

**Particle crossing versus field crossing; a corrective response to
Duff's recent account of string theory**

to the memory of Hans-Jürgen Borchers

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Abstract

Using recent results of advanced quantum field theory, we confute some of M. Duff's claims about string theory which he wrote as an invited paper to the project "Forty Years Of String Theory: Reflecting on the Foundations"

1 Introduction

There are two important conquests in the history of quantum field theory which determined to a large extent its course up to present days: renormalized perturbation theory in QED and the nonperturbative insights into the particle-field relation initiated in the LSZ work [1] on scattering theory as well as the subsequent derivation of the particle analog of the Kramers-Kronig dispersion relations whose later experimental verification strengthened the trust in the causal localization principle of QFT which was put a successful observational test.

Since the perturbative series is known to diverge in all interacting QFTs, the satisfaction about the excellent agreement of its low order terms with experiments is a bit muted, since one does not know whether the subject of ones theoretical interest really exists in an epistemological sense, notwithstanding the fact that perturbation theory is consistent in the much weaker sense of *formal* power series. Whereas in all other areas of theoretical physics one knows the epistemological status of what one wants to approximate, there is in $d=1+3$ dimensional QFT not even a single mathematical controllable interacting model. For these reasons, but even more for practical reasons of what to do in cases where the imprecise conception of asymptotic expansions in the sense of vanishing coupling does not lead to observational agreement (and additional phenomenological ideas are needed), the bootstrap S matrix idea was formulated. Besides the obvious requirement of unitarity and Poincaré invariance there was an (at that time) new requirement: the crossing property. Since its correct formulation and conceptual understanding is pivotal for the present work, a substantial part will be dedicated to the presentation of particle crossing, which is the foundational link between particles and fields (as the generators of local observables). Without such a conceptual investment it is not possible to understand at what point Mandelstam's courageous project to attribute an early constructive role to the S-matrix, (leaving aside all references to Lagrangian/functional quantization and other parallelisms to classical physics) ended in failure.

As already known from the many unavailing attempts to solve the nonlinear Schwinger-Dyson equations, it is difficult to find solutions (and in most cases no solution was found) of nonlinear equations. It is better to try to encode raw principles into an operator formulation which defuses the nonlinear "TNT"¹ or bring it at least into the more amenable form of a spectral representation (example the proven Jost-Lehmann representation or Mandelstam's conjectured two-variable representation for elastic scattering amplitudes). The bootstrap program did not fail because it was build on unreasonable or incorrect postulates, but rather because it was impossible to find any implementation in the form in which they were presented.

In fact in connection with its limitation to elastic scattering (which is only possible in $d=1+1$) and the mathematical classification of "scattering functions" and their Yang-Baxter extensions, it has been possible to set up a bootstrap formfactor project which is not only able to solve the bootstrap program, but also to proof for the first time the mathematical existence of models with realistic short distance behavior (factorizing models) within an extended "bootstrap-formfactor program". Nevertheless nobody was

¹QFT achieves this by obtaining unitarity from the asymptotic behavior of (linear) field operators and their adjoints (the Haag-Ruelle or LSZ scattering theory).

able to defuse the nonlinear aspects of the $d=1+3$ bootstrap program and convert it into a manageable computational scheme on which also students could write their Ph.D. thesis.

On this issue Mandelstam's use of spectral representations [4] was superior, his problem was that Veneziano's guesswork [5] on crossing (using "mathematical experiments" with gamma functions) trapped him too early into the wrong type of crossing (the conformal field crossing) from which there was no escape, given the insufficient understanding of the quantum aspects of causal localization and their relation to the causal locality principle at that time.

If the S-matrix based approach has a long and interesting history, that of local quantum physics (LQP) is even longer and there is still no end insight. It started with Haag's 1957 attempt² [2] to base QFT on *intrinsic principles* instead of subjecting a more fundamental theory via a quantization parallelism to the tune of a less fundamental one. Both the ideas, Mandelstam's as well as Haag's, were top to bottom approaches in the sense that one states initially what properties may be helpful for particle theory and then one tries to obtain the tools to implement them avoiding any quantization parallelism to classical physics. The main difference was that one proposal was based on the S-matrix for which it is notoriously difficult to find sufficiently many characterizing properties (the reason why Heisenberg gave up his previous attempt); on the other hand Haag's local quantum physics approach which was modeled on the enormously successful *action at the neighborhood* principle of Faraday and Maxwell required a lot of modern (partially at that time unknown) mathematics in the attempts to adapt them to the realm of quantum physics. Often the intuitive idea dissipated somewhat into the realm of metaphors while the mathematics became increasingly precise; this seems to be the fate of all foundational concepts, and the idea of *modular localization* which, leads to most of the results in this article, is no exception.

Whereas in the former the S-matrix appeared right from the outset, in local quantum physics (LQP) is considered to be crowning reward to appear at the end of calculations based on the foundational local investment. Indeed. As a metaphorical starting point of a mathematically precise and innovative scheme this was very successful, but its "would be" epistemological content of its foundations melts away if one asks the question of how to realize its localization idea in terms of hardware; even for noncompact regions as Rindler wedges this requires to subject the Minkowski observables to a uniform acceleration (the Unruh Gedankenexperiment). It is the conceptual-mathematical consistency of the formalism which results from an idea, and not the veracity in terms of its hardware arrangements which determines the value of a theory. One concept in which it significantly deviates from the epistemological aspect of quantum mechanics (QM) is that the basic objects are ensembles of operators which share the same spacetime localization region and not individual operators (for more remarks see next section).

In this essay we show that there is a quite unexpected synthesis of the two views. It is based on the recognition that besides its prominence in the large timelike asymptotic behavior of scattering theory, the S-matrix is also a "relative modular invariant of wedge localization" [6]. This leads to new nonperturbative dynamical ideas in LQP into which S_{scat} enters on par with other foundational properties of local quantum physics [7]

²The original version is in French even though most of the talks were in English. Later it was translated back into English [3].

and culminated in the first existence proof [8].

Both attempts tried to avoid the classical-quantum parallelism of Lagrangian quantization which constitutes the basis of perturbation theory; in Lagrangian quantization one starts from the Lagrangian formulation of classical field theory and sees what one gets by following the quantization rules and imposing reasonable interpretations on the computational results. The foundational concept of Mandelstam's approach is similar, but not identical to the of the bootstrap, whereas Haag's local quantum physics, which is conceptually related to Wightman's quantum field- based approach, differs in conceptual-mathematical formulation and physical scope.

My own more recent contribution consists in bringing these two nonperturbative ideas together. This led in the hands of Lechner [8] to an existence proof and the promise of mathematically controlled constructions which are expected to reproduce the formfactors of the extensive bootstrap-formfactor work which has led to explicit expressions for formfactors of many fields (for recent review see [9]) in integrable models. There is also the well-founded hope that an existence proof and a controlled scheme of approximations for the general case may come out of these attempts even in the general case. This and only this would finally define the closure of QFT since neither in case of classical nor for quantum mechanical nonintegrable systems one can hope for more.

2 Particle crossing against crossing from conformal correlation functions

One of the most foundational properties of relativistic QT was (and still) is the particle crossing since it requires to understand the relation between particles and fields (or Mandelstam's S-matrix setting and Haag's local quantum physics) beyond scattering theory. In this essay it will be shown that by connecting these seemingly antagonistic concepts of particle physics in a new "modular" way, one does not only reach a foundational understanding of particle crossing, but one obtains for the first time a new entrance into QFT which unravels the inner workings of particle theory and suggests new nonperturbative constructions.

The particle crossing property was originally an observation on Feynman graphs; it was not limited to the rather trivial statement that each graph has a crossed counterpart, but required rather in the more demanding recognition that the mass shell projections of Feynman amplitudes allows an analytic continuation which *stays on the complex mass shell*; so the complex crossing identity and its analytic continuation to real boundary points can be achieved on-shell. As a property which holds in every order it was accepted (although conceptually not yet understood) as property independent of perturbation theory. The first nonperturbative proof, which was restricted to the elastic 2-particle S-matrix, was based on the complicated theory of several complex variables and used the art of "nose-cutting" (finding holomorphy envelopes, this sport exists only in several variables); the extension of this method was discontinued because it was too unwieldy. The new method reveals that the crossing identity shares its conceptual position with the KMS property related to wedge localization. This does not only place the crossing on par with the thermal aspect of the Unruh effect (localization on a Rindler wedge), but also attributes to it a

central role in a generalization of the famous second quantization functor³ between modular localized subspaces of the Wigner particle-space and localized subalgebras (a local net of algebras) in the absence of interactions. Any interaction leads to a breakdown of this picture; what remains of it (in case that the model admits a complete particle interpretation) is the possibility to emulate wedge-localized interaction free incoming fields within the wedge-localized interacting algebra. This construction and its consequences for QFT, in particular a foundational understanding of particle crossing, will be the main subject of the next section. But some remarks about its history and its intuitive-conceptual content are in order before explaining its mathematical content.

A most fruitful encounter between mathematics and particle physics, which finally led to modular localization, took place in the middle of the 60s at a meeting between mathematical physicist and mathematicians in Baton Rouge (Louisiana, USA). The high point on the mathematical side was the first appearance (still in an incomplete form) of Tomita's theory of a modular theory of operator algebras as a vast generalization of the notion of "modular" in contest of group algebras (unimodularity of Haar measures). Its mathematical imperfections were later "repaired" by Takesaki. The contribution on the side of particle physics was an almost perfect thermal QT theory of "open systems". It replaces the box approximation and the subsequent thermodynamic limit of Gibbs states in which the spacetime symmetries are expected to be recovered by a setting in which one works from the outside with KMS states on the open (no quantization boxes) system. The result was the discovery of an algebra ("hyperfinite type III₁ factor algebra") with remarkably different properties from those of the standard algebras of all bounded operators $B(H)$ of QM. Without the conceptual mathematical content of the analytic KMS formulation coming from the physicists, Connes would have been very hard-pressed to get to his breakthrough in the classification of von Neumann factor algebras.

The realization that these new algebras and their associated KMS states may play a fundamental role in the description of causally localized quantum matter came 10 years after in the application of modular operator theory to wedge-localized algebras in the work of Bisognano and Wichmann [1] and its connection to the Unruh effect and to Hawking radiation. The conceptual conquests of this new structure led to a understanding of the construction recipes for integrable QFT in terms of locality principles of QFT as well to the foundational understanding of particle crossing and the nonperturbative "emulation" setting presented in the sequel.

For experts about the inner workings of QFT this development was somewhat expected as the (at that time) futuristic sounding title "*On revolutionizing quantum field theory with Tomita's modular theory*" of one of Borchers[10] (to whose memory I dedicated this essay) papers reveals. Among recent results, which should even warm the hearts of specialized workers in this area, is a completely new understanding of the Einstein-Jordan conundrum and its aftermath. It started with Jordan's university of Goettingen thesis in which he claimed that Einstein's particle-like (photon) contributions in his famous Gedankenexperiment on fluctuation in localized subvolumes of black-body radiation are, in contrast to Einstein's opinion, not needed in order to explain thermodynamic equilibrium. In Einstein's counter-article he argued that Jordan's arguments, although seemingly

³This term is a misdemeanor and nobody expressed this clearer than E. Nelson by saying: "quantization is an art, but second quantization is a functor".

mathematically correct, are in contradiction to the known absorption coefficient which in turn caused a complete change of Jordan's mind. It did not only convert Jordan to become a originator and fervid defender of "field quantization" for matter and electromagnetism [11][12]. He proposed to show that the new field quantization leads to the same two fluctuation components if the vacuum of his model of a free "d=1+1 photon"⁴ is restricted to a localized subinterval and managed to see in his approximate quantum mechanical calculation the analog to Einstein's matter component in the fluctuation spectrum. His calculation was published as a separate section in the famous Dreimännerarbeit by Born, Heisenberg and Jordan; this is in the opinion of experts [13] *the birth of QFT* [14].

Interestingly enough, his coauthors Born and Heisenberg always felt that there was an aura of incompleteness in Jordan's section. Before Heisenberg published his famous discovery of vacuum polarization in QFT, he challenged Jordan in a correspondence [13] to find an $ln\varepsilon$ contribution from the vacuum polarization near the endpoints of his subinterval (ε = attenuation distance for the polarization cloud i.e. roughness of localization). The KMS thermal aspect of a reduced vacuum state would have cinched the correspondence with Einstein's thermal setting, but the thermal aspects of localization were only seen 5 decades later. The fluctuation in an open subvolume belongs to those problems which even in the simplest case of a free field require approximations; unfortunately quantum mechanical inspired calculations wreck the locality and covariance of this QFT problem and the only known way to maintain these aspects is by checking the thermal aspects. An approximation in which the global vacuum keeps its inside/outside factorization (as in [13]) destroys the holistic aspects which distinguish QFT from QM.

Some years ago Weinberg was asked [15] whether Jordan's field quantization in the context of an infinite collection of quantum mechanical oscillators can be viewed as an early harbinger of string theory. Weinberg answered in the affirmative, but his response remained somewhat incomplete. Since this example is of great pedagogical value for the problem under discussion, it is worthwhile to look at some of the details. Let us do this for a slight generalization of Jordan's model namely instead of taking just one abelian chiral current $j(x)$ we take an n-component abelian current $j_k(x)$ $k = 1, \dots, n$. One can either use this current and its continuum of charge superselection rules in the sense of QFT in which case it is reasonable to ask for local extensions i.e. fields which are formally exponentials⁵ in the potentials of these currents

$$\begin{aligned} \psi_{\vec{q}}(x) &= " : e^{i\vec{q}\vec{\Phi}(x)} : ", \quad \partial\Phi_k = j_k \\ \left[\vec{Q}, \psi_{\vec{q}}(x) \right] &\sim \vec{q}\psi_{\vec{q}}(x), \quad Q_k = \int_{-\infty}^{+\infty} j_k(x)dx \end{aligned} \quad (1)$$

Viewed as a collection of oscillators it represents indeed a *quantum mechanical string*, since any collection of oscillators can be viewed as a string. but this is an interpretation which is not intrinsic since it comes with Born's probabilistic interpretation of the spectral

⁴The reason for the quotation marks is that there is nothing like a d=1+1 photon since the particle concept of QFT, different from QM, depends on spacetime in an essential way. In modern terminology Jordan was dealing with the potential of an abelian chiral current..

⁵In this form the charge conservation has to be added by hand. A more careful limiting procedure starting from exponentials over integrals extended over an interval with one end going to infinity gives the complete commutational rules for these exponentials.

decomposition of the position operator and whether this operator is pictured as a position in actual space or somewhere else does not matter; a quantum mechanical position operator is a global object which by itself has no localization. From the causal localization viewpoint, which is intrinsic to QFT, it is a pointlike localized object in Minkowski spacetime (in Jordan's model a *chiral operator on a lightlike line*). In this context the collection of oscillators is viewed as a holistic object and this view tells us about commutation relations and covariance properties which in perturbative calculations we want to keep. It took many years to find among all possible of quantum mechanical ways of perturbative interactions to find that one which maintains this holistic aspect.

The reason why we do not think about this any more is that renormalized perturbation theory which maintains precisely this holistic aspect has been streamlined. We are only thrown off guard if we come to a new problem as the fluctuation problem in open subvolumes. In this case it is difficult to implement the covariance and localization requirements and what we learned from perturbation theory is of no avail. But we can always check whether our approximation satisfies the thermal and vacuum-polarization consequences of modular localization. Although this aspect of QFT was with us from the 1925 beginning in the Einstein-Jordan conundrum we have evaded it. But there was a high prize to be paid in form of a quite deep confusion brought by string theory.

The holistic way of dealing with the above n-component model goes as follows one first classifies (maximal) extensions taking charge carrying operators of the above exponential type into an enlarged observable algebra which as a result of their lightlike commutation are qualified as observable generating fields. The maximal ways of doing this are classified in terms of even integral lattices L in n-dim. Euclidean space; these extended algebras are rational chiral QFTs since they only have a finite number of superselection sectors (related to the factor classes L^*/L). It turns out that the selfdual which only have one sector which is equivalent to a deep localization property (Haag duality for disjoint intervals). Equivalent relations within the setting of vertex operators which led to interesting relations with the largest "sporadic groups" (moonshine monster) have been before derived [18], but the LQP derivation [19][20] is of particular interest because it shows how deep aspects group theory arise from the realizations of just one and only physical localization principle of QFT.

String theory arises from a quite different use which is more quantum mechanical than holistic. Forgetting the problem of localization in the chiral world of the light cone (or its compactified world $\mathring{R} = S^{(1)}$) one may ask the question whether one can organize the oscillator degrees of freedom in a different way so that a representation of the Poincaré group emerges. Since the n-component current model belongs to the class of non-rational models with continuously many superselection rules there is no reason why only compact groups are allowed (as in higher dimensional QFTs with mass gaps). Indeed one may forget about the holistic requirements in the source space and try to reorganize the oscillator degrees of freedom in such a way that a n-dimensional Poincaré group becomes represented. Whereas this in itself is not surprising, the fact that with the additional requirement of unitarity and positivity of energy there is essentially only one representation (the n=10 superstring and a finite number of M-theoretic variations) calls for attention.

Up to this point the narrative of the string saga is completely correct but there are

two additional points where it moves away from facts. One is that such a positive energy representation of the Poincaré group carries its intrinsic localization property; one only has to confirm that the superstring representation does not contain irreducible parts from the continuous spin Wigner class (which would require string-localization). The pointlike nature follows, either in the form of wave functions generated by applying the sigma model fields $\psi_q(x)$ (1) to the vacuum or directly by computing the commutator of the associated second quantized [16] infinite component quantum field. This new way of organizing the degrees of freedom with respect to a representation of the Poincaré group acting on the target is quantum mechanical since localization does not play a direct role. But since positive energy representations containing massive and massless finite helicity representations always come with pointlike localizability, the latter is a consequence of the nature of the representation.

Any additional operation on these objects which maintains their intrinsic localization would be holistic in the target sense. But the holistic aspects of target and source localizations are different and may not be simultaneously satisfiable. The result of the target construction is an infinite component field with an unbounded "mass/spin tower" of which each finite mass projection represents an ordinary Wightman field (operator-valued Schwartz distribution).

There are two surprising aspects of this construction. On the one hand the wave function space which the $\psi_q(x)$ generate from the vacuum is irreducible i.e. the space is not just a direct sum of irreducible Wigner representation spaces but there are operations which intertwine these representations. This means that the infinite component field is an object as envisaged (but not found) by Barut, Kleinert and others [17] namely a *dynamic* infinite component field. The second surprise is that the construction is unique i.e. there is only one superstring representation (apart from a finite number of "M-theoretic" variations). But should the popular idea *foundational* \rightarrow *unique* ("Weltformel", theory of everything) be inverted to *uniqueness* \rightarrow *foundational* in the sense that our living space-time should be viewed as resulting from a dimensional reduction of the 10-dimensional superstring representation? Should our life take place in the target of a chiral current model? On the answer to this question the believers in the string saga differ sharply from the rest of the particle theory community. With these remarks our attempt to give a more complete answer to the question in Weinberg's lecture [15] our excursion concludes. We will use it in later discussions as a point of reference.

It is a downside of the elegance of renormalized perturbation theory that we forget that its results from inductively implementing the holistic quantum localization principle by starting with a classical pointlike coupling and iterating according to the (Epstein-Glaser) intrinsic quantum localization rules. Unlike approximating (selfadjoint) operators in QM there can be no infinity (ultraviolet divergence), a principle can either be implemented in a particular context or not⁶. What may not be possible to maintain is the bound-

⁶The quantum locality principle is always implementable in the Epstein-Glaser iteration scheme (in general only with an ever increasing numbers of parameters, growing with the perturbative order). The principle of keeping an upper limit of scaling degrees however leads to finite parametric islands (left invariant by renormalization group transformations) within a formal "universal" (useless) master QFT. Whereas the number of pointlike couplings in $d=1+3$ is finite, the use of stringlike (not ST!) is infinite, although not all couplings lead to pointlike observables (subalgebras).

edness of the short distance scaling degree of the resulting fields, in this case the theory contains an ever increasing number of parameters (growing with the perturbative order) which makes the theory less useful for experimental verifications and probably nonexistent from a nonperturbative point of view. The Einstein-Hilbert action certainly violates the boundedness of the scaling requirement. But this does not mean that it also violates the requirement of background independence. The sign that it does not (if treated in a suitably generalized categorical setting) are quite encouraging [21]; in fact one would like to think that there is not just a "third way" to quantum gravity (besides string theory and loop gravity), but that this will be the only path to maintain background independence.

As mentioned to uphold the localization principle in Jordan's highly nontrivial fluctuation problem on a free field [12] requires a different method from that of perturbation theory. The way this is done correctly is to separate the sharp localized ensemble (local observable algebra) from its causal disjoint by an ε -security distance with the help of the "split property" which uses the full power of the Tomita-Takesaki theory of operator algebras and even with its use one only arrives at the leading $\ln\varepsilon$ guessed already by Heisenberg [13]. For the KMS thermal manifestation of the restricted vacuum it is not necessary to any calculation, it is a general structural consequence.

This closer look at Weinberg's answer reveals an important distinction between QM and QFT. QFT is a holistic theory as long as one does not destroy its principle of covariant causal localization by causality violating quantum mechanical oscillator approximations. In a paper entitled: *Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Effective Degrees of Freedom* Hollands and Wald illustrated this holistic property of QFT with the help of the Casimir effect [23]. But what was obviously on their mind was a critique of the quantum mechanical treatment of the cosmological constant problem in which the (global) particle levels in momentum space were subsequently occupied up to an ad hoc cutoff at the Planck mass. This global level-filling is an especially strong violation of the holistic localization principle. Related to this is the issue of localization-entropy [22] which in the simple Jordan model allows a rather explicit treatment [24].

One can only dream about what would have happened if some of the modern insights had been available at the cradle of QFT in the Einstein-Jordan dispute. With individual causally-localized quantum systems being considered as belonging to an *ensemble of individual observables localized in the same region*, there would have been another more intrinsic and natural way that *probability enters QT*. The Born probability (related to the *physical interpretation* of the spectral decomposition theory of the selfadjoint quantum mechanical position operator) is *not* an intrinsic aspect of QT⁷, a fact which caused heated disputes around the measurement process starting from the Copenhagen interpretation and the Schrödinger cat Gedankenexperiment up to more recent somewhat bizarre attempts as *Everett's multiworld interpretation*.

Einstein, who refused to accept this addition of Born to QT, was certainly in peace with the probability which is intrinsic to KMS states on ensembles of statistical thermal systems. It is fascinating to contemplate what would have happened if the vacuum polarization and thermal aspect of the impure state which results from restricting the QFT

⁷Nevertheless it is indispensable for its interpretation and for defining propagation velocities as the quantum mechanical velocity of sound.

vacuum to a localized subalgebra would have been understood already at the genesis of QFT in 1925/26. Einstein almost certainly would have embraced this statistical probability which anyhow was used on his side of the dispute, and the notion of probability in QM (being less fundamental than QFT) would not have taken on the role of a reality-chewing dragon (fought by Einstein throughout his life) but rather that of a toothless tiger in the form of a useful bedside carpet. Without Born's direct assignment of probabilities to individual observables the particle theorists would extract their physical information directly from the ensemble of local observables localized in a specific spacetime region, as done in the Haag-Ruelle scattering theory where the different members only lead to different asymptotic normalization factors but the S-matrix remains unaffected. More specific (off-shell) properties would be filtered out by narrowing the spacetime localization of counters and improving their efficiency, just the way an experimentalist proceeds; in short Haag's idea of local observable idealized as spacetime indexed subalgebras would have been with us from the beginning.

It is not possible to explain the concept of modular localization (whose development took almost 3 decades and which even nowadays is only known to a few of innovative thinkers) in an essay like this which has to stick to what the reader expects from its title. The modular localization of states is somewhat simpler in that it requires no mathematics of operator algebras. It can be defined in any unitary positive energy representation of the Poincare group. The idea is to first define a subspace which is wedge-localized, and then passing to compact or spacelike cone regions (written as intersections of wedges) by intersecting the associated wedge algebras. The emulation idea (next section) secures the well-definedness of the starting point, but the algebraic intersection may be empty (multiples of the identity) which vaguely corresponds to the nonexistence of the quantized analog of a classical Lagrangian field theory.

There exist a series of arguments which do not require the knowledge of conceptional intricate problems. The simplest way to catch the string theorists in flagranti concerning their wrong ideas about localization is the (implicit) conclusions they draw from the Lagrangian of a classical relativistic particle which describes covariant worldlines

$$L_{class} = - \int \sqrt{ds^2} d\tau, \quad \curvearrowright x_{class}^\mu(\tau)$$

$$\nexists \text{ quantized } x_{op}^\mu(\tau)$$

The second line, that this system has no quantization analog, i.e. the nonexistence of a covariant 4-component position operator $x_{op}^\mu(\tau)$ they forget to mention. It would wreck their purpose of using this Lagrangian in support of string theory since there simply exists no frame-independent position operator in any dimension. One would think that this is common knowledge since the times of Wigner's particle classifications, but as the use of the wrong picture in the context of presenting supportive arguments for string theory (page [25]) shows, this assumption is too optimistic, there exists by now a gigantic gap in knowledge about localization concepts. An immediate consequence of this fact is that dynamical variables $X^\mu(\tau)$, as they appear in functional representation of string theories, do not correspond to *embedding* of "quantum world lines" into a hypothetical target space (an internal symmetry space interpreted as spacetime) but lead rather (apart from zero modes) to oscillators in an inner space which should be pictured as a Hilbert space over

a spacetime point and not as its "transversal" extension into a string. This places grave doubts on the veracity of the "derivation of gravity from string theory".

In fact quite generally it is impossible to imbed a lower dimensional QFT in the target space of a higher dimensional one. There are several quite different ways of seeing this. One consists in convincing oneself that quantization of classical fields is only meaningful if the "would be" noncompact indices of the classical field refer to tensorial/spinorial indices associated to the spacetime in which the classical field "lives" (is causally localized). There exists a theorem that in a QFT with a mass gap all internal symmetries must belong to compact internal symmetry spaces [1] hence in this case one is simply overstressing the range of Lagrangian quantization (as in the above counterexample). Another way of seeing this is to argue that if such an embedding would be possible than by dimensional reduction one should return to the original theory. But the theory really obtained in form of a brane in the higher dimensional one *keeps all the degrees of freedom of the higher dimensional one* [26]; its mathematical existence does not save it from being physically pathological⁸. What is however possible is to introduce e.g. stringlike localized generating fields [30], but this has nothing to do with any embedding of a chiral theory into a higher dimensional QFT.

A closely related issue is the use of *nonrational chiral theories* (as the multicomponent chiral current model) which have a continuous set of charge superselection sectors and, as previously mentioned, permit noncompact internal symmetries, in particular the target action of the Poincaré group; but so what? (for more see last section). Even in this case the associated chiral sigma models is pointlike and not stringlike localized in the noncompact target spacetime. In fact the commutator of the free fields associated with this highly reducible representation of the Poincaré group representation on the target space of the sigma model has been correctly computed by string theorists but apparently they thought of their result as a point on a string; what string?

Could it be that the notation X^μ which is reminiscent of a (nonexisting) covariant position operator (instead of a more neutral notation as Φ^μ for the potentials of the chiral current) facilitated string theorists confusion? It is certainly true that on such "target" spaces of multi-component chiral current models one can represent the Poincaré group. Admittedly, it is a bit surprising that even with the severe restriction of unitarity and positive energy this is still possible and even more so that there seems to be precisely one way: the 10 dimensional superstring representation (and its finite number of M-theoretic variations). Infinitely many representations or none at all, would have been more palatable. But should the unexpected rarity (motto: there is nothing else on the market) to realize such a restricted highly reducible Poincaré group representation on the target space of a chiral sigma model be the starting point of a foundational theory of spacetime? Or in other words, can a dynamical⁹ infinite component interaction-free pointlike field play such a fundamental role?

By now it should be visible that the "target crossing" of the dual model had nothing

⁸Since brane constructions are quasiclassical, this degrees of freedom feature is not seen.

⁹"Dynamical" refers to the fact that these fields are not simply the direct sum of a mass/spin tower of free fields, but that the conformal oscillator algebra also links these irreducible components in such that the field becomes irreducible. As it stands it is not an operator-valued Schwartz distribution, but by using projectors onto subspaces of finite $P^\mu P_\mu$ it can be viewed as the limit of such.

to do with the particle crossing which the bootstrap-adherents thought about and which Mandelstam had in mind before he was sucked into the string theoretical confusions. Already from the work [27], in which the Gamma-function recipes of Veneziano were replaced by a chiral operator prescription in chiral models, it could have been seen that the pretended masses appearing in the pole positions of the dual model were (proportional to) scale dimensions of conformal composites which are quadratic in the multicomponent superselected charges of the model which through string theoretic glasses were interpreted as particle momenta. As Mack [26] showed later in a more systematic way (using the formalism of Mellin transformations and the existence of globally convergent operator expansions in conformal QFT), every conformal QFT (independent of its spacetime dimension) can be related in this way to a dual model.

Roughly his argument was as follows. Consider a 4-point function of conformal fields

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4) \rangle \quad (2)$$

$$A_3(x_3)A_4(x_4)\Omega = \sum_k \int d^4z \Delta_{A_3, A_4, C_k}(x_1, x_2, y) C_k(z) \Omega$$

where, as shown in previous work by Mack and other authors the Δ have the property of 3-point functions, the C 's are from an infinite family of conformal fields and, different from a short distance Wilson operator product expansion which only converges asymptotically for short distances, the conformal expansion (written symbolically as 2) converges globally. By using local commutativity, one can apply this expansion simultaneously on bra and ket vacua and in this way obtain three different ways of expanding the 4-point function. The last step consists in Mellin-transforming the 3 different ways of representing the 4-point function. In that case the scale dimensions $\dim C_k$ of the C_k become positions of first order poles of a meromorphic 4-point function; by multiplying them with a number having the dimensions of a mass one obtains a dual model amplitude. The spacetime dimension does not play any role since there are always 3 Mellin variables s,t,u in terms of which the (appropriately normalized) Mellin-transformed 4-point function can be written, but calling them s,t,u does not convert this mathematics into particle physics¹⁰ inasmuch as calling the internal symmetry space of a chiral sigma model "target" space does not create our living spacetime.

It is an interesting hypothetical question whether, giving the present insight, the conceptual guidance at that time of the invention of the dual model was still strong enough in order to have desisted this temptation and avoided the derailment. The gulf between the mathematical correct formalism of crossing properties resulting from Barnes-Mellin transformations of conformal correlators and its wrong interpretation in terms of particle crossing is the rational explanation for string theories bizarre appearance¹¹; it is not necessary (although not entirely incorrect) to follow Phil Anderson and point at the intellectual

¹⁰Unlike Fourier transformation, Mellin transformation has no Hilbert space status i.e. the resulting object does not have a Hilbert space status (the best one can do is selecting those models which have the residuum structure one expects from scattering amplitudes).

¹¹Mellin transforms, unlike Fourier transforms, have no direct connection with the Hilbert space; the only way of checking positivity is to subject the original conformal correlators to the positivity criterion of Wightman functions. The positivity properties of S-matrices and formfactors (even in the perturbative approach) are different.

arrogance of some of its proponents. A crossing which had nothing to do with particle theory was forced to submit to ideas of particle scattering, and when this came apart at the seams, it was used to comply with gravity where it could unfold its truly bizarre appearance, liberated from all observational responsibilities. For those who pushed it into this direction, the bizarre aspect cannot be sensed any more, because hardened by many revolutions, they became blind to what particle physics used to represent. Mathematical unambiguity got confused with physical consistency.

The so-called free string field is a formally pointlike localized field which contains infinitely many irreducible Wigner particle components (those contained in the unitary positive highly reducible "superstring" representation). It does not make sense to impose a stringy tube graph structure on such an object. Its two-point function is more singular than a Schwartz distribution, i.e. it is not a Wightman field but it becomes one by projecting it onto a subspace of finite $P^\mu P_\mu$. It represents a perfect illustration of what in a previously was called *dynamic infinite component field*. Its only relation to the dual model is that the dimensional spectrum of the latter is identical to the (m,s) content of the (pointlike) string field. The dual model on the other hand, being the Mellin transform of a conformal 4-point function after application of the global operator product expansion, is an object which does not have the same Hilbert space status as a scattering amplitude and it therefore makes no sense to "unitarize" it. With other words these two objects do not fit together despite, the fact that they share the spectrum of the superstring representation.

Needless to add that the the dual model first order poles and the (m,s) tower of the string field are identical since they are (up to an added mass parameter, related to what string theorists call the tension) identical to the anomalous dimension spectrum of the (composite) fields associated with the same conformal QFT. The claim that the dual model amplitude is the tree approximation of an interacting string field is however far-fetched since no interaction has been specified.

The conceptual origin of the correct crossing of particle theory has a considerably more demanding derivation; as will be shown in the next section the crossing identity is the rewritten thermal KMS identity resulting from wedge localization. Since particle crossing of formfactors involve interacting operators as well as free fields, the difficult part is the *emulation of wedge-localized interaction-free operators* within the corresponding interacting algebra; this will be taken up in the next section. In the remainder of the present section we will present some remarks about the mathematical formulation of modular localization.

It has been realized, first in a special context in [7], and then in a general mathematical rigorous setting in [28] (see also [29][30]), that there exists a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. The starting point is an irreducible (m>0,s=0) one-particle representation of the Poincaré' group on a Hilbert space H^{12} of wave functions with the inner product

$$(\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p) \varphi_2(p) \frac{d^3 p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{-ipx} \varphi(p) \frac{d^3 p}{2p_0} \quad (3)$$

¹²The construction actually works for arbitrary unitary positive energy representations, not only irreducible ones.

For other (*higher spin*, $m > 0$; *higher helicity*, $m = 0$) representations the relation between the momentum space wave function on the mass shell (or light cone) and the covariant wave functions is more involved as a consequence of the presence of intertwiners $u(p, s)$ between the manifestly unitary and the covariant form of the representation [31].

Selecting a wedge region e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$ one notices that the unitary wedge-preserving boost $U(\Lambda_W(\chi = -2\pi t)) = \Delta^{it}$ commutes with the antiunitary reflection J_W on the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The distinguished role of the wedge region is that they form a *commuting pair* of (boost, antiunitary reflection). This has the unusual and perhaps even unexpected consequence that the unbounded and antilinear operator

$$S_W := J_W \Delta^{\frac{1}{2}}, \quad S_W^2 \subset 1 \quad (4)$$

since $J \Delta^{\frac{1}{2}} J = \Delta^{-\frac{1}{2}}$

which is intrinsically defined in terms of Wigner representation data, is *involutive on its dense domain* and has a unique closure with $\text{ran} S = \text{dom} S$ (unchanged notation).

The involutivity means that the S -operator has ± 1 eigenspaces; since it is antilinear, the $+$ -space multiplied with i changes the sign and becomes the $-$ space; hence it suffices to introduce a notation for just one eigenspace

$$K(W) = \{\text{domain of } \Delta_W^{\frac{1}{2}}, S_W \psi = \psi\} \quad (5)$$

$$J_W K(W) = K(W') = K(W)', \text{ duality}$$

$$\overline{K(W) + iK(W)} = H, \quad K(W) \cap iK(W) = 0$$

It is important to be aware that we are dealing here with real (closed) subspaces K of the complex one-particle Wigner representation space H_1 . An alternative is to directly work with the complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the *graph norm* in terms of the positive operator Δ , the dense complex subspace becomes a Hilbert space H_Δ in its own right. The upper dash on regions denotes the causal disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $\text{Im}(\cdot, \cdot)$ on H .

The two properties in the third line are the defining relations of what is called the *standardness property* of a real subspace¹³; any abstract standard subspace K of an arbitrary Hilbert space permits to define an abstract S -operator

$$S(\psi + i\varphi) = \psi - i\varphi, \quad S = J \Delta^{\frac{1}{2}} \quad (6)$$

$$\text{dom} S = \text{dom} \Delta^{\frac{1}{2}} = K + iK$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group Δ^{it} and an antiunitary reflection which generally have however no geometric interpretation in terms of localization. The domain of the Tomita S -operator

¹³According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

is the same as the domain of $\Delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that for the physical case at hand, this domain is intrinsically determined solely in terms of the Wigner group representation theory.

The K spaces are the real parts of these complex $domS$ (corresponding to the one-particle projection of the real subspaces generated by Hermitian Segal operators). Their symplectic complement can be written in terms of the action of the J operator and croplands to the K-space of the causal disjoint wedge W'

$$K'_W := \{\chi | Im(\chi, \varphi) = 0, \text{ all } \varphi \in K_W\} = J_W K_W = K_{W'}$$

The extension of W-localization to arbitrary spacetime regions \mathcal{O} is done by representing the causal closure \mathcal{O}'' as an intersection of wedges and defining $K_{\mathcal{O}}$ as the corresponding intersection of wedge spaces

$$K_{\mathcal{O}} \equiv K_{\mathcal{O}''} \bigcap_{W \supset \mathcal{O}''} K_W, \quad \mathcal{O}'' = \text{causal completion of } \mathcal{O} \quad (7)$$

These K-spaces lead via (6) and (7) to the modular operators associated with $K_{\mathcal{O}}$.

For those who are familiar with Weinberg's intertwiner formalism [31] in passing from the unitary Wigner to covariant representations in the dotted/undotted spinor formalism it may be helpful to recall the resulting "master formula"

$$\begin{aligned} \Psi^{(A, \dot{B})}(x) = & \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} \sum_{s_3=\pm s} u^{(A, \dot{B})}(p, s_3) a(p, s_3) + \\ & + e^{ipx} \sum_{s_3=\pm s} v^{(A, \dot{B})}(p, s_3) b^*(p, s_3)) \frac{d^3p}{2\omega} \end{aligned} \quad (8)$$

where the a,b operators correspond to the Wigner momentum space wave functions and the u,v are the intertwiners. In our present setting, where these objects are momentum $\Psi^{(A, \dot{B})}(x)$ space wave functions, $\Psi^{(A, \dot{B})}(x)$ is an wave function valued Schwartz distribution i.e. the test function-smearred objects $\Psi^{(A, \dot{B})}(f)$ are Wigner wave functions. Choosing $supp f \subset W$ we obtain the W-localized Wigner wave functions i.e. the $\Psi^{(A, \dot{B})}(x)$ are covariant generators of (all) localized subspaces. The difference to Weinberg's setting is that whereas he uses the computationally somewhat easier covariance requirement, the modular localization method uses causal localization directly and aims directly at modular localization spaces. The above presentation shows how close these two properties are related. The generating pointlike fields are extremely useful in the implementation of perturbation theory. They are the mediators between classical localization which is used when one specifies zero order interactions in form of invariant Wick polynomials (and lets the quantum localization take over in the Epstein-Glaser [32] quantum iteration). Modular localization on the other hand is essential in trying to generalize Wigner's intrinsic representation theoretical approach to the realm of interacting localized observables (next section).

In order to arrive at Haag's setting of local quantum physics, one only has to apply the Weyl functor which maps wave functions into operators and wave function spaces into

operator algebras (or its fermionic counterpart) which leads to the functorial relation

$$K_{\mathcal{O}} \xrightarrow{\Gamma} \mathcal{A}(\mathcal{O}) \quad (9)$$

for interaction-free systems. The functorial map Γ also relates the modular operators S, J, Δ into their "second quantized counterparts" $S_{Fock}, J_{Fock}, \Delta_{Fock}$ in Wigner-Fock space and it is straightforward to check that they are precisely the modular operators of the wedge-localized interaction free algebras i.e. the main theorem of the Tomita-Takesaki theory for local subalgebras

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O})) &\equiv \Delta^{it} \mathcal{A}(\mathcal{O}) \Delta^{-it} = \mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J &= \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned} \quad (10)$$

i.e. the modular unitary generates the modular automorphism group of the subalgebra $\mathcal{A}(\mathcal{O}) \subset B(H)$. One first shows this for the wedge-localized subalgebra and then passes to (9), using that the sharpening of localization commutes with the functor. Wigner-Fock space $J_{\mathcal{O},Fock}, \Delta_{\mathcal{O},Fock}, S_{\mathcal{O},Fock}$ counterparts of the modular operators corresponding to the intersections; since the meaning is in most cases clear from the context we have (10) omitted (and will continue to do so) all subscripts. The only prerequisites for the general (abstract) case is the standardness of the pair (\mathcal{A}, Ω) where "standard" in the theory of operator algebras means that Ω is a cyclic and separating vector with respect to \mathcal{A} , a property which in QFT is always fulfilled for interacting localized $\mathcal{A}(\mathcal{O})$ thanks to the validity of the Reeh-Schlieder theorem. For general localization regions the modular operators have no geometric interpretation (they describe a fuzzy action inside \mathcal{O}) but they are uniquely determined in terms of their W-counterparts, in particular in terms of S_W the intersections in terms of $K_{W,Fock}$.

In the presence of interaction most of this beautiful functorial building will crash, but there is one property which remains; the modular character of the wedge localization is still geometric and the modular unitary is still the same (the interacting theory has the same representation of the Poincaré group), but the comparison of the free J_{in} and its interacting counterpart J are related by the S-matrix $J = S_{scat} J_{in}$; so the $\mathcal{A}(W)$ "feels" the presence of interaction through the S-matrix or the other way around; S_{scat} plays a new role as a relative modular invariant.

Although the functorial relation breaks down in the presence of (any) interactions there is a weak substitute called "emulation" (it emulates W-smearred free field $\Psi(f)$ inside the interacting $\mathcal{A}(W)$). It is extremely powerful in terms of integrable systems and promises to have clout even outside this special family; this will be the main topic of next section.

The modular analysis has some simple consequences on the issue of string localization. There is a whole family of Wigner representations (the "continuous spin" family) for which the intersections $K_{\mathcal{O}}$ vanish for compact \mathcal{O} but not for $\mathcal{O} = \mathcal{C}$ a spacelike cone, a property shared by all unitary positive energy representation. This means that the generating amplitudes are string-localized (the singular core of spacelike cones) thus leading to generating fields which are localized on semiinfinite string-localized. In this case one can show that the intertwiners in (8) depend on the spacelike string direction

e which participates in the homogeneous spacetime transformations [30]. The resulting fields $\Psi^{(A,\dot{B})}(x;e)$ are localized on the semiinfinite line $x + \mathbb{R}_+e$ i.e. to get them into a commuting position it is not sufficient that their endpoints commute rather the full strings have to be relatively spacelike.

The reason why there string-localization was discovered so late, is that they have no Lagrangian characterization and even Weinberg's covariance-based method would have difficulties in covariantizing these representations; the appropriate method for their construction (first of their K_C -spaces and then the core-localized generating wave functions) is modular localization. A string field is a generating field associated to the 10 dimensional superstring representation of the Poincaré group. This highly reducible representation only contains irreducible components of massive and massless finite helicity representations which are all point-localized and therefor cannot be string-localized. The reader should be very suspicious of any claim about Lagrangian descriptions of strings. Only in cases where massless vectorpotentials (which are necessarily string-localized if they act in a Hilbert space [33]) the string-localization of the vectorpotential is transferred to the matter fields while the field strength of the vectorpotentials maintain their point localization.

3 Obtaining generators of interacting wedge algebras by emulation

As explained in the previous section, in the presence of interactions is not possible to find localized operators which create localized particle states with reasonable behavior under translations.

Lemma 1 *Any state $|\psi\rangle \in \text{dom}S_{\mathcal{A}(W)} = \text{dom}S_{\mathcal{A}_{in}(W)} = \text{dom}\Delta^{\frac{1}{2}}$ can be generated from the vacuum by two uniquely determined operators in each of the two algebras*

$$\begin{aligned} |\psi\rangle &= (A)_{\mathcal{A}(W)} |0\rangle = A |0\rangle, \quad A \in \mathcal{A}_{in}(W); \quad (A)_{\mathcal{A}(W)} \in \mathcal{A}(W), \quad \mathcal{A}(W') = \mathcal{A}(W)' \quad (11) \\ [A, A'] |0\rangle &= 0, \quad A' \in \mathcal{A}'_{in}(W), \quad \left[(A)_{\mathcal{A}(W)}, (A')'_{\mathcal{A}(W)} \right] |0\rangle = 0, \quad (A')'_{\mathcal{A}(W)} \in \mathcal{A}'(W) \end{aligned}$$

Here the last line also defines the operators on a dense set (the Reeh-Schlieder property of the commutant algebras) and hence also the closures of the operators $A, (A)_{\mathcal{A}(W)}$ (for which we will maintain the same notation). Note that the A' is from $\mathcal{A}'(W)$ and not from $\mathcal{A}'_{in}(W)$ so that its uniquely related bijective image is not from $\mathcal{A}_{in}(W)$ (as one may naively have expected) but rather from $\mathcal{A}(W)$. The interacting operators $(A)_{\mathcal{A}(W)}$ bijectively related to $A \in \mathcal{A}_{in}(W)$ will be called emulations of free fields; for the special case of the $A = A_{in}(f), \text{supp}f \in W$ this has been previously [35] called PFG (vacuum-polarization-free generator). As will be seen in the sequel, this lemma has some powerful consequences.

This bijection between operators affiliated with different W -localized algebras sharing the same modular unitary Δ^{it} is not an algebraic operator equivalence. Rather it is based on the fact that the infinitely many possibilities of choosing operators from $B(H)$ for creating a prescribed vector state becomes unique if the operator belongs to a localized

algebra and the state belongs to the domain of the Tomita S associated with that algebra¹⁴. So strictly speaking what we call somewhat sloppily *emulation* of operators is a bijection of operators between different algebras which is defined through states from the dense $dom S$. Different from an isomorphism, it does not respect the star-operation since $A^*|0\rangle = S_{\mathcal{A}_{in}(W)}|\psi\rangle$ whereas $B^*|0\rangle = S_{\mathcal{A}(W)}|\psi\rangle$ is a different state, i.e. the star operation does not commute with the operation of emulation.

This has far-reaching consequences, the most important one being a cyclic KMS relation involving one operator $B \in \mathcal{A}(W)$ from the interacting wedge algebra and two operators $A^{(1,2)}$ which originate from $\mathcal{A}_{in}(W)$ via emulation

$$\langle BA^{(1)}A^{(2)} \rangle \stackrel{KMS}{=} \langle A^{(2)}\Delta BA^{(1)} \rangle, \quad \Delta^{it} = U(L(-2\pi t)) \quad (12)$$

$$A^{(1)} \equiv (: A_{in}(f_1)\dots A_{in}(f_k) :)_{\mathcal{A}(W)}, \quad A^{(2)} \equiv (: A_{in}(f_{k+1})\dots A_{in}(f_n) :)_{\mathcal{A}(W)}$$

In words, the restriction of the vacuum to the algebra $\mathcal{A}(W)$ generates a $T = 2\pi$ thermal state which is associated with the Lorentz-boost generator representing the Hamiltonian. Observe that this implies in particular that, different from QM, the pure vacuum state on $B(H)$ is (in the intrinsic characterization of the W-world) highly impure on $\mathcal{A}(W)$. Unruh's Gedankenexperiment shows that the thermal manifestation of localization in terms of hardware is a highly metaphorical nature (but nonetheless of extreme conceptual value). Only in cases in which horizons of causally closed regions are also black hole event horizons of a curved spacetime metric their reality content (observer-independence) increases. The above presentation of the KMS identity is a short hand notation of the existence of a function $F(t) = \langle BA^{(1)}\Delta^{it}A^{(2)} \rangle$ and $G(t) = \langle A^{(2)}\Delta^{-it}BA^{(1)} \rangle$ which are boundary values of an analytic master function which is analytic in the strip $0 < Imt < \pi$. The cyclic KMS identify is the statement that the analytic continuation of one boundary value is identical to the other.

In (12) one can now pass to particles (using the definition of emulation) whenever the emulate acts on the vacuum

$$\langle 0 | BA^{(1)} | \hat{f}_{k+1}, \dots, \hat{f}_n \rangle^{in} = \overset{out}{a.c.} \langle \overline{\hat{f}_{k+1}, \dots, \hat{f}_n} | B | \hat{f}_1, \dots, \hat{f}_k \rangle \quad (13)$$

$$\Delta(A_{in}^{(2)})_{\mathcal{B}(W)}^* | 0 \rangle = \Delta^{\frac{1}{2}} J_0 A_{out}^{(2)} | 0 \rangle \quad (14)$$

where the bra state on the left hand side denotes analytically continued wave functions of outgoing anti-particles come about from pushing $A^{(2)}\Delta$ onto the bra vacuum with the help of the modular relation in the second line. A shorter way of characterizing this state is to say that the outgoing configuration denotes anti-particle wave functions on the backward mass shell.

The crossing identity for the formfactor on the other hand reads as

$$\langle 0 | B | p_1, \dots, p_k, q_{k+1}, \dots, q_{k+l} \rangle = \langle -\bar{q}_1, \dots, -\bar{q}_l | B | p_1, \dots, p_k \rangle_{conn} \quad (15)$$

where the subscript *conn* denotes the absence of contraction terms between bra and ket states (there a non on the left hand side). Formally this relation results from ignoring the

¹⁴This property is closely related to the Reeh-Schlieder property which is sometimes imprecisely referred to as the "state-operator relation".

emulation subscript of $A^{(1)}$, pushing the analytic continuation in the wave function onto the momentum eigenstates and using the density of W-localized wave functions on both sides. One could now think that its derivation from (13) is a fast-selling item, but this would be wrong.

We will sketch some of the points needed to accomplish the derivation of (15) from (13), but we have to leave the details to a separate publication [34].

There are two kinds of theories arising from emulation: integrable and nonintegrable model. The dividing line is the temperateness/nontemperateness of the emulation. Staying with one-particle emulation $(A_{in}(f))_{\mathcal{A}(W)}$, (for reasons of notational simplicity) also called wedge-localized PFGs (vacuum-polarization-free-(one-particle)generators) in [35], the definition of temperate PFGs refers to the existence of a translation invariant domain on which the translated operator is a polynomial bounded operator-valued distribution, just as in the case of Wightman fields apart from the property of covariance. It has been shown in [35] that this requirement leaves no nontrivial S-matrix $S_{scat} \neq 1$ in $d > 1 + 2$ and only elastic S-matrices for $d = 1 + 1$. This by itself is not as surprising as it appears at first sight since the S-matrix is a relative modular invariant and the only dynamical object which characterizes the wedge-localization of $\mathcal{A}(W)$. Neither is it surprising that the generic QFT does not fit into such a restricted setting since the modular construction of emulates only refers to the wedge and its subinvariances. Purely elastic S-matrices are only possible in $d=1+1$ and are solutions of the formfactor-bootstrap program (the only known ones). This is closely related with the kinematical fact that the energy-momentum conserving δ function coalesces with the product of two mass-shell contraction functions and thus elastic interactions cannot be separated from $S_{scat} = 1$ by cluster factorization.

The class of models with temperate PFG's coincides with that of factorizing models and therefore temperateness and integrability are equivalent notions (integrability has the greater intuitive appeal since it has a well-defined meaning also outside QFT) and therefore integrability in QFT can not occur in higher dimensions¹⁵. It has been known for some time that integrable models have (nonlocal) wedge-localized PFGs whose creation/annihilation components fulfill Zamolodchikov-Faddeev commutation relations. Their use allows to derive the "axioms" of the formfactor-bootstrap project. In particular one obtains for the complete physical formfactor of the right hand side of (15) the following expression (written for the special case of $A^{(2)}$ being a one particle operator and using rapidity parametrization)

$$\begin{aligned} \langle \theta_1 | B | \theta_2, \theta_3, \theta_4 \rangle^{in} &= \langle 0 | B | \theta_1 + i\pi, \theta_2, \theta_3, \theta_4 \rangle^{in} + \langle \theta_1 | \theta_2 \rangle \langle 0 | B | \theta_3 \rangle^{in} + \\ &+ \langle \theta_1 | \theta_3 \rangle S_{scat}(\theta_1 - \theta_2) \langle 0 | B | \theta_2, \theta_4 \rangle^{in} + \\ &+ \langle \theta_1 | \theta_4 \rangle S_{scat}(\theta_1 - \theta_2) S(\theta_3 - \theta_1) \langle 0 | B | \theta_2, \theta_4 \rangle^{in} \end{aligned} \quad (16)$$

where the $i\pi$ is the short hand notation in the vacuum.polaritation formfactor Note that the δ -function contraction terms are not identical to those obtained from the application of the LSZ scattering formalism of the books. The reason is that this formalism only

¹⁵There is however the concept of "kinematical integrability" [34] which is not bound to $d=1+1$ and becomes particular interesting with respect to conformal QFT models and their anomalous dimensional spectrum.

applies to nonoverlapping wave functions; as soon as overlapping takes place threshold singularities. come into play which wreck the strong asymptotic approach of incoming states in the Haag Ruelle scattering theory [36][37] and (as it is became for the first time visible through [39]) modify the contact terms which "probe" the multiparticle threshold singularity¹⁶. The extension to more than 3-particle bra states should be obvious [39]. These deviations from the formal LSZ contraction terms have their counterpart in contractions which appear in the way emulated operators in act on incoming particle states. As mentioned the use of integrable wedge generators has led to the first existence proof for models with a realistic noncanonical short distance behavior.

This suggest a strategy for the general case where one also expects a δ -function contact structure. Looking at (16), the two factor product S-matrices can be interpreted as describing a scattering in which particle 1 scatters on two other inert (i.e. no relative interaction without the first particle hitting them) particles. In case of n-particles the highest S-matrix product would describe particle 1 hitting a "swarm" of n-1 inert particles. This can also be written as

$$S(p + k\text{-inert} \rightarrow p + \text{anything}) \equiv \sum_n S_{full}^{-1}(n\text{-particle} \rightarrow \text{anything}) S_{full}(p_1 + k\text{-inert} \rightarrow p_1 + n\text{-particle}) \quad (17)$$

where the particle momentum conservation + Yang-Baxter algebra rules fix the anything in such a way that for n=3 the previous product formula as in (16) reappears. In this form the idea of the momentum preserving "grazing shot" with p onto a inert swarm and activating it without momentum change has a presentation in terms of full scattering matrices.

Such a guess taken serious for the general non-integrable case would allow to write the expressions which multiply the contractions in terms of infinite sums involving S-matrix elements. The full crossing including the contact terms would then realize an shell version of Murphy's law in particle theory¹⁷ i.e. a particular formfactor would communicate with all other formfactors.

But how is one able to proof such a conjecture. In principle its proof is simple; one "only" has to verify that the PFG behind this KMS properties is "wedge local" i.e.

$$\left\langle \psi \left| \left[J \left(A_{in}^{(1)} \right)_{\mathcal{A}(W)} J, \left(A_{in}^{(2)} \right)_{\mathcal{A}(W)} \right] \right| \varphi \right\rangle = 0 \quad (18)$$

$$J = S_{scat} J_{in}, \quad A_{in}^{(1)}, A_{in}^{(2)} \in \mathcal{A}_{in}(W)$$

on the dense set of states mentioned before. But this is easier said than done.

In an operator formulation of crossing in the nonintegrable case it is not possible encode the operator structure into the permutation group. The analytic prerequisite for doing

¹⁶Note that the 2-particle singularity is absent in the rapidity parametrization. This parametrization is therefore a uniformization parameter for integrable models leading to meromorphic S-matrices and correlation functions.

¹⁷"Anything which can couple (according to the rules of superselected charges) actually does couple". QM is (even in its relativistic form [38]) is par excellence the theory which remains outside the range of Murphy's law.

this was the use of θ as a uniformization variable, which breaks down in the presence of inelastic thresholds. Here it is helpful to look at a similar problem in Wightman's theory when in $d=1+2$ the permutation group statistics has to be replaced by the more general braid group statistics. In that case there are cuts in the analytic Bargman-Wightman-Hall domain and the possible ways of reaching the boundary (i.e. the ordering in which the imaginary parts pass to zero) are not sufficient to specify the operator content on the physical boundary; one also must specify the order of paths (crossing cuts). This leaves an infinite number of possibilities instead of the $n!$ permutation group orderings. In fact in the plektonic case these possibilities are parametrized by *words in the braid group*. We conjecture that a similar phenomenon may occur in the setting of emulation [34]; this could significantly simplify the emulation formalism for nonintegrable theories.

In the distant future one could expect that this S-matrix-based setting may lead to an existence proof for an associated local net and to controllable approximation techniques for quantities of physical interests. This then would set the same kind of conceptual closure on QFT and make it akin to any other area of theoretical physics.

4 Comments on the ongoing dispute between followers and opponents of string theory

The S-matrix bootstrap program as well as Mandelstam's subsequently proposed double spectral representation for scattering amplitudes with crossing properties were the first proposals beyond the limitations of the perturbative exploration of Lagrangian quantization. The idea to use the S-matrix and more general on-shell objects already in the computational setup instead of deferring them to the roof of the particle theory project to be obtained at the end of the day were a courageous if not even revolutionary steps notwithstanding their demise. They failed for completely different reasons; in the case of the bootstrap the stew made from unitarity, Poincaré invariance and an incompletely understood crossing amounted to an explosive nonlinear mixture, even less accessible than the nonlinear Schwinger-Dyson equations. Apart from the two-dimensional bootstrap which in the case of a pure elastic scattering function (or rather a Yang-Baxter matrix-valued scattering function) finally led to the bootstrap-formfactor project and its algebraic formulation in terms of wedge algebras presented in the previous sections, there is not a single useful physical concept which arose from it.

Mandelstam's project took a wrong turn when he erroneously accepted the dual model proposed by Veneziano as an implementation as that of particle physics. It dissociated itself increasingly by following a metaphorical idea of string-localization which has nothing to do with that of local quantum physics.

In the present essay we combined the original content of Mandelstam's idea with Haag's project of local quantum physics and constructed in this way a new platform for a different view of QFT in which the S-matrix (in the form of a relative modular invariant for wedge localization) plays a constructive role from the start.

Although the new setting provides an optimistic look into a better future, there is good reason to be pessimistic when it comes to its implementation. The number of physicists familiar with foundational aspects of local quantum physics has not increased since the

time of Mandelstam and the absence of any innovative investment and foundational knowledge within the new globalized communities (which developed into impounding basins of metaphorical thinking) gives no reason for optimism unless a new generation invests into a new start learning lessons from present derailments.

To become aware of the depth and extension of the problem one has to only look at a recent contribution by Duff to a project “Forty Years Of String Theory: Reflecting on the Foundations”. Without wanting to defend Nancy Cartwright’s somewhat extreme points of view on the connection between an emerging unification¹⁸ and the strife for a “Weltformel” i.e. a TOE (a unique theory of everything), it is easy to agree that at least its inverse, namely to conclude from the existence of a unique realization of some idea that it must have foundational physical significance, is not acceptable. But this is exactly what string theorists deduce from the (nearly) unique possibility to represent a unitary positive energy representation on what they call the “target space” of a nonrational chiral sigma model related to a 10 component current with bosonic/spinorial components.

Instead of trying to understand why nonrational chiral theories with their continuously many superselected charges (different from higher dimensional observable algebras which, at least in theories with compactly localizable superselected charges, only permit extensions to charged algebras with compact internal group symmetries), string theorists insist to identify our living spacetime with the noncompact internal symmetry space of a nonrational sigma model by calling it “target space” (field-value space in a classical analogy). Their continuous superselected charge structure behave in such a radically different manner and allow (the representation) Whereas it is certainly true that one even can represent such noncompact groups as the Poincaré group on the target space and one can even show the restriction to positive energy unitary representation leaves only the so-called 10 dimensional superstring representation, to see in this the offer of a 10-dimensional version of actual spacetime is metaphysics for any theoretical physicist who is old enough to have seen better times.

One has all reasons for being somewhat surprised about the near uniqueness of the 10 dimensional superstring representation of the Poincare group on the target space of a nonrational sigma model but why interpret the rarity of such an occurrence and the fact that nonrational chiral theories have not been the subject of systematic studies as the harbinger of a new foundational insight into spacetime.

Admittedly it is a bit surprising that this is possible at all (in higher dimensional theories there are simply no noncompact target spaces on the quantum level), but should this “almost uniqueness” (up to a finite number of M-theoretic variations) be the starting point of foundational ideas about our living spacetime? In view of such a grand design it matters little if one discretely points out that as a result of the special nature of the highly reducible superstring representation of the Poincaré group in which besides massive components only massless finite helicity massless components appear (but not the string-localized [30] “infinite spin” components), the canonically associated free field is infinite-component and pointlike.

Why instead not understand this phenomenon of a 10 component almost unique (up to a finite M-theoretic variation) realization as a property of the family of hardly known

¹⁸I think what she is probably criticising is an enforced unification which does not emerge from the natural flow of improved insights into nature.

nonrational chiral models which have a superselection structure different from rational chiral theories and all higher dimensional QFT? To lift M-theory to the level of representing a key to the understanding of the universe is not much better than the ontological role attributed to the number 42 as an answer to the ultimate question about Life, the Universe, and Everything in D. Adams scientific comedy "the hitchhiker's guide through the galaxy". The mundane alternative to M-theory consist clearly in a better understanding of irrational chiral field theories and their ability to realize noncompact symmetry structures in their "field values" (using the classical terminology of "field value spaces" whose quantum counterpart is often misleadingly referred as "target spaces", as if the chiral sigma model, or any QFT for that matter, could define a source-target embedding).

A closely related remark mentioned in section 2 is that, whereas the principle of classical theory admit dynamical covariant variables $X_\mu(\tau)$ (even in classical mechanics) which parametrize world-lines in any dimension, there are *no covariant quantum position operators which correspond to these classical objects*. Already Wigner knew this fact and not least for this reason preferred a representation theoretical approach to one through quantization (in this case of position operators). This means in particular that "derivations" of gravity from beta function inspired manipulations on actions which contain such variables (e.g. the Polyakov action) are not trustworthy. What was meant as a support of string theory in form of an alleged quantum version of a classical world line [25] reveals itself as part of a misunderstanding of covariant localization in QFT, actually it is also measure of the gigantic misunderstandings of QFT brought about by string theory. It is a deeply degrading experience to the author and most of his critical colleagues in particle theory that in several journals they have to submit their articles under the rubric "Field theory and String theory". This will remain as a mark long after string theory has gone (similar to the 19th century phlogiston). But if it reminds the reader of a future (probably electronic) library that the conquest of the holistic localization concepts required a long time with deep failures on the way, the conceptual struggle would not have been in vain. In fact as often in life, the deepest insights result from the correction of foundational errors (e.g. the dismissal of the ether).

The sociological criticism contained in some well-known books and articles is of no help. Even if at the beginning it was well intentioned, *the distractors of string theory became as problematic as its proponents because they slowly started to live in symbiosis with their opponents as part of a gigantic entertainment industry*¹⁹. Since there is nothing for them to fall back onto, their fate is meanwhile closely tight to that of string theory. For many physicists string theory appeared is the most bizarre subject they ever met. But its bizarre nature is not the result of a computational mistake nor of an easy to discover conceptual misunderstanding. The sophisticated nature of the conceptual mistake which lies on its bottom has to do with an incomplete understanding of QFT which started right at the cradle in 1925/26. It was not fully noticed in renormalized QED because the perturbative formalism automatizes the implementation of the underlying causal localization principle to such a degree that one is not forced to think much about it.

To feel the depth of the crisis into which large parts of particle theory have fallen in

¹⁹The interspaced scientific arguments in my own more sociologically based articles (which already preempted some of the points [6][7]) had the effect that they remained virtually unknown which may save me from such an accusation.

the last 40 years, it is helpful to read a quotation from Einstein's talk in the honor of Planck [41].

In the temple of science are many mansions, and various indeed are they who dwell therein and the motives that have led them thither. Many take to science out of a joyful sense of superior intellectual power; science is their own special sport to which they look for vivid experience and the satisfaction of ambition; many others are found in the temple who have offered the product of their brains on this altar for purely utilitarian purposes. Were an angel of the Lord to come and drive all these people belonging to these two categories out of the temple, the assemblage would be seriously depleted, but there would still be some men, of present and past times, left inside. Our Planck is one of them, and that is why we love him. ...

But where has Einstein's angel of the Lord who protects the temple of science gone in the times of string theory? Reading these lines and comparing them with the content of [40] as well as that of his opponents one cannot help to sense how similar the present Zeitgeist of particle theory has become with that of the financial markets. What will future historians of physics make of such bizarre ideas as "the landscape" or Tegmark's belief that every mathematically correct idea in physics will have a realization in one of the zillions of parallel universes?

Of course the present symbiosis between pro- and sociological opponents will not go on forever since the conservation law: *who moved up with string theory has to come down with it* (unless one can return to something on which one created a reputation before) is still valid. For some people this may never happen because they may have passed away before the great oath of disclosure begins.

It is certainly not true, as Duff insinuates in his article, that all these great names mentioned in his article have endorsed and embraced string theory. He should not have left out the name of Feynman who had, probably driven by the bizarre appearance of string theory pointed out that ST is the first construct in particle theory which is not defended by arguments but by taken recourse to excuses. If he had not have been terminally ill by that time, he could have fought against the emerging calamity in the midst of particle theory by finding some of the scientific Achilles heels of ST beyond its bizarre sociological epiphenomena. and the head of many people would have kept free for conceptual innovations.

Also Duff should not be so sure about counting on Weinberg who on several occasions speculated that string theory may be a camouflaged QFT. In fact this is indeed what the present results show: string theory is a dynamical infinite component QFT where as aforementioned dynamical means that it contains operators which relate the different levels in the infinite mass-spin tower "hovering over one localization point". Concerning most of the other names in Duff's list one should perhaps point out that most scientist of intellectual status have a natural curiosity which leads them to have an unprejudiced look at any not totally reasonable new idea, and nowhere in this essay it was claimed that string theory suffers from a simple-to-recognize mistake. Applying the appropriate unsightliness, this only leaves to hard core string theorists: Gross and Witten. In case of Gross, Duff's connection of string theory being supported by Nobel prize bearers convinces; indeed it would be hard to believe that this theory could exist for more than 40 years without his support. And certainly its present existence is hardly understandable without Witten's

charismatic updates²⁰. It would have been also interesting to the reader to also know what Nobel prize bearers as 't Hooft and Veltman thought about string theory. It would be hard to think that they have remained completely afoot on this subject during the 40 years.

Mentioning Witten brings up another interesting point raised by Duff: the relation of string theory with mathematics. String theoretic pictures have indeed been helpful to generate provable mathematical conjectures but does this mean (as Duff suggests) that it is really such a positive development when a strong autonomous science as particle physics is forced to be happy in a role of a subcontractor of mathematics?²¹ Mathematicians need not to care whether they get their inspirations from (what they conceive as) flourishing landscapes of particle physics or from its ruins. As long as it represents a fertile soil for their imagination and makes them more free to work a bit outside the conjecture-theorem-proof pattern they can profit from this source. With a field medalist among the protagonists of what they conceive as a new golden era in physics they feel less restricted to their traditional way.

In many instances they are even eager to help physicists. One of the populated meeting grounds since the early 70s was and still is geometry. But I hope that the presentation of modular localization in this essay made clear that this means something very different for mathematicians as it does (or rather should do) for particle theorists. The meaning is of course identical in classical field theory but happens to drift apart in its quantized version. The reason is the geometry in QFT rarely appears *without* being burdened with vacuum polarization and thermal KMS properties. A Lagrangian as that topological euclidian construct of what is behind the Wess-Zumino-Witten-Novikov model has nothing to do with the original Wess-Zumino Lagrangian which at least formally complies with Lagrangian quantization. It does not have a localizable QFT behind it and it only serves baptizing a model in the apparent traditional way. Any non-metaphoric computational setting uses representation theoretical methods for the construction of a sigma-model field associated with a current algebra. But of cause this has nothing to do with using formal aspects of field theory for new mathematical insights. Most of the so-called gravitational anomalies in euclidean gravity stand accused of being misleading or irrelevant for the physical properties of gravity [43].

In this context it may be interesting to remind the reader of episodes of a joint physics/mathematics heritage. Concerning quantum physics one may think that the time of the discovery of QM, with such towering figures as John von Neumann and Hermann Weyl, is the best illustration. But this would not be completely correct since Hilbert spaces and the beginnings of spectral theory already existed before QM²². The perfect historical episode where quantum physics and mathematics were totally on par was the completely independent parallel development of what physicists called "statistical mechanics of open systems" and mathematicians "the Tomita-Takesaki modular theory of operator algebras". This meeting of minds had its extension in the Doplicher-Haag-

²⁰A very interesting illustration of "charisma meets uncoolness" can be found on page 299 of [42].

²¹The articles on the Langlands program in hep-th (instead of math-ph) illustrate this problem.

²²Actually Fritz London (during his assitentship at the technical Institute in Stuttgart) was the first who formulated QM in terms of Hilbert spaces (and "rotations" unitary operators therein) in the context of "transformation theory" (see [12])

Roberts superselection analysis where the Markov traces of the Vaughn Jones theory of subfactors and the use of endomorphisms were already preempted. After the Bisognano-Wichmann discovery of the geometric setting of wedge-localization it, begun to gain speed and more recently it arrived at deep connections with a new constructive approach (which in the present work we tried to relate to Mandelstam's pre string S-matrix project). Most of the insights explained in this essay have their beginnings in that fortunate "modular episode".

There is one issue in which Smolin [44] (together with Arnsdorf), standing on the shoulders of Rehren²³ is entirely correct; one of the few instances which have nothing to do with sociology or philosophy of physics. They point towards a kind of conundrum between the string-induced Maldacena view and the Rehren theorem. In a previous paper by Rehren [46] and a subsequent more detailed presentation [47] it was pointed out that Rehren's correspondence, though mathematical correct, has a serious physical shortcoming. One side of this mathematically well-defined correspondence is always unphysical; if one starts from a physical model on the AdS side, the CFT side will have way too many degrees of freedom. This means that although the correspondence respects local commutativity (Einstein causality) it violates what one calls classically the causal propagation (Haag duality) i.e. the algebra of the causal completion of a region $\mathcal{O} \rightarrow \mathcal{O}''$ is larger than that associated with $\mathcal{O} : \mathcal{A}(\mathcal{O}) \subsetneq \mathcal{A}(\mathcal{O}'')$. Thinking of a spatial sphere and its spacetime completion, this means that there are more degrees of freedom at later times than there were in a thin time slice attached to the time-slice extended sphere. For the occupants of this double cone region this is like a "poltergeist" effect; degrees of freedom coming in from nowhere. QFT models accessed by Lagrangian quantization do not have this property and the *time slice postulate of QFT*²⁴ [48] was precisely introduced in order to save from Lagrangian field theory what can be saved in a world outside quantization (as it is needed in the AdS-CFT problem). The latter property is intimately related to the *phase space degrees of freedom* issue which led Haag and Swieca to their result that, different from QM (with or without second quantization) which leads to a finite number of states per cell in phase space, QFT as we know it from quantization requires a compact set (later refined to "nuclear" [49]). It seems that this kind of insight (besides many other deep insights prior to the 80s) was lost in the maelstrom of time. This "mildly" infinite cardinality secures the existence of temperature states for arbitrary temperatures as well as causal propagation. A world outside this requirement can only be realized in one of Tegmark parallel universes.

Ignoring this requirement for a moment before later returning to it, one can ask the question whether one can modify the AdS-CFT setting a bit so that it becomes a bit more sympathetic towards an appropriate reformulated Maldacena's conjecture which in harmony with Rehren's theorem. This is precisely the question Kay and Ortiz asked [50]. Taking their cue from prior work on the correspondence principle of Mukohyama-Israel as

²³It is truly admirable how, in the face of concentrated misunderstandings, Rehren succeeds to maintain his countenance [45].

²⁴In a modern setting the principle of causal localization comprises two requirements on observables: Einstein causality (spacelike commutativity) and Haag duality $\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$ (timelike causal propagation).

well from 't Hooft's brick-wall idea²⁵ [51], these authors start with a Hartle-Hawking-Israel like pure state on an imagined combined matter + (quantum?) gravity (asymptotic) system. They then propose to equate the AdS side of a Rehren hypothetical conformal invariant supersymmetric Yang-Mills model with the restriction of the H-H-I state to a matter subsystem theorem of the combined causally closed subsystem. Their results, although as stated by the authors not rigorous, are sufficiently encouraging in order to be taken serious by the Maldacena community who have confronted us with a tsunami of publications without any tangible result.

But how can one overcome the physical flaw of Rehren's theorem which in the \rightarrow direction leads an overpopulation of degrees of freedom and in \leftarrow to an "anemia". One may hope for a "degree of freedom repair" by some extension of the Kay-Ortiz scheme, but this does not seem plausible. Nevertheless the K-O project is a quite interesting idea which opens a new direction to look into independent of the veracity of the Maldacena conjecture.

Another property which string theorists and therefore also Duff uses to support string theory is supersymmetry. This is a spacetime symmetry which goes beyond the standard direct product structure of *spacetime* \otimes *inner* symmetries. This symmetry appears nowhere in the systematic DHR superselection analysis and therefore one's first thought may be that, although such a symmetry can be realized in the setting of free fields, they it is unstable under interactions. However these fields maintain their supersymmetry in the perturbative approach to appropriately chosen interactions. But do they admit the concept of spontaneous symmetry breaking? A particular illustration of such a symmetry breaking is the Lorentz-invariance in the (thermodynamic limit) of thermal states. Supersymmetric systems in thermal states do however not lead to a spontaneous symmetry breaking but to a "collapse" of that symmetry [54] (the Poincaré subgroup is only spontaneously broken but does not collapse. The attractive mathematical feature of (in particular conformal) supersymmetry however prevent a premature physically motivated dismissal.

In illustrating the "unreasonable power of less than perfect discoveries", Duff points convincingly at the story of antiparticles emerging from Dirac's whole theory. Indeed some discoveries, especially in the beginning of QFT, did not follow straight logical lines. It was not (as one would have expected) Pascual Jordan, the discoverer of QFT and the positivistic advocate of quantizing everything (Maxwell fields, matter fields) which permits to be quantized, who first saw the charge symmetry leading to antiparticles as an intrinsic property of the underlying causal localization aspect of QFT, rather this was shown later first by G. Lüders and in a more general context (on solely structural grounds) by R. Jost.

But it was Dirac whose particle hole theory gave first rise to the idea of antiparticles. His philosophical setting for QT was quite different from Jordan's positivism, since, fol-

²⁵This idea seems to be very similar to the use of thermal aspects originating from the restriction of a pure global state (e.g. the vacuum or a Hartle-Hawking-Israel state) to a causally closed subsystem within the setting of modular localization. In case it is it would mean that the underlying problem already was there at the very birth of QFT in form of the Einstein-Jordan conundrum [52]. Further research on this interesting matter is warranted-

lowing the classical guide, he used wave quantization only for classical Maxwell waves and described massive matter in terms of Heisenberg's QM. It was somewhat artistic to see antiparticles in the context of hole theory; in fact this setting was later abandoned after it became clear that it becomes inconsistent as soon as vacuum polarization comes into play²⁶. Dirac only came around to embrace universal field quantization in the early 50s but he did discover antiparticles long before.

As far as stressing old-fashioned virtues in particle theory, there is no problem to agree with Duff. This also includes his refutation of a time-limit on string theory research as expressed in the papers of Woit and Smolin. If string theory really would be what it claims to be, namely a theory which goes beyond QFT it has the right to take as much time as it needs to settle this problem; but the point is that it isn't.

Duff forgot to mention what he considers to be the string theoretic analog of Dirac's discovery. Also his mentioning of the Higgs mechanism and gauge theory in connection with string theory warrants some correcting remarks. The present day view of massive vectormesons by a Higgs symmetry breaking and the Higgs particle playing the role of "God's particle" (giving masses to the other particles) is what the maelstrom of time left over from a much richer picture which in the 70ies was referred to as the Schwinger-Higgs screening mechanism [53]. The Higgs model is nothing else than the charge-screened mode of scalar electrodynamics. Whereas the quantum mechanical Debye screening only generates a long-range effective interaction, the QFT screening is more drastic in that it affects also the particle spectrum. Charge screening means that the integral over the charge density vanishes, whereas a (spontaneous) symmetry breaking (as a result of the Goldstone mechanism) brings about a divergence of this integral (as a result of its bad infrared behavior caused by the coupling of the conserved current to the massless Goldstone boson). The observable result of the computation based on the "Higgs symmetry breaking" is the same but this should not justify its conceptual means.

With the idea of string localization, also the question of alternatives to the Schwinger-Higgs screening are re-opened. Spinor-QED has a massive counterpart [55] without introducing S-H screening via an additional Higgs degree of freedom, the so-called massive QED which in the pointlike formalism needs an intermediate BRST ghost formalism in order to lower the scaling dimension of the effective vectorpotential from 2 to 1. It may be interesting to try to substitute the BRST formalism by using string-localized free vector fields which also have short distance dimension $d = 1$. This should be no problem in massive QED which hitherto was only formulated in the indefinite metric setting [55]. For Yang-Mills theory there is a "perturbative theorem" that the consistent use of the BRST formalism requires the presence of additional physical degrees of freedom (the scalar Higgs). This is the reason which generated that desperation about finding a Higgs particle at LHC.

String theory worsens the situation by stating that a Higgs would not be enough, one also should see supersymmetry. But what if the no-Higgs situation in the abelian case has a counterpart in YM in the sense that a treatment which start with string-localized massive gluons (which have $\text{dim}=1$ and hence are formally renormalizable) and maintains pointlike physical subobservables? In that case the main purpose of string

²⁶It was still used in the first textbooks by Wenzel and Heitler but did not survive renormalized perturbation theory where vacuum polarization became important.

localization would have been to lower the field dimension so that one can slip in below the power-counting limit. It would serve as a kind of catalyzer for being able to stay below the powercounting limit (using the dimension-reducing aspects of string-localization) but have no bearing on localization which remains pointlike. Whereas the indefinite metric (Gupta-Bleuler, BRST) formalism seems to be limited to vectorpotentials, the idea of string-localization could have a much larger range.

String theory, far from having contributed anything to LHC relevant problems has and hence different from what Duff claims, has to the contrary contributed to the stagnation of vital parts of particle physics²⁷. What does Duff (or anybody else) expect from a theory which is already misleading in the terminology of its name?

It will be a long lasting task for the coming new generations to remove all the metaphoric rubble it leaves behind in order to be fit to extract anything useful from this last generation of experiments at LHC. But the immense progress one could expect from getting back on track by correcting these (by no means trivial) conceptual errors could be much more than a consolation for the lost decades.

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²⁷The conceptual confusing situation caused by string theory hinders young people with brilliant computational abilities (Zvi Bern and several others) to reach their true innovative potential.

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