ISSN 0029-3865

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS
Rio de Janeiro

Notas de Física

CBPF-NF-004/09
December 2009

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Ministério da Ciência e Tecnologia
BRASIL UM PAÍS DE TODOS
GOVERNO FEDERAL

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# On a CP Anisotropy Measurement in the Dalitz Plot


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**Abstract**

We describe a novel use of the Dalitz plot to probe **CP** symmetry in three-body modes of $B$ and $D$ mesons. It is based on an observable inspired by astronomers' practice, namely the significance in the difference between corresponding Dalitz plot bins. It provides a model independent mapping of local **CP** asymmetries. We illustrate the method for probing **CP** symmetry in the two complementary cases of $B$ and $D$ decays: in the former sizable or even large effects can be expected, yet have to be differentiated against leading Standard Model contributions, while in the latter one cannot count on sizable effects, yet has to deal with much less Standard Model background.


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# Contents

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# 1 Prologue

While the announcement of the 2008 Nobel Prize in Physics has made official the status of KM theory as the main source of **CP** violation as observed in $K$ and $B$ decays, it does not close the chapter on it for three main reasons [1]:

- We know baryogenesis in our Universe requires New Physics with **CP** violation.

- A host of largely theoretical arguments suggests – persuasively in our view – that New Physics exists around the TeV scale with rich dynamical structures. In general those can provide several novel sources of **CP** violation. In that context one uses the high sensitivity of **CP** studies as a tool to search for New Physics and hopefully infer some of its salient features. One should keep in mind that observable **CP** asymmetries can be *linear* in a New Physics amplitude with the Standard Model (SM) providing the other one; therefore one achieves sensitivity to small contributions.

- The LHCb experiment [2] is poised to acquire large sets of high quality data on the decays of $B$ and $D$ mesons.

The stage for **CP** studies has recently become wider with the observation of $D^0 - \bar{D}^0$ oscillations [3, 4]. Fortunately we can expect to continue our quest for **CP** violation with the continuing work of the Belle Collaboration, the hoped for realization of a Super-B Factory [5] and in particular with the beginning of the LHCb experiment.

We will focus on three-body final states in the decays of $B$ and $D$ mesons and a novel strategy to probe **CP** symmetry in their Dalitz plots. Those two classes of transitions offer complementary challenges both on the experimental and theoretical side.

- KM dynamics is expected to generate large **CP** asymmetries – to the tune of, say, 10 - 20 % – in modes like $B \to \pi\pi\pi$, $K\pi\pi$, $Kp\bar{p}$. Since those command very small branching ratios, we still need very large samples of $B$ mesons, as will be produced by the LHCb experiment. While we can be confident that such effects will be observed (supporting the experimentalists' enthusiasm), the real challenge at that time will be how to interpret a signal: does it reveal the presence of New Physics or is it consistent with being from KM dynamics alone? This will represent a non-trivial conundrum, since all the available evidence points to New Physics mustering no more than *non-leading* contributions in $B$ decays; this is often referred to as the "Flavour Problem of New Physics". We will have a realistic chance to answer this question only if we have accurate as well as comprehensive data.

- On the other hand in most *charm* decays experimental bounds tell us we can hope for at best moderate size asymmetries even in the presence of New Physics. The redeeming feature is that the SM can generate **CP** violation at most at the $10^{-3}$ level in the 'best' cases and even significantly less in others. However we view the latter as good news: for almost any asymmetry observed in the near future would represent strong evidence for the intervention of New Physics. Using the language of 'signal-to-noise' familiar from experimental studies: while we expect significantly smaller asymmetries in $D$ than in $B$ decays, the *theoretical* 'noise' or 'background' – i.e. contributions from the SM – is even more reduced in the former than the latter. Thus we conjecture

$$\left.\frac{\textbf{CP}\text{ asymmetry}}{\text{SM contribution}}\right|_{\textbf{D}\text{ decays}} > \left.\frac{\textbf{CP}\text{ asymmetry}}{\text{SM contribution}}\right|_{\textbf{B}\text{ decays}} \tag{1}$$

The charm branching ratios are typically sizable and we have already acquired a great deal of experience in describing them and their Dalitz plots. The central challenge then is to control systematics to a degree that allows probing asymmetries down to $10^{-2}$ or even better.

So far no **CP** asymmetry has been *established* on the five sigma significance level. However we expect that to change soon and actually predict Dalitz studies to become one of the central tools for **CP** probes.

- *Accurate* measurements of **CP** asymmetries will be a necessary (though probably not sufficient) condition for deciding whether they reveal the intervention of New Physics or not. In the case of charm decays the anticipated small size of effects constitutes the main challenge; in $B$ decays, on the other hand, we have to learn how to subtract the presumably leading SM contribution. Dalitz plot descriptions with their many correlations yield *over*constraints, providing reliable validation tools. Tracking the time evolution in $D^0$, $B_d$ and $B_s$ transitions characteristic of oscillations will further illuminate the underlying dynamics.

- Establishing the intervention of New Physics in **CP** studies will of course represent a seminal achievement, yet we want to do even better. For our goal has to be to infer salient features of that anticipated New Physics. Some of those can be read off the flavour structure of the final states. Yet to infer the Lorentz structure of the underlying operator we have to go beyond final states consisting of two pseudoscalar or one pseudoscalar and one vector meson; for their amplitudes are described by a single number. However, once we have three pseudoscalar mesons or a baryon-antibaryon pair plus a meson, the kinematics is no longer trivial, and final state distributions can tell us whether spin-zero or spin-one couplings are involved in the transition operator. This important feature will be illustrated below.

Fortunately pioneering work has been done by Belle and BaBar: Based on a Dalitz plot study they have extended their probe of **CP** invariance to quasi-two-body channels involving resonances and indeed found intriguing *evidence* for a direct **CP** asymmetry in the mode $B^\pm \to K^\pm \rho^0$ [6, 7, 8], which – if established – would be a first.

Relying on mass projections is only one way to use the dynamical information contained in a Dalitz plot, and it cannot be expected to harness the full potential of Dalitz studies.

It is the *interference* between two *neighbouring resonances* that presumably provides the most sensitive **CP** probe. For a direct **CP** asymmetry to surface one needs the interplay of a weak and a strong phase with the former in contrast to the latter changing signs under a **CP** transformation. For the latter one usually takes the strong phase shifts, which cannot be calculated from first principles. Yet when one deals with a finite-width resonance, its Breit-Wigner parameters can provide the required strong phase, which varies with the mass bin in a characteristic and largely predictable way. This feature can provide further validation for the experimental findings. Alas – a full-fledged Dalitz plot description requires huge statistics and considerable theoretical 'overhead' in selecting the transitions deemed relevant and parametrizing their amplitudes. It has to be the ultimate goal to develop such a complete description with as much accuracy as possible, yet that will be a long term task, and it is not clear what irreducible model dependence will remain. A full Dalitz plot description would help in the extraction of the CKM phases [9, 10]. There are, however, some theoretical issues concerning the large phase space of the B decay into three light mesons [11] that need to be understood.

To avoid such model dependence one can divide the Dalitz plot into bins, and then directly compare the **CP** conjugate Dalitz plot regions in a bin-by-bin basis. Yet results based on studying the ratio between the difference over the sum of the populations are quite vulnerable to fake effects from statistical fluctuations. Therefore we suggest a refinement of such a direct comparison, namely to study the *significance* of the difference. This proposal has been inspired by what has become standard routine in astronomy when analyzing light sources in the sky [12]. Its main values lies in three aspects:

- As illustrated later it provides a model-independent and robust method to determine **CP** asymmetries already with limited statistics and identify the regions of a Dalitz plot, where they occur.
- This is particularly important when dealing with small or even tiny effects as expected in charm decays.
- Its findings provide powerful constraints on any full Dalitz plot model to emerge.

In talking about 'limited' statistics we do *not* mean *small* statistics – a situation addressed in [13]. Since our method involves analyzing *distributions* even in sub-domains of the Dalitz plot, it requires substantial data sets.

It has been estimated that LHCb will collect very sizable data sets of three-body decays already in one nominal LHC year:

- about $10^6$ singly Cabibbo suppressed $D^\pm \to \pi^\pm\pi^+\pi^-$ and $10^5$ doubly Cabibbo suppressed $D^\pm \to K^\pm\pi^+\pi^-/K^\pm K^+K^-$;
- around $10^5$ $B^\pm \to K^\pm\pi^+\pi^-$, $B^\pm \to K^\pm K^+K^-$ and $B^\pm \to \pi^\pm\pi^+\pi^-$;
- more than $10^4$ $B^\pm \to \pi^\pm K^+K^-$ and the baryonic modes $B^\pm \to K^\pm p\bar{p}$, $\pi^\pm p\bar{p}$.

The paper will be organized as follows: in Sect. 2 we will briefly review the basics of Dalitz plot analyses, introduce a novel observable for probing **CP** symmetry there and comment on the isobar model; in Sect. 3 we apply it to $B$ mesons decaying into three light mesons and present Monte Carlo studies; in Sect. 4 we discuss analogous $D$ decays where one has to face rather different challenges; in Sect. 5 we present expectations about in which direction and to which degree relevant theoretical tools might get refined in the near future before summarizing and giving an outlook in Sect. 6.

# 2 Basics and Virtues of the Dalitz Plot

## 2.1 Basics

It is of course a mathematical triviality that *local* asymmetries are bound to be larger than fully integrated ones. Yet a Dalitz plot description translates such a general qualitative statement into a much more concrete one. For it exhibits all that can be learnt directly from the data on final states of three stable particles and their dynamics. Since the phase space density of the Dalitz plot is constant, any observed structure reflects the dynamics of the decay. Enhanced populations in certain mass regions can reveal the presence of a strong resonance and indicate their widths. The angular distributions characteristic for the spin of the resonances modulate the mass bands. Distorted or twisted mass bands point to the interference between resonances. These observations can be cast into a quantitative treatment by making an ansatz for the final state amplitude consisting of terms describing the moduli and complex phases of the contributing resonances and the non-resonant contribution. These entities contain a great deal of subtle dynamical information. Comparing them for **CP** conjugate transitions provides a very powerful probe of **CP** invariance. While

**CP** violation has to enter through complex phases on the *fundamental* level of the underlying dynamics, it can manifest itself in the Dalitz plot through differences in both the aforementioned moduli of the hadronic resonances and their phases for conjugate transitions. Since there are typically several resonances contributing to a decay, ample opportunities arise for **CP** violation to surface in a Dalitz plot. Hadronic 'complexities' thus represent good news for the *observability* of **CP** asymmetries. They become a challenge only, when one undertakes to *interpret* a signal in a *quantitative* way. Yet even there a Dalitz analysis provides essential assistance: the reliability of Dalitz plot parametrizations can be inferred from the amount of *over*constraints they manage to satisfy.

However Dalitz studies still retain a measure of model dependance due to the choices one makes concerning the resonances to be included and their parameterization and also due to the treatment of the non-resonant contribution; the S-wave is the largest source of systematics due to strong dynamics. The greatly different phase space available in $B$ and $D$ decays makes for an almost qualitative difference in how to treat them. We will comment on it later.

While we maintain that such model dependencies can be reduced considerably with increasing data sets and, more important, with future theoretical insights, we want to propose a novel method for searching for **CP** asymmetries in three-body final states that is robust in two respects: it requires no model assumptions and provides an effective filter against effects due to statistical fluctuations. Yet first we will make a few rather technical remarks on how various phases enter the interference between neighbouring resonances.

## 2.2 Phases with Breit-Wigner Resonances

Due to **CPT** invariance **CP** violation can express itself only via a complex and presumably weak phase. For it to become observable, we need the interference between two different, yet still coherent amplitudes. Oscillations can provide such a scenario – as can hadronization in general. The latter case is usually expressed by stating that the two amplitudes have to exhibit different weak as well as strong phases: $\mathcal{M} = e^{i\phi_1^{we}} e^{i\delta_1^{st}(f)}|\mathcal{M}_1| + e^{i\phi_2^{we}} e^{i\delta_2^{st}(f)}|\mathcal{M}_2|$, with $(\phi_1^{we}, \delta_1^{st}(f)) \neq (\phi_2^{we}, \delta_2^{st}(f))$. We can write it also in terms of phases that combine the weak and strong phases:

$$\mathcal{M} = e^{i\delta_1(f)}|\mathcal{M}_1| + e^{i\delta_2(f)}|\mathcal{M}_2| \; , \quad \delta_i(f) \equiv \delta_i^{st}(f) + \phi_i^{we} \tag{2}$$

$$\overline{\mathcal{M}} = e^{i\bar{\delta}_1(f)}|\mathcal{M}_1| + e^{i\bar{\delta}_2(f)}|\mathcal{M}_2| \; , \;\; \bar{\delta}_i(f) \equiv \delta_i^{st}(f) - \phi_i^{we} \tag{3}$$

It is often implied that the strong phases $\delta_i^{st}(f)$ carry a fixed value for a given final state $f$. This does not need to be true. More specifically it will definitely not hold when the final state contains a resonance. The Breit-Wigner excitation curve for a resonance $R$ reads

$$F_R^{\rm BW}(s) = \frac{1}{m_R^2 - s - im_R\Gamma_R(s)} \; , \tag{4}$$

introducing a sizable phase as expressed through

$$\mathrm{Im}F_R^{\rm BW}(s) = \frac{m_R\Gamma_R(s)}{(m_R^2 - s)^2 + (m_R\Gamma_R(s))^2} \; , \tag{5}$$

where $\Gamma_R(s)$ denotes the energy dependent relativistic width. In our discussion of $B^\pm \to K^\pm\pi^+\pi^-$ we will focus on the interference between $\rho^0$ and $f_0$ to generate a **CP** asymmetry. The relevant amplitude components for $B^+$ and $B^-$ are:

$$\mathcal{M}_+ = a_+^\rho e^{i\delta_+^\rho} F_\rho^{\rm BW} \cos\theta + a_+^f e^{i\delta_+^f} F_f^{\rm BW} \tag{6}$$

$$\mathcal{M}_- = a_-^\rho e^{i\delta_-^\rho} F_\rho^{\rm BW} \cos\theta + a_-^f e^{i\delta_-^f} F_f^{\rm BW} \tag{7}$$

The $\delta_\pm$ contain both the fixed weak and the strong phases with the Breit-Wigner functions $F^{\rm BW}$ introducing additional mass dependent strong phases as sketched above. For the $f_0$ we have followed BaBar's treatment [6] using the Flattè representation, which reflects the proximity of the $K\bar{K}$ threshold and the ensuing distortion of the resonance curve. In Eqs.(6,7) above $\theta$ is the angle between the $\pi^-$ and the $K^+$ momenta, measured in the $\rho$ rest frame. This angle describes the angular distribution of a vector meson.After taking the modulus square of these amplitudes one reads off that a **CP** asymmetry will arise, when there are non-zero *weak* phases.

Since charm decays proceed in an environment of virulent final state interactions, an absence of strong phase shifts in $D$ decays is the least of our concern, since it would happen only 'accidentally'. Yet in the presence of hadronic resonances it becomes even a 'mute' point, since the resonance provides a mass dependent strong phase that is predictable in most cases and thus actually helps to validate a signal. Resonances then create the more favourable scenario.

Ideally we would apply the method proposed by us (see below) to real primary data. Unfortunately we do not have access to those. Therefore we start out by using models that are *consistent* with *existing* data to create a Monte Carlo Dalitz plot; for $B^\pm \to K^\pm\pi^+\pi^-$

we have been thus 'inspired' by Babar [6] and for $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ by E791[14]. Then we create by hand a single 'seed' for a **CP** asymmetry and analyze whether our method can uncover it; subsequently we vary that single seed.

We will employ the *isobar* model [15] for constructing Dalitz plots. The amplitude for resonant sub-processes is expressed through Breit-Wigner functions multiplied by angular distributions as determined by their spins. The amplitudes of all contributing sub-processes are combined coherently with complex coefficients. The latter represent free parameters that are fixed from the data using a maximum likelihood fit: the magnitudes of the complex coefficients are related to the fractional contributions of each sub-channel and their relative phases reflect the final state interactions between the resonances and the 'bachelor' particles. In Eqs. (6,7) we have exemplified the general procedure by writing down the amplitude for $B^{\pm} \to K^{\pm}\rho^{0}/K^{\pm}f_0$. These relative phases are treated as constant, since they depend only on the total mass of the system, which in this case amounts to the mass of the decaying heavy meson. The non-resonant three-body contribution is usually assumed to be flat over the Dalitz plot or at least described by a smooth distribution.

## 2.3 The Novel Proposal

The challenge we have to deal with in comparing Dalitz plot populations is one of unbiased pattern recognition. It is thus analogous to one faced routinely by astronomers: they often search for something they do not quite know what it is – at least initially – at a priori unknown locations and having to deal with background sources that are all too often not really understood. This sounds like a hopeless proposition, yet astronomers have been successful in overcoming these odds. Thus we should be eager to learn from them.

The Pierre Auger observatory has already adopted the same method for statistical weighting in their searches for cosmic ray sources, and we propose to follow suit in defining a search strategy for **CP** asymmetries in Dalitz analyses: rather than study the customary *fractional* asymmetry

$$\Delta(i) \equiv \frac{N(i) - \bar{N}(i)}{N(i) + \bar{N}(i)} \tag{8}$$

in particle vs. anti-particle populations $N(i)$ and $\bar{N}(i)$ for each bin $i$, respectively, one should analyze the *significance*

$$^{\mathrm{Dp}}S_{CP} \equiv \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}} \ , \tag{9}$$

which amounts to a standard deviation for a Poissonian distribution [1]. We will demonstrate below through Monte Carlo studies of $D$ and $B$ decays that analysis of the significance $\sigma$ provides a more robust probe of **CP** symmetry. We will illustrate how the observable ${}^{\rm Dp}{\rm S}_{CP}$ is highly effective in filtering out genuine asymmetries from statistical fluctuations.

A final technical comment concerning binning size: in the studies presented below we have required bins to contain at least twenty events. This number appears 'reasonable', but is somewhat ad-hoc. Applying our method to real primary data in the future should shed light on the appropriateness of this lower bound.

### 2.4 First Summary of the Advantages of Our Proposal

Analyses of Dalitz plots have so far not 'bagged' any success in *establishing* **CP** violation. Even so we expect them to become central probes of **CP** invariance due to the following features:

- Local asymmetries are bound to be larger than integrated ones thus facilitating the task of controlling systematic uncertainties.
- The latter – either due to production asymmetries or to detection inefficiencies – can be probed and controlled through the analysis of ratios of particle yields.
- The bin observable ${}^{\rm Dp}{\rm S}_{CP}$ defined in Eq.(9) does not suffer from any model dependance and allows a robust search for asymmetries that are small or in relatively small samples.
- This procedure does not represent a diversion on the (long) path to the ultimate goal, namely to arrive at a complete Dalitz plot description and all it can teach us. On the contrary – it will accelerate our progress on that journey providing us with increasingly powerful pointers for where to focus our attention and constraints for the Dalitz parametrizations.

In the following we will present case studies of $B$ and then $D$ decays to illustrate the general method.

[1] We will refer to analyzing ${}^{\rm Dp}{\rm S}_{CP}$ instead of $\Delta(i)$ as adopting the 'Miranda' procedure or as 'mirandizing' the **CP** search.

# 3 $B$ Decays

## 3.1 General Remarks

Decays $B \to h_1 h_2 h_3$ with $h_i = \pi$, $K$ exhibit a pattern in their Dalitz plots that at first sight might look surprising: the bands near the edges are crowded while the interior is sparsely populated. Yet on second thought this is as expected. For the phase space available in $B$ decays is quite large, in particular for non-charm final states. Those will typically consist of significantly more than three stable mesons. For three meson final states the two primary $\bar{q}q$ clusters produced in the $B$ decays have to recede from each other quickly with untypically low masses; thus they generate the pattern sketched above.

That final states consisting of just two or three pseudoscalar mesons are a rather untypical subset of nonleptonic $B$ decays can be seen also in another way: it has been firmly established that the lifetime of charged $B$ mesons exceeds that of neutral ones [16]: $\tau(B^+)/\tau(B_d) = 1.071 \pm 0.009$ — in agreement with already the first fully inclusive theoretical treatment based on the operator product expansion, which traces this difference back mainly to a *destructive* interference in nonleptonic $B^+$ decays [17]. Yet when one sums over the $B \to D\pi$, $B \to D^*\pi$ and $B \to D\rho$ channels one finds that there the $B^+$ width exceeds that for $B_d$ by about a factor of two! This is in marked contrast to the case of $D$ mesons where the sum of the partial widths for $D \to K\pi$, $K^*\pi$ and $K\rho$ already exhibit the same pattern as the total widths.

There are many modes that carry considerable promise to reveal **CP** violation and shed light on the underlying dynamics. We will focus on just one $B$ (and later on just one $D$) mode in this note for two reasons: the pedagogical one that we do not want to 'over-feed' the reader; and the very practical one that so far little experimental information exists about these $B$ decays. In this spirit we will discuss $B^\pm \to K^\pm \pi^+ \pi^-$. This channel is predicted to have a large component from the Penguin operator. Since that operator is derived from a loop process — i.e., a pure quantum effect — it represents a wide gateway for New Physics. One should also note that the neutral two-body counterpart $B_d \to K^+ \pi^-$ has already shown a direct **CP** asymmetry [18].

Observing a **CP** asymmetry here is unlikely to be the main challenge – that role is reserved for the question whether an observed signal is generated by CKM forces alone or requires the intervention of New Physics that probably provides merely a non-leading contribution. We know of no model-independent way to settle this issue and thus have to rely on theoretical treatments that are based on more than just basic features of QCD, yet still require model assumptions not (yet) derived from QCD.

## 3.2 $B^\pm \to K^\pm\pi^+\pi^-$

We will describe this case in considerable detail, since it commands a relatively large branching ratio compared to other charmless final states and there is strong evidence for a direct **CP** asymmetry associated with the $B^\pm \to K^\pm\rho^0(770)$ sub-channel [6, 7]. It also provides a clear illustration of the power of our method.

The moduli and phases of its amplitudes are 'inspired' by BaBar's results [6]. We include five resonant and one non-resonant contribution; the latter is assumed to be flat over the Dalitz plot purely for reasons of convenience and the lack of a specific alternative. We analyze two versions each with a *single seed* of **CP** violation, namely one with a **CP** asymmetry in the overall phase for the $\rho^0(770)$ and the other one for the $f_0(980)$. To provide a clear demonstration of our method we start out by assuming the phase of the $B^+ \to K^+\rho^0(770)$ relative to $B^- \to K^-\rho^0(770)$ to be large, namely $60^o$, which is still allowed by the data [6]. Then we analyze two cases with a significantly smaller phase difference, namely $20^o$ and $10^o$, respectively. In the latter two cases neither a visual inspection of the Dalitz plot nor using the fractional difference $\Delta(i)$ suffice to establish the resulting **CP** asymmetry. Yet an analysis of the significance ${}^{\rm Dp}{\rm S}_{CP}$ allows even to locate the origin of the asymmetry in the Dalitz plot.

### 3.2.1 Model "$\rho^0$"

The specifics of this version are listed in Table 1. For diagnostic clarity we pick two sets of amplitudes for $B^+$ and $B^-$ decays, shown in Table 1, that differ in a single parameter only, namely the *phase* of the $\rho(770)K$ amplitude, while all moduli of the amplitudes are the same.

With these parameters the signal amplitudes for the $B^+$ and $B^-$ are integrated over the Dalitz plot, yielding a direct **CP** asymmetry of about $3 \times 10^{-3}$. For a sample with 300K $B^+ \to K^+\pi^+\pi^-$ decays this corresponds to 298K $B^- \to K^-\pi^+\pi^-$ events. At first one might think that the fully integrated rate can show no difference for the $B^+$ and $B^-$ channels, when the only seed of **CP** violation planted into the Monte Carlo model is a

| mode | $a^+$ | $\delta^+$ | $a^-$ | $\delta^-$ |
|---|---|---|---|---|
| $K^*(890)\pi$ | 1.0 | 0.00 | 1.00 | 0.00 |
| $K(1430)\pi$ | 2.1 | 6 | 2.1 | 6 |
| $\rho(770)K$ | 0.9 | **-34** | 0.9 | **26** |
| $f_0(980)K$ | 1.0 | 132 | 1.0 | 132 |
| $\chi_c K$ | 0.3 | -143 | 0.3 | -143 |
| $NR$ | 0.6 | -109 | 0.6 | -109 |

Table 1: Magnitudes and phases, in degrees, of the amplitudes defining Model "$\rho^0$" for our toy Monte Carlo sample. The difference in the $\rho(770)$ *phase* for the $B^+$ and $B^-$ channels provides the only source for a genuine **CP** violation.

difference in the overall phase of the $\rho$ contribution. Yet the small direct **CP** asymmetry is due to the interference of the triangle – pun intended – of $K\rho$, $Kf_0$ and $K^*\pi$ amplitudes.

For the $B^+$ and $B^-$ samples we assume a background of about 200K events. The resulting Dalitz plots are shown in Fig.1. They do not look quite the same. To make their differences more explicit we have plotted the fractional asymmetry $\Delta(i)$ of Eq.8 bin for bin in Fig.2. The resulting display is a very noisy one with many bins showing sizable differences, both in the $\rho - f_0$ interference region, where our model has to yield a genuine asymmetry, and in the central region, where it can*not*.

The 'eager' eye might notice that the differences in the former follow a slightly more systematic pattern than in the latter, yet it could not be called compelling, in particular if we did not know the underlying dynamical structure.

The effect of the statistical fluctuations can be ilustrated by the following exercise. We plot in Fig.3 the significance distribution for a situation where the $B^+$ and $B^-$ Dalitz plots were generated with exactly the same set of parameters. In this case only statistical fluctuations are observed. The upper display shows most bins to exhibit some differences; yet the fact that the ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ distribution is completely consistent with a pure Gaussian pattern, as shown in the lower display, reveals them to be consistent with mere statistical fluctuations.

After our method has successfully passed this null test we return to the model defined

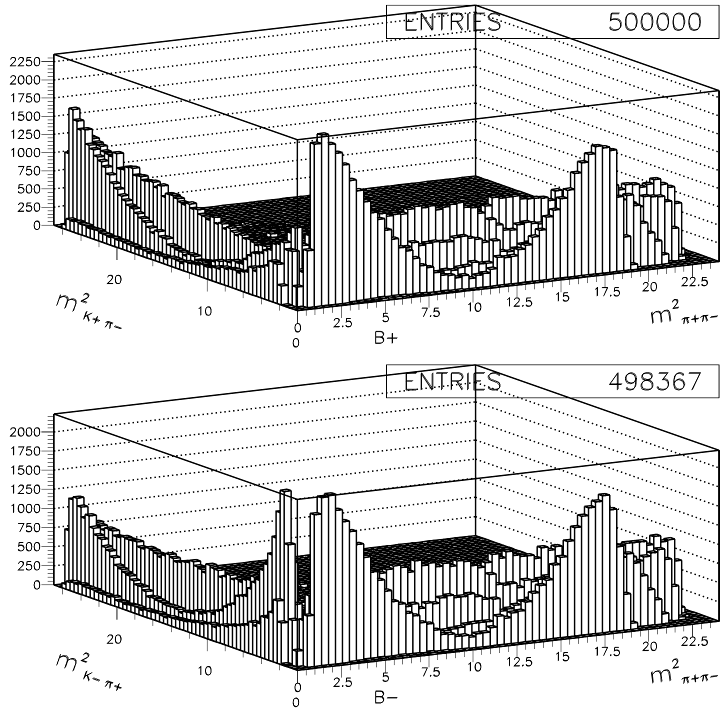


Figure 1: Dalitz plot distribution for $B^+ \to K^+\pi^+\pi^-$ (top) and $B^- \to K^-\pi^+\pi^-$ (bottom) in Model "$\rho^0$".

by Table 1 and Fig 1. To obtain a clearer picture we 'mirandize' our analysis, i.e. turn to the significance ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ defined in Eq.9. We plot the resulting values for ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ in the upper display of Fig.4 and its distribution together with a Gaussian fit in the lower display.

The Dalitz plot of the significance ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ shows a considerably less noisy pattern than before with an obvious asymmetry surfacing in the $\rho$ - $f_0$ interference region. The fact that a genuine **CP** asymmetry has surfaced in the Dalitz plot is demonstrated in the lower display: there is no acceptable Gaussian fit to the ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ distribution meaning the asymmetries are considerable larger than can be generated by statistical fluctuations.

Our ambition has of course to go further than just knowing that somewhere in the Dalitz plot there is a true **CP** asymmetry – we want to determine in which subdomain(s) it resides and whether it is due to an interference between neighbouring resonances or due to different widths of two **CP** conjugate resonances. For that diagnosis we divide the Dalitz plot region into subdomains. The choice of these subdomains has to be informed by our understanding

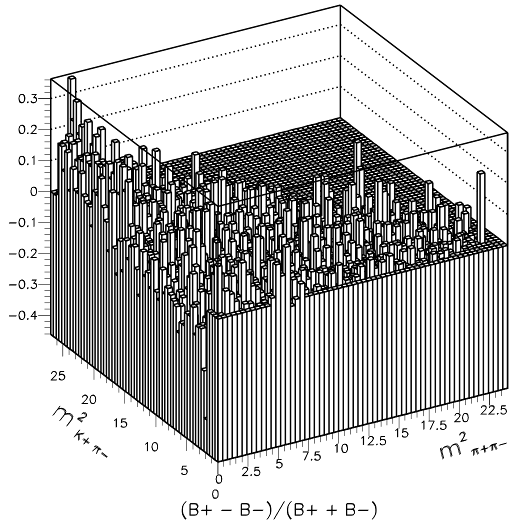


Figure 2: Asymmetry in the Dalitz plot bins for Model "$\rho^0$" as defined by Table 1.

of the significant subprocesses. In the case of $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ we divide it into the four regions shown in Fig.5: I and II containing the $\rho(770)$ resonance, III with the $K\pi$ resonances and IV populated mainly by background. In Fig.6 we have plotted the $^{\mathrm{Dp}}\mathrm{S}_{CP}$ distributions separately for these sub-domains. The results are very telling: the plot clearly reveals that the asymmetry resides in regions I and II, while III and IV show no trace of a genuine **CP** asymmetry – in full agreement with the underlying model chosen to generate these Dalitz plots.

### 3.2.2 Model "$f_0$"

In this version we use the same model parameters as above (see Table 1) with two essential differences: $\delta^+ = \delta^- = -34^o$ for the $\rho^0(770)$; $\delta^+ = 132^o \neq \delta^- = 69^o$ for the $f_0(980)$, i.e., a phase difference of 63 $^o$. Again such a difference is quite compatible with BaBar's findings [6].

The $B^+$ and $B^-$ Dalitz plots are shown in Fig. 7. One can see that the two plots are different. Turning to a plot of the fractional asymmetry $\Delta(i)$ shows there are many bin-by-bin asymmetries, yet those exhibit again a rather noise pattern, see Fig. 8a. Once again 'mirandizing' the display, i.e., plotting $^{\mathrm{Dp}}\mathrm{S}_{CP}$ instead of $\Delta(i)$, leads to a more organized message, shown in the upper display in Fig. 8b. In particular, when looking at the $^{\mathrm{Dp}}\mathrm{S}_{CP}$

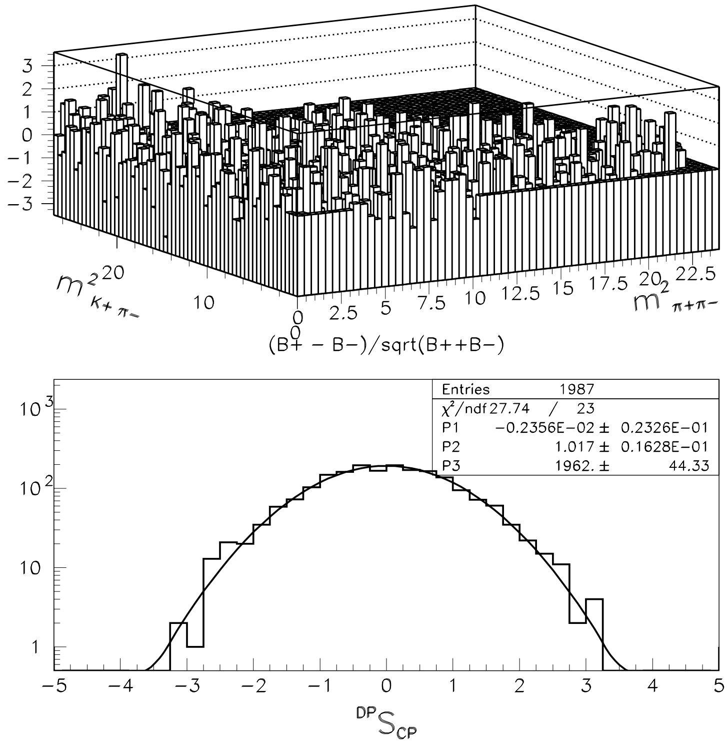


Figure 3: Top: Significance $^{\mathrm{Dp}}\mathrm{S}_{CP}$ plot for two CP conserving 300K signal + 200K background samples for CP symmetric decays. Bottom: Gaussian fit for the $^{\mathrm{Dp}}\mathrm{S}_{CP}$ distribution; $P1$, $P2$ and $P3$ denote the fit values for the central value, width and normalization parameter, respectively.

distribution of Fig. 9 we see that over and above the statistical fluctuations there is a genuine **CP** asymmetry.

As before its location can be narrowed down further by dividing the Dalitz plot in the four regions of Fig. 5 and plotting the $^{\mathrm{Dp}}\mathrm{S}_{CP}$ distributions separately for them, see Fig. 9. It clearly identifies regions I and II as the main origin of the asymmetry. That is as it has to be, since the interference between the $K\rho$ and $Kf_0$ amplitudes, which is the "engine" of **CP** violation in our model, takes place mainly there.

### 3.2.3 Comparing the "$\rho^0$" and "$f_0$" Models

The preceding discussion has shown that the $^{\mathrm{Dp}}\mathrm{S}_{CP}$ observable and its distribution provides a powerful tool that in a model independent way allows to establish the existence of a genuine

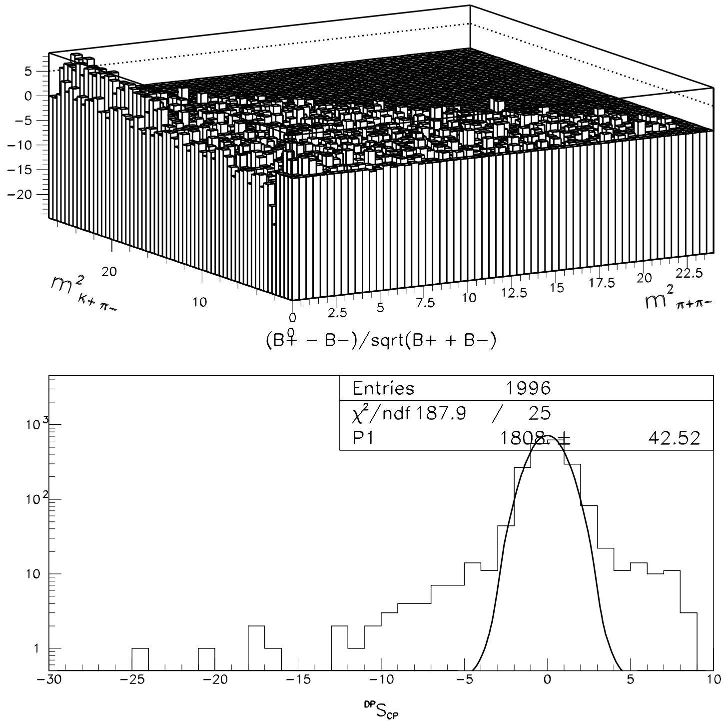


Figure 4: Top: Significance $^{\rm Dp}{\rm S}_{CP}$ plot for $B^{\pm} \rightarrow K^{\pm}\pi^{\mp}\pi^{\pm}$ for model "$\rho^0$". Bottom: $^{\rm Dp}{\rm S}_{CP}$ for the bins in Top Figure that pass the statistical cut, fit to a centred Gaussian with unit width. P1 is the normalization parameter.

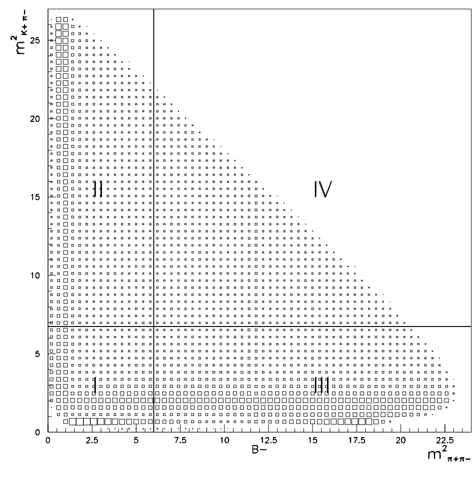


Figure 5: $B^{\pm} \rightarrow K^{\pm}\pi^{\mp}\pi^{\pm}$ Dalitz plot for model "$\rho^0$" divided into regions.

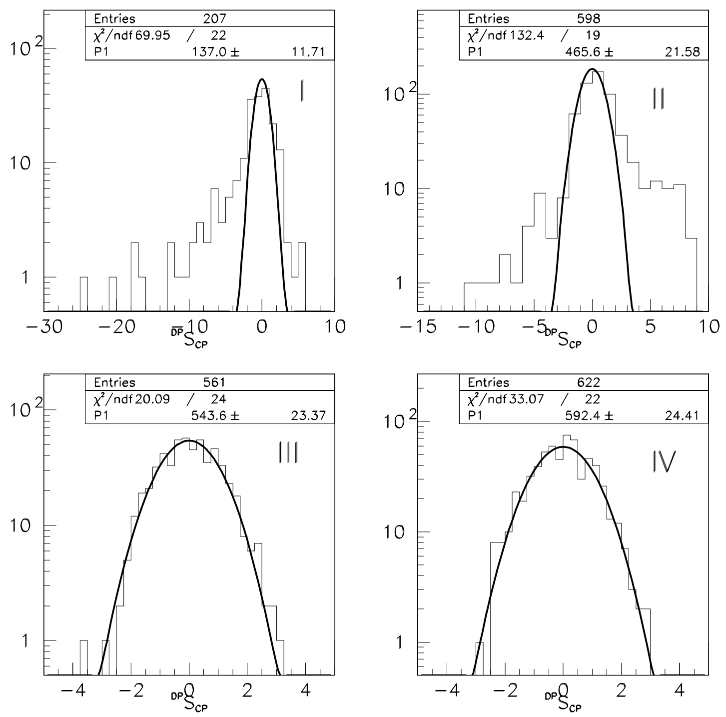


Figure 6: Distribution of figure 4 divided in the regions shown in figure 5. P1 is the normalization parameter.

**CP** asymmetry over and above statistical fluctuations and even determine the subregion(s) of the Dalitz plot, where it originates. For both the two Dalitz models employed above it was mainly the $\rho - f_0$ interference domain.

In addition, a closer analysis allows to distinguish the cases where the asymmetry is driven by a difference in the $K\rho$ and in the $Kf_0$ phase, respectively, for the $B^+$ and $B^-$ decays, see Figs. 6 and 9b. The discriminator is provided by the interference with the 'silent' partner, the $K^*\pi$ amplitude. This ability would provide important diagnostics about the underlying dynamics: for it would enable us to decide whether the **CP** odd operator generating the asymmetry carries vector or scalar quantum numbers.

### 3.2.4 The case of a smaller $\rho$ phase difference of $20^o$ and $10^o$

The overall phase differences we have assumed for the two models employed above were rather large, although still consistent with present data. Consequently even an unsophisti-

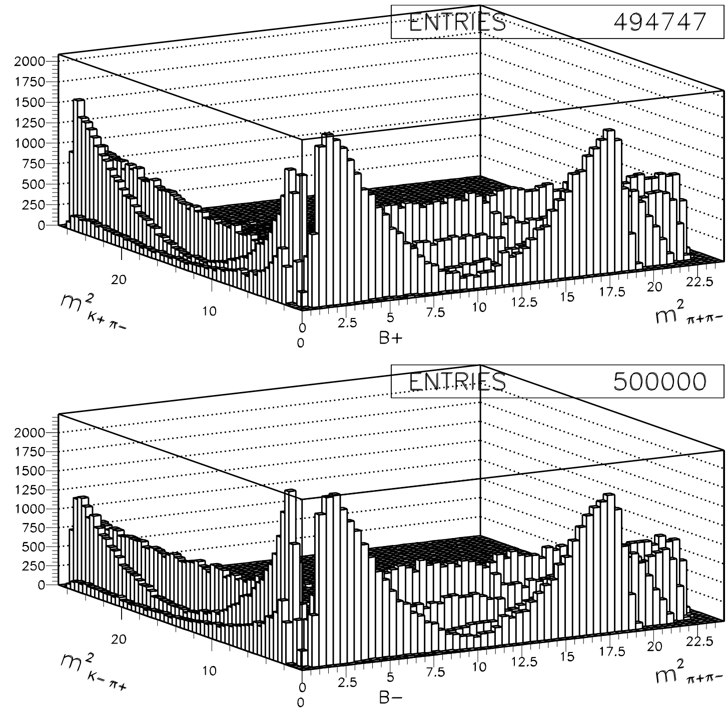


Figure 7: Dalitz plot distributions for $B^+ \to K^+\pi^+\pi^-$ (top) and $B^- \to K^-\pi^+\pi^-$ (bottom) in Model "$f_0$".

cated 'look' at the conjugate Dalitz plots suggested the existence of a true **CP** asymmetry. Yet for smaller and presumably more realistic values of these phase differences one needs the more refined analysis outlined above. Instead of the $60^o$ phase difference in the $\rho^0$ amplitude we had assumed above in our model "$\rho^0$", we now assume a phase difference of just $20^o$ and $10^o$, respectively, while leaving the other parameters as listed in Table 1. Figs. 10 and 11 show the resulting Dalitz plots separately for the $B^+$ and $B^-$ decays; they look very much the same now. In Figs. 12 and 13 the lower and upper displays show the $\Delta(i)$ and ${}^{\rm Dp}{\rm S}_{CP}$ plots, for the two scenarios of a $20^o$ and $10^o$ phase difference, respectively. The lower displays of $\Delta(i)$ are very noisy with no clear message. The upper display of ${}^{\rm Dp}{\rm S}_{CP}$ shows a systematic deviation from zero for the $20^o$ case, while that can be hardly said for the $10^o$ case. The existence of a genuine asymmetry is demonstrated by the ${}^{\rm Dp}{\rm S}_{CP}$ *distribution* of Fig. 14a.

Even better, one can localize the region of origin for the **CP** asymmetry as the one where

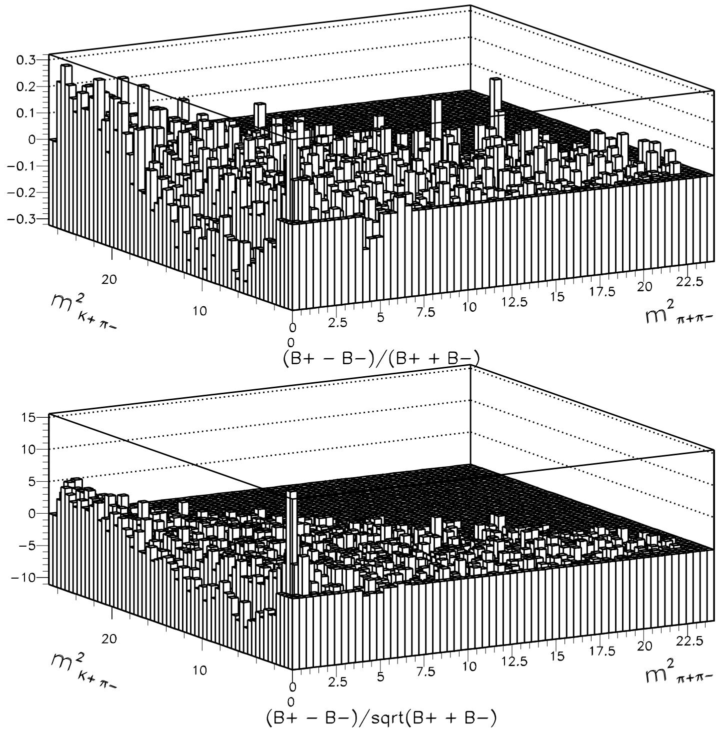


Figure 8: Top: Asymmetry in the Dalitz plot bins for Model "$f_0$". Bottom: Plot of the significance ${}^{\rm Dp}{\rm S}_{CP}$ for $B^{\pm} \to K^{\pm}\pi^{\mp}\pi^{\pm}$.

$\rho^0$ and $f_0$ interference takes place, see Fig. 14b.

The situation for the $10^o$ case is more delicate. The ${}^{\rm Dp}{\rm S}_{CP}$ plot in Fig. 15a shows there is no good Gaussian fit to it: the distribution is a bit wider than a Gaussian expression can yield, but still symmetric around its maximum. Yet plotting the ${}^{\rm Dp}{\rm S}_{CP}$ distributions separately for the four regions as before – Fig. 15b – reveals a clear message: there is a true **CP** asymmetry in regions I and II where $\rho^0$-$f_0$ interference takes place, but none in regions III and IV.

We want to stress that for the last two scenarios – a small phase difference of $20^o$ and $10^o$, respectively, the sophistication provided by an analysis in terms of the significance ${}^{\rm Dp}{\rm S}_{CP}$ was essential in revealing the underlying dynamics.

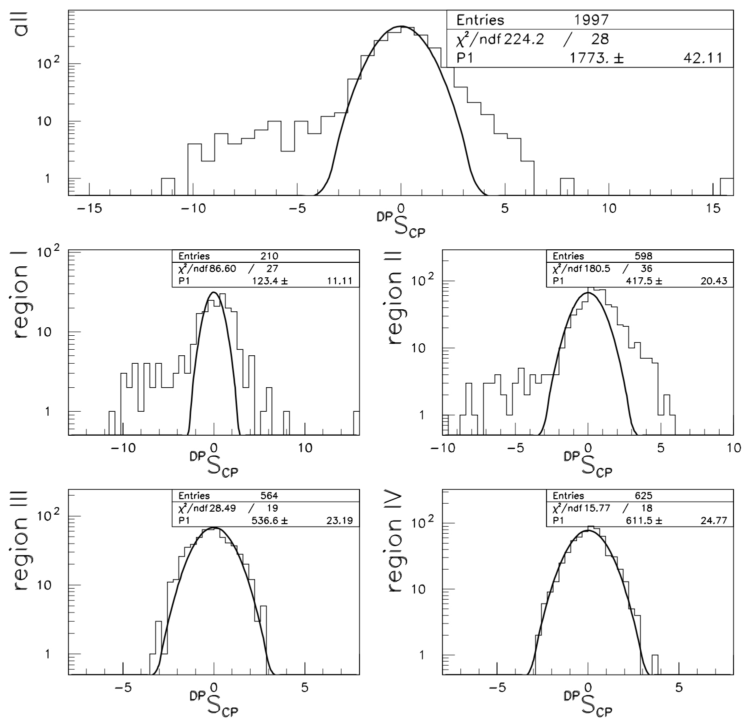


Figure 9: Top row: $^{DP}\mathcal{S}_{CP}$ for the bins in Fig. 8b that pass the statistical cut, fit to a centred Gaussian with unit width for model "$f_0$". P1 is the normalization parameter. Bottom two rows: Distribution of top row divided into the regions shown in Fig. 5. P1 is the normalization parameter.

## 3.3 Future $B$ Studies

We already mentioned there are several other modes that can be studied with high statistics by LHCb:

- $B^\pm \to \pi^\pm\pi^+\pi^-$: like $B^\pm \to K^\pm\pi^+\pi^-$ it receives contributions from tree as well as Penguin operators, yet with the weight of the former enhanced. It thus represents a nicely complementary process.

- The more unconventional channels $B^\pm \to \pi^\pm p\bar{p}$, $K^\pm p\bar{p}$ : the presence of the meson allows us to measure the proton and anti-proton polarization, probing for a **CP** asymmetry, otherwise impossible in two-body decays like $B_d \to p\bar{p}$.

- $B_d - \bar{B}_d$ oscillations would lead to Dalitz plots for $B_d \to K_S\pi^+\pi^-$, where the weight of

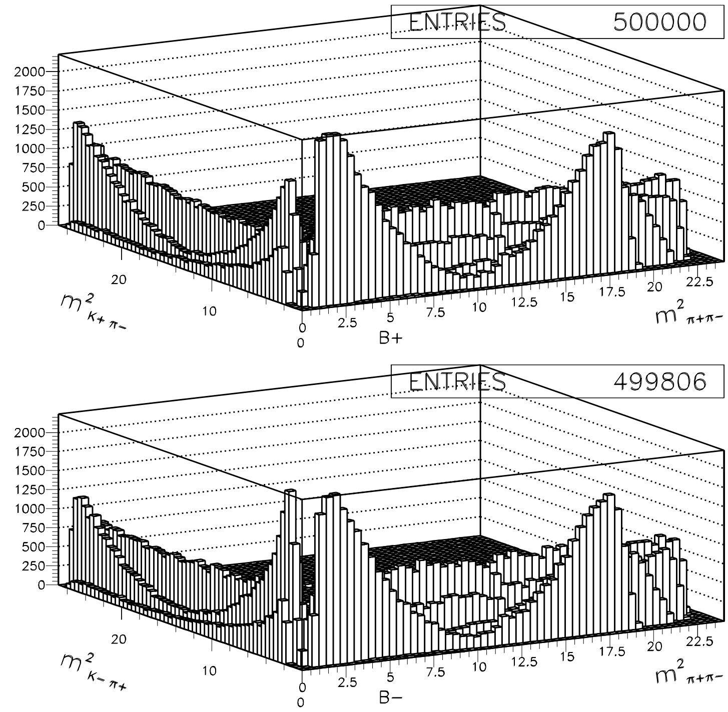


Figure 10: Dalitz plot distributions for $B^+ \to K^+\pi^+\pi^-$ (top) and $B^- \to K^-\pi^+\pi^-$ (bottom) in a model "$\rho^0$" with a $20^o$ phase difference.

different components would shift with the time of decay thus producing time dependent Dalitz plots.

- The same will happen for $B_s \to K_S K^-\pi^+$, $K_S K^+ K^-$, albeit with a much faster oscillation rate.

We will address these transitions in future work.

In this note we have shown how mirandizing the analysis of Dalitz plots – i.e., studying the 'significance' distributions – can act as a powerful filter against statistical fluctuations. Yet real data are also vulnerable to systematic experimental uncertainties. For a full validation of our method someone has to apply it to real primary data, to which we have at present no access.

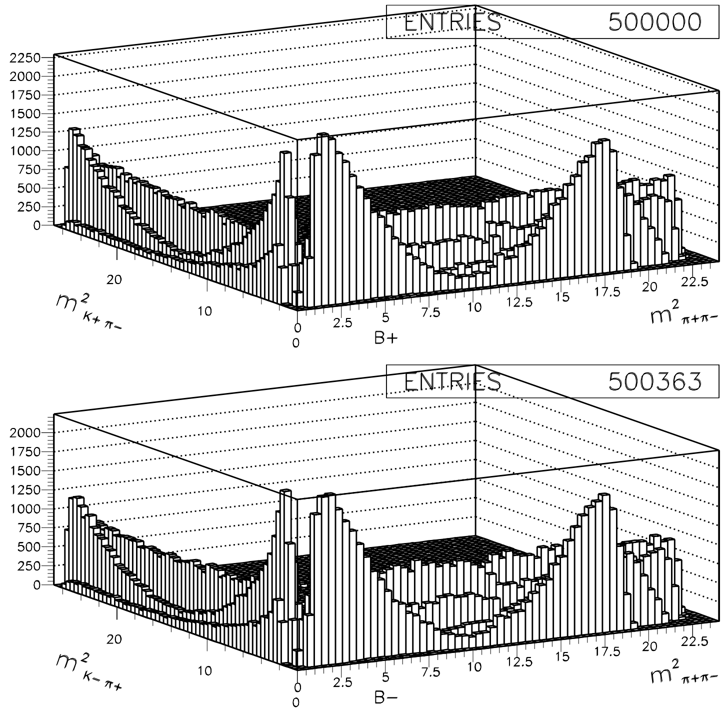


Figure 11: Dalitz plot distributions for $B^+ \to K^+\pi^+\pi^-$ (top) and $B^- \to K^-\pi^+\pi^-$ (bottom) in a model "$\rho^0$" with a $10^o$ phase difference.

# 4 $D$ Decays

The SM generates a relatively dull weak phenomenology for charm transitions: 'slow' $D^0 - \bar{D}^0$ oscillations and tiny **CP** asymmetries; this, however, makes it a promising landscape to search for New Physics [19, 20, 21]. At the same time we have to analyze more closely how slow is 'slow' quantitatively and how tiny is 'tiny'. One has to concede that SM dynamics might saturate the observed size of $x_D = \Delta M_D/\overline{\Gamma}_D$ and $y_D = \Delta\Gamma_D/\overline{\Gamma}_D$, and that CKM forces can produce **CP** asymmetries on the $\mathcal{O}(10^{-3})$ level in singly Cabibbo suppressed (SCS) modes. Furthermore, ignoring $D^0 - \bar{D}^0$ oscillations, purely Cabibbo allowed (CA) and doubly suppressed (DCS) channels (i.e. those with*out* a $K_S$ or $K_L$) cannot exhibit direct **CP** violation. Any such effect in DCS modes and one on the about 0.01 or larger level in SCS decays will thus establish the intervention of New Physics. Basing such claims on somewhat smaller effects will require theoretical progress that appears quite feasible.

The phase space available in $D$ decays is significantly smaller than in $B$ decays. Two- and

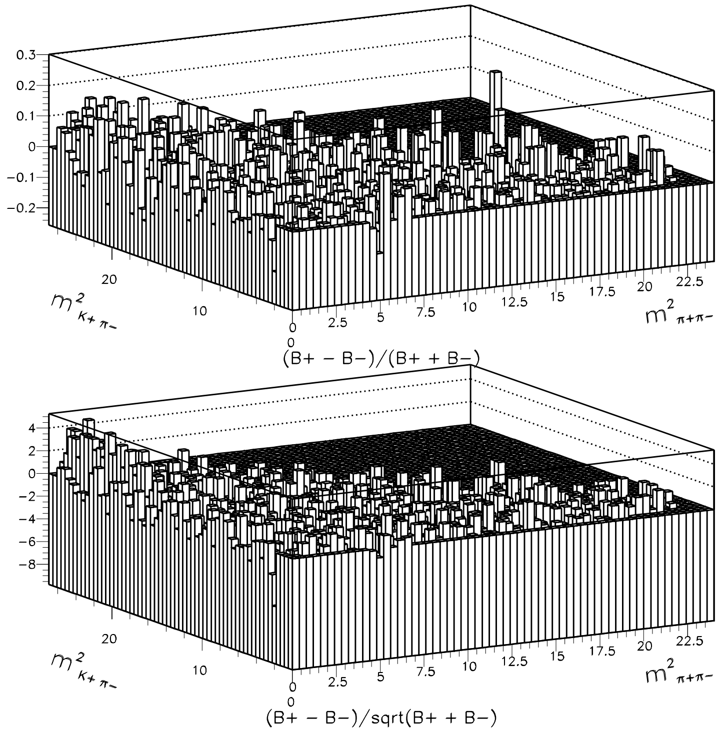


Figure 12: Plot of $\Delta(i)$ (top) and ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ (bottom) for $B^+ \to K^+\pi^+\pi^-$ and $B^- \to K^-\pi^+\pi^-$ in a model "$\rho^0$" with a $20^o$ phase difference.

quasi-two-body channels make up more than half of the full nonleptonic width. Furthermore the Dalitz plots are populated more thorougly than for $B$ decays. Determining the impact of individual contributions therefore amounts to a more delicate task.

## 4.1 $D^+ \to \pi^+\pi^+\pi^-$

In the SM there are already two different amplitudes contributing to these SCS transitions, and they carry a relative weak phase, albeit a tiny one $\sim \mathcal{O}(\lambda^4) \sim 10^{-3}$. Finding **CP** asymmetries significantly larger than $10^{-3}$ would provide strong prima facie evidence for the presence of New Physics. Searching down to asymmetries as small as $10^{-3}$ requires huge statistics as well as excellent control over systematics; the method proposed by us should be a powerful tool in taking up such a challenge. Probing the whole Dalitz plot with its various structures should allow us to make the case for New Physics even compelling making use also of the anticipated theoretical refinements sketched below.

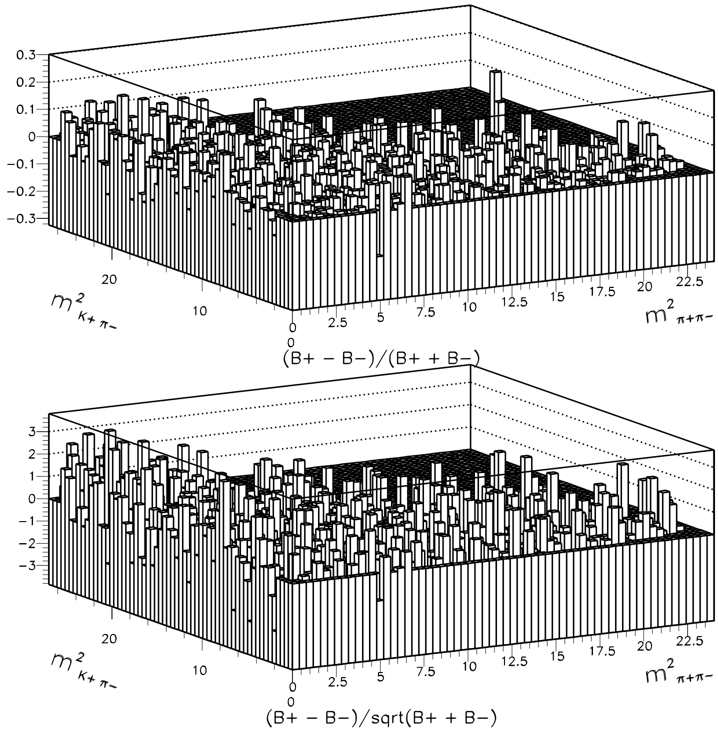


Figure 13: Plot of $\Delta(i)$ (top) and ${}^{\mathrm{Dp}}\mathrm{S}_{CP}$ (bottom) for $B^+ \to K^+\pi^+\pi^-$ and $B^- \to K^-\pi^+\pi^-$ in a model "$\rho^0$" with a $10^o$ phase difference.

It should be noted that New Physics scenarios like the Littlest Higgs Model with T parity could have an observable impact here through new heavy states appearing as virtual particles in Penguin diagrams [22].

We adopt a decay model containing four components, namely

- $D^\pm \to \rho^0\pi^\pm$,
- $D^\pm \to \sigma^0\pi^\pm$.
- $D^\pm \to f_0\pi^\pm$
- a uniform non-resonant $D^\pm \to \pi^+\pi^-\pi^\pm$.

We have used the values obtained by Fermilab experiment E791 [14] for the magnitudes and phases of the amplitude coefficients.

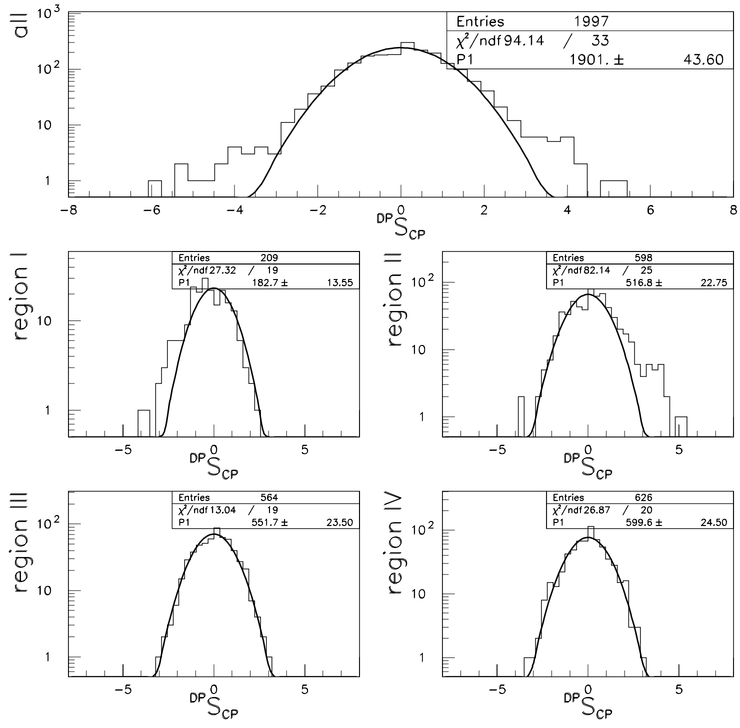


Figure 14: Top row: Distribution of ${}^{\rm Dp}{\rm S}_{CP}$ that pass the statistical cut, fit to a centered Gaussian with unit width; P1 is the normalization parameter. Bottom two rows: Distribution of ${}^{\rm Dp}{\rm S}_{CP}$ divided into the regions shown in Fig. 5 in a model "$\rho^0$" with a $20^o$ phase difference. P1 is the normalization parameter.

A difference in the $\sigma$ phase – a very conceivable scenario – will affect many parts of the Dalitz plot and induce **CP** asymmetries, since the $\sigma$ possesses a very sizable width relative to the phase space available in $D$ decays. The resulting complexities are very intriguing and will be analyzed in a separate paper.

The case of the $f_0$ amplitude having a different phase in $D^+$ and $D^-$ decays is very interesting for another reason: as long as it has any $\bar{u}u$ or $\bar{d}d$ component it will contribute in this final state. We have found that this relatively small contribution can still produce a clear signature in **CP** asymmetries mainly due to the narrow width of the $f_0$. Details of the required analysis will also be given in the future paper.

In this pilot study we will focus on one scenarios, which leads to clean signatures, namely those with a 1% (equivalent to $3.6^o$) phase difference in $\rho^0$ amplitude. We have selected much

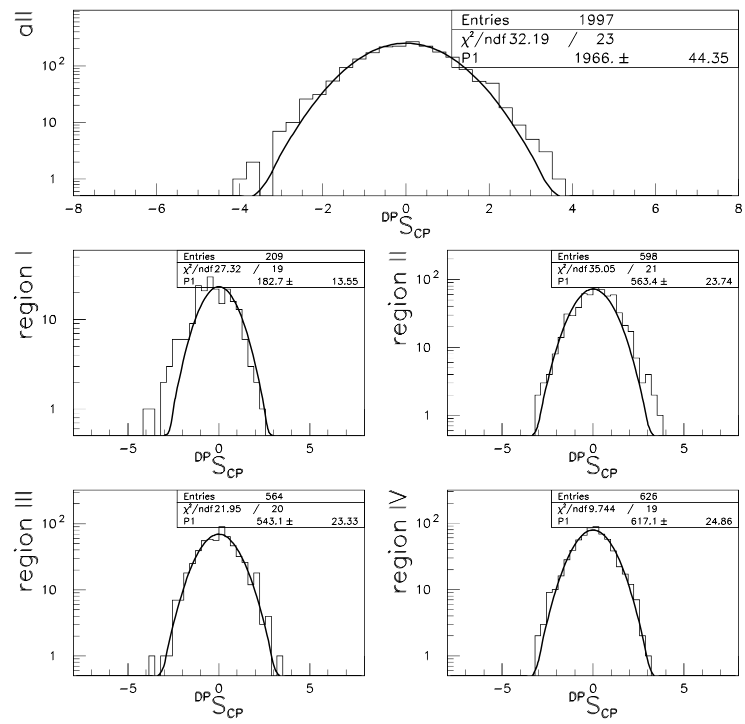


Figure 15: Top row: Distribution of $^{\rm Dp}{\rm S}_{CP}$ that pass the statistical cut, fit to a centred Gaussian with unit width; P1 is the normalization parameter. Bottom two rows: Distribution of $^{\rm Dp}{\rm S}_{CP}$ divided into the regions shown in Fig. 5 in a model "$\rho^0$" with a $10^o$ phase difference. P1 is the normalization parameter.

smaller phase differences here than for our discussion of $B$ decays above for two reasons: (i) They represent much more realistic New Physics scenarios. (ii) Due to the considerably smaller phase space and thus shrunk Dalitz plot areas one can expect such effects to be still observable.

In Fig. 16 we display the Dalitz plot for this model and the $^{\rm Dp}{\rm S}_{CP}$ distribution for the whole plot as well as for the two regions I and II. The overall $^{\rm Dp}{\rm S}_{CP}$ distribution unequivocally reveals the existence of a **CP** asymmetry, since it does not at all follow a Gaussian fit. The distributions for the two regions I and II exhibit a very telling pattern, namely a rather asymmetric distribution of the bin-wise **CP** asymmetries. That is how it has to be for the line dividing I and II chosen to go through the gap between the two $\rho$ 'lobes', as can be seen by straightforward arithmetic. Applying Eqs. (6,7) to $D^\pm \to \pi^\pm\pi^+\pi^-$, with

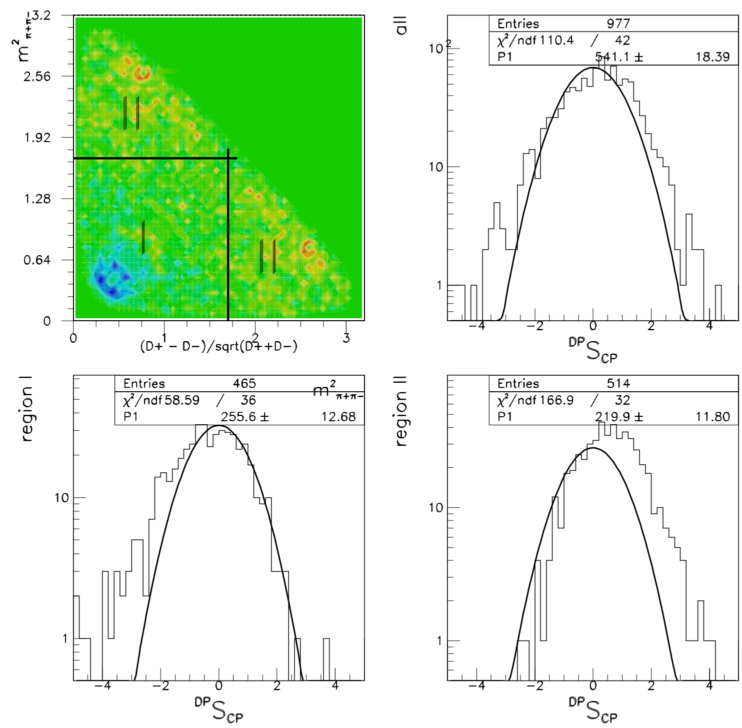


Figure 16: Dalitz plot for $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ in a model with a 1% ($3.6^{o}$) phase difference in the $\rho^{0}$ amplitude with sub-domains I and II; distributions of the significance ${}^{\rm Dp}{\rm S}_{CP}$ for the whole plot and the two sub-domains I and II; P1 is the normalization parameter.

an analogous, though smaller $\rho$ - $f_0$ interference used in those equations leads to following difference in the $D^{+} \to \pi^{+}\pi^{+}\pi^{-}$ - $D^{-} \to \pi^{-}\pi^{+}\pi^{-}$ amplitude squared:

$$
\begin{aligned}
\Delta\mathcal{M} \;=\; & |\mathcal{M}_{+}|^{2} - |\mathcal{M}_{-}|^{2} = [(a_{+}^{\rho})^{2} - (a_{-}^{\rho})^{2}]|F_{\rho}^{\rm BW}|^{2}\cos^{2}\theta + [(a_{+}^{f})^{2} - (a_{-}^{f})^{2}]|F_{f}^{\rm BW}|^{2} \\
& +2\cos\theta|F_{\rho}^{\rm BW}|^{2}|F_{f}^{\rm BW}|^{2} \times \\
& \{[(m_{\rho}^{2} - s)(m_{f}^{2} - s) - m_{\rho}\Gamma_{\rho}m_{f}\Gamma_{f}][a_{+}^{\rho}a_{+}^{f}\cos(\delta_{+}^{\rho} - \delta_{+}^{f}) - a_{-}^{\rho}a_{-}^{f}\cos(\delta_{-}^{\rho} - \delta_{-}^{f})] \\
& -[m_{\rho}\Gamma_{\rho}(m_{f}^{2} - s) - m_{f}\Gamma_{f}(m_{\rho}^{2} - s))[a_{+}^{\rho}a_{+}^{f}\sin(\delta_{+}^{\rho} - \delta_{+}^{f}) - a_{-}^{\rho}a_{-}^{f}\sin(\delta_{-}^{\rho} - \delta_{-}^{f})]\}
\end{aligned}
\tag{10}
$$

The term quadratic in $\cos\theta$ is responsible for the parabolic shape of the spin one resonance seen in Fig.1. Yet the interference generates a term linear in $\cos\theta$. Therefore the interference is destructive in region I of Fig. 16– thus implying fewer events for $D^{+}$ than $D^{-}$ – and the opposite in region II.

This example illustrates the power of the mirandizing procedure to unequivocally uncover even a small asymmetry and track its local origin in the Dalitz plot.

### 4.2 Future $D$ studies

As before with $B$ decays many promising channels await careful study:

- The more complex scenarios in $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$, where the seeds for **CP** violation reside in the $f_0$ and $\sigma$ amplitudes deserve detailed analysis.
- The doubly Cabibbo suppressed modes $D^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, $K^{\pm}K^{+}K^{-}$ could reveal a new source of direct **CP** violation [23].
- With the observation of $D^0 - \bar{D}^0$ oscillations one expects time dependent Dalitz plots to emerge in $D^0 \to K_S\pi^{+}\pi^{-}$. This time evolution will allow to differentiate between direct and indirect **CP** asymmetries.

## 5 On Refining the Theoretical Tools

Even lattice QCD does not allow to treat final state interactions as a matter of principle, except for kaon decays, where elastic unitarity can be assumed. Elastic unitarity makes little sense for $B$ decays; it might be an approximation of some value in $D$ decays, but we have no reliable even semi-quantitative estimate for how good an approximation it might be.

We should be able to clarify the picture at least somewhat by adopting 'theoretical engineering' [24]: One considers $D_{(s)} \to PP$ ($P$ = pseudoscalar meson) on all Cabibbo levels for $D^0$, $D^+$ and $D_s^+$ mesons. Relying on a modicum of theoretical judgement one selects diagrams deemed relevant for these processes and expresses their amplitudes in terms of the known CKM factors and radiative QCD corrections and the a priori unknown moduli and strong phases of their matrix elements. Fitting these expressions to a comprehensive body of well measured branching ratios one fixes these moduli and strong phases. The resulting *over*constraints provide a check on the reliability of such a fit. The analogous procedure is then applied to $D_{(s)} \to PV$ ($V$ = pseudoscalar meson). While such an analysis cannot replace a full Dalitz plot description, it can provide valuable constraints on the latter.

Alternatively we should be able to develop some framework where we can have a semi-quantitative treatment of the interference of a narrow resonance with a broad non-resonant contribution that to first approximation can be considered even as flat.

# 6 Summary and Outlook

So far Dalitz plot studies have not established any **CP** violation with at least five sigma significance — yet we are confident this period will soon come to an end. We actually expect such studies to become a central tool for obtaining a more detailed picture of and perspective on limitations of **CP** invariance. An acceptable description of the Dalitz plot usually has to satisfy a sizable number of *over*constraints, which provides a powerful validation tool to control systematics. Furthermore — and maybe even more importantly — it provides us with information about the Lorentz structure of the underlying transition operator that cannot be inferred from partial rate asymmetries in two-body final states.

A full fledged Dalitz plot description thus represents the 'holy grail' in our quest for mapping out **CP** violation in $B$, $D$ and maybe even top quark decays. The journey there will however require a substantial amount of time, as it is with all 'holy grails'. It also remains to be seen to which degree there will arise uncertainties due to an irreducible model dependance. The method we have proposed in this note for searching for **CP** asymmetries in the populations of Dalitz plots is *not* meant to replace Dalitz plot parametrizations:

- The proposed method will allow to establish the existence of **CP** asymmetries with *more limited statistics* and identify their topography in the Dalitz plot in a *robust and model independent* way.
- Furthermore isospin sum rules [25] can already be applied to its findings.
- It will speed up the construction of the full Dalitz plot description and provide powerful validation for it.

In this paper we have described a model independent method for establishing the existence of a **CP** asymmetry in a Dalitz plot and inferring its location. To fully gauge its power it is important to apply it to high statistics primary data with their experimental systematic uncertainties. If it passes that test, then one can study how to extract maximal information about its parameters, in particular the weak phase producing it. The relevant expression is given in Eq.(10). Various methods can be employed to achieve such a goal; finding the optimal one requires future detailed analysis. [2] Various features can be employed to discriminate between SM and potential New Physics effects. In $B \to K\pi\pi$ the SM can generate a significant weak phase only through its $(V-A)\times(V-A)$ currents, since the $b \to s$ Penguin operator does not carry a weak phase. New Physics thus could make its presence felt through producing a weak phase for a scalar state like the $f_0$.

Clearly a large amount of also theoretical work is required. While we should not count on theorists achieving miracles, we can expect a positive learning curve for them.

**Acknowledgments:** This work was supported by the NSF under the grant number PHY-0807959 and by CNPq. One of us (IB) thanks the CBPF for the kind hospitality extended to him, while this work was completed. We have benefitted from constructive criticism by T. Gershon and the referee. We are also grateful to João Torres and Chris Howk for clarifying conversations about issues related to astronomy.

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