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OHM'S LAW AND DEFINITION OF METALLIC STATE*

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1. INTRODUCTION

The phenomenon of electric conductivity consists in the transformation of electromagnetic field energy (e.g. of the magnetic energy $\frac{1}{2}L.J^2$ of a D.C.) into mechanical energy of the internal vibrations of a metallic lattice. It is, therefore impossible to treat this problem without reference to the electromagnetic field.

In what follows, we shall first derive a simultaneous solution of the fundamental equations of mechanics and electrodynamics analogous to Lorentz' theory of dispersion. This solution, however, will not be found in agreement with experimental evidence. Nevertheless, it permits to study the essential factors which intervene in the problem and permits to derive the correct solution from the particular way in which our first attempt fails.

The first step consists in the simultaneous solution of the equations

$$\mathrm{rot}\,\vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j} \quad ; \quad \mathrm{rot}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$m.\dot{\vec{v}} + \eta.\vec{v} = e.\vec{E} \tag{2}$$

$$\vec{j} = n.e.\vec{v} \tag{3}$$

In (3) n denotes the number of electrons per unit volume, the viscosity coefficient η represents, in (2), schematically the coupling between the electrons and the lattice vibrations.

2. THE SOLUTION OF THE SYSTEM

In order to use ordinary notations, we write (2) by means of (3) in the form

$$\sigma\Lambda\dot{\vec{j}} + \vec{j} = \sigma.\vec{E} \tag{2'}$$

with

$$\sigma = \frac{ne^2}{\eta} \quad ; \quad \Lambda = \frac{m}{ne^2} \tag{4}$$

In (4) σ denotes the coefficient of specific conductivity, Λ shall be called, in what follows, the Lorentz'-London inertia constant.

Equation (2') contains two well known limiting cases. With $\Lambda = 0$ (charges without inertia) it leads to Ohm's law

$$\vec{j} = \sigma.\vec{E} \tag{2''}$$

with $\sigma \rightarrow \infty$ we obtain one of the equations which have been proposed by F. London in order to describe the Meissner effect

$$\Lambda \frac{\partial \vec{j}}{\partial t} = \vec{E} \qquad (2''')$$

(2') mixes, therefore all necessary elements which intervene in the problem of conductivity, but we shall see that it mixes them in a wrong way. Nevertheless, we may conclude from (2''') that the Meissner effect represents a simple, classical effect of inertia.

It is much more difficult to understand the limit $\Lambda = 0$, which leads to Ohm's law (2''). It means that charges move as if they had no mechanical inertia. We know phenomena which give direct evidence of the specific electron inertia m/e in metals. On the other hand Ohm's law holds in metals with remarkable accuracy. There must, therefore, exist very general types of motion which are, apparently, inertiafree. We shall find below, that classical mechanics ignores such types of motion, while quantum theory accounts for them. Contrarily to current views, we arrive, therefore, to the conclusion that supraconductivity is, basically, a classical phenomenon, while normal conductivity represents an essential quantum effect.

In order to see the difficulties to which (1) and (2') lead, we derive, with div $\vec{E} = 0$, the following relations:

$$\left(1 + \sigma\Lambda\frac{\partial}{\partial t}\right)\left(\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\right) = \frac{4\pi\sigma}{c^2}\frac{\partial\vec{E}}{\partial t} \qquad (5)$$

$$\text{rot}\,\vec{j} = -\sigma\frac{1}{c}\frac{\partial}{\partial t}\left(\vec{H} + \Lambda\cdot c\cdot\text{rot}\,\vec{j}\right) \qquad (6)$$

which become in the limit $\sigma \longrightarrow \infty$ for any periodical phenomenon of arbitrary frequency

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{4\pi}{\Lambda c^2}\vec{E} \qquad (7)$$

$$\Lambda\cdot c\cdot\text{rot}\,\vec{j} = -\vec{H} \qquad (8)$$

(7) and (8) are well known relations of London's theory.

(5) describes a superposition of **two effects**: the exponential penetration of a field into a metal, without absorption, due to the inertia Λ and the damped oscillating penetration with absorption due to conductivity σ. Neither in the case of supraconductivity nor in the normal case (5) leads to correct results. In the case of supraconductivity London's equations (7) are only valid for small frequencies, while, according to (5), they should hold for any frequency. In the case of normal conductivity (5) leads for sufficiently high frequencies (e.g. in the infrared) to a dominating influence of the terms containing Λ and, therefore to practically total reflexion of light at the surface of a metal, while experimental evidence shows, that we obtain the correct answer only if we disregard the terms with Λ.

3. THE RESONANCE CONDITION

In order to avoid the above mentioned contradictions, we consider a classical damped oscillator under the influence of an external, periodical force $F = F_o . e^{i\omega_o t}$. Denoting by ω the proper frequency of the oscillator, we obtain

$$m. (\omega^2 - \omega_o^2). \xi + \eta . \dot{\xi} = F \tag{9}$$

According to (9) the inertia term $-m.\omega_o^2 . \xi$ becomes reduced by the acting elastic force and vanishes in the case of resonance

$$\omega_o = \omega \tag{10}$$

This circumstance permits us to replace our above statement, that electrons move in a normal metal as if they had no mechanical inertia, by the more suitable conjecture that the metal behaves as if it was in resonance with any of the considered waves.

Before we proceed to investigate how such a behaviour can be explained, we shall formulate our condition axiomatically:

We shall call the behaviour of a substance metallic, if in a given range of parameters (e.g. of temperature) there exists a frequency range in which any electromagnetic wave is in resonance with the substance.

We have, now, to prove that a substance, under certain conditions can satisfy our resonance condition and that our condition is sufficient in order to remove the contradictions above stated.

4. REALIZATION OF THE RESONANCE CONDITION

Within the reach of classical mechanics the resonance condition can hardly be sensefully satisfied. In quantum theory our condition requires first the existence of a continuous energy spectrum. This necessary, but not sufficient condition is satisfied by most substances at sufficiently high energies or temperatures. In the case of metals it is satisfied already at energy zero or, at least, at very small energies above the ground state.

We have further to require, that there exist radiative resonance transitions between any two states of the continuous spectrum. This condition can be satisfied by electrons which move under the influence of external forces. It cannot be satisfied by an ideal electron gas. An ideal electron gas can, therefore, not conduce to metallic behaviour.

We consider, now, a plane electromagnetic wave, which consists of a large, but not well defined number of photons and which has a fairly well defined phase. If this wave becomes reflected by the surface of a metal, every incident photon becomes absorbed by the metal and almost every photon becomes subsequently reemitted by coherent scattering in the direction of reflexion. We will find, on the surface of the metal a resonance equilibrium between incident and reflected photons and certain excited states of the electrons.

The (quasi-)stationary eigenfuction which describes the resonance equilibrium belonging to a given initial state of the electrons, ψ_0, can be written in the form

$$\sum_n d_n \cdot \psi_n \cdot \Gamma_n \qquad (11)$$

Γ_0 refers to to the field configuration if no electron is excited. It represents a wave packet of fairly well defined phase. ψ_n are eigenfunctions of electron states which can be reached from ψ_0 by resonance absorption of n photons. Analogously, Γ_n describes the field which results from Γ_0 through absorption of n photons. Since the Γ_n are no eigenfunctions, but wave packets, they will not be orthogonal between themselves. The superposition of the different terms of (11) localises the reflexion process on the surface of the metal and destroys the field in the interior of the metal. The coefficients d_n satisfy a system of linear equations and depend on the particular radiative transition matrix elements of our system. They satisfy the condition $\sum_n |d_n|^2 = 1$.

Practically only a finite number of terms, of the order of the average number of photons present in the field, will contri bute to the sum (11). (11) represents only an approximate, not a rigorous solution of our problem, because it neglects virtual dispersion transitions and, in particular incoherent scattering processes, which, even in the case of absence of excitations of lattice vibrations reduce slightly the intensity of the reflected light. The inclusion of incoherent emission of photons by interaction with the lattice vibrations accounts, in our picture, for the effect of conductivity.

Since, in (11), the Γ_n are not orthogonal (known phase), observable, time dependent transition densities of the type

$$\psi_n^* \psi_{n+1}$$

result from (11). These are the charge and current densities which are required by Maxwell's theory in order to account for the reflexion process. Those are interference terms between two eigenfunctions, terms which are ignored by classical theory. We can, now, also answer the question concerning the origin of the elastic force which we had postulated ad hoc in (9). Those forces are due, according to (11) to the Schrödinger or Dirac tensions, which are characteristic for quantum mechanics. It is the transition terms of those tensions which compensate the inertia of the electron motion and assure the validity of Ohm's law in metals.

5. THE T - ν - DIAGRAM OF SUPRACONDUCTIVITY

A supraconductor, in which we have seen inertia effects to be present, cannot satisfy our resonance condition and, according to our definition, does not represent a metallic state.

Though we ignore up to now, how supraconductivity can be interpreted by a detailed model, we know that the supraconducting state is restricted to small energies of the order.

$$k.T_s$$

(T_s = transition temperature).

If a wave of small frequency

$$h.\nu < k.T_s \tag{12}$$

is reflected by a supraconductor, equations of the type of (5) must hold, since the resonance condition is not satisfied. We know from experience, that, indeed, supraconductivity is found at low frequencies. If, however, the frequency is high

$$h.\nu > k.T_s \tag{13}$$

resonance absorption will, in general become possible and the supraconductor will behave like a metal. This is again in agreement with experience. M. v. Laue has described these conditions by supposing, ad hoc, two independent mechanisms of conductivity[1]. In our representation the failure of London's equations at high frequencies becomes a natural consequence of our basic assumptions. In the T- ν - diagram, supraconductivity does not occupy the whole band of low temperatures but only the narrow corner determined by (12).

(1) Max Von Laue, Theorie der Supraleitung, Berlin und Göttingen, 1947.