+n^2)(q+n^2)\Delta^2 \; \ell_f^2} \left\{ \exp\left[n\varphi(\alpha)\right]\left[\frac{n}{6}(n^2-15v^2+4)\cos\left(\frac{n}{2}\log\frac{v+1}{v-1}\right) + \right.\right.$$

$$+ \left(-n^2 v + \frac{5}{2}v^3 - \frac{3}{2}v\right)\sin\left(\frac{n}{2}\log\frac{v+1}{v-1}\right)\Bigg] - \exp\left[n\varphi(\beta)\right]\left|\frac{n}{6}(n^2-15u^2+4)\cos\left(\frac{n}{2}\log\frac{u+1}{u-1}\right) + \right.$$

$$+ \left(-n^2 u + \frac{5}{2} u^3 - \frac{3}{2} u\right) \sin\left(\frac{n}{2} \log \frac{u+1}{u-1}\right) \Big] \Big\}^2$$

where

$$v = (\ell_f \Delta)^{-1} \left(\alpha^2 + \frac{1}{4}\Delta^2 + \ell_f^2\right)$$

$$u = (\ell_f \Delta)^{-1} \left(\beta^2 + \frac{1}{4}\Delta^2 + \ell_f^2\right)$$

and

$$\varphi(\alpha) = \tan^{-1}\left(\frac{2\alpha\ell_f}{\frac{1}{4}\Delta^2 + \alpha^2 - \ell_f^2}\right) \qquad (9)$$

with an anologous expression for $\varphi(\beta)$.

Figure 6 shows the charge-exchange differential cross-section for incident meson momentum $q_o$ = 200 MeV/c, and $R_1 = -\ 0.3338 \times \times 10^{-13}$cm, $R_o = 0$. In the curve $C_1$ the Coulomb interaction of the two protons was taken into account, and in the curve $C_2$ it has been neglected. We see that the shape of the angular distribution at this energy is not much altered by the Coulomb effect, but the value of the cross-section is more or less uniformly increased at all angles. The increase in the calculated total cross-section, obtained by including the Coulomb interaction, is of 20% for $q_o$ = = 200 MeV/c; 13% for 350 MeV/c; 7.5% for 530 MeV/c; and 6% for 642 MeV/c.

The general form of the differential charge-exchange scattering cross-section in the lab system is

$$\frac{d\sigma_{ex.}}{d\Omega} = N^2 M^2 q_o \left[k^2(m^2+k^2)^{\frac{1}{2}} (M^2+k^2)^{\frac{1}{2}} (1+k^2R_1^2)(1+k^2R_o^2)\right]^{-1} \cdot$$

$$\cdot\ (R_1 - R_o)^2\ H(q_o, \theta) \qquad (10)$$

In Fig. 7 the function $H(q_o, \theta)$ is plotted for several values of the incident momentum $q_o$.

With the assumption made, that only S-waves contribute to the meson-nucleon interaction in both isotopic spin states, we cannot play with parameters to modify the form of the angular distribution. The only effect that the choice of parameters may have is that of multiplying the curves by a constant factor. We must choose the values of $(1+k^2 R_1^2)(1+k^2 R_o^2)(R_1 - R_o)^2$ so that the total cross--sections reproduce the experimental values in Table I.

In Figures 8, 9 and 10 the differential cross-sections given by Eq. (10) are compared with the experimental points[1]. For $q_o$ = = 642 MeV/c the theoretical curves fit the experimental points well. For $q_o$ = 530 MeV/c the agreement is still fairly good, but the curves for 530 and 350 MeV/c together show that there may be a systematic deviation of the experimental points from the theoretical curves. This may be due to three-body effects in our multiple scattering model, which we would expect to be stronger at the lower energies. Also it may be due to a P-wave contribution in the $K^+n$ interaction but we would expect this to be also strong in the higher energy (642 MeV/c) where the agreement is so good with S-waves only. Of course there could be a strong P-wave interaction in the I = 0 state for incident mesons of momentum about 400 MeV/c.

Beside an improvement in the statistics in the measurements at the lower energies, it seems desirable to have the experimental determination of the differential exchange cross-section at 810 MeV/c. We expect the independent particle model to be valid at this energy, as it seems to be at 642 MeV/c.

Integrating Eq. (10) over the solid angle we obtain the total exchange cross-section

$$\sigma_{ex}(q_o) = (1+k^2R_1^2)^{-1}\,(1+k^2R_o^2)^{-1}(R_1-R_o)^2h(q_o) \tag{11}$$

The values of f, g, h in table II show that as the incident energy increases, the total inelastic and exchange cross-sections tend to the values that we would obtain if the proton and neutron were free and non-interfering sources of scattering. The differences come mainly from the Pauli principle (in case of charge exchange scattering) and the final state interaction. For example, at 642 MeV/c the charge exchange cross section is 25 per cent smaller than that of the scattering by free neutrons.

Assuming a value of 16 mb for the $K^+$-p total cross section at all energies, and using the fact that the interference with the Coulomb potential is constructive, we obtain for $R_1$ (using Eq. (3)) the values listed in Table III. Using Eq. (11) with the experimental values of $\sigma_{exch.}$ we solve for $R_o$, obtaining two solutions, $R_o^+$ and $R_o^-$, for each energy. Each of the two solutions predicts a different value for the inelastic (without charge-exchange) cross section. These values are indicated in table III. The experimental errors in $\sigma_{exch.}$ cause uncertainties in the values of $R_o$ and $\sigma_{inel}$, which are also indicated in the table.

The two groups of solutions for $\sigma_{inel}$ are very well separated, so that the experimental decision on which is the right one should not be difficult. To this end we can use data obtained in emulsions. Assuming that in the emulsion material we have approximately the same number of neutrons and protons and that the cross sections are additive (the nucleons being incoherent sources of scattering) we have

$$X = \frac{\sigma_{ex.}}{\sigma_p+\sigma_n} = \frac{(R_1-R_o)^2}{4(1+k^2R_o^2)R_1^2+(R_1+R_o)^2+4k^2R_1^2R_o^2} = \frac{\sigma_{exch.}^{\text{emulsions}}}{(\sigma_{inel}+\sigma_{elastic})^{\text{emulsions}}} \qquad (12)$$

TABLE III

Values of the scattering parameters $R_1$ and $R_o$ in the I = 1 and I = 0 states obtained using the value of 16 mb for the $K^+p$ cross section, the fact that the $K^+p$ potential is repulsive, and the experimental values of the charge exchange cross sections in Table I. Two solutions, called $R_o^+$ and $R_o^-$, are obtained for $R_o$ at each energy. The values of $\sigma_{inel}$ predicted by these two groups of values are shown in the last column.

| $q_o$ (MeV/c) | $R_1$ ($10^{-13}$cm) | $R_o$ ($10^{-13}$cm) | $\sigma_{inel}$ (mb) |
|---|---|---|---|
| 350 | - 0.3874 | $R_o^- = 0.0037^{+0.02}_{-0.017}$ | $12.20^{-0.25}_{+0.21}$ |
| | | $R_o^+ = -0.9515^{+0.05}_{-0.04}$ | $28.71^{+0.43}_{-0.46}$ |
| 530 | - 0.4317 | $R_o^- = 0.1593^{+0.05}_{-0.03}$ | $13.69^{+0.04}_{+0.07}$ |
| | | $R_o^+ = -4.891^{-4.1}_{+1.6}$ | $30.92^{-0.30}_{+0.20}$ |
| 642 | - 0.4754 | $R_o^- = 0.3216^{+0.16}_{-0.08}$ | $15.66^{+2.63}_{-0.80}$ |
| | | $R_o^+ = 1.2419^{-0.46}_{+0.57}$ | $23.76^{-2.90}_{+1.65}$ |

where $\sigma_p$ is the $K^+p$ scattering cross section, $\sigma_n$ is the cross section for the process $K^+n \longrightarrow K^+n$, and $\sigma_{exch.}$ that for $K^+n \longrightarrow K^0p$. In Eq. (12) it is assumed that shadow effects affect equally the two terms of the fraction. The approximations involved in writing Eq. (12) tend to be better at higher incident energies. The cross sections obtained experimentally using emulsions[3] at energies between 295 and 373 MeV (incident momentum between 615 MeV/c and 712 MeV/c) are 167 mb for charge exchange, 240 mb for inelastic (non-charge exchange) and 128 mb for elastic (small angles excluded to account for Coulomb effects). This gives $X = 0.45 \pm 0.1$. For the solution $R_0^-$ at 642 MeV/c we obtain $X = 0.36$, while using $R_0^+$ we obtain $X = 0.27$. This, together with a continuity argument applied to the values of $R_0^{\pm}$ at different energies, seems to indicate that the $R_0^-$ solutions are the right ones.

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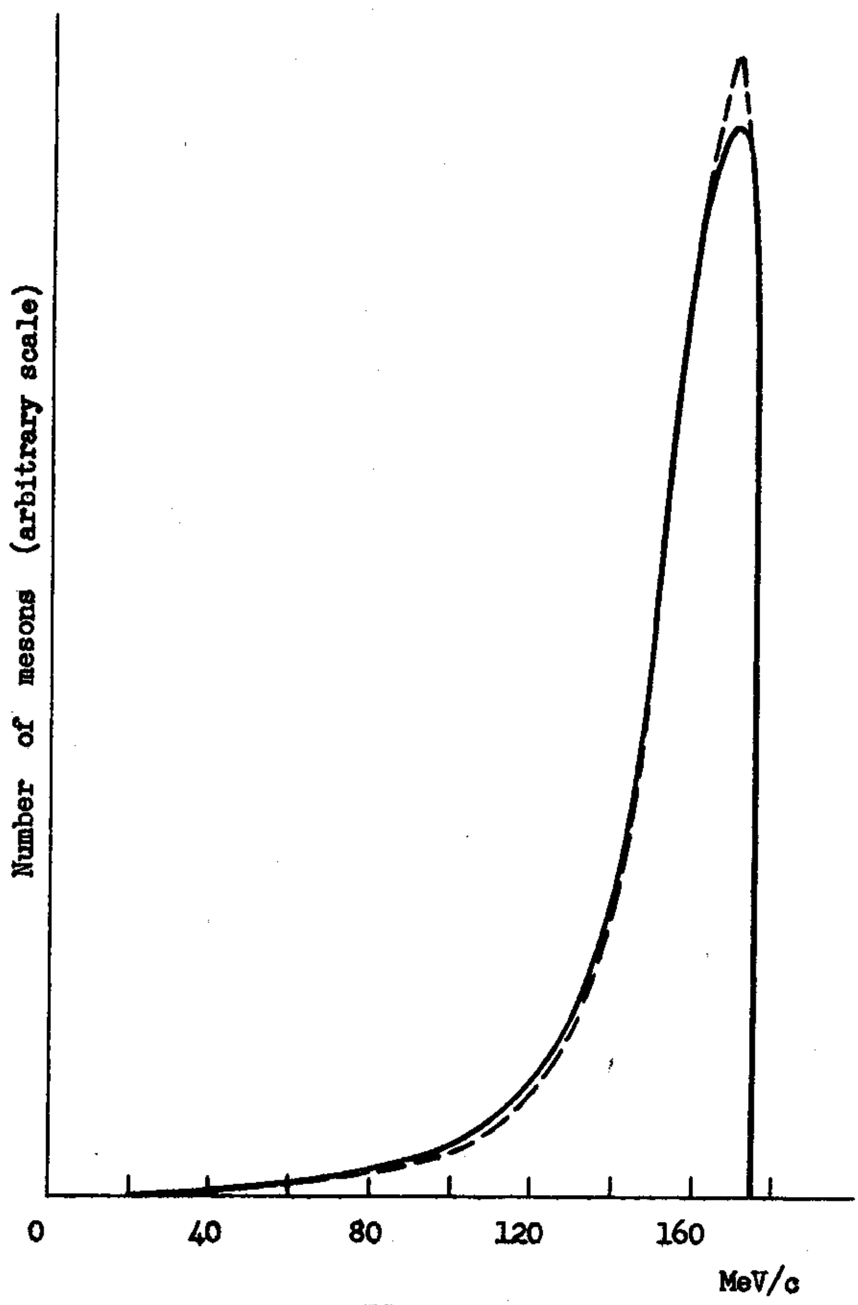



Fig. 1 - Energy spectrum of the $K^+$ mesons emerging from the inelastic process $K^+d \longrightarrow K^+np$ at an angle of $45^o$ with respect to the incident meson. Incident meson momentum $q_o$ = 200 MeV/c. $R_1 = -0.3338 \times 10^{-13}$ cm. Solid curve traced for $R_o = 0$. In dashed curve $R_o = -0.1 \times 10^{-13}$ cm.

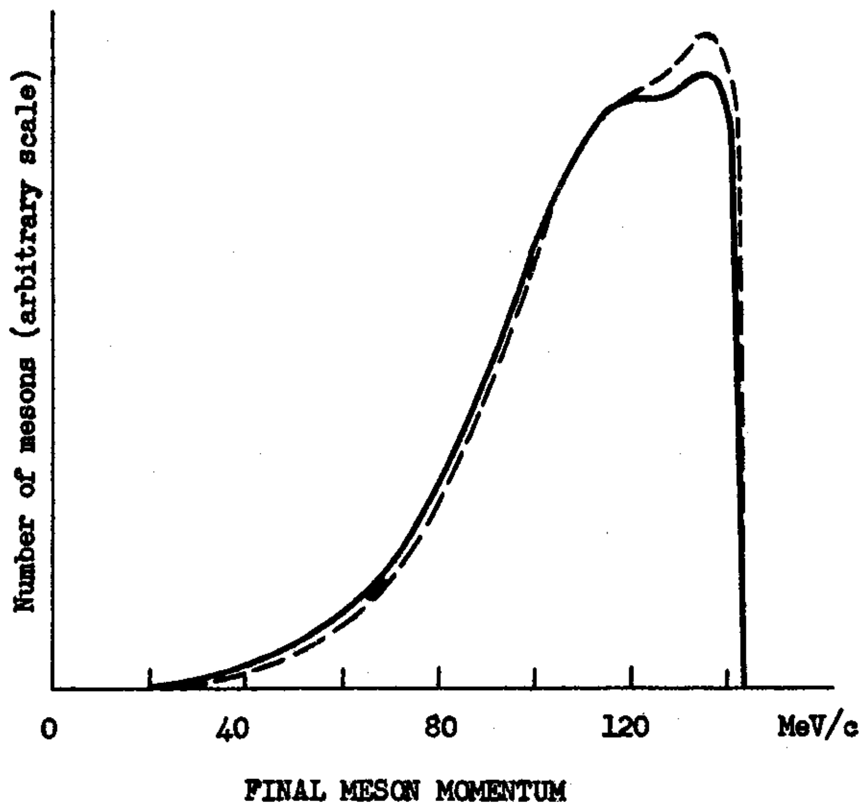



Fig. 2 - Energy spectrum of the $K^+$ mesons coming out the process $K^+d \longrightarrow K^+np$ at an angle of $90^o$ with respect to the incident meson. Incident meson momentum $q_o = 200$ MeV/c. $R_1 = -0.3338 \times 10^{-13}$ cm. Solid curve, $R_o = 0$. Dashed curve, $R_o = -0.1 \times 10^{-13}$ cm.

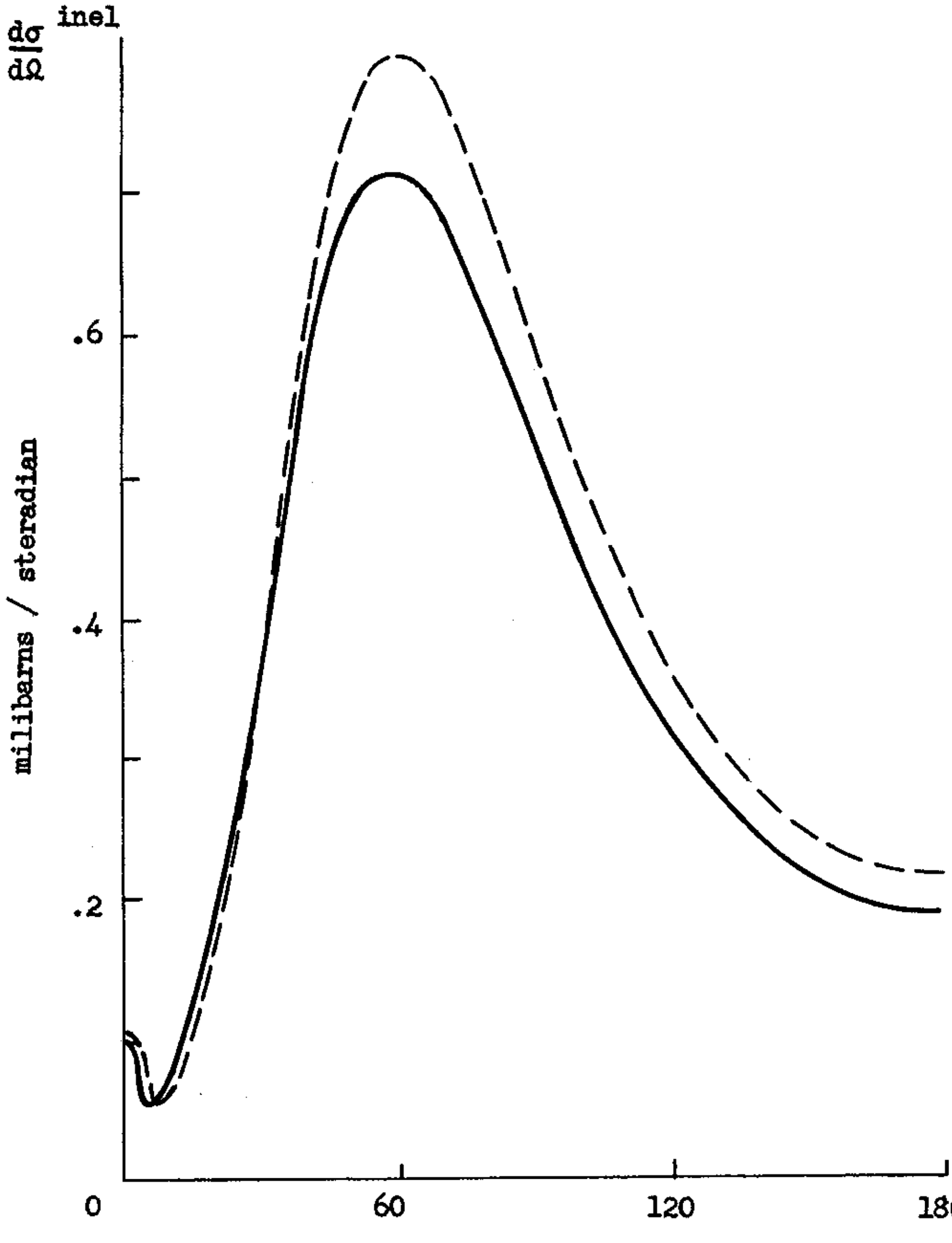



Fig. 3 - Differential cross sections for $K^+d \longrightarrow K^+np$. The angle is that between the incident and the outgoing mesons. The scattering parameter in the isotopic spin I = 1 state is taken as $R_1 = -\ 0.3338 \times 10^{-13}$ cm. The solid curve is obtained for $R_o = 0$, and the other for $R_o = -\ 0.1 \times 10^{-13}$ cm.

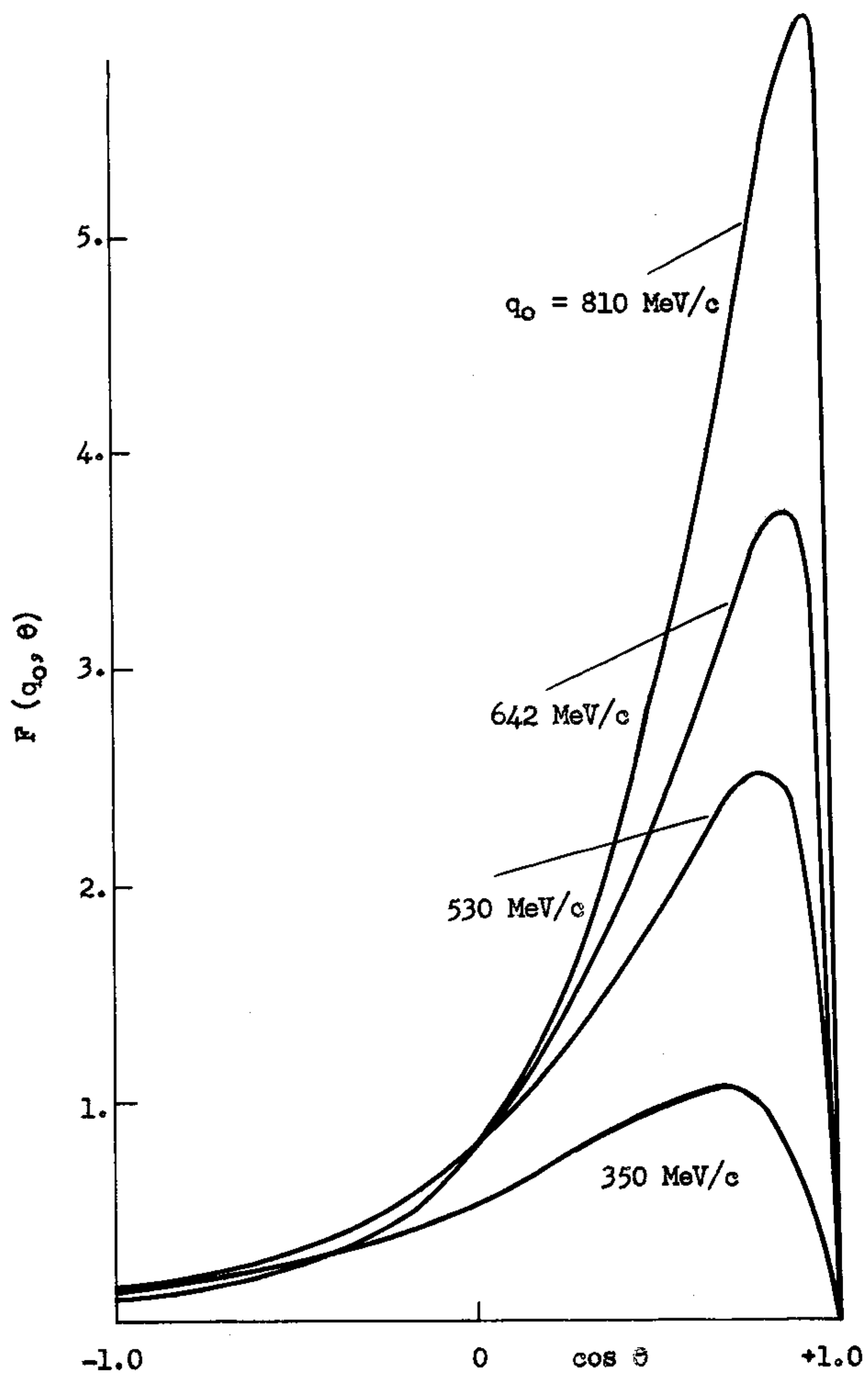



Fig. 4 - Plots of the functions $F(q_o,\theta)$ for several values of the incident meson momentum $q_o$. $\theta$ is the angle between the incident and the outgoing mesons in lab. system.

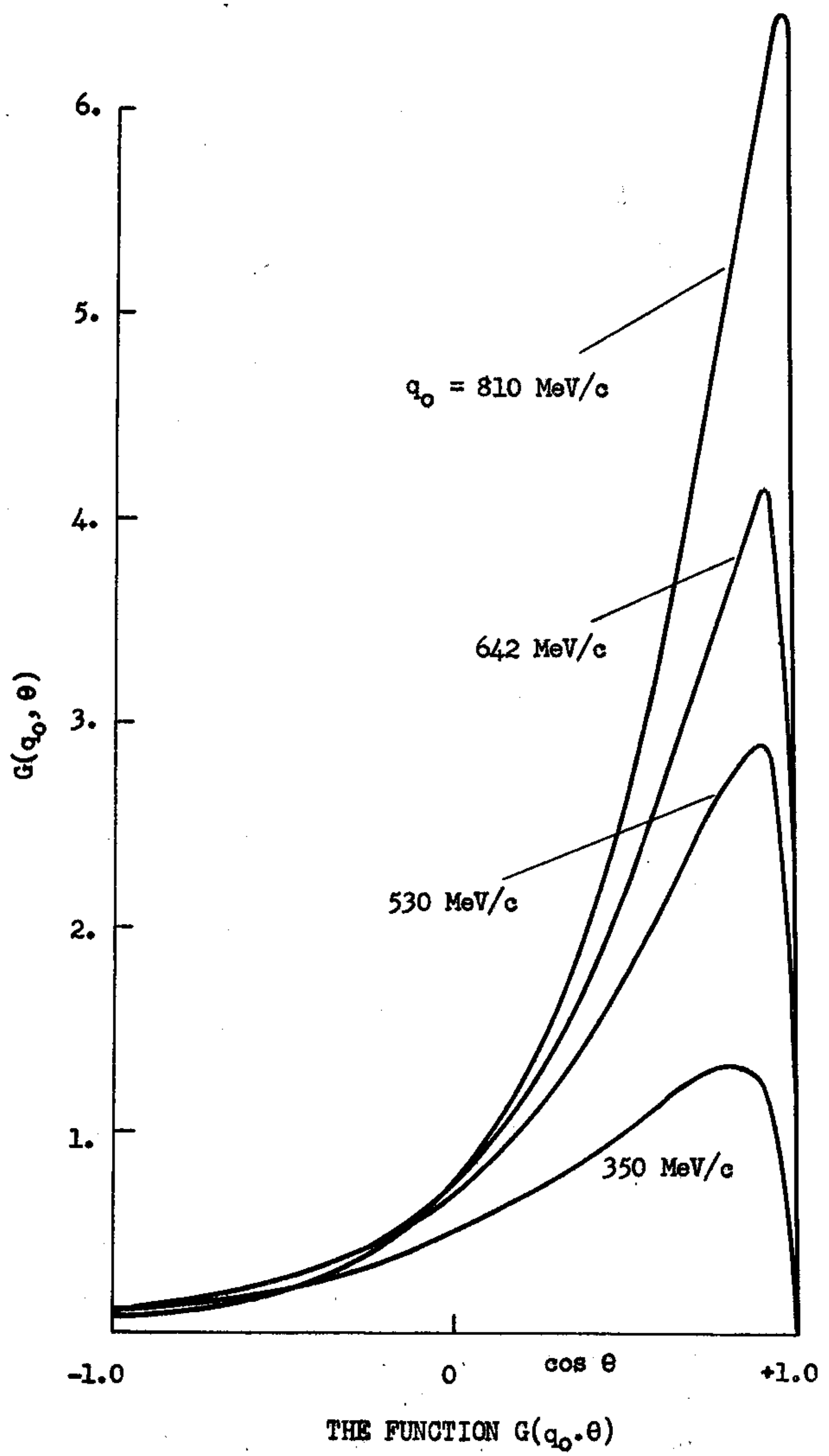



Fig. 5 - Plots of the functions $G(q_o, \theta)$ for several values of the incident meson momentum $q_o$. $\theta$ is the angle between the incident and the outgoing mesons.

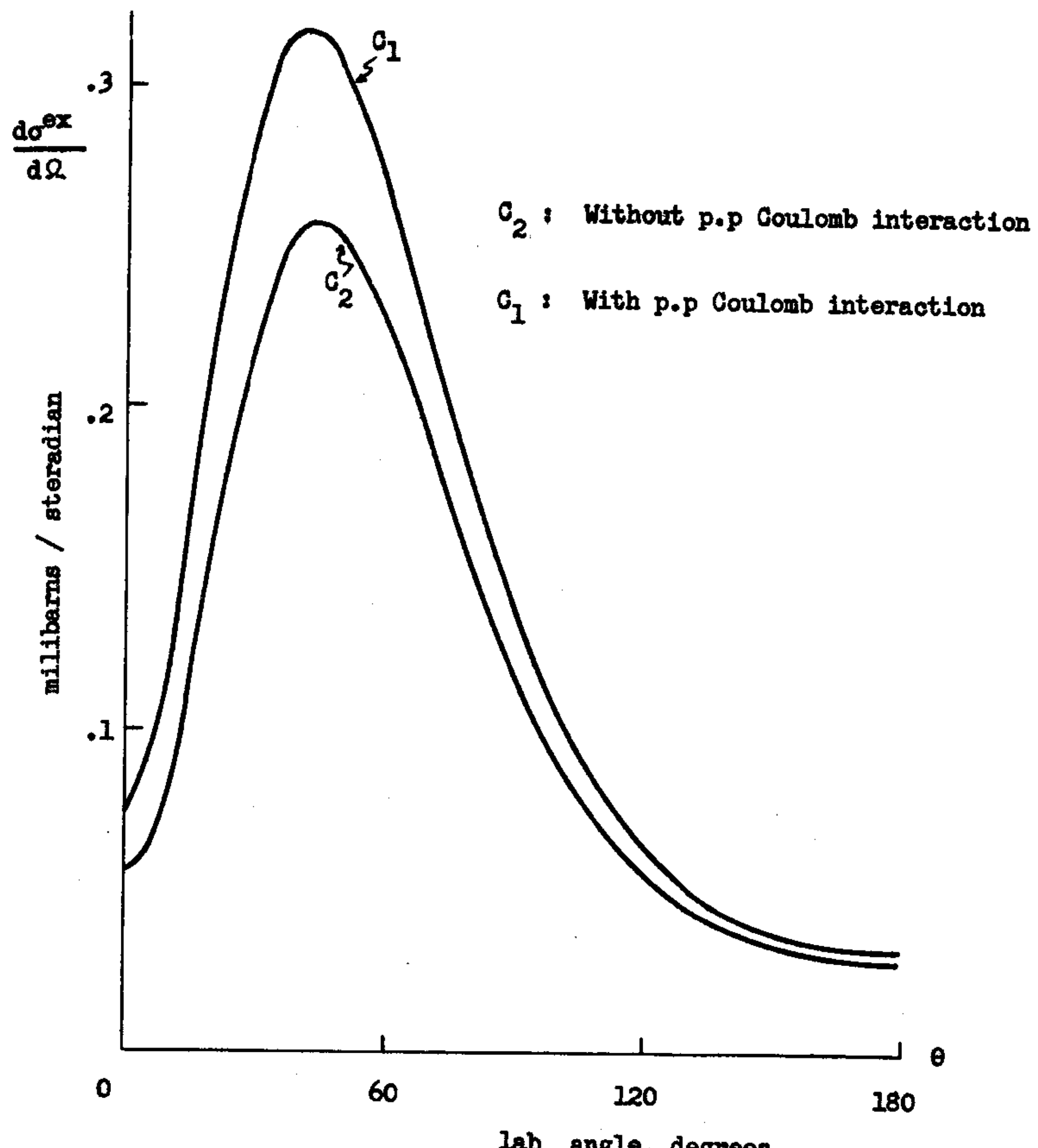
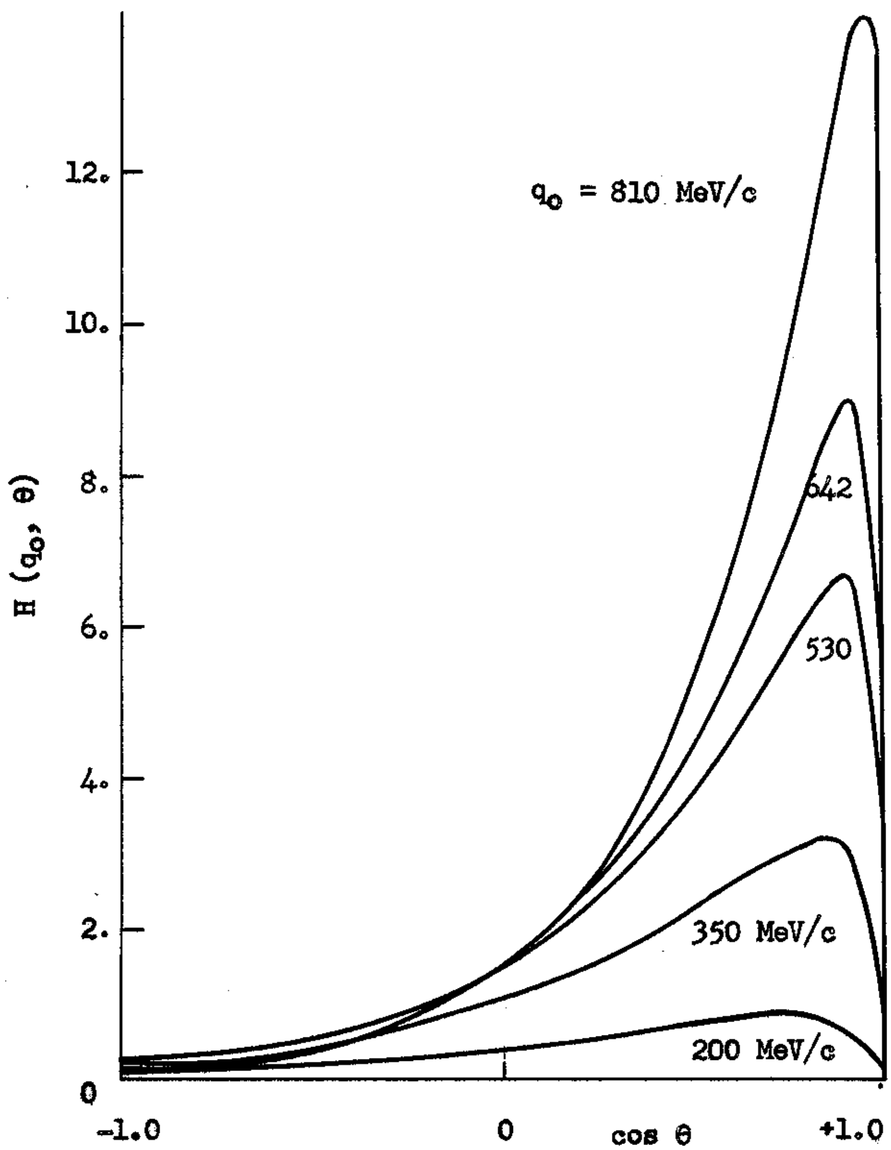



Fig. 6 - The differential charge-exchange cross-section ($K^+d \rightarrow K^0pp$) for an incident meson momentum $q_o$ = 200 MeV/c. The angle is between the incident and the outgoing mesons. The Coulomb interaction of the two protons in the final state increases the total cross section by about 20 percent. The values of the parameters are $R_1 = -0.3338 \times 10^{-13}$ cm and $R_o = 0$.




THE FUNCTION $H(q_o, \theta)$

Fig. 7 - Plots of the functions $H(q_o,\theta)$ for several values of the incident meson momentum $q_o$. The angle $\theta$ is between the incident and the outgoing mesons. The function H appears in the expression for the charge exchange differential cross section.

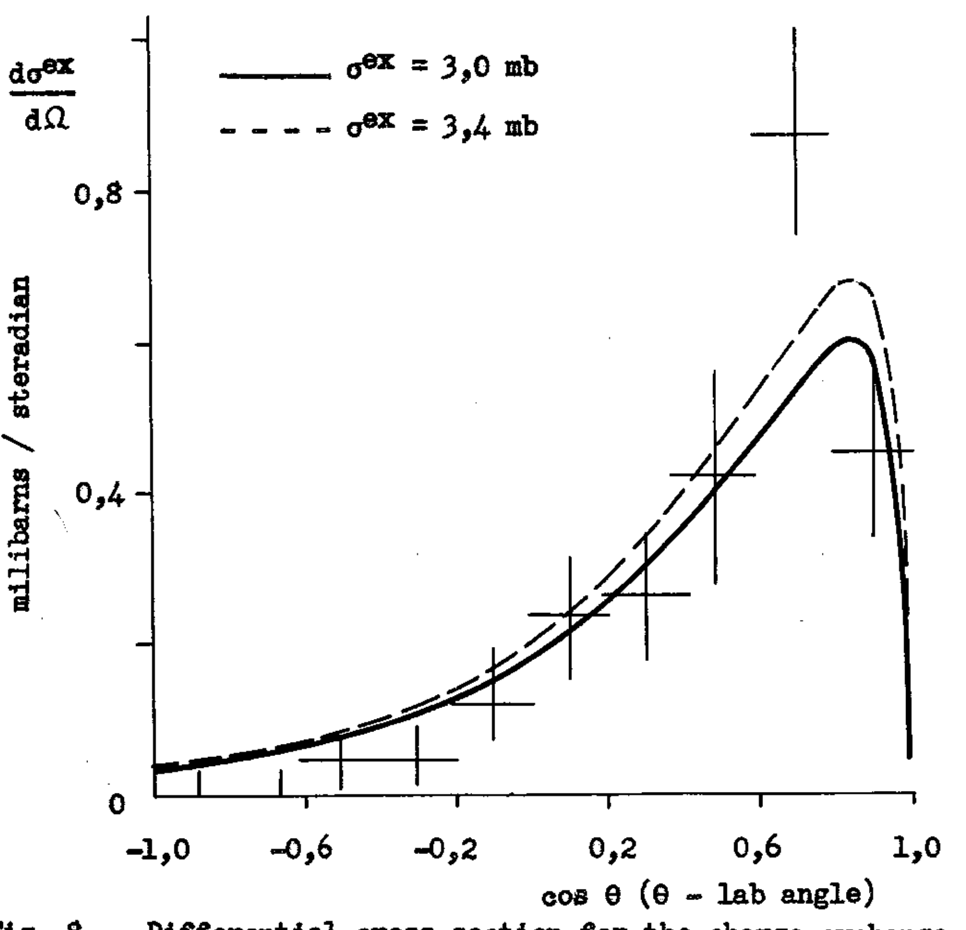



Fig. 8 - Differential cross section for the charge-exchange scattering $K^+d \to K^o pp$ for incident meson momentum $q_o$ = 350 MeV/c. The experimental points are obtained in reference (1). θ is the angle between $K^o$ and $K^+$ in lab system. Theoretical curves obtained assuming only S-waves in the K--nucleon interaction in both isotopic spin states.

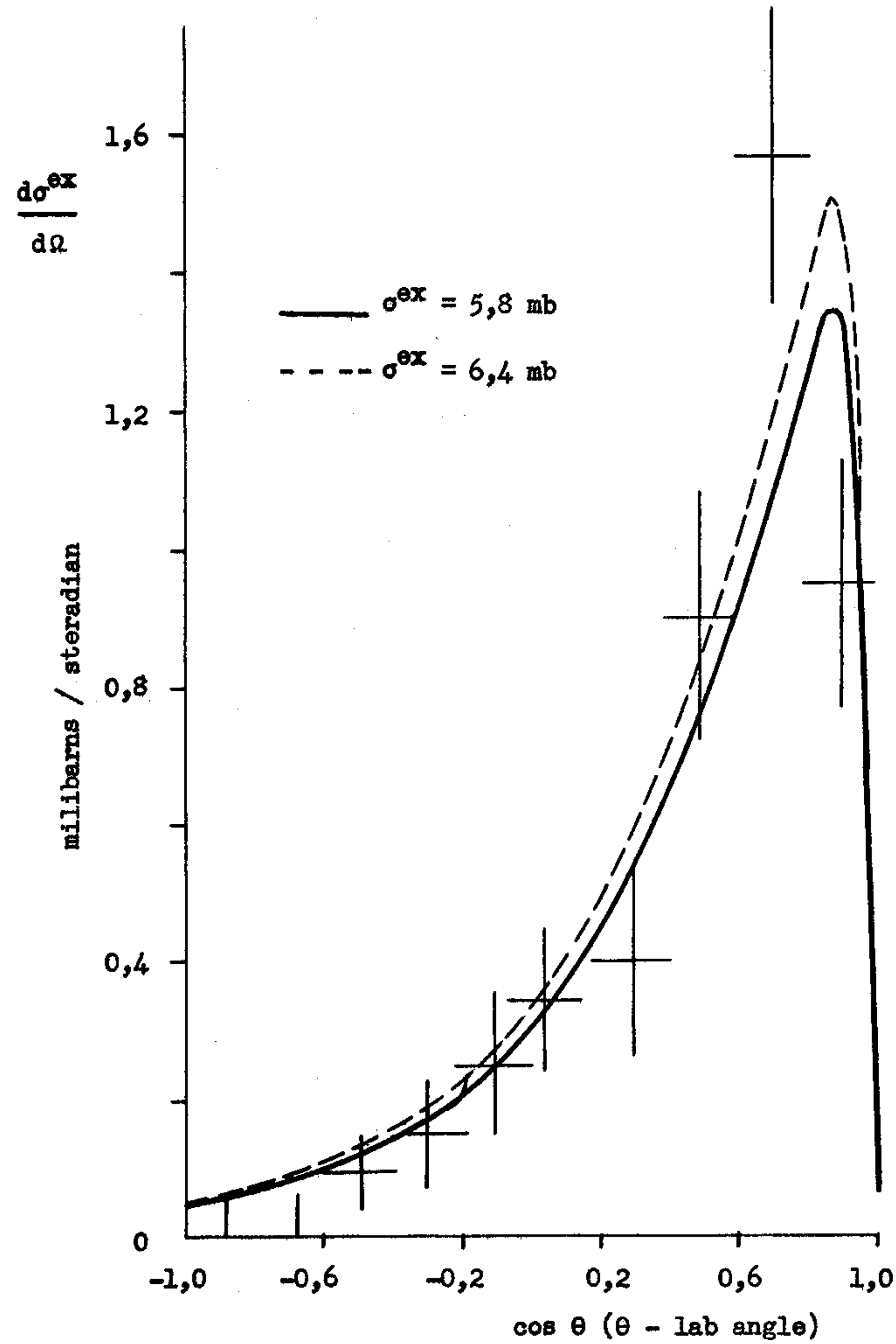



Fig. 9 - Differential cross-section for $K^+d \rightarrow K^0pp$ in lab system. Incident meson momentum $q_o$ = 530 MeV/c. Experimental points as given in reference 1. Theoretical curves obtained assuming only S-waves contributing to K--nucleon interaction in both isotopic spin states.

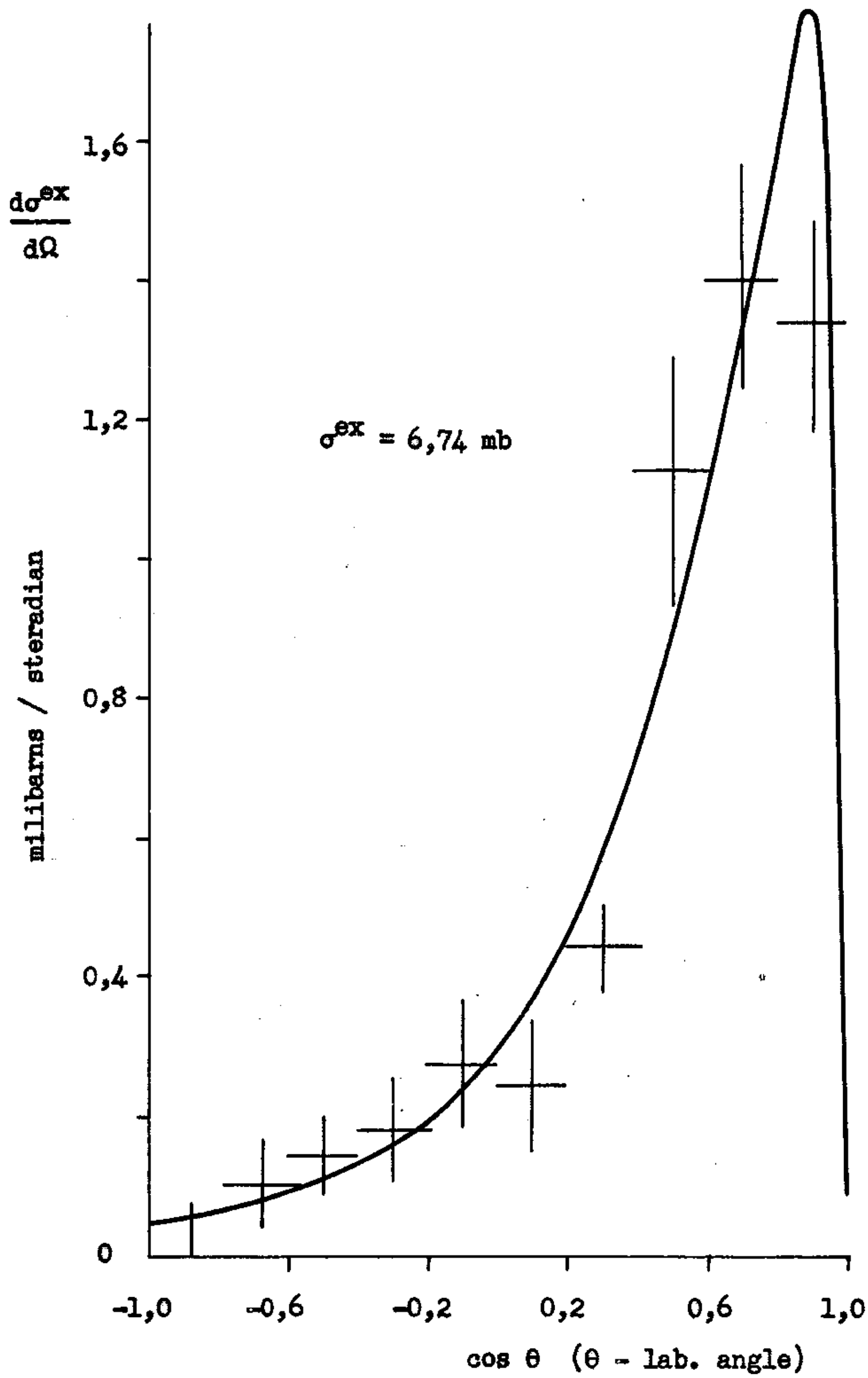



Fig. 10 - Differential cross section for the charge exchange scattering $K^+d \rightarrow K^o pp$ for incident meson momentum $q_o$ = 642 MeV/c. Experimental points as given in reference 1. θ is the lab angle between the incident and the outgoing mesons. The theoretical curve is obtained assuming only S-waves contributing to K-nucleon interaction in both isotopic spin states.