GEOMETRIC THEORY OF DYONS

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ABSTRACT

We present a model of geometrization of electrodynamics of doubly charged particles in the continuation of the Einstein programme of unification.

1. INTRODUCTION

More than fifty years ago Einstein proposed to treat the gravitational phenomena as a manifestation of the dynamical structure of space-time. The success of Einstein's General Theory of Relativity brought the hope that one could interpret all known long range interactions (e.g., gravitation and electrodynamics) as some special geometric properties.

Unfortunately all tentatives of realization of such complete classical geometrization have failled. In order to understand the reasons of such failure in the case of electrodynamics we should look for the reasons by which the geometrization of gravity has succeeded.

It is not difficult to convince one self that the main support of Einstein's geometrical theory of gravity rests on the equivalence principle, as it appears in Eötvos experiment. This shows that there is an universal constant relating all existing particles which consists in the fact that the ratio of the inertial mass to the gravitational mass for any substance turns to be completely independent of the substance. This simple, precise and direct experimental fact is on the basis of the success of the geometrical scheme of the gravitational interaction.

It seems to me that the difficulty in a corresponding geometric treatment for the electromagnetic field is related to the fact that an explicit universal relation for charged particles has not yet been pointed.

Recently(6) I gave some arguments which could change such situation. The main point rests in the simple observation made by some authors(3) which tell us that electrodynamics is not incompatible with the existence of doubly charged particles. These particles which have both electric (e) and magnetic (b) charges were called dyons by Schwinger[2]. We go one step further and assume that all existing particles are true dyons.

In order to conciliate such property with experimental observations[3] there must be an universal constant, call it γ, which is given by the ratio of the magnetic charge to the electric charge of any real particle[*].

If we give, in our theory, to the constant γ the same status that the inertial to gravitational mass ratio have in Einstein's theory, then it seems that necessary conditions to undertake the geometrization of electrodynamics are created.

Let me stress out here that our present geometric scheme is strongly dependent on the fundamental constant γ.

Of course, the existence of a universal γ is not a matter of convention but it is a subject which should be decided only by experiment.

In Section II we make a short review on Electrodynamics of doubly charged particles.

In Section III we present the Cabibbo-Ferrari description of magnetic and electric charge particles which uses two four-vector potentials to describe the electromagnetic field.

Section IV introduces the GCR manifold which is a special restriction on all possible geometries of Cartan type which admits a torsion tensor in the affine connection.

Section V presents a model for a dynamic of the

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[*] We remark that the universality is required for real accesible particles. It makes no restriction on the analogous ratio for hidden particles, like quarks for instance.

Restricted Cartan Geometry which is equivalent to Maxwell's electrodynamics of dyons.

We end with Section VI in which some comments on the present work is made and its possible relations with others models is envisaged.

2. ON ELECTRODYNAMICS

Some years ago Dirac (1931) contemplated the possibility of extending the dual symmetry of electric and magnetic field in the presence of charges by postulating the existence of a new particle carrying only magnetic charge. The main consequences of this hypothesis was the creation of a simple and precise explanation of the quantization of the electric charge.

Such spectacular result has inhibited the appearance of alternative models of extending the symmetry of Maxwell's electrodynamics.

It was only at the sixties that new forms of looking at the invariance properties of electrodynamics has appeared.

In Dirac's approach the electromagnetic field $F_{\mu\nu}$ obeys the modified set of equations

$$F^{\mu\nu}{}_{|\nu} = e\ J^{\mu} \tag{2.1a}$$

$$\overset{*}{F}{}^{\mu\nu}{}_{|\nu} = b\ K^{\mu} \tag{2.1b}$$

in which the introduction of a particle which carries pure magnetic charge b is postulated in order to preserve dual symmetry in the presence of charge. The dual map makes $F_{\mu\nu}$ to go into $\tilde{F}_{\mu\nu}$ given by

$$\tilde{F}_{\mu\nu} = \cos\theta\ F_{\mu\nu} + \sin\theta\ \overset{*}{F}_{\mu\nu} \tag{2.2}$$

in which the dual $\overset{*}{F}_{\mu\nu}$ is defined as usually: $\overset{*}{F}_{\mu\nu} = \frac{1}{2}\ \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ in which $\varepsilon_{\mu\nu\rho\sigma}$ is the completelly anti-symmetric Levi-Civita symbol. Invariance of equations (2.1) under the dual map is obtained if the charges suffer a corresponding rotation

$$\tilde{e} = \cos\theta\ e + \sin\theta\ b$$
$$\tilde{b} = -\sin\theta\ e + \cos\theta\ b \qquad (2.3)$$

Nevertheless, instead of postulating a new particle, one could interpret the above symmetry in the context of doubly charged particles (dyons). Indeed, it is not difficult(3) to show that the observed charge in the case of a dyon of magnetic charge b and electric charge e is given by $q = (e^2+b^2)^{1/2}$. Thus, for a dyon which obeys the symmetric equations (2.1) with $K^\mu \equiv J^\mu$ it becomes a matter of convention to say that a given particle has only electric charge or only magnetic charge. This is related to an arbitrary choice of the dual angle θ. In this vein we can interpret the electron as a particle with double charge (e,b) and by fixing the angle $\theta = \text{arc tg}\ b/e$ we regain the usual pure electrically charged particle.

This is the case for a single particle. Now, let us turn to a collection of interacting doubly charged particles (e_i, b_i). It is not difficult(3) to show that in order to reconciliate the idea that all existing particles are true dyons with observation, we must require that the ratio $(b/e)_i$ for any particle i must be the same. We will call this universal constant as γ:

$$\left(\frac{b}{e}\right)_i = \left(\frac{b}{e}\right)_j = \cdots = \gamma\ .$$

Harrison et al., in the early sixties, argued that a limit on the examination on the earth's magnetic field gives for the ratio b/e for the electrons and protons the result that $\left|\left(\frac{b}{e}\right)_{electron} - \left(\frac{b}{e}\right)_{proton}\right| < 10^{-24}$ (in Gaussian units), which is a rather good point in favor of the existence of a constant γ .

Recently, Strazhev(9) presented an analysis of the quantum effects of an electrodynamics with doubly charged particles. He shows that Lorentz invariance imposes that equivalence of the theory of usual one-charged particles and the theory of dyons is obtained only if there is an universal ratio among the charges, that is, only if γ is a true constant for all existing particles. This corroborates the classical analysis made previously by Harrison et al.

In the present paper we will show that it is possible to geometrize the theory of doubly charged particles in the realm of a Cartan geometry. Before to elaborate this point let us discuss the question of electromagnetic potentials.

3. THE POTENTIALS

In the traditional way electrodynamics is described by means of a four-vector potential which is related to the field $F_{\mu\nu}$ through the expression

$$F_{\mu\nu} = W_{\mu|\nu} - W_{\nu|\mu} \tag{3.1}$$

Either in Dirac's model or in the electrodynamics of dyons, it is no more possible to maintain equation (3.1). The reason is simple that the dual $\overset{*}{F}^{\mu\nu}$ is no more divergence free.

Dirac and others have solved this problem by considering the existence of strings, that is, a line of singularities for the potential.

However there is an alternative way to define potentials without introducing such unusual features envolving strings. The idea can be better explained if we uses the language of differential forms.

Let F be a 2-form which in a natural basis is written as $F = \frac{1}{2} F_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu}$. We identify $F_{\mu\nu}$ with the electromagnetic field. Let us limit our analysis, at this stage, to the case in which the support of F is compact. Then, there is a theorem due to Hodge and de Rham which states that it is possible to find a one-form W and a three-form Z such that, up to an harmonic two-form, we can write

$$F = dW + \delta Z \tag{3.2}$$

in which d is the exterior derivative and δ is defined by $\delta Z = *d*Z$. The $*$ operation maps the three-form Z into its corresponding one-form dual $*Z$ (7,8) .

This fact is on the basis of a suggestion by Cabibbo and Ferrari to describe the electromagnetic field in terms of two potentials W_{μ} and Z_{μ}. They set

$$F_{\mu\nu} = W_{\mu|\nu} - W_{\nu|\mu} + \varepsilon_{\mu\nu}{}^{\rho\sigma} Z_{\rho|\sigma} \tag{3.3}$$

This expression admits an extended gauge given by the map:

$$\begin{aligned} W_{\mu} &\rightarrow W_{\mu} + A_{\mu} \\ Z_{\mu} &\rightarrow Z_{\mu} + B_{\mu} \end{aligned} \tag{3.4}$$

in which A^{μ} and B^{μ} satisfy the null field condition:

$$A_{\mu|\nu} - A_{\nu|\mu} + \varepsilon_{\mu\nu}{}^{\rho\sigma} B_{\rho|\sigma} = 0 \tag{3.5}$$

From equation (2.3) we conclude that in order to annihilate the magnetic charge we have to do a dual rotation of an angle θ = arc tg (b/e). The gauge corresponding to this choice for the dual angle will be called the Maxwell gauge (M-gauge, for short).

In the framework of the two potential theory dual rotation is achieved by setting

$$\begin{aligned} W_\mu \to \tilde{W}_\mu &= \cos\theta\ W_\mu + \sin\theta\ Z_\mu \\ Z_\mu \to \tilde{Z}_\mu &= -\sin\theta\ W_\mu + \cos\theta\ Z_\mu \end{aligned} \tag{3.6}$$

In the M-gauge, it must be possible to reduce the two potentials to one single four-vector, that is we can make $\tilde{Z}_\mu$, for instance, to vanish. Such reduction must be independent of any special charged particle and thus it is possible only in the case the ratio b/e = γ is an universal constant. In this case we obtain

$$Z_\mu = \gamma\ W_\mu \tag{3.7}$$

We conclude that the one-potential theory is the limit of the two-potentials theory if these vectors are parallel, and if we can choose a gauge, independently of any charged particle , such that the factor of proportionality of the two potentials measures the universal constant b/e $\equiv \gamma$. This fact will be the main point of contact between the electrodynamics of dyons and the Cartan geometry, as we will see later on.

4. RESTRICTED CARTAN GEOMETRY

There has been an increasing interest on Cartan geometry in the last years. The main reason for this seems to be related to the possible role of torsion in avoidance of singularities of the geometry. It has been argued that in the framework of standard Cartan-Kibble-Sciama theory, in which torsion is algebraically related to the spin density of matter, cosmological models of Friedmann type with a minimum value for the radius of the Universe can be elaborated. Some explicit simple configurations for the spin density of the global matter has been presented which exhibits such property.

In this theory torsion does not propagate: in the absence of matter with spin, it vanishes.

Others models, using Cartan geometry, have been proposed either as a modification in the coupling of matter with gravitation or as some new theory of gravity.

In a very different context in the present work, we advocate a model in which torsion is the key property to geometrize electrodynamics of dyons.

Let us review some basic useful properties of Cartan geometry.

A covariant derivative (noted by double bar $\|$) is defined by means of an affine connection $\Gamma^{\alpha}_{\mu\nu}$. We write, for an arbitrary co-variant vector B_{α}:

$$B_{\mu\|\nu} = \frac{\partial B_{\mu}}{\partial x^{\nu}} - \Gamma^{\varepsilon}_{\nu\alpha} B^{\alpha} = B_{\mu;\nu} - K^{\varepsilon}_{\nu\alpha} B^{\alpha} \qquad (4.1)$$

The affine connection is split into the Christoffel symbol $\{{}^{\varepsilon}_{\nu\alpha}\}$ plus a tensor $K^{\varepsilon}_{\nu\alpha}$:

$$\Gamma^{\varepsilon}_{\nu\alpha} = \{{}^{\varepsilon}_{\nu\alpha}\} + K^{\varepsilon}_{\nu\alpha} \qquad (4.2)$$

The derivative in the Riemannian associated space is denoted by;

Defining the torsion as the anti-symmetric tensor

$$\tau^{\varepsilon}_{\nu\alpha} = \Gamma^{\varepsilon}_{\nu\alpha} - \Gamma^{\varepsilon}_{\alpha\nu} \tag{4.3}$$

we obtain

$$K^{\rho}{}_{\lambda\nu} = g^{\rho\mu}g_{\varepsilon\nu}\tau^{\varepsilon}{}_{\mu\lambda} + g^{\rho\mu}g_{\varepsilon\lambda}\tau^{\varepsilon}{}_{\mu\nu} + \tau^{\rho}{}_{\lambda\nu} \tag{4.4}$$

Remark that due to the conservation of lengths in a parallel transport we must have the identity

$$K_{\rho\lambda\nu} + K_{\nu\lambda\rho} = 0$$

in which $K_{\rho\lambda\nu} \equiv g_{\rho\mu}K^{\mu}{}_{\lambda\nu}$.
The torsion tensor $\tau^{\alpha}{}_{\mu\nu}$ has 24 degrees of freedom. It is possible to decompose it in the irreducible parts and write

$$\tau^{\alpha}{}_{\mu\nu} = L^{\alpha}{}_{\mu\nu} + \frac{1}{3}\left(\delta^{\alpha}_{\mu}\tau_{\nu} - \delta^{\alpha}_{\nu}\tau_{\mu}\right) - \frac{1}{3}\,\varepsilon^{\alpha}{}_{\mu\nu\lambda}\Sigma^{\lambda} \tag{4.5}$$

in which $L^{\alpha}{}_{\mu\nu}$ is a trace-free tensor (with 16 independent components), the trace τ_{μ} and the pseudo-trace Σ_{μ} are defined by

$$\tau^{\alpha}{}_{\alpha\nu} = \tau_{\nu}$$

$$\tau^{\alpha}{}_{\overset{*}{\alpha\nu}} = \Sigma_{\nu}$$

We remark that if we do not want to introduce new fields besides electromagnetic and gravity, we have to freeze some of the 24 degrees of freedom of the torsion tensor. As we saw in the previous section we should permit at most two four-vectors (besides the metric tensor $g_{\mu\nu}$) and thus it is natural to set

$$L^{\alpha}{}_{\mu\nu} = 0 \tag{4.6}$$

When this condition is satisfied the geometry will be called a Cartan Restricted Geometry (GCR). In this case the contortion tensor reduces to the form

$$K^{\rho}{}_{\lambda\nu} = \frac{2}{3}\left[\delta^{\rho}_{\lambda}\tau_{\nu} - \tau^{\rho}g_{\lambda\nu}\right] - \frac{1}{3}\,\varepsilon^{\rho}{}_{\lambda\nu\alpha}\,\Sigma^{\alpha} \,. \tag{4.7}$$

Let us make some comments on this geometry.

If we are going to identify the trace τ^{μ} and the pseudo--trace Σ^{μ} with the two four-vector potentials W^{μ} and Z^{μ} then we should be able to demonstrate that coupling the electron, for instance, with the electromagnetic field reduces to an extension of the equation of the electron in a flat Minkowskii space-time to the generalized corresponding equation in a curved geometry of the type GCR. Let us show that this is precisely that happens to occur. We remark here that we will follow the approach of the modern gauge theory and start with a massless electron. Its mass will appears as a consequence of a Higgs mechanism. Thus we consider here only the electrodynamics of massless particles.

The extension of Dirac's equation to a curved geometry assumes the form

$$\gamma^{\alpha}D_{\alpha}\Psi = 0 \tag{4.8}$$

in which the co-variant derivative $D_{\alpha}\Psi$ is given by

$$D_{\alpha}\Psi = \frac{\partial\Psi}{\partial x^{\alpha}} - \Gamma_{\alpha}\Psi \tag{4.9}$$

or displaying the complete set of indices

$$D_{\alpha}\Psi^{A} = \frac{\partial\Psi^{A}}{\partial x^{\alpha}} - \Gamma_{\alpha}{}^{A}{}_{B}\Psi^{B}(x)$$

The internal connection Γ_{α} can be obtained by a slight modification of the Fock-Ivanenko coefficients.

We define a local tetrad system $e^A_{\ \mu}$ and extend as usual the elements of Clifford algebra of the matrices γ^α to space-time dependent functions $\gamma^\alpha(x)$. Then we obtain the metric tensor $g_{\mu\nu}(x)$ by the anti-commutating relation

$$\left\{\gamma_\mu(x)\ ,\ \gamma_\nu(x)\right\} = 2g_{\mu\nu}(x)\ \mathbb{1}$$

in which $\mathbb{1}$ is the identity of the Clifford algebra.
The conservation of length's condition $g_{\mu\nu\|\lambda} = 0$ can be obtained by setting(10)

$$D_\mu\gamma_\nu(x) \equiv \frac{\partial\gamma_\nu(x)}{\partial x^\mu} - \Gamma^\varepsilon_{\mu\nu}\gamma_\varepsilon(x) + \gamma_\nu(x)\Gamma_\mu(x) - \Gamma_\mu(x)\gamma_\nu(x) = \tag{4.10}$$

$$= 0$$

From this condition, and following Fock-Ivanenko we obtain the internal connection $\Gamma_{\nu\ B}^{\ A}$:

$$\Gamma_\nu = \frac{1}{8}\left[\frac{\partial\gamma_\mu}{\partial x^\nu}\gamma^\mu - \gamma^\mu\frac{\partial\gamma_\mu}{\partial x^\nu} + \Gamma^\varepsilon_{\nu\mu}(\gamma^\mu\gamma_\varepsilon - \gamma_\varepsilon\gamma^\mu)\right] \tag{4.11}$$

or, using the decomposition (4.2) and the Fock-Ivanenko symbol $\Gamma_\nu^{(grav.)}$ we can re-write eq. (4.11) under the form

$$\Gamma_\nu = \Gamma_\nu^{(grav)} + \frac{1}{8}K^\varepsilon_{\ \nu\mu}(\gamma^\mu\gamma_\varepsilon - \gamma_\varepsilon\gamma^\mu) \tag{4.11'}$$

The equation of motion for the Ψ-field of the electron is obtained from the Lagrangian

$$L = \sqrt{-g}\ \ \bar{\Psi}\gamma^\mu D_\mu\Psi \tag{4.12}$$

The coupling of the Ψ-field with the Cartan geometrical objets τ_μ and Σ_μ is contained in the new term of the Lagrangian given by

$$L_{int} = - \frac{i}{8} \overline{\Psi} \gamma^\mu K^\varepsilon{}_{\mu\alpha} (\gamma^\alpha \gamma_\varepsilon - \gamma_\varepsilon \gamma^\alpha) \Psi \qquad (4.13)$$

Using (4.7) into expression (4.13) a straightforward calculation reduces L_{int} into the form

$$L_{int} = i \overline{\Psi} \gamma^\mu \Psi \tau_\mu + \frac{i}{2} \overline{\Psi} \gamma_5 \gamma_\alpha \Psi \Sigma^\alpha \qquad (4.13')$$

in which we have defined γ_5 by the expression:

$$\gamma_5 = \frac{i}{4!} \varepsilon_{\alpha\beta\rho\sigma} \gamma^\alpha \gamma^\beta \gamma^\rho \gamma^\sigma$$

We remark that the interaction (4.13') does not violate parity once the pseudo-vector Σ^α couples only to the pseudo current.

Equation (4.13') induces us to identify τ^μ and Σ^μ with the potentials W^μ and Z^μ of the electromagnetic field. From dimensional considerations we conclude that we must introduce a constant β with the dimension $M^{-1/2}L^{-1/2} \equiv (\text{mass})^{-1/2}(\text{length})^{-1/2}$. We set

$$\tau_\mu = \frac{3}{4} \beta W_\mu$$
$$\Sigma_\mu = \frac{3}{2} \beta Z_\mu \qquad (4.14)$$

If the constant β is not a new independent one but should be constructed from others constants, then a good candidate is given by

$$\beta = a \frac{\text{velocity of light}}{\text{charge of the electron}} = a \frac{c}{e}$$

in which a is an adimensional constant.

With this choice, the Lagrangian (4.13') reduces to the usual theory of a doubly charged electron. Let us turn now to the dynamics for the torsion tensor.

5. DYNAMICS

In the previous section we were conducted to identify the trace and the pseudo-trace of the torsion with the potentials of the electromagnetic field. The next step into the geometrization scheme should be the description of dynamics.

In the search of a Lagrangian which describes electrodynamics and gravidynamics two remarks are very important. The first one is related to the fact that although the curvature tensor $R_{\alpha\beta\mu\nu}$ contains terms which are quadratic in the affine connection $\Gamma^{\alpha}_{\mu\nu}$, its contraction $R_{\mu\nu}$ does not contain mixed terms of the trace and the pseudo-trace. Indeed, a straightforward calculation gives:

$$R_{\mu\nu} = \dot{R}_{\mu\nu} + \frac{4}{3}\tau_{\mu;\nu} + \frac{2}{3}\tau^{\alpha}{}_{;\alpha}\, g_{\mu\nu} - \frac{8}{9}\tau_{\mu}\tau_{\nu} + \frac{8}{9}\tau^{2}\, g_{\mu\nu} - \frac{2}{9}\Sigma^{2}\, g_{\mu\nu} + \frac{2}{9}\Sigma_{\mu}\Sigma_{\nu} + \frac{1}{3}\varepsilon^{\alpha\beta}{}_{\mu\nu}\,\Sigma_{\alpha|\beta} \tag{5.1}$$

in which $\dot{R}_{\mu\nu}$ is the Riemann curvature tensor constructed only with $g_{\mu\nu}$ and its derivatives, the symbol; as before represents co-variant derivative in the associated Riemann space. The anti--symmetric part of (5.1) gives:

$$R_{\mu\nu} - R_{\nu\mu} = \frac{4}{3}\left(\tau_{\mu;\nu} - \tau_{\nu;\mu} + \frac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma}\Sigma_{\rho|\sigma}\right) \tag{5.2}$$

If we compare this form (5.2) with the previous decomposition of the electromagnetic tensor $F_{\mu\nu}$ given by (3.3), then we conclude that the previous identification of the two traces (τ_μ, Σ_μ) with the two potentials (W_μ, Z_μ) induces to the relation

$$F_{\mu\nu} = (\text{constant})(R_{\mu\nu} - R_{\nu\mu}) \tag{5.3}$$

From (5.3) we note that the general form of the Lagrangian to describe the dynamics of the electromagnetic field should contain, at least, quadratic terms on the curvature tensor. Substituting (4.14) into (5.1) we write

$$R_{\mu\nu} = \dot{R}_{\mu\nu} + \beta \left[W_{\mu;\nu} + \frac{1}{2} W^{\alpha}{}_{;\alpha}\, g_{\mu\nu} - \frac{1}{2} W_\mu W_\nu + \frac{1}{2} W^2 g_{\mu\nu} - \right.$$
$$\left. - \frac{1}{2} Z^2 g_{\mu\nu} + \frac{1}{2} Z_\mu Z_\nu + \frac{1}{2} \varepsilon^{\alpha\lambda}{}_{\mu\nu}\, Z_{\alpha|\lambda} \right] \tag{5.4}$$

and contracting to obtain the scalar R:

$$R = \dot{R} + \beta \left[3W^{\alpha}{}_{;\alpha} + \frac{3}{2} W^2 - \frac{3}{2} Z^2 \right] \tag{5.5}$$

In the search for the Lagrangian for the geometrical objects of the GCR one should use an analogy with the corresponding terms in the Riemannian case. There are three possible terms, in a Riemann space, which are quadratic in the curvature tensor:

$$\begin{aligned} L_a &= \dot{R}^2 \\ L_b &= \dot{R}^{\mu\nu}\dot{R}_{\mu\nu} \\ L_c &= \dot{R}^{\mu\nu\rho\sigma}\dot{R}_{\mu\nu\rho\sigma} \end{aligned} \tag{5.6}$$

However, under a variational principle these three terms are not independent. They satisfy an identity which reduces the most general form to a combination of two of these Lagrangian. We must have the identity:

$$\delta \left[\sqrt{-g}\; L_c\right] = \delta \left[\sqrt{-g}\; (4L_b - L_a)\right] \tag{5.7}$$

Thus the most general Lagrangian has the form

$$L = mL_a + nL_b$$

in which m and n are constants.

We write, for the total quadratic Lagrangian with source:

$$L = \sqrt{-g}\; (m\dot{R}^2 + n\dot{R}^{\mu\nu}\dot{R}_{\mu\nu}) + \sqrt{-g}\; L_m$$

With the stress-energy tensor $T_{\mu\nu}$ defined by $T_{\mu\nu} = \delta L/\delta g^{\mu\nu}$, it is straightforward to obtain that the trace of the stress tensor satisfies the relation

$$T = (m + 3n)\; \dot{\Box}\, \dot{R} \tag{5.8}$$

Thus, if the source of the field contains massless particles (radiation) then either $\dot{\Box}\, \dot{R} = 0$ or $m + 3n = 0$.

We see that the combination $\dot{R}^{\mu\nu}\dot{R}_{\mu\nu} - \frac{1}{3}\dot{R}^2$ constitutes a very special combination, in the Riemann space, associated to radiation.

Such result suggests the investigation of an analogous combination of the quadratic Lagrangian in the Cartan space.

From (5.4) and (5.5) we obtain

$$R_{\mu\nu}R^{\mu\nu} - \frac{1}{3} R^2 \doteq \dot{R}^{\mu\nu}\dot{R}_{\mu\nu} - \frac{1}{3}\dot{R}^2 + \beta^2 \Big[W^{\mu;\nu} W_{\mu;\nu} -$$

$$- (W^{\alpha}{}_{;\alpha})^2 - \frac{1}{4} (Z_{\alpha;\beta} - Z_{\beta;\alpha})^2 - \dot{R}^{\mu\nu} W_\mu W_\nu + \dot{R}^{\mu\nu} Z_\mu Z_\nu +$$

$$+ \frac{1}{2} W^{\mu}{}_{;\mu} Z^2 + W_{\alpha;\beta} Z^{\alpha} Z^{\beta} + \frac{1}{2} Z^2 W^2 - \frac{1}{2} (Z_{\mu} W^{\mu})^2 \Big] \qquad (5.9)$$

in which the symbol $\doteq$ means that these terms are equal up to a total divergence term.

At this point one should proceed and evaluate the equations of motion from (5.9). However, in order to simplify our exposition, let us work in the Maxwell gauge and assume that the two vectors are parallel (reducing the two potentials to the usual one potential theory) by setting as in (3.7) $Z_{\mu} = \gamma W_{\mu}$. In this case (5.9) reduces to

$$R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \doteq \dot{R}^{\mu\nu} \dot{R}_{\mu\nu} - \frac{1}{3} \dot{R}^2 + $$
$$+ \left(\frac{1}{2} - \frac{\gamma^2}{4}\right) (W^{\mu;\nu} - W^{\nu;\mu})(W_{\mu;\nu} - W_{\nu;\mu}) + \gamma^2 \dot{R}^{\mu\nu} W_{\mu} W_{\nu}$$

Set $f_{\mu\nu} \equiv W_{\mu;\nu} - W_{\nu;\mu}$ to obtain

$$R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \doteq \dot{R}^{\mu\nu} \dot{R}_{\mu\nu} - \frac{1}{3} \dot{R}^2 + $$
$$+ \left(\frac{1}{2} - \frac{\gamma^2}{4}\right) f^{\mu\nu} f_{\mu\nu} + \gamma^2 \dot{R}^{\mu\nu} W_{\mu} W_{\nu} \qquad (5.10)$$

If we define

$$F_{\mu\nu} \equiv \phi (f_{\mu\nu} - \gamma f^{*}_{\mu\nu})$$

in which ϕ and γ are constants, we have

$$F_{\mu\nu} F^{\mu\nu} \doteq \phi^2 (1-\gamma^2) f_{\mu\nu} f^{\mu\nu} + \text{total divergence}$$

and thus

$$R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \doteq \dot{R}^{\mu\nu} \dot{R}_{\mu\nu} - \frac{1}{3} \dot{R}^2 + \frac{1}{4} \frac{(2-\gamma^2)}{\phi^2 (1-\gamma^2)} F_{\mu\nu} F^{\mu\nu} + $$
$$+ \gamma^2 \dot{R}^{\mu\nu} W_{\mu} W_{\nu} \qquad (5.11)$$

Finally, the dynamics of the electromagnetic and the gravitational field is given by:

$$L = \frac{1}{k} \sqrt{-g} \left\{ R + \sigma^2 \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) \right\} + eJ^{\mu} W_{\mu} + bJ^{\mu} Z_{\mu} + L_M \tag{5.12}$$

in which σ is a constant with dimension of a length.

In the case $\dot{R}_{\mu\nu} = 0$ we obtain for the $f_{\mu\nu}$-field:

$$\begin{aligned} f^{\mu\nu}{}_{;\nu} &= e_0 J^{\mu} \\ \overset{*}{f}{}^{\mu\nu}{}_{;\nu} &= 0 \end{aligned} \tag{5.13}$$

in which we have defined the renormalized charge $e_0 = 2 \left(\frac{2e+\gamma b}{\gamma^2 - 2} \right)$
Equivalently, using the variable $F_{\mu\nu}$ instead of $f_{\mu\nu}$ we have:

$$\begin{aligned} F^{\mu\nu}{}_{;\nu} &= \phi\, e_0\, J^{\mu} \\ \overset{*}{F}{}^{\mu\nu}{}_{;\nu} &= \gamma\, \phi\, e_0\, J^{\mu} \end{aligned} \tag{5.14}$$

Thus, we conclude that the geometrization scheme is possible if all dyons have the constant ratio $\frac{\text{magnetic charge}}{\text{electric charge}} = \gamma$.

Note that the choice of variable $f_{\mu\nu}$ or $F_{\mu\nu}$ is arbitrary because they are related by a dual rotation and the system is invariant under such transformation.

Remark furthermore that the meaning of the constant γ which measures the proportionality of the trace and the pseudo--trace of the torsion in GCR is nothing but the universal constant ratio of the magnetic charge with respect to the electric charge. The existence of such constant and the assumption (to be proved by experiment) that γ is independent of any dyon supports the geometrization scheme presented in this paper.

Let us come back to the Lagrangian (5.12). We have introduced the constant σ for dimensional considerations. In order to obtain the correct Maxwell limit (in the case the associated Riemann curvature $\dot{R}_{\mu\nu}$ vanishes) we impose

$$\frac{\sigma^2 \beta^2}{k} = c^2$$

in which β is the constant introduced in (4.14). Using the tentative value $\beta = a \frac{c}{e}$ (a is an adimensional constant) we obtain

$$\sigma^2 = \frac{1}{a^2} k e^2$$

We see then that the linear Lagrangian is coupled to the gravitational constant k and the quadratic Lagrangian is coupled to the electromagnetic constant (the unit of charge) e.

6. CONCLUSION

The description of Electromagnetic field by means of two four-vector potentials is an almost direct consequence of the dual symmetry of the theory.

The usual one potential model can be re-gained by a choice of the dual angle (which we have called Maxwell-gauge) which annihilate the magnetic charge and leaves a pure electric charge for any dyon.

Such description could well be accomodated in a Restricted Cartan Geometry, which contains only two independent vectors: the trace and the pseudo-trace of the torsion. In order of this identification to be independent of any particular dyon, the ratio of the magnetic charge to the electric charge of any dyon must be an universal constant. Such universality, that is the main support of the geometrization scheme, makes the difference between electric and magnetic charge a pure matter of convention. This indistinguishibility has the same formal content as that which occurs between the inertial and the gravitational mass and which is on the basis of the geometrization of the gravitational field.

Let us make now a remark on the equation of the electron (4.8). We have shown that the coupling of the electron with the electromagnetic field is obtained as the equation of a free particle in a curved Cartan restricted geometry. One should pehaps ask what happen if the particle described by equation (4.8) does not acquires a mass, like the neutrino, which does not have a direct electromagnetic interaction. This implies that the total observed charge of the neutrino vanishes. It does not imply that both the electric and the magnetic charge of the neutrino vanishes but only that $q_\nu = (e_\nu^2 + b_\nu^2)^{1/2} = 0$. In order to conciliate this observed fact ($q_\nu = 0$) with our model we have to require that the neutrino ratio of the magnetic to the electric charge is a pure imaginary number. Thus we are conducted to state that there are two classes of dyons, which are characterized by the ratio

$$\text{(a)} \qquad \left\{\frac{\text{magnetic charge}}{\text{electric charge}}\right\}^2 = 1$$

$$\text{(b)} \qquad \left\{\frac{\text{magnetic charge}}{\text{electric charge}}\right\}^2 = -1$$

in which we have normalized the universal constant γ.

The fact that the magnetic charge of the neutrino (for instance) is an imaginary number is not in contradiction with observation once only the total charge q is a direct observable. One may pehaps be unsatisfied with the introduction of an apparently unobservable. The unique defense in favour of this,. that I can say is that: it works.

Using this value for the neutrino total charge into the equation (4.13') it is possible to annihilate the interacting term and thus eliminate the influence of the torsion into the neutrino equation.

Let us make another remark with respecto to the Lagrangian (5.12). The linear term on the scalar of curvature is introduced in order to reproduce Einstein's theory as a first approximation. If the torsion is a constant then this term reduces to $\dot{R} + \Lambda$ in which the constant factor Λ equals $\frac{3}{2}\beta(1-4\gamma^2)W^2$, for W^2 = constant. Thus a constant torsion would renormalized the cosmological constant.

It seems worthwhile to remark that the model we present here induces a correction on the usual Maxwell theory by the presence of the non-minimal interacting term $\dot{R}^{\mu\nu}W_\mu W_\nu$ on the Lagrangian (5.12). A similar term ($RW_\mu W^\mu$) has been analysed recently[6] and it has been shown that in a conformal flat universe of Friedmann type, this factor induces a non-singular behaviour of the Cosmos by allowing the radius of the Universe to attain a non-zero minimal value.

Finally let us emphasize that we have learned from the last fifty years that the structure of space-time depends on the universality of the basic characteristics of the existing interactions. This has been noted the first time by Einstein with respect to the universal equivalence of inertial and gravitational mass and sketched out here in the case of the constancy of the ratio of the magnetic and the electric charge, for Electrodynamics.

These two universals do not exhaust the reality, but they contain a firm and clear basis on the observation which can be rejected or confirmed by experiments.

Any further tentative of modification of the structure of space time should exhibit *ab initio*, a new corresponding universal. This is the continuation of the Einstein programme.

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