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Causality and dispersion relations and the role of the S-matrix in the ongoing research

To the memory of Jaime Tiomno (1920-2011)

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Abstract

The adaptation of the Kramers-Kronig dispersion relations to the causal localization structure of QFT led to an important project in particle physics, the only one with a successful closure. The same cannot be said about the subsequent attempts to formulate particle physics as a pure S-matrix project.

The feasibility of a pure S-matrix approach are critically analyzed and their serious shortcomings are highlighted. Whereas the conceptual/mathematical demands of renormalized perturbation theory are modest and misunderstandings could easily be corrected, the correct understanding about the origin of the crossing property demands the use of the mathematical theory of modular localization and its relation to the thermal KMS condition. These concepts which combine localization, vacuum polarization and thermal properties under the roof of modular theory will be explained and their use in a new constructive (nonperturbative) approach to QFT will be indicated. The S-matrix still plays a predominant role, but different from Heisenberg's and Mandelstam's proposals the new project is not a pure S-matrix approach.

1 Introduction to the various causality concepts along historical lines

Analytic properties of scattering amplitudes which arise as consequences of causal propagation properties in the setting of classical optics in dielectric media appeared first under the name *dispersion relations* in the late 20s in the work of Kramers and Kronig[1]. The mathematical basis on which this connection was derived amounted basically to an application of Titchmarsh's theorem: a function (more generally a distribution) f(t) which is supported on a halfline, is the Fourier transform of a function $a(\omega)$ which is the boundary

[1]Here and in the following we refer references to the bibliography in [1] wherever it is possible. This monography is a competent and scholarly written account of the subject, though it does not contain the QFT theory derivation which is based on the Jost-Lehmann-Dyson representation, the latter can be found in [2].

value of a function whose analyticity domain is the upper half-plane. With appropriate restrictions on the increase at infinity, this analytic behavior can be recast into the form of a dispersion, the best form for experimental checks of causality, which in the simplest case is of the form

$$a(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Im\ a(\omega')d\omega'}{\omega - \omega'}, \quad Im\omega > 0$$

Only after world war II this idea of relating spacetime causality with analyticity in the form of dispersion relations found its way into quantum theory (QT). Schützer and Tiomno [3] were among the first who worked out conditions under which a dispersion relation can be derived for scattering amplitudes of elementary processes in the setting of quantum mechanic (QM); later there appeared other contributions with a different adaptation of the notion of causality and slightly different restrictions on the two-particle interaction potentials. These considerations can in principle be extended to a more recent relativistic generalization of QM called "*Direct Particle Interaction*" (DPI) [4], a theory which is solely build on particles without the use of fields or algebras of local observables. In such a setting in which the Poincaré group is unitarily represented and the S-matrix comes out to be Poincaré-invariant there is no implementation of micro-causality, similar to the work of Schützer and Tiomno one can only implement macro-causality which includes the spacelike cluster-factorization and Stueckelberg's *causal rescattering* requirements [5][6]. The additional difficulty in the DPI case is that the naive addition of pair potentials would be in contradiction with the multiparticle representation of the Poincaré group representation and those macro-causality requirements.

The main problem in passing from classical optics to QM is that the latter has no finite limiting velocity and admits no wave fronts since their formation is prevented by the positivity of energy. Wave packets dissipate instantaneously in such a situation; a finite velocity, as e.g. the speed of sound, in a quantum mechanical medium (e.g. idealized as a lattice of oscillators) arises only as an "effective" velocity of a disturbance i.e. as an asymptotically defined (large time) mean value in wave packet states. In that case localization is defined in terms of the spectral theory of the *position operator* $\mathbf{x}_{op}$ and described in terms of a wave functions $\psi(\mathbf{x})$ which is a square integrable function on its spectrum. There is no way to talk about localized observables; the obvious attempt to go to the second quantized Schrödinger formalism, and define $\psi_{op}(\mathbf{x})$ and its local functions as pointlike local generating fields, or to introduce region-affiliated observables by smearing, does not work because these objects loose this property immediately since, unlike for wave function propagation, there is no stable meaning of "effective" on the level of localized observables.

In QFT the energy positivity and the resulting instantaneous dissipation of wave packets of course persists; the only change (due to Poincaré covariance) is that the effective velocity is limited by the speed of light. On the algebraic side one finds a very precise definition of causal locality in terms of spacetime-indexed operator algebras $\mathcal{A}(\mathcal{O})$; it consists of two parts

$$[A,B] = 0,\ A \in \mathcal{A}(\mathcal{O}),\ B \in \mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})',\ \textit{Einstein causality} \tag{1}$$
$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''),\ \textit{causal shadow property},\ \mathcal{O}''\ \textit{causal completion of}\ \mathcal{O}$$

Here the first requirement is the algebraic formulation of the statistical independence of spacelike separated observables; the upper dash on the spacetime region denotes the spacelike disjoint region, whereas on the algebra it stands for the commutant algebra. The second line is the local version of the "time-slice" property [7] where the double causal disjoint $\mathcal{O}''$ is the causal completion (causal shadow) of $\mathcal{O}$ i.e. the area of total dependence on $\mathcal{O}$ i.e. a kind of quantum counterpart of classical hyperbolic propagation. The two causality requirements are not independent. If the Einstein causality can be strengthened to Haag duality[2]

$$\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})', \ \textit{Haag duality}$$

There are other physically less agreeable reasons why QFTs could violate the causal shadow property[3]. For example there may be many more degrees of freedom in the causal shadow than in $\mathcal{O}$. In this case an observer would have the impression that the additional degrees of freedom in the causal completion $\mathcal{O}''$ to those which he observed in $\mathcal{O}$ must have entered in some mysterious "sideway" manner instead of being determined by the initial data. This phenomenon occurs in holographic projections or dimensional restrictions of higher dimensional QFT to lower ones, the only exception are holographic projections onto null-surfaces.

The causal shadow property does not lead directly to restriction on single operators, but is expected to play a prominent role in securing a complete particle interpretation of a QFT. We will return to this problem later on. In its global form, namely for $\mathcal{O} = \mathbb{R}^3$ at fixed time or a time-slice $0 < t < \varepsilon$, it has been termed *primitive causality* or time slice property[4]. Whereas in its global form it exists also in QM, its local (causal shadow) counterpart is specific for QFT.

It is of paramount importance for the physical interpretation of QFT that the two notions of localization, the particle-based *Born-Newton-Wigner localization* of wave functions and the algebraic notion of *causal localization* of local observables, which for finite times behave in an antagonistic way, come harmoniously together in the timelike asymptotic region of scattering; this implies in particular that the *frame-dependent BNW- localization* becomes *asymptotically frame-independent*. Without this peaceful asymptotic coexistence there would be no scattering probabilities and hence no particle physics. Instead of lamenting about the lack of a particle concept for finite times in QFT, it is better to emphasize the asymptotic particle/field harmony which is after all everything one needs (the half glass full against the half glass empty view).

It is interesting to note that the Born probability notion entered the already existing formalism of QT in 1926 in the very special setting of the Born approximation for the scattering amplitude where it is most needed in order to get to cross sections; its use for

[2]The exceptional cases where Haag duality for local observables breaks down are of special interest. For the local algebras generated by the free Maxwell field this happens for $\mathcal{O}$ =toroidal spacetime region (the full QFT version of the semiclassical Aharonov Bohm effect [25]).

[3]The holographic projection onto a timelike lower dimensional spacetime (AdS-CFT correspondence) and the restriction to such a spacetime provide illustration of this violation [8], whereas the holographic projections onto null surfaces avoids this pathological behavior (loss of information by projection onto horizon).

[4]The meaning of primitive causality in [1] is slightly different.

Schrödinger wave functions appeared a short time later in Pauli's articles. Newton and Wigner adapted this localization to the slightly different normalization of relativistic wave function, being aware that strictly speaking there is no frame dependent observable called "position operator" among the causally localized observables of QFT[5]. This implies that there is also no conceptual basis for the derivation of Heisenberg's uncertainty relation (for a QFT substitute see last section).

In addition to the above spacelike commutativity and causal completion property, the spacetime-indexed local algebras fulfill a set of obvious consistency properties which result from the action of Poincaré transformations on the localization regions. This is automatically fulfilled if these spacetime indexed operator algebras are generated by finite-component Poincaré covariant fields $\Psi^{(A,\dot{B})}$ fields where our notation refers to the well-known dotted/undotted spinorial formalism.

As a result of the "ultraviolet crisis"[6] of QFT, which started in the 30s and lasted up to the beginnings of renormalization theory at the end of the 40s, local QFT became discredited and the introduction of an elementary length into QFT, as well its total abandonment in favor of a ultraviolet-finite unitary S-matrix, were seriously contemplated. Although QFT enjoyed a strong return, the longings for a S-matrix theory or a nonlocal (often by inferring nonlocality by non-commutativity) never disappeared even though all these attempts during more than half a century remained unsuccessful

With such doubts about the validity of the principle of causal locality, and in spite of the observational success of perturbative QED in which these principle is realized, there was a strong desire to directly check the validity of the forward dispersion relation, which was done in 1967 [10] for $\pi - N$ scattering at up to that time highest available highest energies. This was the first (and the last) time a model-independent fundamental principle of a relativistic QT was subjected to a direct experimental test in the strong interaction region where perturbation theory[7] was not applicable. The importance of this successful test results from the rigorous and profound mathematical-conceptual work which connected the causality principle to the dispersion relation; for an unproven conjecture such a concerted effort could hardly have been justified. Therefore the precise and detailed mathematical work was a valuable investment in particle physics. The link between the principle and its analytic consequences was a spectral representation for the particle matrix element of two fields, the so called Jost-Lehmann-Dyson representation [2] which generalizes the Källén-Lehmann representation for the two-point function to particle matrix elements of commutators of two fields. As a side remark, the JLD spectral representation has been almost exclusively used for the derivation of analytic S-matrix properties; I only know of one quite different use: the Ezawa-Swieca proof [12] of the Goldstone's conjecture based on the generalization of a property of a concrete Lagrangian model stating that a conserved current yields a divergent global charge (spontaneous symmetry breaking) only if

[5]Although Wigner's positive energy representations laid the particle foundations of QFT, he deprived himself the entrance into QFT since he saw no frame independent concept of localization. The modern concept of "modular localization" was not available at that time and Haag's attempt to convince Wigner remained apparently without success [9].

[6]It was really a crisis in the thinking about QFT and not of QFT itself.

[7]A perturbative account of dispersion relations and momentum transfer analyticity was presented in [11]. At that time the divergence of the perturbative serious was only a suspicion but meanwhile it is a fact.

a zero mass particle (the Goldstone boson) prevents the large distance convergence [8].

The formulation of the dispersion relations and their experimental verification was a remarkable achievement in several respects. Besides the aforementioned aspect of a confrontation of a principle directly with an experiment without the intercession of a model, it is the only "mission accomplished" achievement in high energy physics. This is because after the problem had been formulated within the setting of QFT and worked out theoretically, it underwent a successful observational test; the physicists who participated in this unique endeavour could afterwards turn their interest to other problems with the assurance and feeling of satisfaction of having contributed to the closure of one important problem.

It is important not to keep the dispersion relation project separate from later attempts to base particle theory solely on the construction of an S-matrix. The first such attempt was by Mandelstam in 1957 [27]. I remember from my attending seminars at the University of Hamburg that Lehmann Jost and Källén were extremely unhappy about accepting unproven representations for problems of observational relevance. They must have had a premonition that such methods, once they become acceptable, may lead to a metaphorical derailment of particle theory.

Naturally a successful concluded project as the unravelling of the connection between causality and dispersion relation invites to look out for extensions into other directions. There were at least two good reasons for this. One was that the successful perturbative approach in QED could not be expected to work in strong interactions (at that time $\pi - N$ interactions). The other is more profound on the theoretical side and relates to the growing suspicion that the renormalized perturbative series may always diverge in QFT; a suspicion which later on became a disturbing fact, since it meant that the only known way to access Lagrangian interactions did not reveal anything about the mathematical existence. This insight relativises the success of QED somewhat, because to realize that the only remaining possibility, namely asymptotic convergence for infinitesimally small couplings has no useful mathematical status, is a sobering experience. An experimental comparison with perturbation theory is only fully successful if the theory has a mathematical-conceptual existence status. This deficiency of the presently only calculational access distinguishes QFT from any other physical theory and the tacit assumption underlying all quantum field theoretical research is of course that this a temporary shortcoming of our capabilities and not a flaw in our characterization of QFT.

The S-matrix boostrap project, which was vigorously proposed, notably by Chew, was based on the conjecture that a unique S-matrix, primarily of strong interactions, can be determined on the basis of three principles unitarity, Poincaré invariance and the crossing property, where the last requirement extends the analyticity of the dispersion relations. Viewed in retrospect, it is not these three requirements which would cause raised eyebrows but rather the idea that one can use them to "bootstrap" one's way into finding *the* S matrix of strong interactions. Such ideas about the existence of a unique particle physics "el dorado" which can be found by juxtaposing the right concepts have arisen several times in particle physics. One of the postulates is usually highly nonlinear as the unitarity requirement in the present case. In such a situation pedestrian attempts

[8]The other alternative, namely that the global charge vanishes [13] is related to the Schwinger-Higgs mechanism of charge screening.

to join linear requirements with nonlinear ones lead in most cases to a explosive nonlinear batch, of which no solution can be found by computational tinkering.

But this negative result is often not the end of the story since it may nourish the hope (as in the similar case of the nonlinear Schwinger-Dyson equations) that if this nonlinear batch has any solution at all, it should be rather unique; so maybe there is only one solution. In this way the dream of a theory of everything (TOE, apart from gravity) was born. A few prominent physicists, among them Dyson, initially supported this project. QFT avoids such head on encounters with the nonlinear unitarity, by deriving the latter from the asymptotic convergence of Hermitian fields in the setting of scattering theory.

The invigorated QFT in the form of Yang-Mills gauge theories, and the lack of concrete computational bootstrap problems caused a shift away from the bootstrap project; but unfortunately without bringing it to a critical conclusion. As we nowadays know, there is nothing wrong with those postulates, the error lies in the expectation that they can be directly used in this raw form to implement calculations.

The few remaining adherents of an S-matrix approach who did not convert fully to gauge theory, turned to more phenomenological motivated problems as Regge trajectories; this led them to the class of *dual resonance models*, which afterwards culminated in string theory. With discrepancies in the scattering results involving high momentum transfer, the phenomenological support disappeared. The whole setting was far too sophisticated for phenomenological applications and there arose the idea to marry the orphaned mathematical formalism with more fundamental physics. The proposal that this fundamental physics should be the observational most inaccessible region of gravity at the Planck mass was the assurance that never again (or at least not for the foreseeable future) there should be any harm coming from the observational side; this is where string theory remained ever since. In retrospect it is a bit surprising that this found the immediate support of the dual model/string community because at that time the critical tradition in particle theory had not yet completely disappeared and there was no common accepted viewpoint of what to make with the Gamma functions formalism of the dual model, its identity to the Mellin transformation formalism of conformal QFT was only noticed much later [29][8]. There were and still are those who have completely different viewpoints about quantum gravity than those coming from string theory. But in the modern globalized world there is hardly any in-depth critical dialogue between these different monocultures concerning Planck mass gravity, the desire to sell one's own brand is much stronger.

The theoretical challenge of such a proposal is primarily not whether it can be experimentally tested, but rather whether it is in agreement with the causal localization principles of relativistic QT. In the present context we are therefore not worried by its lack of observational success, this worry expired anyhow with the move from strong interaction phenomenology to string theory where it became wrapped into its new protective Planck range shield. Rather it is our intention to draw attention to a serious theoretical shortcoming, namely that the dual model as well as string theory contains a fundamental irreparable misunderstanding about causal localization, which is the central principle of QFT and the main topic of this paper. An error about such a subtle property is not something which one should lightly forget by ignoring it and moving to the next theory, which is what one usually does in such a situation. In that case one would loose the chance for a significant gain of knowledge and also the answer to the question: how can it be that so many among us allowed themselves to be misled for more than 5 decades?

Before addressing these points it is interesting to report on another much less ostentatious development which was strongly influenced by the bootstrap project but took a completely different turn. This was incredibly successful in its own terms, since it accomplished all its self-posed aims. But as a result of its more modest ambition and lack of propagandistic presentation, it is probably little known to the majority. This setting of *factorizing models* (necessarily 2-dimensional), often referred to as integrable field theories, came into being through the observation that certain quasiclassical aspects, first noted by Dashen, Hasslacher and Neveu on some 2-dimensional QFTs (the most prominent was the Sine-Gordon equation) suggested two-particle elastic S-matrices which were exact solutions of the bootstrap requirements [14]. The existence of an infinite family contradicted of course the TOE uniqueness aspect of the bootstrap project. Even worse for the anti-QFT ideology of the S-matrix bootstrap, each of the purely elastic two-dimensional S-matrices which resulted from this bootstrap classification was associated to the system of formfactors of a unique (in an appropriate sense) QFT [15]. This time the true (nonperturbative) conceptual-mathematical existence of interacting models could be shown [16], the crucial insight which led to this result was the existence of very simple generators of wedge algebras in factorizing models [17]. This was a significant progress on the most important issue of QFT: its existence and its construction. After 80 years absence of mathematically control here was a class of ("factorizing") two-dimensional models with noncanonical short distance behavior for which all doubts about their existence could be removed and many of the objects one is interested in were computed. The computational difficulties turn out to be opposite to those of perturbation theory, namely the more one moves off mass-shell the more extensive the calculations become: the S-matrix and the formfactors require less computational work whereas the correlation functions remain prohibitively complicated. Fortunately the existence proof does not require such explicit calculations. This is certainly a respectable success, even if the limitation to d=1+1 still reminds us of the enormous work ahead which will be necessary in order to secure the existence of realistic d=1+3 models.

Some of the ideas, especially those about the conceptual origin of the crossing property of formfactors, combined with the progress in local quantum physics (LQP), have led to a surprising connection between crossing and the thermal aspects of modular localization. This gave rise to a new setting of QFT in which the S-matrix plays a distinguished role, but it becomes incorporated into setting of formfactors. It turned out that their analytic properties, in particular the crossing property, can be understood in terms of the modular operator theory of the wedge algebra [8][18]. The aforementioned setting of two-dimensional factorizing models reappears in this new setting as a special case. This new project could be considered as a heir to the old S-matrix ideas since the crossing property plays a crucial role in both, but now not as a God-given rule abstracted from Feynman diagrams but rather as a fundamental consequence of the thermal properties of modular localization, more concretely from the thermal KMS properties of the vacuum state restricted to the wedge algebra. The conclusions are however very different; as already mentioned the new setting destroys the dream of the existence of a unique S-matrix theory which can be "bootstrapped" from some postulates, however it does render the S-matrix and particle states important concepts to be used right at the beginning of constructions of QFT which goes somewhat (but not completely) against Heisenberg's comment "the S-matrix is the roof of the theory and not its foundation" [9] with which he distanced himself from his 1946 S-matrix proposal[9]. This is very different from the present way of dealing with QFTs (e.g. perturbation theory) in which the vacuum correlations of fields and only afterwards their momentum space mass shell projection (or their large time asymptotic limits) are constructed. It should be seen as the heir of the Causality-Dispersion relation project of the 60s which, although working on-shell, never considered itself a pure S-matrix project.

2 Macro- and Micro-causality

The first S-matrix proposal for the construction of relativistic QTs in 1943/46 by Heisenberg [19] was motivated by the desire to overcome the reputed ultraviolet problem as well as to avoid the conceptional difficulties of introducing a short distance behavior improving elementary length into QFT; in a pure global S-matrix setting one would have gotten rid of the two problems. Heisenberg's idea was that one may find sufficiently many properties of S directly without having to "interpolate" the incoming and the outgoing particles in a scattering process by interacting (off-mass-shell) pointlike local fields. There was no problem to account for the obvious properties as unitarity, Poincaré invariance and the cluster factorization for large spacelike separation

$$S = e^{i\eta},\ example : \eta = \int \eta(x_1, ..x_4) : A_{in}(x_1)..A_{in}(x_4) : d^4x_1..d^4x_4$$

$$\eta(...)\ conn. \curvearrowright \lim_{a\to\infty} S(g_1..g_{k+1}^a..g_n^a; f_1..f_{l+1}^a..f_m^a) = S(g_1..g_k; f_1..f_l)S(g_{k+1}..g_n; f_{l+1}..f_m)$$

Unitarity is satisfied by writing the S-matrix in form of a Hermitian phase operator η, the operational Poincaré invariance follows from that of the coefficient functions η (in general an infinite series), and the cluster property is a consequence of the connectedness of the $\eta's$. But as Stueckelberg pointed out some years later [5], such an Ansatz lacks the macro-causality property which he identified with an S-matrix property called "causal rescattering" [6]. This property gives no direct restriction on the two-particle amplitude. In its simplest version it states that the 3-3 S-matrix should contain a particle pole contribution which corresponds to a two-step process: first two of the particles interact and then one of the outgoing particles interacts with the (up this point) noninteracting third incoming particle.

That the second process happens an (in the S-matrix idealization) infinite time *afterwards* means that there is a pole term corresponding to the timelike connection between the two 2-particle processes which has the same $i\varepsilon$ prescription as the Feynman propagator; the only distinction is that in the present case the latter has only asymptotic validity (in momentum space near the pole). This causal re-scattering is one of the additional requirements on S introduced by Stueckelberg which apparently cannot be implement by

[9]Only in factorizing theories their purely elastic S-matrices (only vacuum polarization no on-shell particle creation through scattering) can be computed through the bootstrap requirements. In higher dimensions there are no theories with only elastic scattering (Áks theorem), real particle creation and vacuum polarization go together.

hand while maintaining unitarity at the same time. This shows that a pure S-matrix theory without using a field-like mediator between incoming and outgoing scattering states is not a realistic goal, a conclusion that also Heisenberg reached some years later [9]. But the first S-matrix attempt was not totally in vain, because by Stueckelberg's suggestive ad hoc simplification of using the Feynman propagator also outside the timelike asymptotic region and assuming that the interaction region can be shrunk to a point, he independently obtained the Feynman rules through overidealizing macro-causality. So if Feynman would not have found a more operational setting for their derivation, we would not have been left completely empty-handed since there would have been a pertubative suggestion by Stueckelberg; however to prove perturbative on-shell unitarity without an operational formalism is not an enterprise whose successful accomplishment is guarantied. According to an article by Wanders (in [5]) Stückelberg actually found an iterative causal unitarization starting with a Hermitian Wickproduct of fields and invoking micro-causality in every iterative order in order to restrict the freedom to the stricture of counterterms. "Unitarization" by itself is not a well-defined procedure. At the moment one invokes microcausality one has left the realm of a pure S-matrix theory. The perturbative S-matrices one obtains this way are therefore identical to those from a QFT, in fact the Epstein-Glaser perturbation theory is the mathematically polished form of the Stueckelberg causal unitarization.

Macro-causal structures in scattering amplitudes (without their micro-causal counterparts) are automatically fulfilled in theories with a spacetime dynamics e.g. a Hamiltonian or an equation of motion. The "primitive causality" in Nussenzveig's presentation of non-relativistic scattering problems [1] is based on the same physical idea, except that (unlike causal re-scattering) one cannot remain within a pure S-matrix setting; the definitions of Schützer and Tiomno [3] use the interaction dynamics for all times. Such macro-causality concepts are quite efficient if one wants to show that proposals of "modifications by hand", as e.g. the introduction of the Lee-Wick complex poles into the Feynman rules, lead to time precursors and in this way violate primitive causality [20].

These causality properties can be *formulated* in terms of particles, but can they also be computational *implemented* in a pure particle setting i.e. in a dynamics which is formulated only in terms of particles? There exists a little known quantum mechanical relativistic multiparticle scheme[10] which leads to interacting multi-particle representations of the Poincaré group and fulfills all the causality properties which one can formulate in terms of interactions between particles only: the setting of *direct particle interaction* (DPI) [4]. Assuming for simplicity identical scalar Bosons, the c.m. invariant energy operator is $2\sqrt{p^2+m^2}$ and the interaction is introduced by adding an interaction term v

$$M = 2\sqrt{\vec{p}^2+m^2} + v, \quad H = \sqrt{\vec{P}^2+M^2} \tag{2}$$

where the invariant potential v depends on the relative c.m. variables p, q in an invariant manner i.e. such that M commutes with the Poincaré generators of the 2-particle system which is a tensor product of two one-particle systems.

[10] Although Dirac introduced important concepts based on his project of a relativistic particle theory his implementation of a particle-hole theory led to inconsistencies in perturbative orders in which vacuum polarization entered.

One may follow Bakamijan and Thomas (BT) [21] and choose the Poincaré generators in a way so that the interaction only appears explicitly in the Hamiltonian. Denoting the interaction-free generators by a subscript 0, one arrives at the following system of two-particle generators

$$\begin{aligned} \vec{K} &= \frac{1}{2}(\vec{X}_0 H + H\vec{X}_0) - \vec{J} \times \vec{P}_0 (M+H)^{-1} \\ \vec{J} &= \vec{J}_0 - \vec{X}_0 \times \vec{P}_0, \end{aligned} \tag{3}$$

where the two particle operators $\vec{X}_0, \vec{P}_0, \vec{J}_0$ with the subscript zero are just the sum of the corresponding one-particle operators. The interaction v may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum p in the c.m. system; but since the scheme anyhow does not lead to local differential equations, there is not much to be gained from such a choice. The Wigner canonical spin $\vec{J}_0$ commutes with $\vec{P} = \vec{P}_0$ and $\vec{X} = \vec{X}_{.0}$ and is related to the Pauli-Lubanski vector $W_\mu = \varepsilon_{\mu\nu\kappa\lambda} P^\nu M^{\kappa\lambda}$.

As in the nonrelativistic setting, short ranged interactions v lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian

$$\Omega_\pm(H, H_0) = \lim_{t\to\pm\infty} e^{iHt} e^{-H_0 t} \tag{4}$$

$$\begin{aligned} \Omega_\pm(M, M_0) &= \Omega_\pm(H, H_0) \\ S &= \Omega_+^* \Omega_- \end{aligned} \tag{5}$$

The identity in the second line is the consequence of a theorem which say that the limit is not affected if instead of M one takes take a positive function of M (5) as $H(M)$, as long as H_0 is the same function of M_0. This insures the the asymptotic *frame-independence* (P-invariance) *of asymptotic objects as the Møller operators and the S-matrix,* but not that of semi asymptotic operators as formfactors of local operators between *ket* in and *bra* out particle states. Apart from this *identity for operators and their positive functions* (5), which seems to plays no role in the nonrelativistic scattering, the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property is the statement that $\Omega_\pm^{(2)} \to \mathbf{1}$, $S^{(2)} \to \mathbf{1}$, i.e. the scattering formalism is identical. In particular the two particle cluster property, which says that for short range interactions the S-matrix approaches the identity if one separates the center of the wave packets of the two incoming particles, holds also for the relativistic case. Having a representation theory of the two-particle Poincaré group does not amount to covariance at finite spacetimes, but together with the short range requirement they secure the existence of a unitary Poincaré invariant two particle S-matrix.

There is no problem in finding restrictions on the interaction v which correspond to those e.g. which Schützer and Tiomno [3] used in the nonrelativistic setting. It is however nontrivial to generalize this setting to *multiparticle* interactions since the representation theory of the Poincaré group prohibits a trivial implementation of cluster factorization by adding up two-particle interactions as in the nonrelativistic case. The Coester-Polyzou formulation of DPI shows that this is possible [4]. The proof is inductive and passes the clustering of the n-particle S-matrix to that of the n-particle Poincaré group representation which than in turn leads to the clustering of the (n+1)-particle S-matrix etc. There always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix*. Such transformations, which are unfortunately not unique, are called *scattering equivalences.* They were first introduced into QM by Sokolov [22] and their intuitive content is related to a certain insensitivity of the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is reminiscent of the insensitivity of the S-matrix against local changes in the interpolating field-coordinatizations in QFT[11] in QFT by e.g. using composites instead of the Lagrangian field. From the construction it is clear that this relativistic DPI has no fundamental significance. Its theoretical use consists in providing counterexamples to incorrect conjectures as e.g. the claim that Poincaré invariance of the S-matrix and cluster factorization requires QFT. The comprehension of its existence sharpens the recognition of the importance of the causal localization structure in the particle-field dichotomy.

3 Analyticity as a starting point for a particle theory?

The two-fold limitation of perturbation theory, on the one hand its divergence of the perturbative series and the ensuing doubts about the ontological status of interacting QFT, and on the other hand its unfitness to describe nuclear interaction (in those days the $\pi - N$ interactions) led to a return of the S-matrix idea. In many aspects the new bootstrap ideas went beyond what people learned from the Heisenberg program and its criticism by Stueckelberg, but not in all. The nontrivial macro-causality properties as the spacelike clustering and the timelike causal rescattering property do not occur in the bootstrap list. Unimportant or forgotten in the maelstrom of time? It is characteristic of the three S-matrix projects that the subsequent one added a new idea but ignored some of the old messages. As will be seen in more details, the newly added crossing property of the 50s was simply read off from Feynman graphs, but the lack of understanding its root-causes carried the seeds of a conceptual derailment whose consequences are strongly influencing the present situation.

Crossing is most clearly formulated in terms of formfactors, the crossing for scattering amplitudes is a consequence of the formfactor crossing and the LSZ reduction formula. Our proof in the formfactor setting (see last two sections) is very different from the proof of the 60s which was limited to the elastic component of the scattering amplitude [23][24] and used analytic techniques of functions of several complex variables. It contains an element of surprise, since the so-called *crossing identity* turns out to be a somewhat camouflaged KMS identity from the restriction of the global vacuum to the wedge localized algebra. That the restriction of the global vacuum to the wedge algebra leads to an impure thermal state which is KMS with respect to the wedge preserving Lorentz boost is of course known from the *Unruh and Hawking thermal manifestations of localization* of quantum matter behind causal- black hole event- horizons[12], but that the thermal KMS identity

[11] In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class) or in the setting of AQFT by picking another operator from a local operator algebra.

[12] Contrary to popular opinion it is not the curvature but rather the localization which generates the

also manifests itself in the for half a century allegedly well-known region in the center of high energy theory in form of the would be God-given *crossing identity* including its analytic continued connection vacuum polarization components with formfactors is new and totally unexpected. This explains the subtitle of the second subsection below[13] in which we will present some details leading to this observation. The title of the section is a characterization of the beliefs at the time of the Chew-Mandelstam S-matrix setting; I do not think that analyticity is a physical principle, rather it is a consequence of the spectrum property and causal localization principle, but sometimes the path from the principles to analytic properties is subtle and demanding; as shown in the derivation of the high energy physics dispersion relation from the JLD representation and of the crossing properties from the properties of wedge-localized operator algebras.

We will subdivide this section into three subsections. The first, entitled a cul-de-sac, critically reviews the post bootstrap S-matrix project which started with a concrete conjecture, in particular the Mandelstam representation, and became more phenomenological oriented with the attempted incorporation of Regge poles and their trajectories. At the time of the dual resonance model it failed on the observational as well as the conceptual side. The ultimate step to string theory resulted from an attempt to lend conceptual physical importance to a mathematically tempting formalism[14] by simply forgetting the failed observational connection and postulating the yet unknown gravitational physics at the Planck scale as its new range of application. Our interest in this paper is limited to its problematic relation with the localization concepts of QT, in particular whether it really has the localization property which string theorist ascribed to it [8].

The second subsection explains why the vacuum state restricted to a spacetime localization region $\mathcal{O}$ turns into an impure thermal KMS state[15]; the most interesting case is $\mathcal{O} = W$ (the wedge region). It also provides additional insight into what physical string-localization really means and why the objects of string theory are not string- but rather point-localized. It also places big marks of doubts on the string S-matrix proposal resulting from pure prescriptive manipulation of functions which by decree are promoted to be scattering amplitudes. Without being able to formulate these recipes in terms of states and operators they are totally unconvincing; freely roaming sophisticated looking rules which ignored the lesson from Stueckelberg's iterative S-matrix construction by unitarization (see next section) and which simply replace the grapical "worldlines" for particle propagators by tubes (worldsheets) without backing up the graphical pictures by Hilbert space operators fall back behind Stueckelberg's work and do not pass the minimum requirement which any credible conjecture in QT has to fulfill.

Even if this (after more than 4 decades) would still work by some overlooked magic, there is still the conceptual problem of attributing a meaning to a "stringy" S-matrix; an S-

thermal aspect. The event horizon attributes to the localization in front of the Schwarzschild horizon a physical reality whereas the causal horizon of a Rindler wedge has a more fleeting existence.

[13]An ironical way of coping with such situations and unfortunately not translatable without loosing its flavour.

[14]Although the formalism originated by playing with properties of gamma functions, the result obtained first by Veneziano looked as waiting in a holding loop for conceptual rather than phenomenological applications.

[15]This may be seen as the metaphor-free aspect of the "broiling vacuum polarization soup" of the books on QFT.

matrix is a *global* object which a priory does not contain any information about spacetime localization. In fact for the only known case of *genuine string-localization* as the best possible localization, namely electrically charged matter fields [25][26], the consequence of the weaker than pointlike localization is the (since Bloch and Nordsiek well-known) phenomenon of infrared divergences in scattering theory which leads to the abandonment of the S-matrix in favour of photon-inclusive cross sections (for which unfortunately no elegant LSZ like representation in term of spacetime correlations has yet been found).

The third subsection "the constructive future of QFT" presents a new constructive setting in which an algebraic version of crossing and *analytic exchange* (explained there) is the starting point of a new approach, which in d=1+1, as a result of a simplification of the vacuum polarization of generating operators for the wedge algebra, leads to the before mentioned first existence proofs and model constructions for the class of factorizing models. In a certain sense this success vindicates some aspects of the old dream of the bootstrap community and in particular of Stanley Mandelstam [27], even though its implementation requires quite different concepts and a return to QFT after the S-matrix setting having lost its claim to uniqueness. Nevertheless the use of the S-matrix as a basic computational tool (and not just the roof of local particle physics) is shared with the S-matrix attempts of the 50s and 60s which now, together with formfactors of fields and a new much more subtle role of the crossing property, forms the backbone of the new approach.

3.1 A cul-de-sac with important lessons

The first two attempts to avoid fields in favour of a pure S-matrix approach failed basically for the same reasons. The nonlinear unitarity of the S-matrix together with other linear physically motivated restrictions results in a rather unwieldy computational batch which unfortunately (in the eyes of some) created the impression that if such a system of requirements admits any solution at all, then it should be rather unique. In this way an early version of a theory of everything (TOE still without gravity) was born. But the disparity between high dreams and the difficulty to translate them into credible computations led to an early end of the projects. This discrepancy was most discouraging for young newcomers who were looking for doable and at the same time credible computations. They were much better served by the new nonabelian gauge theories for weak and strong interactions. The dream of uniqueness of the bootstrap ended in the middle of the 70s with the construction of the infinitely large family of d=1+1 QFT, the so-called *factorizing models* whose different S matrices all fulfills all the bootstrap requirements. The uniqueness of a QFT associated to each of those S-matrices (the uniqueness of the inverse problem of scattering theory) was however strengthened[16].

In this situation Stanley Mandelstam [27] proposed a representation of the elastic scattering amplitude which contained the dispersion relation as well as the up to that time known momentum transfer so-called t-analyticity. In contradistinction to the nonlinear bootstrap program it seemed more susceptible to computational ideas. For the project of

[16] An S-matrix does however not distinguish a particular field, rather it associates to a local equivalence class (Borchers class) or more compactly to a unique net of local operator algebras of which those fields are different generators.

establishing the observational validity of the (model-independent) causality principle underlying QFT via an experimental check of the dispersion relations, this guessed but never proven representation would however have had little interest; in this case the rigorously established Jost-Lehmann-Dyson representation was the link between the principle and its observable consequence and in this way clinched the *causality and dispersion relation project*.

There was a second more phenomenological motivated train of thought, the idea of analytic continuation in the angular momenta and the connection of the related Regge trajectories (particles, resonances) which offered tempting observational correspondences with the real world of strong interactions. In this situation Gabriele Veneziano, guided by the Mandelstam representation, produced a formula which combined infinitely many particle poles into a trajectory in such a way that the relation between Mandelstam's s and t channels appeared as a implementation of the crossing property; as a result the model was called the dual model. Veneziano achieved this by using mathematical properties of Gamma and Beta functions in an ingenious way, so that the result appeared to some as a profound confirmation of the underlying phenomenological ideas and to others too mathematical for the phenomenological use.

This construction, which appeared at the beginning to be unique, admitted several similar solutions [28]. In fact, as we know nowadays, the correct interpretation of these magic constructions has nothing to do with the world of S-matrix of massive particles realizing the crossing identity [8] but are rather the result of the fact that the appropriately normalized *Mellin-transforms of correlation functions of conformal covariant fields produce the various dual models [29] whose duality properties are vaguely reminiscent of crossing properties* but have entirely different conceptual physical origins. All dual models which hitherto had been introduces in terms of rather involved ad hoc mathematical tricks were now united under a unified mathematical principle. The convergent global spacetime operator expansions (the global form of the only asymptotically convergent short distance expansion) which exist only in conformal QFT pass under Mellin transformation of spacetime correlation functions to the pole series in the Mellin variables. The anomalous dimensions of the conformal fields have the appearance of squares of particle masses in the Mellin transform, and in the canonical quantization of the Nambu-Goto Lagrangian the Mellin pole spectrum has indeed been converted into the mass/spin "tower" of an infinite component field. But there is no relation of the duality in the s-t Mellin variables to the origin of the correctly understood crossing property in QFT as it holds for formfactors and scattering amplitudes (next two sections) except that both properties arise from causal localization[17].

I dare to conjecture that the dual model as a description of scattering of particles would not have arisen if these insights would have been available at the end of the 60s. What looked as a impressive cooperation of ingenuity and luck at that time, appears in retrospect as a the result of clever tinkering without conceptual guidance. This is corroborated by the correct derivation of the crossing property from the thermal property of localization which leads to a *holistic* interplay of all states and has no separate realization on one-particle states without the participation of the scattering continuum (last section).

[17]This is a rather empty statement because all properties in QFT arise from the mathematical and conceptual precise formulation of "modular localization".

The remaining misunderstandings are also related to localization; and since this is the most central and at the same time most subtle principle in relativistic QT, also their analysis is profitable and interesting. From the point of view of Poincaré covariant localization any quantum theory which admits algebras of local observables can be considered a QFT, not just those models which are expected to be behind the finite number of renormalizable interactions associated to the Lagrangian- or functional- quantization. This includes string theory, since it is also thought of as a covariant quantum theory with the difference that, according to the expectations of the ST community, it is not pointlike but rather stringlike generated. The dual models are in the eyes of their protagonists the starting (lowest order) amplitudes for a new kind of iterative approach to a unitary S-matrix. Hence from a conceptual point of view it would not be necessary to assign a localization to this model, such a property could even create confusion with the global nature of an S-operator. Strangely enough, this is precisely what dual model supporters were doing when they view the model as resulting from an embedding of a one dimensional "source" chiral theory into an n-dimensional "target" spacetime. Whereas the embedding of a lower dimensional localizable quantum theory into a higher dimensional one is not possible, the restriction or holographic projection to a lower dimensional one makes mathematical sense but generally the lower dimensional one is overloaded with degree of freedoms leading to pathological physical behavior. The lighthearted game with spacetime dimensions which arose in the orbit of string theory may be consistent with classical, theory but has no support from local quantum theory.

A closer examination reveals that this not possible; rather in figurative terms the infinite oscillator degrees form an internal Hilbert space over a point in an n-dimensional target space and the n-component zero mode of the chiral theory gives rise to the momentum on which the Poincaré group acts in the standard way. At this level the scheme is still outside quantum physics since the representation space of the Poincaré group is not yet a Hilbert space; actually the internal symmetry space (=target) in low dimension QFT does not have to be a Hilbert space, since internal symmetry groups in chiral models may be noncompact.

All this can be derived in a formally simpler and more standard way by canonically quantizing the Nambu-Goto Lagrangian[18], which also indicates what one has to do in order to return to Hilbert space quantum physics: the superstring representation for n=10. This is the only way to obtain a Poincaré group representation on the inner symmetry space[19] of a 10 component chiral current algebra. String theorists base their terminology on a classical picture of fields, the classical value space passes to "target" space. All these concepts have a very doubtful meaning in QFT, the precise quantum terminology close to those quasiclassical concepts would be "inner symmetry space". The only word which is totally misplaced here is "string", because the resulting object is a pointlike localized infinite component field [8].

I am not a string theorist and I do not know what is on peoples minds when they

[18] In the original geometric (square root) form the Labrangian describes an integrable system whose quantization is inequivalent to the canonical result [30].

[19] String theorists base their terminology on a classical picture of fields, the classical value space passes to "target" space. All these concepts have a very doubtful meaning in QFT, the precise meaning would be "inner symmetry space".

present their results in this terminology; I would expect them to say, if asked about those points, that there has been indeed a lot of misplaced terminology resulting from the rapid development of this area, but what is really meant here is that the S-matrix obtained from unitarization of the dual model (imitating the graphology of Feynman, but taking world-sheets instead of world-lines), is that of "stringy interactions". This leads to the question what does it mean to assign "stringyness" to an S-matrix i.e. how can one reconstruct spacetime properties from a global object. The second more series problem is what is the meaning of unitarization if

1) The lowest order, namely the Mellin transform of a conformal QFT, has in contrast to the Fourier transform no Hilbert space presentation in terms of "Mellin correlators".

2) Since the beginnings of these S-matrix ideas almost 5 decades ago there has been no clarification of that "unitarization" in terms of operators and states, which are the only appropriate concepts. All attempts, even those by the best minds in ST, ended without an result. There is also no physical reason why a Mellin transform, which by itself has no Hilbert space presentation, can be used as the starting point of a unitarization towards an S-matrix.

Reminding the reader of the before mentioned similarities between Stueckelberg's and Feynman's ideas, it is interesting to mention that Stueckelberg's approach[5] actually amounts to an iterative unitarization process for an S-matrix starting with the first order

$$S = 1 + \sum_{k=1}^{\infty} S_k, \quad S_1 + S_1^* = 0 \tag{6}$$
$$S_2 + S_2^* = -S_1 S_1^*,\ S_3 + S_3^* = -(S_1 S_2^* + S_2 S_1^*),\ etc$$

. He started (in modern terminology) from a anti-Hermitian S_1 in terms of a spacetime integral over a scalar Wick-ordered polynomial in free fields. Since in each order only the Hermitian part is determined by the previous orders he imposed micro-causality even though this has no cogent reason in an S-matrix setting. He then showed that this restricted unitarization scheme determines S up to local counterterms and the result is identical to Feynman's, so imposing the micro-causality on the operator densities of the various S_k is equivalent to perturbative QFT [20]. But what does it mean to obtain the string theory S by unitarizing the dual model as its first start. Where is the first order operator in a Hilbert space which is the string analog of Stueckelberg's S_1? And what controls the higher order anti-Hermitian parts? And what does "stringy" for the first order mean, assuming that it entered the theory through the dual model and why should calculations of functions based on splitting and fusing tube graphs which for 50 years resisted its presentation in terms of operators and states have anything to do with QT?

Particles described by pointlike interacting fields lead to formfactors which are anything but pointlke delta functions, they rather show a rich extended structure i.e. the

[20] The iteratively constructed S(g) is Stueckelberg-Bogoliubov-Shirkov operator functional which depends on space-time dependent coupling function. The S-matrix (formally for constant g's) is related to this functional by the "adiabatic limit" which is as difficult to establish as the asymptotic convergence of fields in the LSZ scattering theory (and in most interesting cases does not exist)-

underlying pointlike nature cannot be seen by naively looking at physical obervable. But as long as the generators of local algebras are pointlike fields, the theory is called "pointlike". The causal localization principle only requires that the local observables are pointlike generated but charged fields may instead of pointlike also stringlike. The only theory in nature which needs stringlike generators is QED. The covariant semi-infinite string localization of the photon potential $A_\mu(x,e)$ which results from having a vector-potential in Hilbert space i.e. avoiding indefinite metric extensions and paying the prize by allowing a weaker localization, transfer the stringlike localization through the QED interaction to the charged matter field. Whereas the nonlocality in the photon description is removed by passing to the pointlocal field strength, there is no way to remove the string-localization of the charged particle with the result that the ensuing infrared behavior destroys the large time convergence which is necessary for the existence of the S-matrix [25]. In fact string-localization as an alternative to the local gauge setting is an important idea for making progress in gauge theory [26] and for this reason the misunderstandings about causal localization caused by string theory is much more problematic than other errors which already vanished in the maelstrom of time.

3.2 Modular Localization and its thermal manifestation

In diesem Fall und ueberhaupt, kommt es ganz anders als man glaubt. (W. Busch*)*

Many properties in QFT allow a more profound understanding (beyond the mere descriptive presentation) in a formulation in which one deals with space-time indexed systems of operator algebras rather than with their generating point- or string-like generating fields. An illustration was given within the algebraic formulation of causality, in which the *causal shadow property* is more natural then its formulation for individual fields. This is in particular true about the restriction of the vacuum to such subalgebras, since the holistic [31] properties of states can be better defined as positive functionals on an operator algebra than on individual operators in the algebra; in fact in order to identify states in an intrinsic way, the type of operator algebra enters as the dual structure of states in an essential way. Since the structure of local algebras in QFT is very different from its global counterpart[21], this algebraic point of view is particularly important in the present context where impure states resulting from restrictions and their characterization in terms of a thermal KMS ensemble comes into play.

In the following we will present some important results from local quantum physics [32]; the reader who is unfamiliar with these concepts should consult the literature [6]. Spatial separation of a quantum mechanical global algebra into two mutually commuting spatially separated (inside/outside) subalgebras leads to a *tensor factorization* of the quantum mechanical vacuum state and the phenomenon of *entanglement* for general states. Nothing like this happens for local subalgebras in QFT. Rather they radically change their algebraic properties; instead of being equal to the algebra of all bounded operators on a smaller Hilbert space, (also called type I_∞ factor algebra) as in QM, the localized subalgebras of QFT belong to a different type called hyperfinite type III_1 factor algebras (in the

[21] The global algebras (without their local substructure) have have standard form of algebras in QM namely B(H) (all bounded operators acting on a Hilbert space). This continues to be true in QM for the local algebras but changes in QFT.

Murray-von Neumann-Connes classification) for which among many other changes the above tensor-factorization breaks down. For reasons which will become clear later on, we will call this operator algebra type shortly a *monad,* so every localized algebra of QFT is a *monad* [6].

Although the division into a spacetime region and its causal complement by the use of Einstein causality leads to the mutual commutativty, and in spite of the fact that the algebra associated to a spacetime region and its commutant together generate the full global algebra $B(H)$, the *vacuum does not factorize* and the prerequisites for entanglement are not fulfilled. In fact a monad has no pure state at all, all states are impure in a very singular way, i.e. they are not density matrix states as impure states in QM. In fact the vacuum restricted to a local monad turns into a singular KMS state. Such states appear in QM only in the thermodynamic limit of Gibbs states on box quantized operator algebra systems in a volume V. Later we will turn to the mathematics behind these observations which is *modular operator theory* and more specific *modular localization.*

Usually people are not interested[22] in an intrinsic description of the thermodynamic limit state (called "statistical mechanics of open systems"), but if they were, as Haag, Hugenholtz and Winnink in 1967 [32], they would find that the limiting state ceases to be a density matrix state and becomes instead a singular KMS state i.e. a state which has lost its tracial property and hence has its Gibbs representation property (volume divergence of partition function); instead it fulfills an analytic relation which first appeared as a computational trick (to avoid computing traces) in the work of Kubo, Martin and Schwinger and later took on its more fundamental significance [32] which it enjoys in physics and mathematics. Whereas monads with singular KMS states appear in QM *only* in the thermodynamic limit of finite temperature Gibbs states, their occurrence in QFT is abundant since the reduction of the vacuum onto every localized subalgebra (whose causal complement is nontrivial) leads to such an impure state which is KMS "thermal" with respect to the modular Hamiltonian on a monad (see below).

With a "split" of size ε between the subalgebra and its causal complement[23], one can to a certain extend return to the quantum mechanical tensor product situation, but the states which factorize do not include the vacuum state whose restriction continues to be impure, but now as a bona fide density matrix state. Whereas one can not associate a finite localization-entropy with a sharply localized operator-algebra, this becomes possible after the ε-split. The result [33] is a logarithmically modified area law which for an n-dimensional double cone reads $(\frac{R}{\Delta R})^{n-2} ln(\frac{R}{\Delta R})$ with R the radius of the double cone and $\Delta R = \varepsilon$ the split distance which may be pictured in more physical terms as a kind of attenuation length for the vacuum polarization cloud which for $\varepsilon \to 0$ would diverge in a analogous way as in the the limit $V \to \infty$ the entropy diverges with the n-1 dimensional volume factor. This relation between the sharpness ε of the localization boundary and

[22]For example the tensor factorization formalism known as "thermo-field formalism" is lost in the thermodynamic limit (the open system description) i.e. this formalismus is not suited to describe "open systems".

[23]The possibility of doing this is called "the split property". Wheres the standard box quantization does not allow to view the boxed system as a subsystem of a system in a larger spacetime, the splitting achieves precisely this at the price of vacuum polarization at the boundary. The physics based on splitting is called "open system" setting.

the localization entropy/energy replaces the uncertainty relation of QM.

The connection between modular localization and thermal aspects may be little known, but there is one system which is almost part of folklore: the before mentioned Unruh Gedankenexperiment which is nearly as old as the close related Hawking effect. In both cases quantum fields become localized, in the first case behind the observer-dependent causal shadow region of a causal horizon of a wedge (i.e. the wedge itself) and in the second case the localization is less fleeting and more physical because it is the event horizon of the Schwarzschild metric. In the first case the important question, which Unruh answered by the construction of a Gedankenexperiment, was what is the physical meaning of being localized in a wedge W of Minkowski spacetime? In this case the modular Hamiltonian is the generator of the W-preserving Lorentz boost i.e. the W-localized observable/counter/observer must be uniformly accelerated in order not to trespass the horizon of the wedge; for him the inertial frame Hamiltonian of Minkowski spacetime is irrelevant, his Hamiltonian is the spectral symmetric boost generator. This requires to pump energy to accelerate an observable i.e. the Unruh is not an perpetuum mobile for creating heat and the vacuum on the global algebra of all operators in Minkowski spacetime remains of course in its ground state.

In order to remove the last vestige of mystery from the connection of localization with KMS property of the reduced vacuum and the ensuing effects of vacuum polarization, we will now show how the crossing property has its explanation in the KMS property of the wedge-restricted vacuum. To do this we need one property from the modular operator theory applied the wedge algebra $\mathcal{A}(W)$ which denotes the operator algebra formally generated by smeared fields whose smearing function support is contained in $suppf \subset W$. For this algebra acting on the vacuum Ω the Tomita S-operator is well-known to have the form [32]

$$def.\ SA\Omega = A^*\Omega, \quad A \in \mathcal{A}(W), \quad S = J\Delta^{\frac{1}{2}} \qquad (7)$$
$$\Delta^{i\tau} = U(\Lambda(-2\pi\tau)),\ J = S_{scat}J_0,\ S_0 = J_0\Delta^{\frac{1}{2}}$$

Here the modular unitary Δ^{it} and the antiunitary J_0 which appear in the polar decomposition of S are determined by the representation of the Poincaré group; J_0 represents the reflection on the edge of the wedge (TCP apart from a π-rotation along the wedge). The S-matrix S_{scat} plays the role of a relative modular invariant between the free (incoming) and the interacting wedge algebra. The equality of the dense domains of the interacting S with that of the free S_0 i.e. $domS = domS_0 = dom\Delta^{\frac{1}{2}}$ implies that there is a dense set of states (namely those in $dom\Delta^{\frac{1}{2}}$) which can be generated both in the interaction free algebra $\mathcal{A}_{in}(W)$ generated by the W-smeared incoming fields and operators from the interacting algebras $\mathcal{A}(W)$. In other words this equality of domains together with the domain of S in terms of the associated algebra, generates a bijective relation between a dense set of operators affiliated with two very different algebras which only share the localization region and the Poincaré group representation and hence the domains of their different Tomita S involutions.

There is one more important fact: the global vacuum is a KMS states on $\mathcal{A}(W)$ where the Hamiltonian K is the generator of the modular unitary $\Delta^{i\tau} = e^{-iK\tau}$ (for convenience B for interacting and A for free operators) which is shared between the interacting and free algebras [8] (for more mathematical details [18])

$$\langle B_1 B_2 \rangle = \left\langle B_2 e^{-K} B_1 \right\rangle, \quad B_1, B_2 \in \mathcal{A}(W) \tag{8}$$
$$\langle A_1 A_2 \rangle = \left\langle A_2 e^{-K} A_1 \right\rangle, \quad A_1, A_2 \in \mathcal{A}_{in}(W) \; or \; A_{out}(W)$$
$$SB\Omega = B^*\Omega, \;\; SA_{in}\Omega = S_{scat}S_0 A_{in}\Omega = A_{out}^*\Omega \tag{9}$$

The previous bijection between a dense set of operators affiliated to $\mathcal{A}(W)$ and $\mathcal{A}_{in}(W)$ leads to a generalized KMS relation for mixed products

$$\left\langle B(A_{in}^{(1)})_{\mathcal{B}} A_{in}^{(2)} \right\rangle = \left\langle A_{out}^{(2)} \Delta B A_{in}^{(1)} \right\rangle \tag{10}$$

here the $(A_{in}^{(1)})_{\mathcal{B}}$ denotes those operator of the interacting $\mathcal{B}(W)$-algebra which is bijective related to the operator $A_{in}^{(1)}$ from the $\mathcal{A}_{in}(W)$ algebra. If one would start from the localized algebra of bounded operators $\mathcal{A}_{in}(W)$, the bijectively related set of operators would be a set of unbounded operators associated with the interacting algebra $\mathcal{B}(W)$; modular localization theory attribute an important physical role to domains of certain unbounded operators which they do not have in Lagrangian QFT. In order to get from this generalized KMS property to the crossing identity in terms of particle states one needs one more nontrivial step, namely one has to translate the meaning of Wick-ordered incoming states parametrized in terms of momentum space rapidities into bijective related B-operators[24] [35] for which we will abandon the inconvenient $(A_{in}^{(1)}(f))_{\mathcal{B}}$ in favor of simply $B(f)$

$$A_{in}(f)\Omega = B(f)\Omega, \; suppf \subset W, \int A_{in}(\mathbf{p},\theta)\Omega\check{f}(\mathbf{p},\theta)d\mathbf{p}d\theta = \int B(\mathbf{p},\theta)\Omega\check{f}(\mathbf{p},\theta)d\mathbf{p}d\theta \tag{11}$$
$$density \; of \; wave \; functions \curvearrowright \; A_{in}(\mathbf{p},\theta)\Omega = B(\mathbf{p},\theta)\Omega, \; A_{in}(f) \overset{bijec}{\sim} B(f)$$

This admits a generalization to wedge-localized n-particle states : $A_{in}(f_1)...A_{in}(f_n)$: Ω. A Wick product does not exist for the B-operators since in addition to the contraction delta functions $\delta(\theta-\theta')\delta(\mathbf{p}-\mathbf{p}')$ the B's have in general an unknown algebraic structure which, even in the simplest case of temperate PFG's (15) is quite different from that of the creation/annihilation components of free fields. Although they are on-mass-shell they are not free and certainly not pointlike local. The only product which maintain their symmetry of $n-B$ states in the presence of interactions are the θ-ordered $\mathcal{T}$-products. Using again the density of the wave function which are boundary values of functions analytic in the $i\pi$ strip of the θ-plane we obtain

$$: A_{in}(\mathbf{p}_1,\theta_1)...A_{in}(\mathbf{p}_n,\theta_n) : \Omega = \mathcal{T}B(\mathbf{p}_1,\theta_1)...B(\mathbf{p}_n,\theta_n)\Omega \tag{12}$$
$$TB(\mathbf{p}_1,\theta_1)...B(\mathbf{p}_n,\theta_n) = B(\mathbf{p}_{i_1},\theta_{i_1})...B(\mathbf{p}_{i_n},\theta_{i_n}), \; \theta_{i_1} > ... > \theta_{i_n}$$
$$A_{in}(\mathbf{p}_1,\theta_1) = a^*(p,\theta) \; or \; a(p,\theta) \; on \; the \; neg.mass \; shell$$

For ordered θ-configurations there are no contractions (since they are local in θ-space) and hence in this case the ordering with respect to positive/negative energies can be

[24] For the bijectively related state vectors the density argument permits to omit the wave functions $\check{f}$ but this is not allowed for the bijectivrly related operators.

omitted. Knowing the scattering interpretation of the θ-ordered product does not imply the knowledge of their unordered products. Modular theory does however relate the anti θ-ordered product of B's to the bijective connected product of outgoing fields. This ordering relation allows to derive the sequential composition rule which is isomorphic to that of Wick-products

$$\begin{aligned} &\mathcal{T}B(\mathbf{q}_1,\vartheta_1)...B(\mathbf{q}_m,\vartheta_m)\mathcal{T}B(\mathbf{p}_1,\theta_1)...B(\mathbf{p}_n,\theta_n)\Omega = \\ &= \mathcal{T}B(\mathbf{q}_1,\vartheta_1)..B(\mathbf{q}_m,\vartheta_m)B(\mathbf{p}_1,\theta_1)...B(\mathbf{p}_n,\theta_n)\Omega \;\; if\; \vartheta_i > \theta_j \end{aligned} \tag{13}$$

To avoid any misunderstanding, from these these momentum rapidity states one can only return to the dense set of W-localized states; the $B(\mathbf{p},\theta)$ are only useful for the description of states, the bijective related operators are the $B(f) \equiv (A_{in}(f))_{\mathcal{B}(W)}$ the letter only result from them by integration with mass-shell projected wedge-supported test functions.

Hence the bijective related operators in the middle of the extended KMS relation (10) can in case of the fulfillment of the required rapidity ordering be applied to the incoming state in order to increase the number of particles. The *extended KMS identity* passes under these conditions into the *crossing identity*

$$\begin{aligned} \langle 0\,|B|\,\mathbf{p}_1,\theta_1,..\mathbf{p}_n,\theta_n\rangle^{in} &= {}^{out}\langle -\mathbf{p}_1,\theta_1 - i\pi;..-\mathbf{p}_n,\theta_n - i\pi\,|B|\,\mathbf{p}_{l+1},\theta_{1+1};..\mathbf{p}_n,\theta_n\rangle^{in}_{c.o} \\ &= {}^{out}\langle -p_1;..-p_l\,|B|\,p_{l+1};..p_n\rangle^{in}_{c.o}, \quad \Delta^{\frac{1}{2}}J_0\,|p\rangle = |-p\rangle \end{aligned} \tag{14}$$

where the c. o. means omission of contraction terms between in and out states (in agreement with the absence of contractions on the left hand side). In the general case the bra states are antiparticle states on the backward mass shell. This is the result of the action of the Δ operation from the KMS property on the bra states of the form $S\,|p_1,..p_l\rangle^{in}$ which leads because of $\Delta S = \Delta^{\frac{1}{2}}J_oS_{scat}$ to the crossing relation, since S_{scat} transforms in into out and the remainder the forward shell particles into the backward shell antiparticles. The analyticity needed to relate the unphysical backward mass shell to physical momenta is precisely that known from the work of Kubo Martin and Schwinger.

There exists a proof of the crossing property for elastic scattering amplitude based on the on-shell restriction of the Fourier transformed 4-point function of local fields [23]. The method presumably allows an extension to the more general case of formfactors[25]. Although both methods are founded on causal locality the method of this paper reprocesses this property into the modular theory of the wedge-localized algebra. This is advantageous in several respects. On the one hand, as our derivation shows, there is no necessity to use any particular interpolation field. In fact it only uses $\mathcal{A}(W),\Omega$ and the free field algebras associated with scattering theory. The second important aspect which has no counterpart in the old derivation is the KMS property resulting from the restriction of the vacuum to $\mathcal{A}(W)$. This together with the new bijective related operators between the interacting and the free wedge algebras which is a consequence of their shared modular unitary Δ^{it} leads to the crossing dainty which is nothing but the reformulated KMS identity. Unlike in the Chew-Mandelstam S-matrix setting here the analyticity has an operational function which leads for the first time to a physical interpretation: the crossing identity is equal to the KMS identity of the wedge algebra with the modular Hamiltonian being equal to

[25] Using the LSZ reduction formalism, scattering amplitudes can be expressed in terms of formfactors.

the wedge-preserving Lorentz boost generator and the analyticity region a multi θ strip $(0 < Im\theta < \pi)$.

So the crossing property shares with the Unruh relation the KMS state which results from restricting the Minkowski space vacuum to the Rindler wedge; both suffer the same consequences from modular wedge-localization In the Unruh case one has the problem to think (in terms of an Gedankenexperiment) about the macroscopic realization of wedge-localized hardware and the measurement of temperature non-inertial systems. The interest in the crossing relation lies in the fact that the k,l formfactors are related to the coefficients of the vacuum polarization.

Even if some of the arguments are subtle and the result unexpected it should be clear that *crossing is something very different from what it was thought to be in the dual model and string theory* where it was identified with the properties of the suitably normalized Mellin transforms of conformal 4-point functions with an obvious extension to conformal n-point function [29].

The remaining open question is: can an already 50 year lasting conceptual derailment of particle theory caused by the misplaced hope for a pure S-matrix theory be brought to an end?

3.3 A revolutionary new operational setting: modular localization

The aftermath of the dispersion relation, which started with Mandelstam's conjectured two-variable representation, and moved via the use of the conformal Mellin transforms in the dual model to the canonically quantized Nambu-Goto Lagrangians and its d=1+9 dimensional supersymmetric infinite component QFT called "superstring" fell way behind the original expectations and even after more than 50 years left little more than conceptual dumbness and wildly diverging opinions[26]. Their remains of course the achievement of the dispersion relation project in its original form as an outstanding shiny success of modesty and precision, one in which the experimental efford matched the conceptual mathematical dedication.

This setback should however not be seen as an argument against the use of S-matrix concepts and analyticity in particle momenta in constructive approaches to models of particle physics. Whereas it would be foolish to claim the feasibility of a pure S-matrix approach after its spectacular 3 times failure (Heisenberg, bootstrap, string theory), field theoretic constructions in which on-shell particle properties including the S-matrix play an important role are not affected and are in fact strengthened by recent results. This new direction is based on the new insight, that in addition to the its historical role in scattering theory, S_{scat} is a *relative modular invariant* (between the incoming and the interacting wedge algebra). We have seen that this leads to a considerably more subtle less global and more holistic connection between particles and fields than that via scattering theory. For the first time we have a theory in which geometric aspects of localization are encoded into domain properties of individual operators and range properties of subalgebras acting

[26]There will be inevitably the accusation that my presentation of string theory is outmoded. But presenting a more sophisticated version of a fundamentally flawed theory makes its conceptual content only more sophisticated but not less flawed.

on the vacuum, before domain properties were only a formal nuisance for novices of QFT. This renders causal localization in QFT, in contrast to the Born localization associated to the spectrum of the quantum mechanical position operator, a fully holistic concept (for more details see [31]). In general the above bijection between W-localized free fields and their interacting W-localized counterparts obtained from modular localization theory has the unexpected aspect that even if the operators initially have translation invariant domain[27] this property is not preserved in the bijection. Intuitively speaking this happens because the bijection is associated entirely to the region W and transformations which lead out of W should not be expected to preserve this it. This encoding of regions into domains of operators is not part of QM, it arises through the modular localization property of QFT.

In d=1+1 there is in addition to the obvious simplification due to the absence of a transverse **p**-dependence

$$A_{in}(f)\Omega = B(f)\Omega,\ A_{in}(f) \in \mathcal{A}_{in}(W),\ B(f) \in \mathcal{B}(W),\ B(f) = \int \check{f}(\theta)B(\theta)d\theta$$

the additional possibility of having much better behaving B's which have a translation invariant domain which can even be extended to a Poincaré invariant domain[35]. Such $B(\theta)$ have been called *temperate vacuum-polarization-free* (PFG) since, although they are operators from the interacting algebra, they create a one-particle state without vacuum polarization admixture [35]. In d>1+1 this is not possible, all PFGs are *non-temperate*. The only known models with temperate PFGs are the d=1+1 factorizing models [15]

Such temperate PFGs have a number of remarkable properties which make them fascinating objects of a new "theoretical laboratory". We use this terminology for interacting low-dimensional models which permit a complete mathematical control including an existence proof and the explicit construction of formfactors. Although they themselves do not describe realistic particle physics, their construction gives valuable nonperturbative insights into the inner workings of QFT which justifies calling them theoretical labs. Their wedge generators, for which we will from now on use the letter $Z(\theta)$ instead of $B(\theta)$,are not only temperate, but also obey simple commutation relations close to free fields. In the simplest case we have

$$\Phi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(Z^*(\theta)e^{ipx} + h.c.\right)\frac{d\theta}{2},\quad p = m(ch\theta, sh\theta) \tag{15}$$
$$Z(\theta_1)Z(\theta_2) = s(\theta_1-\theta_2)Z(\theta_2)Z(\theta_1),\ Z(\theta_1)Z^*(\theta_2) = s(\theta_1-\theta_2)Z^*(\theta_2)Z(\theta_2) + \delta(\theta_1-\theta_2)$$

The field is pointlike local if the s function is a constant; if it is a nonconstant function of modulus one (with some additional easy to satisfy analytic properties), the field is wedge-localized i.e. $\Phi(f), suppf \subset W$ is affiliated to $\mathcal{A}(W)$ [17][16]. These commutation relations (15) are of the Zamolodchikov-Faddeev type. Although these fields are, like free fields, particle number conserving and the nontrivial S-matrix (related to the function s) inherits this property, the generators of sharper localized double cone algebras which are

[27] In the standard formulation of QFT [34] the domains are Poincaré invariant so the bijectively related fields are not standard.

obtained by intersecting wedge algebras create the infinite vacuum polarization, clouds which remind us that we are confronting genuine interacting QFT and not relativistic QM.

The formfactors of these "factorizing models" (factorization of elastic n-particle $S_{scat}^{(n)}$ into two-particle amplitudes $S_{scat}^{(2)}$) are complicated rich function and have been explicitly computed for all families of factorizing models for which the bootstrap S-matrices have been determined previously. After the formfactors of the Z(N) and SU(N) family have recently been computed by Babujian, Foerster and Karowski [36][37], the same authors are hopeful that they will soon finish their formfactor calculations for the most subtle family: the O(N) models (private communication). In this context it is worthwhile to mention that in the 70's there was a very active collaboration on these matters between Germany and Brazil. Swieca's concretization of the principle of "nuclear democracy" in the context of the Z(N) [38] and SU(N) models [39] in the form of "*the antiparticle as the bound state of N-1 particles*" plays an important role in the computation of the formfactors of these models. From the recent above work one notices that although the volume of that collaboration went down, its quality is as impressive as ever.

The direct use of the formfactors in terms of an expansion of local fields into PFG generators $Z(\theta)$ fails on one point, namely one does not know the convergence status of these series (there are indications in favor of convergence). Therefore a different more abstract algebraic way has been taken; there the nonexistence would show up in the triviality of the double cone intersection $\mathcal{B}(\mathcal{O}) = C\mathbf{1}$ whereas the pointlike generation would reflect itself in the "standardness" of $\mathcal{B}(\mathcal{O})$ (acts cyclic on the vacuum and contains no annihilators). Successful proofs have been carried out by Lechner [16]. For the first time in the 90 year history one has the mathematical certainty about the existence of some QFT[28]. For the confirmation of the nontriviality of the intersection it was necessary to analyze the cardinality of degrees of freedom in the wedge algebra and establish that its cardinality is "nuclear" [16]. This problem is closely related to localization; already in the 60's Haag and Swieca [13] showed that the "finite number per phase space cell" known from QM does not hold in QFT based on causal localization; in the second case the cardinality is mildly infinite (the Haag-Swieca compactness criterion) and the later *nuclearity* criterion is a sharpening of this property [32]. Formally these criteria are fulfilled in Lagrangian quantization, but in holographic projections or restrictions to timelike submanifolds (branes) they are violated [8].

This modest success in the case of temperate PFGs suggests to return to the general case of nontemperate PFGs and ask the question of what can be added to the crossing property which could lead to unique formfactors without falling back to temperateness. We have no definite answer but it seems to be worthwhile to point out that even in the temperate case, the crossing property is not the sole analytic requirement since it only relates the n-component vacuum polarization components of an operator with the k-l (k+l=n) formfactors. As the KMS relation for thermal correlation of operators it says

[28]For fields with the same short distance dimensions as their free field counterparts (superrenormalizable interactions) existence proofs were given based on functional analytic methods known from QM. These are much further removed from the realistic case than the anomalous short distance dimension carrying factorizing models.

nothing about what happens if e.g. one interchanges two adjacent operators (two adjacent θ) by analytic continuation (and not by Bose statistics) since it only governs cyclic KMS changes. In the analytic setting of Karowski et al [40] the elastic two-particle S-matrix appears in such an exchange. As explained there the idea of analytic exchange goes back to the Watson theorem which states that the two-particle formfactor of a current has an analytic structure such that the different values on both sides of its elastic cut are determined by the elastic part of the S-matrix. The higher cuts (which do not exist in factorizing models) are expected to have similar connections with higher scattering contributions. Hence in contrast to crossing this *analytic exchange* property could be the additional property which should follows from the algebraic structure. It does so in the case of temperateness (15), where the commutation relation of Zs accounts fully for the analytic exchange and the crossing identity.

If the Watson argument has a generalization to inelastic cuts, one would expect the analytic change in the order of the rapidities to not maintain the same n of an n-particle formfactor but rather cause a branching to all the other $m \neq n$. This would be very remarkable because then "Murphy's law" of QFT which says *what can be coupled (i.e. allowed by superselection rules), is always coupled* and which holds in the off-shell field regime [31] would have an extension to formfactors. This would provide a much better picture about the vacuum and its polarization than the existing metaphoric one which views the vacuum as a broiling soup of particle/antiparticle pairs which for a short time outfox the energy conservation with the help of the uncertainty relation. Too hope for exact constructions of QFT outside of factorizing models is perhaps a bit too naive, although unlike for the theorems for the divergence of off-shell perturbative series, there exist no such statements for formfactors. It also could happen that, as in the case of [16] one finds an existence proof which uses solely the cardinality of degrees of freedom of wedge-localized algebras.

The question arises why did the S-matrix program, which once was considered the legitimate heir of the immensely successful but more modest dispersion relation project, go so wrong ? Why did the former find a successful closure after less then a decade whereas the latter, even after 5 unsuccessful decades, still roams over our heads like the proverbial homeless flying dutchmen? The final answer will have to be left to historians. But any particle theoretician of a sufficient age and independence will find it is hard to disagree on the following observations.

The eras of the QED renormalization, the dispersion relations, and the beginnings of the standard model are characterized by a *critical tradition* which, far from being just the cherry on the particle physics pie, was the necessary counterbalance against the unavoidable speculative aspects on its research frontiers. Later these critical voices[29] did not only disappear, but precisely those kind of individuals who, by their scientific achievements and reputation in earlier times would have been the standard bearers of this tradition, played increasingly the role of salesmen as can be seen by their slogans like "string theory is a gift of the 21 century which by luck fell upon the 20 century" or "string theory is the only game in town".

The dependency of novices of particle theory on their advisors and their globalized

[29]One of the most prominent representatives of the Streitkultur of the old world was Res Jost; his critical repudiation [41] of the presumptuousness in papers as [42] are high points of that culture.

communities whose primary concern is to foster the monoculture to which they subscribe and by which they themselves have been formed cements the hegemony at the cost of real scientific innovations. The immense pressure and the very narrow area of professional expertise limits their capabilities of forming a creative individual line of research. This may explain why flawed theories propagandized by reputable people have a chance to spread and stay for decades as in no times before, especially if the conceptual error is not easily identifiable with standard computational recipes of QFT. The globalization of errors is of course not limited to science, those made in globalized capitalism are more spectacular and harmful to our species.

It is interesting to note that young string theorist sometimes perceive that there are strange things happening in their community. For example a conjecture which becomes the theme of a community dialogue may end after many turns as a community-excepted theorem, even though nothing substantial has happened apart from several passages of the conjecture through the community. It still has the same form as at the beginning, nevertheless the growth of community confidence has transformed a conjecture into a theorem. Even though a young PhDs or postdocs in particle theory may not have the necessary conceptual-mathematical insight to analyze the scientific significance of such a situation, he can still look at the collateral sociological aspects and wonder about what's going on here [43].

Without the corrective element of the Streitkultur as it existed at the time of Pauli[30], Jost, Lehmann, Källén, Oppenheimer, Feynman, Landau,... particle theory would have entered collective misunderstandings or have moved towards intelligent and entertaining science journalism at a much earlier time; there were always speculative issues around which, if left to roam freely, could have derailed particle theory already in the 60s or 70s. The disappearance of that tradition of a Streitkultur was not really felt before the 80s. Many physicists of the older generation who did not make critical remarks in public (as e.g. Steven Weinberg) voted meanwhile with their feet and looked for other potentially more promising and healthy areas outside of high energy physics, which led to an additional loss of critical power.

Nowdays many particle theorists, including those string theorists who have become weary of the "no other game in town" kind of propaganda, now look forward to observational results at the LHC; but it is not clear whether experimental results can rescue a theoretical situation which was muddled by theoreticians; to take a community out from a self-inflicted dead end, especially if its ideas became entrenched at the Planck length is probably outside their abilities. Experiments have never been used for this purpose before, they mainly served to discriminate between competing internally consistent proposals within an at the time accessible energy range. There is a good chance that the LHC experiments cannot fulfill the high theoretical expectations.

I have criticized the various S-matrix projects of which the last one, namely the dual model/string theory, can be traced back to ideas of Mandelstam. But there is some irony in my criticism, because the approach in local quantum field theory which I advocated here does not follow the pure doctrine of local quantum physics, rather the S-matrix and on-shell formfactors enter in an essential way; in fact besides its obvious role in scattering

[30]There are still arguments among the older surviving members of that glorious epoch about whether Pauli overdid it in certain occasions [9].

theory S_{scat} plays a more discrete and harder to assess role as a *relative modular invariant* for the wedge region which is totally specific for QFT. To avoid being misunderstood, a pure S-matrix theory without using at the same time formfactors will in my opinion never be possible[31], even if the uniqueness of the inverse problem of QFT can be established (for which there are very good reasons even outside factorizing models). But I cannot deny that in the approach I am advocating the S-matrix, which often has been called the roof of a relatistic quantum theory, is used from the beginning as a constructive tool, together with formfactors which are associated with wedge-localization objects.

According to the tenets of pure local quantum physics [32], a QFT should be constructed by starting with the structure of its local "germs" (the putative rigorous concept replacing the Lagrangian interaction density). But the results emerging from the modular wedge localization seem to favor a compromise between Mandelstam's radical nonlocal on-shell doctrine [44] and Haag's foundational idea around local "germs" [32]. Although full and partial on-shell quantities as the S-matrix and formfactors have, in contrast to compactly localized objects as germs, simpler properties in particular when it comes to vacuum polarization, one needs the "germs" for the conceptual understanding of the properties which relate formfactors among themselves and with the S-matrix, in particular because, as the study of factorizing models has shown, the cardinality of phase space degrees of freedom which is necessary for a physically acceptable QFT seems to require the study of the local germs.

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