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SHEAR–FREE AND HOMOLOGY CONDITIONS FOR SELF–GRAVITATING DISSIPATIVE FLUIDS

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Abstract

The shear free condition is studied for dissipative relativistic self–gravitating fluids in the quasi–static approximation. It is shown that, in the Newtonian limit, such condition implies the linear homology law for the velocity of a fluid element,only if homology conditions are further impossed on the temperature and the emission rate.It is also shown that the shear free plus the homogeneous expansion rate conditions are equivalent (in the Newtonian limit) to the homology conditions. Deviations from homology and their prospective applications to some astrophysical scenarios are discussed, and a model is worked out.

Key-words: Gravitation; Relativity; Hydrodynamics; Stellar dynamics; Radiation mechanics; General; Diffusion.

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1 Introduction

As it is well known the shear plays an important role in general relativistic and cosmological models (see [1] and references therein).

In the case of slowly evolving (quasi–static) non–dissipative systems, it can be shown that in the Newtonian limit, the shear–free condition leads to the homologous contraction (or expansion) law for the velocity [2]. However this is not necessarily the case in the presence of a heat flow vector [3] and/or free streaming radiation (see below). This fact, and the great relevance of homology conditions in astrophysics (see [4]) provide the main motivation for this work.

It is our purpose here to explore deeper the link between these two conditions and present some astrophysical scenarios where departures from homologous evolution (keeping the shear free condition) migth drastically change the whole picture of the system.

Accordingly, we shall consider dissipative systems. Indeed, dissipation due to the emission of massless particles (photons and/or neutrinos) is a characteristic process in the evolution of massive stars. In fact, it seems that the only plausible mechanism to carry away the bulk of the binding energy of the collapsing star, leading to a neutron star or black hole is neutrino emission [5].

In the diffusion approximation, it is assumed that the energy flux of radiation (as that of thermal conduction) is proportional to the gradient of temperature. This assumption is in general very sensible, since the mean free path of particles responsibles for the propagation of energy in stellar interiors is usually very small as compared with the typical length of the object. Thus, for a main sequence star as the sun, the mean free path of photons at the centre, is of the order of $2\,cm$. Also, the mean free path of trapped neutrinos in compact cores of densities about $10^{12}\,g.cm.^{-3}$ becomes smaller than the size of the stellar core [6, 7].

Furthermore, the observational data collected from supernovae 1987A indicates that the regime of radiation transport prevailing during the emission process, is closer to the diffusion approximation than to the streaming out limit [8].

However in many other circumstances, the mean free path of particles transporting energy may be large enough as to justify the free streaming approximation. Therefore we will include simultaneously both limiting cases of radiative transport (diffusion and streaming out), allowing for describing a wide range situations.

As mentioned before homologous evolution, an assumption widely used in astrophysics [4], is known to be equivalent, in the non–dissipative case, to the shear–free condition in the Newtonian limit [2]. As we shall see here, the presence of dissipative terms requires further the assumption of homology conditions on temperature and emission rate, in order to keep the homologous linear dependence of the velocity. It will also be shown that imposing the rate of expansion to be independent of the radial coordinate (together with shear free condition) amounts to the full set of homology conditions.

Although deviations from the homologous evolution are shown to introduce extremely small modifications in the expression for the velocity, these terms might be relevant in some very specific situations which we will discuss later.

It is also worth mentioning that although the most common method of solving Einstein's equations is to use commoving coordinates (e.g. [9],[10]) we shall use noncomoving coordinates, which implies that the velocity of any fluid element (defined with respect to a conveniently chosen set of observers) has to be considered as a relevant physical variable [10].

The plan of the paper is as follows. In Section 2 we define the conventions and give the field equations and expressions for the kinematical and physical variables we shall use, in noncomoving coordinates. In Section 3 we give the general expression for the velocity and evaluate the dissipative terms. A very simple model is presented in Section 4. Finally a discussion of results is presented in Section 5.

2 Relevant Equations and Conventions

2.1 The field equations

We consider spherically symmetric distributions of collapsing fluid, which for sake of completeness we assume to be locally anisotropic, undergoing dissipation in the form of heat flow and/or free streaming radiation, bounded by a spherical surface Σ.
The line element is given in Schwarzschild–like coordinates by

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \tag{1}$$

where $\nu(t,r)$ and $\lambda(t,r)$ are functions of their arguments. We number the coordinates: $x^0 = t$; $x^1 = r$; $x^2 = \theta$; $x^3 = \phi$.
The metric (1) has to satisfy Einstein field equations

$$G^{\nu}_{\mu} = -8\pi T^{\nu}_{\mu}, \tag{2}$$

which in our case read [11]:

$$-8\pi T_0^0 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right), \tag{3}$$

$$-8\pi T_1^1 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right), \tag{4}$$

$$\begin{aligned} -8\pi T_2^2 = -8\pi T_3^3 = & - \frac{e^{-\nu}}{4}\left(2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})\right) \\ & + \frac{e^{-\lambda}}{4}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right), \end{aligned} \tag{5}$$

$$-8\pi T_{01} = -\frac{\dot{\lambda}}{r}, \tag{6}$$

where dots and primes stand for partial differentiation with respect to t and r, respectively. In order to give physical significance to the T^μ_ν components we apply the Bondi approach [11].

Thus, following Bondi, let us introduce purely locally Minkowski coordinates (τ, x, y, z)

$$d\tau = e^{\nu/2}dt\,; \qquad dx = e^{\lambda/2}dr\,; \qquad dy = rd\theta\,; \qquad dz = r sin\theta d\phi.$$

Then, denoting the Minkowski components of the energy tensor by a bar, we have

$$\bar{T}_0^0 = T_0^0\,; \qquad \bar{T}_1^1 = T_1^1\,; \qquad \bar{T}_2^2 = T_2^2\,; \qquad \bar{T}_3^3 = T_3^3\,; \qquad \bar{T}_{01} = e^{-(\nu+\lambda)/2}T_{01}.$$

Next, we suppose that when viewed by an observer moving relative to these coordinates with proper velocity ω in the radial direction, the physical content of space consists of an anisotropic fluid of energy density ρ, radial pressure P_r, tangential pressure $P_\perp$, radial heat flux $\hat{q}$ and unpolarized radiation of energy density $\hat{\epsilon}$ traveling in the radial direction. Thus, when viewed by this moving observer the covariant tensor in Minkowski coordinates is

$$\begin{pmatrix} \rho + \hat{\epsilon} & -\hat{q} - \hat{\epsilon} & 0 & 0 \\ -\hat{q} - \hat{\epsilon} & P_r + \hat{\epsilon} & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\perp \end{pmatrix}.$$

Then a Lorentz transformation readily shows that

$$T_0^0 = \bar{T}_0^0 = \frac{\rho + P_r\omega^2}{1-\omega^2} + \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}} + \epsilon, \tag{7}$$

$$T_1^1 = \bar{T}_1^1 = -\frac{P_r + \rho\omega^2}{1-\omega^2} - \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}} - \epsilon, \tag{8}$$

$$T_2^2 = T_3^3 = \bar{T}_2^2 = \bar{T}_3^3 = -P_\perp, \tag{9}$$

$$T_{01} = e^{(\nu+\lambda)/2}\bar{T}_{01} = -\frac{(\rho + P_r)\omega e^{(\nu+\lambda)/2}}{1-\omega^2} - \frac{Qe^{\nu/2}e^{\lambda}}{(1-\omega^2)^{1/2}}(1+\omega^2) - e^{(\nu+\lambda)/2}\epsilon, \tag{10}$$

with

$$Q \equiv \frac{\hat{q}e^{-\lambda/2}}{(1-\omega^2)^{1/2}} \tag{11}$$

and

$$\epsilon \equiv \hat{\epsilon}\frac{(1+\omega)}{(1-\omega)}. \tag{12}$$

Note that the coordinate velocity in the (t, r, θ, ϕ) system, dr/dt, is related to ω by

$$\omega = \frac{dr}{dt}\, e^{(\lambda-\nu)/2}. \tag{13}$$

Feeding back (7–10) into (3–6), we get the field equations in the form

$$\frac{\rho + P_r\omega^2}{1-\omega^2} + \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}} + \epsilon = -\frac{1}{8\pi}\left\{-\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)\right\}, \tag{14}$$

$$\frac{P_r + \rho\omega^2}{1-\omega^2} + \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}} + \epsilon = -\frac{1}{8\pi}\left\{\frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)\right\}, \tag{15}$$

$$\begin{aligned} P_\perp = -\frac{1}{8\pi}\Bigg\{ & \frac{e^{-\nu}}{4}\left(2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})\right) \\ & -\frac{e^{-\lambda}}{4}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right)\Bigg\}, \end{aligned} \tag{16}$$

$$\frac{(\rho + P_r)\omega e^{(\nu+\lambda)/2}}{1-\omega^2} + \frac{Qe^{\nu/2}e^{\lambda}}{(1-\omega^2)^{1/2}}(1+\omega^2) + e^{(\nu+\lambda)/2}\epsilon = -\frac{\dot{\lambda}}{8\pi r}. \tag{17}$$

Outside of the fluid distribution, the spacetime is that of Vaidya, given by

$$ds^2 = \left(1 - \frac{2M(u)}{R}\right) du^2 + 2dudR - R^2 \left(d\theta^2 + sin^2\theta d\phi^2\right), \tag{18}$$

where u is a coordinate related to the retarded time, such that $u = constant$ is (asymptotically) a null cone open to the future and R is a null coordinate ($g_{RR} = 0$). It should be remarked, however, that strictly speaking, the radiation can be considered in radial free streaming only at radial infinity.

The two coordinate systems (t, r, θ, ϕ) and (u, R, θ, ϕ) are related at the boundary surface and outside it by

$$u = t - r - 2M\, ln\left(\frac{r}{2M} - 1\right), \tag{19}$$

$$R = r. \tag{20}$$

In order to match smoothly the two metrics above on the boundary surface $r = r_\Sigma(t)$, we first require the continuity of the first fundamental form across that surface. Which in our notation implies (see [12])

$$e^{\nu_\Sigma} = 1 - \frac{2M}{R_\Sigma}, \tag{21}$$

$$e^{-\lambda_\Sigma} = 1 - \frac{2M}{R_\Sigma}. \tag{22}$$

Where, from now on, subscript Σ indicates that the quantity is evaluated at the boundary surface Σ and $R = R_\Sigma(u)$ is the equation of the boundary surface in (u, R, θ, ϕ) coordinates. And

$$[P_r]_\Sigma = \left[Q\, e^{\lambda/2}\left(1 - \omega^2\right)^{1/2}\right]_\Sigma, \tag{23}$$

expressing the discontinuity of the radial pressure in the presence of heat flow, which is a well known result [13].

Next, it will be useful to calculate the radial component of the conservation law

$$T^\mu_{\nu;\mu} = 0. \tag{24}$$

After tedious but simple calculations we get

$$\left(-8\pi T^1_1\right)' = \frac{16\pi}{r}\left(T^1_1 - T^2_2\right) + 4\pi\nu'\left(T^1_1 - T^0_0\right) + \frac{e^{-\nu}}{r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right), \tag{25}$$

which in the static case becomes

$$P_r' = -\frac{\nu'}{2}\left(\rho + P_r\right) + \frac{2\left(P_\perp - P_r\right)}{r}, \tag{26}$$

representing the generalization of the Tolman–Oppenheimer–Volkof equation for anisotropic fluids [14].

2.2 The kinematical variables

The components of the shear tensor are defined by

$$\sigma_{\mu\nu} = u_{\mu;\nu} + u_{\nu;\mu} - u_\mu a_\nu - u_\nu a_\mu - \frac{2}{3}\Theta P_{\mu\nu}, \tag{27}$$

where

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu\,; \qquad \Theta = u^\mu_{;\mu}\,; \qquad a_\mu = u^\nu u_{\mu;\nu}, \tag{28}$$

denote the projector onto the three space orthogonal to u^μ and the expansion, respectively. A simple calculation gives

$$\Theta = \frac{e^{-\nu/2}}{2\left(1-\omega^2\right)^{1/2}}\left(\dot{\lambda} + \frac{2\omega\dot{\omega}}{1-\omega^2}\right) + \frac{e^{-\lambda/2}}{2\left(1-\omega^2\right)^{1/2}}\left(\omega\nu' + 2\omega' + \frac{2\omega^2\omega'}{1-\omega^2} + \frac{4\omega}{r}\right), \tag{29}$$

$$\sigma_{11} = -\frac{2}{3\left(1-\omega^2\right)^{3/2}}\left[e^{\lambda}e^{-\nu/2}\left(\dot{\lambda} + \frac{2\omega\dot{\omega}}{1-\omega^2}\right) + e^{\lambda/2}\left(\omega\nu' + \frac{2\omega'}{1-\omega^2} - \frac{2\omega}{r}\right)\right], \tag{30}$$

$$\sigma_{22} = -\frac{e^{-\lambda}r^2\left(1-\omega^2\right)}{2}\sigma_{11}, \tag{31}$$

$$\sigma_{33} = -\frac{e^{-\lambda}r^2\left(1-\omega^2\right)}{2}\sin^2\theta\sigma_{11}, \tag{32}$$

$$\sigma_{00} = \omega^2 e^{-\lambda}e^{\nu}\sigma_{11}, \tag{33}$$

$$\sigma_{01} = -\omega e^{(\nu-\lambda)/2}\sigma_{11}, \tag{34}$$

and for the shear scalar σ

$$\sigma = \sqrt{3}\left(\frac{\Theta}{3} - \frac{e^{-\lambda/2}}{r}\frac{\omega}{\sqrt{1-\omega^2}}\right). \tag{35}$$

2.3 The Weyl tensor

The model to be presented in Section 4 is obtained from the assumption of conformal flatness. Furthermore, since the publication of Penrose's work [15], there has been an increasing interest in studying the possible role of Weyl tensor (or some function of it) in the evolution of self-gravitating systems [16]. This interest is reinforced by the fact that for spherically symmetric distribution of fluid, the Weyl tensor may be defined exclusively in terms of the density contrast and the local anisotropy of the pressure (see below), which in turn are known to affect the fate of gravitational collapse [17].

Therefore it is worthwhile to include here some expressions for the Weyl tensor. Thus, using Maple V, it is found that all non–vanishing components of the Weyl tensor are proportional to

$$W \equiv \frac{r}{2} C^3_{232} \quad = \quad W_{(s)} + \frac{r^3 e^{-\nu}}{12} \left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2} \right) \tag{36}$$

where

$$W_{(s)} = \frac{r^3 e^{-\lambda}}{6} \left(\frac{e^\lambda}{r^2} - \frac{1}{r^2} + \frac{\nu'\lambda'}{4} - \frac{\nu'^2}{4} - \frac{\nu''}{2} - \frac{\lambda'}{2r} + \frac{\nu'}{2r} \right), \tag{37}$$

corresponds to the contribution in the static (and quasi–static) case.

Also, from the field equations and the definition of the Weyl tensor it can be easily shown that (see [18] for details)

$$W = -\frac{4\pi}{3} \int_0^r r^3 \left(T^0_0 \right)' dr + \frac{4\pi}{3} r^3 \left(T^2_2 - T^1_1 \right). \tag{38}$$

2.4 The slowly evolving approximation

In this work we shall consider exclusively slowly evolving systems. That means that our sphere changes slowly on a time scale that is very long compared to the typical time in which it reacts on a slight perturbation of hydrostatic equilibrium, this typical time is called hydrostatic time scale. Thus our system is always in hydrostatic equilibrium (very close to) and its evolution may be regarded as a sequence of static models linked by (6).

This assumption is very sensible because the hydrostatic time scale is very small for almost any phase of the life of a star. It is of the order of 27 minutes for the sun, 4.5 seconds for a white dwarf and 10^{-4} seconds for a neutron star of one solar mass and $10 Km.$ radius [4].

Let us now express this assumption through conditions for ω and metric functions.

First of all, slow contraction (or expansion) means that the radial velocity ω measured by the Minkowski observer, as well as time derivatives are so small that their products as well as second time derivatives can be neglected. Thus we shall assume

$$\ddot{\nu} \approx \ddot{\lambda} \approx \dot{\lambda}\dot{\nu} \approx \dot{\lambda}^2 \approx \dot{\nu}^2 \approx \omega^2 \approx \dot{\omega} = 0 \tag{39}$$

Then, it follows from (6) and (10) that Q and ϵ are, at most, of order $O(\omega)$.

In this approximation, (25) becomes

$$(P_r + \epsilon)' + (\rho + P_r + 2\epsilon)\frac{\nu'}{2} - 2\frac{P_\perp - P_r - \epsilon}{r} = 0 \tag{40}$$

which is the equation of hydrostatic equilibrium for an anisotropic fluid radiating a null fluid of energy density ϵ.

Thus, as mentioned before, the system, although evolving, is in hydrostatic equilibrium (up to order $O(\omega)$), this allows for a very simple extension of any static solution to the slowly evolving case.

3 Shear–free and homology conditions

As mentioned before the only relevant component of the shear tensor is σ_{11} given by equation (30).

Evaluating this last equation in the slowly evolving approximation, we obtain

$$\sigma_{11} = -\frac{2}{3}e^{\lambda}\left(e^{-\nu/2}\dot{\lambda} + e^{-\lambda/2}(\omega\nu' + 2\omega' - \frac{2\omega}{r})\right) \tag{41}$$

Next, using (17) and

$$P_r + \rho = \frac{e^{-\lambda}}{8\pi r}(\nu' + \lambda') - 2\epsilon \tag{42}$$

easily obtained from (14) and (15),one gets

$$\sigma_{11} = -\frac{2\sigma_{22}}{r^2}e^{\lambda} = -\frac{2\sigma_{33}}{r^2 sin^2\theta}e^{\lambda} = -\frac{4}{3}e^{\lambda/2}\left(\omega' - \frac{\omega\lambda'}{2} - \frac{\omega}{r} - 4\pi r Q e^{3\lambda/2} - 4\pi r\epsilon e^{\lambda}\right) \tag{43}$$

.

We can solve (43) for ω, to obtain

$$\omega = \omega_\Sigma\left(\frac{r}{r_\Sigma}\right)e^{(\lambda-\lambda_\Sigma)/2} - 4\pi r e^{\lambda/2}\int_r^{r_\Sigma}\left(Qe^{\lambda} + \epsilon e^{\lambda/2} - \frac{3}{16\pi}e^{-\lambda}\frac{\sigma_{11}}{r}\right)dr \tag{44}$$

From the above equation we we find that in the non–dissipative, shear free case we obtain

$$\omega = \omega_\Sigma(\frac{r}{r_\Sigma})e^{(\lambda-\lambda_\Sigma)/2} \tag{45}$$

In the Newtonian limit we have $M(u) \approx \lambda \approx \nu \approx 0$ and we recover the well known linear expression, typical of the homologous evolution [4]

$$\omega_{Newt.} = \frac{\omega_\Sigma}{r_\Sigma} r \tag{46}$$

Also, from (35) evaluated in the slowly evolving approximation, it follows that in the shear–free motion

$$\Theta = (\frac{3\omega}{r})e^{-\lambda/2} \tag{47}$$

which of course is valid also in the dissipative case. Using (45), we can write

$$\Theta = (\frac{3\omega_\Sigma}{r_\Sigma})e^{-\lambda_\Sigma/2} \tag{48}$$

Implying that even in the general (relativistic) case, the expansion rate is homogeneous (independent of r) for the slow, and dissipativeless shear–free motion.

Let us now consider the dissipative shear–free case.

¿From the relativistic Maxwell-Fourier law, we have

$$q^\mu = \kappa P^{\mu\nu}\left(T_{,\nu} - Ta_\nu\right) \tag{49}$$

or

$$q^1 = Q = -\kappa e^{-\lambda}\left(T' + \frac{T\nu'}{2}\right) \tag{50}$$

where T is the temperature and κ denotes the coefficient of conduction. It should be reminded that in the quasi–static approximation, the system is assumed to be relaxed at all times (the relaxation time is zero) and accordingly, any hyperbolic transport equation reduces to (49).

Then feeding back (50) into (44) and using (22) together with the shear–free condition, we obtain

$$\omega = \left[\frac{\omega_\Sigma}{r_\Sigma}\left(1 - \frac{2M(u)}{r_\Sigma}\right)^{1/2} + 4\pi\kappa\left(T_\Sigma - T\right) + 2\pi\kappa\int_r^{r_\Sigma} T\nu' dr - 4\pi\int_r^{r_\Sigma} \epsilon e^{\lambda/2} dr\right] e^{\lambda/2} r, \tag{51}$$

which in the Newtonian limit yields

$$\omega_{Newt.} = \frac{\omega_\Sigma}{r_\Sigma} r - 4\pi r\int_r^{r_\Sigma} \epsilon dr + 4\pi\kappa\left(T_\Sigma - T\right) r. \tag{52}$$

Also, it follows that the expansion (47) with (51) can be written

$$\Theta = \Theta_\Sigma + 3\left[4\pi\kappa\left(T_\Sigma - T\right) + 2\pi\kappa\int_r^{r_\Sigma} T\nu' dr - 4\pi\int_r^{r_\Sigma} \epsilon e^{\lambda/2} dr\right] \tag{53}$$

Thus, unlike the non-dissipative case (see also [3]), the shear-free collapse in the Newtonian limit does not yield the linear law of homologous contraction [4], unless we impose further homology conditions on T and ϵ, i.e. unless we assume that for any given fluid element, all along the evolution

$$\frac{T}{T_\Sigma} = constant$$
$$\frac{\epsilon}{\epsilon_\Sigma} = constant.$$

¿From (51) we observe that the sign of ω for any value of r, is not necessarily the same as that of ω_Σ (as is the case in the non-dissipative evolution). In particular, for sufficiently large (negative) gradient of temperature and/or sufficiently large (positive) ϵ term, we may have $\omega_\Sigma > 0$ and $\omega < 0$. The same conclusion of course applies to Θ.

In other words the system may be evolving in such a way that inner shells collapse, whereas outer ones expand.

This effect, which we have called "thermal peeling" [3], is also present in the relativistic regime, provided the third term in the right side of (51) is not too large. It represents the analog of the "cracking", however whereas the later takes place, under some conditions, when the system abandons the state of equilibrium or quasi–equilibirum [19], the former occurs while the systems is evolving quasi–statically.

However, observe that expressing variables in c.g.s. units, we have that,

$$\kappa T \sim 10^{-59}\,[\kappa][T] cm^{-1}$$

where $[\kappa]$ and $[T]$ denote the numerical values of these quantities as measured in $erg\, s^{-1}\, cm^{-1}\, K^{-1}$ and K respectively. Therefore extremely high conductivities and/or $\triangle T$ are required for thermal peeling to be observed in Newtonian regime. Also, we have

$$\epsilon \sim 10^{-59}\,[\epsilon] cm^{-2}$$

where $[\epsilon]$ denotes the numerical values of this quantity as measured in $erg\, s^{-1}\, cm^{-2}$.

Before closing this section, it is worth mentioning that, in general, such high thermal conductivities are associated to highly compact, degenerate objects where Newtonian limit is not reliable.

Also, it should be noticed that in (52) it has been assumed that terms of order $O(M/r_\Sigma)$ and higher are negligible with respect to $\kappa\left(T_\Sigma - T\right)$. This of course is not always true, as commented above, in which case eq.(52) is not valid. Finally, it is worth noticing that demanding Θ to be homogeneous, we are lead to the homologous contraction, implying thereby that (in the Newtonian limit), the shear free and homogeneous expasion rate conditions are equivalent to the whole set of homologous conditions.

4 A model

In order to illustrate the point raised in Section 3, let us present a very simple model based on the assumption of conformal flatness and shear–free condition. Also, since local anisotropy does not enter explicitly in (51) we shall assume $P_r = P_\perp$

Thus assuming $W = 0$, it follows from (38)

$$\rho' = \frac{3\epsilon}{r} \tag{54}$$

Next, taking for simplicity $Q = 0$ (pure free streaming dissipation) and

$$\epsilon = \beta r(1 - \frac{r}{r_\Sigma}) \tag{55}$$

with $\beta = \beta(t)$ one obtains

$$\rho = 3\beta r(1 - \frac{r}{2r_\Sigma}) + \gamma(t) \tag{56}$$

with $\gamma = \rho(0, t)$. Observe that with this choice of ϵ (if we assume $Q = 0$, i.e the dissipation takes place at the free streaming approximation exclusively), the evolution proceeds adiabatically (the total mass is constant) even though $\epsilon \neq 0$ within the sphere.

Next, from the definition of the mass function [2]

$$m(r, t) = 4\pi \int_0^r r^2 T_0^0 dr = \frac{r}{2}(1 - e^{-\lambda}) \tag{57}$$

and junction conditions, it followws

$$m = \frac{4\pi r^3}{3}\left(3\beta r + \gamma - \frac{3\beta r^2}{2r_\Sigma}\right). \tag{58}$$

and

$$\beta = \frac{2}{3r_\Sigma}\left(\frac{3M}{4\pi r_\Sigma^3} - \gamma\right). \tag{59}$$

with $M = m_\Sigma$. As expected, if we put $\beta = 0$ we recover the well known interior Scwarzshild solution (evolving quasi–statically). The remaining of the metric and physical variables may now be easily obtained from field equations. Feeding back (55) into (44) one sees that playing with β it is possible (at least in principle) to obtain $\omega < 0$ for some values of r, even though ω_Σ is assumed possitive (peeling).

5 Conclusions

We have seen how dissipative terms affects the radial dependence of ω and the expansion rate, in the shear–free case, if we relax the homology conditions on dissipative variables. We have also seen that the dissipative terms may lead to a "peeling". However these contributions appear to be extremely small and therefore it is pertinent to ask if there exist astrophysical scenarios where dissipative contributions might have some effect on ω, and in particular if they could produce a "peeling".

Assuming the highest values for luminosity at the last stages of stellar evolution, of the order of 10^5 times the sun luminosity, produced at a shell of radius of 1/10 of solar radius, we only get

$$\epsilon \sim 10^{-36}\, cm^{-2}.$$

A more promising case is provided by the Kelvin–Helmholtz phase of the birth of a neutron star [20]. Indeed in this phase, during tens of seconds, some 10^{53} ergs are radiated away. If this energy is transported via difussion to the surface, then assuming [21]

$$\kappa \approx 10^{23}[\rho/10^{14} g\, cm^{-3}][10^8 K/T] erg\, s^{-1}\, cm^{-1}\, K^{-1} \tag{60}$$

we see that the corresponding contribution to (51) is still too small.

However if we assume that part of this 10^{53} ergs are propagated in the free streaming regime, then the last term in (51) for sufficiently small r (as compared to r_Σ) is of the order of $\frac{1}{r_\Sigma}10^{52}$. Therefore for positive surface velocities of the order of $30 m/s$ there may be a peeling ($\omega < 0$ for $r < r_\Sigma$).

Finally, let us mention that in a pre–supernovae event, values of the order of 10^{13} and 10^{37} have been estimated for $[T]$ and $[\kappa]$ respectively [22]. With these values, it is clear that peeling is also possible, in particular for sufficiently large values of r_Σ

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