NOTAS DE FÍSICA
VOLUME XXIII
Nº 3

STATIC SCALARLY CHARGED DUST

by
A.F. da F. Teixeira, M.M. Som and Idel Wolk

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS
Av. Wenceslau Braz, 71 - Botafogo - ZC-82
Rio de Janeiro, Brazil
1974

Notas de Física - Volume XXIII - Nº 3

STATIC SCALARLY CHARGED DUST *

A.F. da F. Teixeira, M.M. Som and Idel Wolk
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro, Brazil

(Received 13th August 1974)

ABSTRACT

In this paper we show that when one has a static regular distribution of incoherent matter charged in scalar sense the matter density must be numerically equal to the charge density. The exterior field for such a charged dust sphere has also been studied. A solution which is analogous to *Reissner-Nordstrom* one is presented for a mass point charged scalarly.

* To be submitted for publication to Canadian Journal of Physics

I. INTRODUCTION

During fifties considerable interest has been focussed on a set of field equations in which a scalar field is coupled with gravitational field (SZEKERES 1955, BERGMANN and LEIPNIK 1957, YILMAZ 1958). Later DAS (1962) studied the coupled gravitational and scalar fields in the presence of incoherent matter charged in scalar sense. To obtain exact solutions he introduces beside the static condition two additional assumptions: 1) a functional relationship between g_{oo} and the scalar potential, and 2) equality of the densities of charge in scalar sense and matter.

In the present paper we show that one obtains the above mentioned functional relationship from the field equations alone, and the equality of densities follows directly if the distribution is regular. We also study the field due to a mass point charged in scalar sense; this solution is analogous to Reissner-Nordstrom solution for charged mass point.

II. GENERAL SYSTEMS

We start with the static line element

$$ds^2 = e^{2\eta}\, dx^{o^2} + g_{ij}\, dx^i\, dx^j \quad , \tag{2.1}$$

with η and g_{ij} functions of x^k only; we are using coordinates

$$x^\mu = (x^o, x^i) \; , \quad i = 1, 2, 3 \; . \tag{2.2}$$

In Einstein equations

$$R^{\mu}_{\nu} = -8\pi \left(T^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu} T\right) \tag{2.3}$$

we consider $c = G = 1$ and

$$T^{\mu}_{\nu} = \rho u^{\mu} u_{\nu} - \frac{1}{4\pi}\left(S^{,\mu} S_{,\nu} - \frac{1}{2}\delta^{\mu}_{\nu} S^{,\lambda} S_{,\lambda}\right); \tag{2.4}$$

here $\rho(x^i)$ is mass density,

$$u^{\mu} = \delta^{\mu}_{o} e^{-\eta} \tag{2.5}$$

is the four-velocity field of ρ, and $S(x^i)$ is the long range scalar field satisfying

$$S^{;\mu}_{;\mu} = 4\pi\alpha \quad , \tag{2.6}$$

where $\alpha(x^i)$ is the source density of S. Comma(,) and semicolon (;) denote partial and covariant derivatives, respectively. The divergence of equation (2.3) yields with the help of (2.4) and (2.6)

$$\rho u^{\mu}_{;\nu} u^{\nu} - \alpha S^{,\mu} = 0 \ . \tag{2.7}$$

Now for $\mu = \nu = 0$ in equation (2.3) we get

$$(\sqrt{-g}\,\eta^{,i})_{,i} + 4\pi\sqrt{-g}\,\rho = 0 \quad , \tag{2.8}$$

and from (2.6) and (2.7) we obtain

$$(\sqrt{-g}\cdot S^{,i})_{,i} - 4\pi\sqrt{-g}\,\alpha = 0 \qquad \text{and} \tag{2.9}$$

$$\rho\eta_{,i} + \alpha S_{,i} = 0 \quad . \tag{2.10}$$

This last equation implies that S, η and ρ/α are functionally related. Writing $S = S(\eta)$ we have from (2.10)

$$\rho = -\,\alpha\, S' \quad , \tag{2.11}$$

where

$$S' = dS/d\eta \quad . \tag{2.12}$$

Substitution of this in (2.8) and use of (2.9) gives the divergence relation

$$\left[\sqrt{-g}\,\eta^{,i}\,(1 - S'^2)^{1/2}\right]_{,i} = 0 \quad . \tag{2.13}$$

We define the function

$$\xi(\eta) = \int (1 - S'^2)^{1/2}\, d\eta \quad ; \tag{2.14}$$

if the region is singularity-free then from (2.13)

$$\int \xi^{,i}\,\sqrt{-g}\, ds_i = 0 \quad , \tag{2.15}$$

where the integral is taken over an arbitrary closed surface inside the region. And again from (2.13) we obtain

$$\int \xi\xi^{,i}\,\sqrt{-g}\, ds_i = \int \xi^{,i}\,\xi_{,i}\,\sqrt{-g}\;\; dv \quad . \tag{2.16}$$

We can choose as surface of integration an η = const surface, then left hand side vanishes due to (2.15); now since $\xi^{,i}\ \xi_{,i}$ has everywhere the same sign the right hand side can only vanish when $\xi_{,i} = 0$, that is,

$$S'^2 = 1 \quad . \tag{2.17}$$

We then get from (2.11) and (2.12)

$$\alpha = \pm\, \rho \qquad \text{and} \tag{2.18}$$

$$S = \mp\, \eta \quad ; \tag{2.19}$$

in the last equation we choose the integration constant as zero for convenience.

These results exactly coincide with the non-relativistic analogous ones, in which η is the Newtonian potential: the gravitational attraction is balanced by the "long range" scalar repulsion produced by the source α of density numerically equal (in our units) to the mass density .

Following Das (1962) we next define a new 3-space metric

$$\overline{g}_{ij} = -e^{2\eta}\, g_{ij} \tag{2.20}$$

and evaluate the corresponding Ricci tensor $\overline{R}^i_j$; then from the equations for R^o_o and R^i_j in (2.3) and from $S^2 = \eta^2$ we get that $\overline{R}^i_j = 0$; since in 3-space this implies that also the Riemann-Christoffel tensor $\overline{R}^i_{jkl} = 0$, we have a flat 3-space with metric $\overline{g}_{ij}$. So the only possible solution of such systems is

$$\alpha = \pm \rho \quad , \tag{2.21}$$

$$S = \mp \eta \quad , \tag{2.22}$$

$$ds^2 = e^{2\eta}\, dx^{o^2} - e^{-2\eta}\, \bar{g}_{ij}\, dx^i\, dx^j \tag{2.23}$$

and

$$e^{2\eta}\bar{\Delta}\eta = 4\pi\rho \tag{2.24}$$

where the Laplacian $\bar{\Delta}$ corresponds to the flat metric $\bar{g}_{ij}$.

III. SPHERICALLY SYMMETRIC EXTERIOR SOLUTIONS

We consider now the case $\rho = \alpha = 0$. Our concerning equations are then only (2.1) to (2.5). We choose the line element

$$ds^2 = e^{2\eta}\, dx^{o^2} - e^{2\zeta}\, dr^2 - r^2\, e^{\zeta-\eta}(d\theta^2 + \sin^2\theta d\phi^2) \tag{3.1}$$

with η and ζ functions of radial coordinate r alone. Einstein equations give now

$$\eta_{11} + 2\eta_1/r = 0 \quad , \tag{3.2}$$

$$r^2(\eta_{11} + \zeta_{11}) + 2r(\eta_1 + \zeta_1) + 2(1 - e^{\eta+\zeta}) = 0 \quad \text{and} \tag{3.3}$$

$$2\zeta_{11} + 3\,\eta_1^2 - \zeta_1^2 - 2\eta_1\,\zeta_1 - 4\eta_1/r = 4S_1^2 \quad , \tag{3.4}$$

where the subscript 1 means d/dr.

For $g_{oo} = - g_{rr} = 1$ at infinity and $S = 0$ there, we get from (3.2) (3.3) and (3.4) respectively

$$\eta = - a/r \quad , \tag{3.5}$$

$$e^{\zeta} = (b/2r)^2 \sinh^{-2}(b/2r)\ e^{a/r} \tag{3.6}$$

and

$$S = s/r \quad , \tag{3.7}$$

with a, b and s integration constants satisfying

$$b^2 + 4s^2 - 4a^2 = 0 \quad . \tag{3.8}$$

From (3.8) we find that there are only two independent constants; these two constants may be interpreted as parameters representing gravitational and scalar charges. Then this solution is analogous to Reissner-Nordstrom solution. Solution with $s = 0$ corresponds to Schwarzschild solution in our coordinate system: in this case the parameter a is the Schwarschild mass.

If this external solution is to represent the field due to a scalarly charged dust sphere the $g_{\nu\mu}$'s and S are to satisfy the continuity conditions at the boundary; so we have from (2.22) (3.5) and (3.7) $a^2 = s^2$; then (3.8) gives $b = 0$ so we have from (3.6) $\zeta = - \eta$. Under these conditions the exterior metric is

$$ds^2 = e^{-2a/r}\, dx^{o^2} - e^{2a/r}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad , \tag{3.9}$$

with the scalar field given by $S = \pm a/r$.

ACKNOWLEDGEMENTS

On of the authors (M.M.S.) acknowledges financial support from Conselho Nacional de Pesquisas, Brazil.

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