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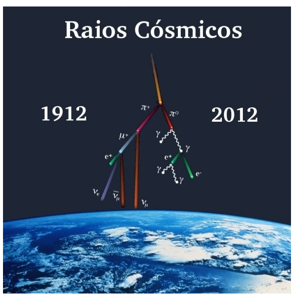

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The foundational origin of integrability in quantum field theory[1]

dedicated to Raymond Stora on the occasion of his 80th birthday

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Abstract

There are two foundational model-independent concepts of integrability in QFT. One is "dynamical" and generalizes the solvability in closed analytic form of the dynamical aspects as known from the Kepler two-body problem and its quantum mechanical counterpart.

The other, referred to as "kinematical" integrability, has no classical nor even quantum mechanical counterpart; it describes the relation between so called *field algebra* and its local *observable* subalgebras and their discrete inequivalent representation classes (the DHR theory of superselection sectors). In the standard case of QFTs with mass gaps it contains the informations about the representation of the (necessary compact) internal symmetry group and statistics in form of a tracial state on a "dual group". In Lagrangian or functional quantization one deals with the field algebra and the division into observable /field algebras does presently not play a role in constructive approaches to QFT. "Kinematical" integrability is however of particular interest in conformal theories where the observable algebra fulfills the Huygens principle (lightlike propagation) and lives on the compactified Minkowski spacetime whereas the field algebra, whose spacetime symmetry group is the universal covering of the conformal group lives on the universal covering of the compactified Minkowski spacetime. Since the (anomalous) dimensions of fields show up in the spectrum of the unitary representative of the center of this group , the kinematical structure contained in the relation fields/Huygens observables valuable informations which in the usual terminology would be called "dynamical".

The dynamical integrability is defined in terms of properties of "wedge localization" and uses the fact that modular localization theory allows to "emulate" interaction-free wedge-localized operators in a bijective manner with the wedge localized interacting algebra. Emulation can be viewed as a generalization of the functorial relation between localized subspaces of Wigner particle spaces and localized subalgebras of the global algebra of all operators on Wigner-Fock space which does not require a classical-quantum quantization parallelism. Its extension to interacting QFTs leads to a profound understanding of integrability versus nonintegrability and of the crossing property of particle theory. Integrable models with nontrivial scattering amplitudes can only occur in d=1+1 where they are only consistent with elastic S-matrices. The associated field theories are the so-called "factorizing models"; there existence can be (and in many cases has been) established by methods of modular localized operator algebras.

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1 Integrability in classical and quantum theory

Integrability in celestial mechanics had originally the obvious meaning of being able to solve the equation of motion in closed analytic form. The prototype model was the Kepler two-body system, which in case of three or more celestial bodies can only be numerically approximated with arbitrary accuracy. For the non-integrable case the terminology does not just refer to the fact that an attempt to find a solution in closed analytic form remained without success, but rather that there exists a proof that such a solution does not exist.

As the mathematical sophistication evolved, physicists and mathematicians developed model independent criteria for integrability. The modern definition which is sufficiently general to cover all of classical mechanics was in terms of the existence of a *complete set of conservation laws in involution* [1].

This definition could be extended from mechanics to include *classical field theory* where, according to Noether's theorem, a symmetry in the Lagrangian setting leads to a conserved current and integrability means that there exists an infinite complete set of conserved currents in involution. Quantum mechanics is basically what is obtained from classical mechanics by "quantization"; the fact that this process is not an isomorphism but a more artistic correspondence (thinking e.g. of the problem of ordering of operator products) did not affect the inference of quantum integrability via quantization from its classical counterpart. The most famous illustration is the quantum analog of the Kepler problem i.e. the hydrogen atom. In this case the conservation laws which lead to integrability can even be elegantly presented in terms a spectrum-setting $O(4)$ group symmetry. The quantum mechanical integrability is inherited from its classical counterpart by quantization; anomalies need the presence of infinitely many degrees of freedom and hence can only occur in quantum field theory (QFT).

There are many models of QFT which have remained outside the Lagrangian quantization formulation (most of the so-called d=1+1 factorizing models), in fact the "scaling Z(N) Ising model" [2] is representative (as the authors emphasize in the introduction of their paper) for a class of well defined QFTs with very nontrivial short distance properties beyond that of the unrealistic superrenormalizable canonical models to which the quantum mechanical functional methods of Glimm-Jaffe [3] are applicable. Only a few integrable models (including the famous Sine-Gordon model) allow a Lagrangian representation; the first indication about field theoretic integrability came from the famous quasiclassical observations on the Sine-Gordon model by Dashen-Hasslacher and Neveu [4]. But even in those fortunate cases where the Lagrangian quantization setting provides a renormalized perturbation series, the latter is known to diverge. Lagrangian quantization, although being able to "baptize" a theory with a name from classical field theory, does not provide a proof of existence of a QFT; it only leads to a formal power series which is mathematically consistent only in the formal sense of a power series. Hence the surprises about the numerical success of renormalized QED and the subsequent observational achievements of the standard model did not yet reach their closure.

The importance of the dynamically integrable d=1+1 factorizing models does not result from insights obtained through the classical parallelism of Lagrangian- or functional-quantization. It rather has been revealed by extending a method which was first introduced in 1939 by Wigner[2] as a means to obtain an unique and intrinsic classification of relativistic particles (instead of having to cope with an ever-growing number of quantized field equations leading to equivalent descriptions). In the context of QFT, representation theoretical methods were first used in connection with current algebras and chiral conformal QFTs. A link between those methods and the quasiclassical DHN observations was the first indication that the quasiclassical Sine-Gordon spectrum was the exact "nuclear democracy" particle spectrum in a d=1+1 adaptation of a bootstrap setting [6] (later bootstrap-formfactor setting [7]). This project reached its present (preliminary) closure when it was realized that

- Elastic two-particle scattering functions fulfilling unitarity and the crossing property can be classified [8][9] and lead to n-particle elastic S-matrices through a combinatorial product formula. The Zamolodchikovs formal algebraization added a useful facilitating tool [10] in the implementation of this (originally analytic) classification project.

- The bootstrap-formfactor program associates to each such scattering function[3] explicitly computable particle formfactors of local covariant fields from the (Borchers) local equivalence class of (composite) fields. The relation between the scattering function to an associated model of QFT is unique (uniqueness of the solution of the inverse scattering problem) and the formfactors of (composite) fields which have been computed in the bootstrap-formfactor project are formfactors of generating fields in this unique QFT. For the solution of the existence problem one uses algebraic methods.

- The creation and annihilation operators of the Zamolodchikov-Faddeev algebra turn out to be the Fourier components of covariant nonlocal vacuum-polarization-free generators (PFGs) of interacting wedge-localized algebras [11][12]. They are special objects within the theory of "modular localization" which permits to "emulate" wedge-localized products of free fields *inside the associated wedge-localized interacting algebra.*

- The action of translations on a wedge-localized algebra together with that of the modular reflection (in d=1+1 the TCP operation) generate a net of right and left directed wedge algebras whose double cone intersections are compactly localized algebras which act cyclic and separating on the vacuum [14][15]. This (Reeh-Schlieder) property establishes the nontrivial existence of factorizing models in the setting of algebraic QFT. The local equivalence class of fields (of which the bootstrap-formfactor program already determined their formfactors) are different "coordinatizations" of the local net of operator algebras. As expected such generators inherit

[2]Wigner always maintained a philosophical stance; his critical position towards QFT did not result from ignoring its great successes in renormalized QED, but rather from his philosophical conviction that a more fundamental theory should not dance to the classical tunes of a less fundamental. An intrinsic quantum understanding of causal localization as the recent "modular localization" [5] was still far away, and it is well-known that he remained critical with respect to the frame-dependent Born-Newton-Wigner probabilistic localization.-

[3]In the presence of backward scattering and/or inner symmetry indices the scattering function is a matrix function which fulfills the Yang-Baxter equation [13].

all the technical problems of pointlike singular fields (operator-valued distributions) which, through misunderstandings of the inherent singular nature of objects obtained through field quantization, were erroneously interpreted as an internal inconsistency of QFT ("ultraviolet catastrophe") during the first three decades of its existence.

- A sharp division between "temperate" and non-temperate generating PFG's (section 4) of wedge-localized algebras gives rise to a dichotomy of integrable/nonintegrable QFTs. Modular localization relates this division directly to the foundational causal locality principles of algebraic QFT, thus avoiding more indirect definitions via complete sets of conserved currents in involution based on Lagrangian quantization. In contradistinction to classical and quantum mechanics, integrability in QFT is strictly tied to the spacetime dimension d=1+1. An existence proof in the (generic) case of nonintegrable models does not yet exist, but as the three and more particle systems of celestial classical- and quantum- mechanics such models (including the standard model) will never be solved but at best approximated with increasing accuracy.

- The DHR superselection theory [16] which constructs a full QFT from its local observables leads to a different kind of "partial (or kinematical) integrability" which is of a more kinematical nature. It is based on the fact that theories which are nonintegrable in the previous (dynamical) sense may still contain integrable (fully computable) substructures which characterize the positioning of their observable subalgebras to their full field algebras. In the standard case of QFT with mass-gaps the Doplicher-Haag-Roberts (DHR) superselection theory extracts this structure from the local representation theory of the net of observables [16] in the form of a tracial state on the dual of a compact group (including the statistics data). Although this is crucial for understanding the origin of inner symmetries from the causal localization structure of the observables (by definition invariant), its computational advantage is limited by the fact that the Lagrangian approach bypasses the observables in favor of the charge-carrying fields. It becomes important for conformal QFT where the observable algebras are Huygens-invariant. Even if they are not explicitely known, they contain sufficient structure in order to be able to analyze their local representation classes in terms of tracial states (Markov traces) on discrete algebras (Hecke algebras) containing representations of the braid group. The resulting operator algebras of type II_1 are special cases of those appearing in the Jones subfactor theory [17]. Unlike the algebra $B(H)$ of all operators on a Hilbert space and the causally localized subalgebras of QFT $\mathcal{A}(\mathcal{O})$ ("monades", see later) they are too "small" for the realization of causal localization but they can take care of the difference between left/right as it occurs in braiding. What makes this algebraic structure particularly interesting is that this algebraic structure contains the unitary representing the center of the covering group which contains informations about the spectrum of the anomalous dimensions (last section).

The new results in the above list concern the spacetime origin of integrability and constitute the main part of this paper. Although the important unifying concept of *modular localization* within the modular theory of operator algebras exist for more than a decade, I am perfectly aware that most of the readers to whom I want to communicate these findings have not become aware about them before. In order not to loose those readers, I will present in the next section some of them in an intuitive reader-friendly way whenever this is possible (related papers are [18][49][59]).

The foundational role of modular localization is best accounted for by first showing its importance in a variety of familiar problems of particle physics, before presenting in section 4 the resolution of the integrability problem. In this way it will become clear that this problem cannot be solved without a radical re-orientation with respect to the principles of QFT.

The natural setting for the modular theory is that of "local quantum physics" (LQP) [16] which appeared in a rudimentary form already in 1957 in Haag's first attempt [19] to formulate QFT in an intrinsic way (without using any parallelism to the classical Lagrangian fields). This setting competed with Wightman's at that time already existing more mathematically based setting interpreting fields as operator-valued distributions. Both approaches were strongly influenced by Wigner's 1939 particle classification in terms of group representation concepts instead of quantization of classical field equations. Haag's basic idea was very simple and almost naive: measurements of local observables in a causally closed spacetime region $\mathcal{O}$ which have a certain duration in time (the duration of the activation of a particle counter) and spatial extension within $\mathcal{O}$ should be members of an algebra ("ensemble" after the recognition that the restricted vacuum becomes an impure thermal state). As an experimental physicist does not really know the internal structure of his measuring tools, he can only obtain more precise informations by sharpening the localization of the effective part of his counters and by using them in coincidence and anti-coincidence arrangements. For extracting scattering data it is not necessary to know the detailed properties of an individual $\mathcal{O}$-localized observable, the information that it belongs to a localized ensemble $\mathcal{A}(\mathcal{O})$ is sufficient since the individual differences only show up in adjustable normalization factors of asymptotic operators.

As many ideas which do not result from the extension of an already existing formalism, but rather from philosophical contemplations about a new setting, they are not to be taken literally; but their metaphorical aspects remain harmless if they lead to a consistent mathematical formulation, as it is the case with local quantum physics (LQP). Already for the localization inside a noncompact spacetime region as a Rindler wedge, only a uniform acceleration can keep a particle counter (observer) inside the causal horizon of the wedge; the counter must be uniformly accelerated in the wedge direction in order to prevent an escape through the wedge's causal horizon (the Unruh Gedankenexperiment). There is no harware blueprint to localize quantum hardware inside a compact spacetime localization region, not even in form of a Gedankenexperiment. The localization within a laboratory and the duration of the registering counting are only approximate realizations of the metaphoric idealized localization; the precise realization of modular localization, i.e. the analog of what in case of the wedge corresponds to unform acceleration, remains in the dark. In spite of this, the description of a local observable as a member of a local ensemble forming a localized algebra $\mathcal{A}(\mathcal{O})$ has turned out to be one of the most fruitful nonperturbative ideas, notwithstanding its metaphoric origin. To its many already known successes, this paper adds the foundational understanding of integrability which enriches the known bootstrap-formfactor project with an existence proof for the d=1+1 factorizing models.

The illustration in terms of the Unruh Gedankenexperiment also exposes another aspect of causal localization which remained unresolved for a long time and even nowadays appears somewhat mysterious to many physicists: the vacuum state on the global algebra $B(H)$ restricted to an ensemble of $\mathcal{O}$ localized observables turns into an impure thermal KMS state. This is accompanied by an (entropy-creating) vacuum-polarization cloud at the causal horizon. Whereas in QM the vacuum factorizes upon inside/outside partition of the global $\mathbb{R}^3$ into a tensor product vacuum, the situation in QFT is radically different [18]. There is a widespread misconception that these manifestations of localization only exist in connection with quantum matter localized behind event horizons of black holes associated with appropriate curved spacetimes, whereas the main difference to the observer-dependent "fleeting" Unruh-type Gedankenexperiment consists in the realization that the dividing horizon is predetermined by the *observer-independent properties of the spacetime metric*. In other words the observer's proper time is identical to the Killing time in a black hole world since the localization is predetermined by metric properties and this time is also the one which is related to the measured temperature.

Causal localization in the sense of Haag's LQP [16] is a metaphoric start of a mathematically solid and physically very fruitful conceptional setting which stands fully on its own feet. The history of QFT would have probably developed differently if these manifestations had been noticed already at the time of Jordan's 1925 discovery of QFT in his attempt to understand the analogy between Einstein's thermal fluctuations in open subsystems of black-body radiation and his proposed fluctuations of quantum field subsystems obtained by restricting the vacuum of the quantum photon field to a spacetime subvolume. Although the two fluctuation components (the wave- and particle- component) in Einstein's Gedankenexperiment on which he based his photon hypothesis were identified by Jordan, the complete unravelling of the *Einstein-Jordan conundrum*[4] was only possible with the help of the recent modular localization property of LQP. In fact the presentation of the full equivalence with Jordan's d=1+1 "photon" model (really a chiral conformal current model) has only been given recently [21][32].

Fluctuation calculations in open subsystems only allow approximate solutions, and as a result pose a conceptual problem. Whereas in renormalized perturbation theory for pointlike localized fields the covariance and localitity aspects have been "streamlined" into an elegant formalism (in the Epstein-Glaser iteration scheme [24] this is most visible), this elegance is of no avail in the E-J conundrum. It is difficult to maintain the local covariance principle in such approximations and even in recent detailed and careful presentations of Jordan's calculation by historians of physics [22], there is a conceptional error on this subtle but important point. The only known way to uphold the localization principles of QFT throughout the calculation in Jordan's model is to find an approximation for the vacuum reduced to an interval I_ε where ε is an "attenuation length" (roughened surface) conceded to both sides of the interval I within which the vacuum-polarization cloud at the boundary attenuates. The mathematical formalism of modular localization offers a canonical way (the split property) to do this [32]. Although it is impossible to describe

[4]For a presentation in a modern setting see [21] and references therein.

the associated Gibbs state in closed form, one can see that its trace diverges logarithmically and that the appropriately normalized limiting state is a singular KMS state whose associated Hamiltonian is the generator of the dilation which leaves the interval invariant. The fluctuations of the energy-momentum tensor in such a state are the searched for thermal fluctuation, but this time the thermal aspects are entirely caused by modular localization.

One may only speculate that Einstein, in his dispute with Jordan, could have given up his philosophical resistance against Born's addition of the probabilistic interpretation of QT. The probability which inevitably comes with the thermal manifestation of the intrinsic modular quantum localization referring to an ensemble of objects sharing the same localization is something with which Einstein would have had no problems; what worried him philosophically was to attribute probabilities to measuring individual observables as done in QM. Since the more fundamental QFT in the form of LQP extracts all measurable results from localized ensembles, the question arises whether one cannot do particle theory solely based on the natural probability which enters in the notion of ensembles of local observables and their surprising intrinsic thermal properties. In this way the Born probability would have not been linked with an individual particle and the presentation of the measurement problem starting from the Copenhagen interpretation up to Everett's multi-world formalism idea would have taken a different turn more compatible with Einstein's philosophical realism.

Perhaps with this additional insight Einstein could have supported Jordan against the critical resistance of Born and Heisenberg (in order to obtain an extra section in the famous "Dreimännerarbeit"). Unimaginable what turn the conceptual development of QFT could have taken! The full conceptual arsenal of QFT would have been available starting from the Dreimännerarbeit in 1925, most of the (still ongoing) fights about the measurement issue would have been superfluous, and even the post wwII covariant renormalized perturbation theory would have appeared as a natural perturbative implementation of causality principles without being aggrieved by an ultraviolet catastrophe. Fundamental laws of statistical mechanics would have been behind the behaviour of localized quantum matter (with or without spacetime curvature) a long time before Ted Jacobson proposed connections between general relativity and the fundamental law of thermodynamics [23].

In a relativistic DPI (direct particle interactions) scheme [18] based on interacting multiparticle representations of the Poincaré group there are no covariant local observables. The velocity of light emerges in such a setting as a limiting asymptotic velocity of the center of mass of wave packets. Like the velocity of sound in QM this is an effective velocity which in the acoustic case depends on the exitation of the ground state and in DPI refers to the exitations of the QM vacuum. An important fact in QFT, where one has in addition to the covariant modular localization also the frame-dependent Born-Newton-Wigner localization of states, consists in realizing that these two concepts coalesce in the asymptotic time-like region; this is the basis of the Poincaré invariance of the S-matrix. Hence the DPI is primarily a pure S-matrix property which only can take care of those properties which allow a formulation entirely in terms of particles (cluster-factorization, Stückelberg's causal rescattering,..) [59]. The fact that localized wave packets also possess superluminal components does not contradict causality (the the discussion of Fermi's Gedankenexperiment in [18] and feferences therein).

Many of the geometric aspects of local Lagrangians which were proposed in the 70s are not consistent with causal localization (and its vacuum polarization and thermal aspects). An example is the Wess-Zumino-Witten-Novikov Lagrangian which was proposed as a Lagrangian formulation for the construction of group valued sigma-model fields from the representation theory of chiral currents. Notwithstanding its mathematical appeal, this topological Lagrangian is in conflict with localization properties of QFT which also explains why it does not possess a perturbation theory based on locality (the original Wess-Zumino Lagrangian does not have this problem).

Closely related is the question whether the Chern-Simons Lagrangian can really define a localizable QFT. Whereas the knot and 3-mf invariants have been extracted from a Chern-Simons functional integral representation, using a quasiclassical approximation of the C-S functional integral [28], it remains doubtful that the quantization of this Lagrangian provides a sufficient cardinality of phase space degrees of freedom for implementing causal localization. The separation of such subalgebras of intertwiners of superselected charges forming Hecke-type algebras with representations of the braid group, knot invariants and 3-manifold invariants from the localization properties of the full theory is the main tool of the DHR construction. It leads to type II_1 algebras as they appear in the subfactor work of Vaughn Jones [17] but these algebras are too small for implementing causally localized subalgebras and the close relation with the quasiclassical Chern-Simons construction suggests that a Chern-Simons-based Lagrangian or functional setting suffered from the same limitation concerning the lack of localization. This small nonlocalizable subtheory of a full localizable QFT is what is called its "kinematically integrable" part in the last section. It should not be confused with the (dynamical) integrability which only occurs in d=1+1 and excludes dynamically integrable subtheories in higher spacetime dimensions.

A recent mathematical rigorous algebraic construction [29] in the framework of local quantum physics combined the localization aspects with the plektonic properties (braid group representation and its inexorable connection to knot theory and three-manifolds) under one conceptual roof and it is totally implausible that this richness can be related to a C-S functional integral. The authors come to the surprising conclusion that the Bargman-Wigner representations of d=1+2 particles with anomalous spin have no free field realization but only admit interacting nonintegrable field theoretic extensions[5]. The ensuing braid group representation is an extension of the infinite permutation group representations resulting from the DHR superselection analysis of standard (bosonic, fermionic) QFT (for more see the last section).

The modular localization structure leads to the necessity to rethink many popular pictures and "results". Since the 60s geometric aspects of differential geometry played an increasingly role, but whereas geometrical structures in mathematics have realizations in very different contexts (example: Riemann surfaces as embedded in 3-dimensional space, as the form of Fuchsian groups, ...), geometrical aspects of spacetime localization in QFT are inexorably accompanied by thermal manifestations and entropy-causing vacuum polarization. In chiral conformal QFT one may associate Riemann surfaces with particular

[5]In a way these particle representations of the Bargman infinite covering group associated with the d=1+2 spacetime lead to QFTs which have no classical counterpart from which they could arise by quantization.

models. But in contrast to simple-minded geometrical visualizations they never represent the "living space" of a chiral conformal field theory but refer to conformal QFTs in global non-vacuum (thermal KMS-like) states (the best studied case is a torus).

There is hardly any mathematical structure which is in bigger antagonism to modular localization as string theory. To see the problem in a nut-shell, look at the string theorist's use of the Lagrangian of a classical relativistic particle describing covariant world-lines as a "warm up" to string theory [37]:

$$\begin{aligned} \mathcal{L}_{class} = \sqrt{ds^2} \ \curvearrowright \ cov.\ worldline\ x_\mu(\tau) \\ but\ \nexists\ quantized\ cov.\ X_\mu^{op}(\tau) \end{aligned} \tag{1}$$

What the protagonists of such an idea forget to state is the second line. Acceptance of such a wrong suggestion by the string community has thrown part of particle physics back behind Wigner's representation theoretical approach. There are simply no covariant spacetime position operators in any dimension which describe the covariant embedding of a string (or the spacetime embedding of a chiral theory into a target space).

The potentials Φ_i of an n-component abelian chiral current $j = \partial\Phi$ has been used as an analog of a (nonexisting) relativistic position operator by string theorists. Thanks to the fact that such a current theory has continuously many representations labeled by an n-component continuous charge[6] one may see this charge (zero mode) in analogy to an n-component momentum vector whereupon the anomalous dimensions (which are quadratic in the charges) correspond to mass squares. One can strengthen this analogy by using the degrees of freedom contained in chiral multicomponent current as a *representation space of a Poincaré group* so that the charge components and the zero mode components become the momentum and localization variables on which this group acts.

Up to this point everything is exactly as string theorists want it to be. The error is in the claim about the localization coming with this representation, which according to string theorist is string-like wheras in reality it is pointlike. This follows from the fact that wave function spaces which carry irreducible positive energy Wigner representation not including the infinite spin representations (a requirement which is fulfilled by the reducible superstring Wigner representation) is pointlike generated and so is its associated "second quantized" string field theory.

It is therefore of no surprise that the explicit calculation of the (graded) commutator of string fields (for technical reasons up to an arbitrary bounded value of the κ^2 in their Källen-Lehmann representation) confirms this pointlike nature [38]. What has happened is that by enforcing the representation of a Poincaré group, the oscillator variables in $\Phi(x)$, after removing the zero mode (the would be localization point in target space), become quantum mechanical oscillators in an internal space over this localization point. In other words these oscillators, which in the chiral current model played a holistic role of building up a pointlike chiral field, are now just quantum mechanical operators acting in an internal space over a localization point in a space on which the Poincaré group acts [30].

[6] The abelian current model belongs to the non-rational chiral theories which constitute the only known QFTs with a continuous set of superselection secctors. This is the prerequisite for interpreting the inner symmetry space as a target space on which noncompact inner groups can act.

With the X_μ-variables in Polyakov's Lagrangian referring (apart from the zero mode) to the internal structure of a pointlike object, the claim that string theory leads to a description of gravity becomes questionable.

To avoid misunderstandings, *string-localized fields do exist*; they are typically of the form $\Psi(x,e)$ and are localized on spacelike semilines $x+\mathbb{R}_+e$, e a spacelike unit vector (see next section), but different from the objects of string theory (e.g. the Polyakov string) they are not solutions of Euler-Lagrange equations. In higher spacetime dimension there are of course Lagrangians with inner noncompact classical symmetries, but unless the classical field space is identical to that of the classical tensor/spinor space associated to the spacetime living space of these objects, there exists no standard QFT associated via quantizations to those Lagrangians. theory with mass gaps (since those must gave compact inner symmetries). Here "standard" means QFT with mass gaps, which with their necessary compact internal symmetry group structure [16] cannot support noncompact target spaces. Our discussion shows that even in those d=1+1 cases where there exist a continuum of superselection sectors and the noncompact target space requirement is fulfilled, the localization remains pointlike.

For later use (last section) we also add an remark about regularizations which are not supported by modular localization. Whereas the dimensional regularization in interactions of scalar particles do not seem to cause problems[7] since scalar particles have a quite trivial dependence of spacetime dimensions, the application of this regularization to theories involving $(m, s \geq 1)$ fields is dubious since already the Wigner one-particle representations depend in an essential way on spacetime dimensions. We will return to this point in the last section (in connection with the beta=0 arguments for interacting conformal theories).

Hoping that the previous remarks may have aroused the reader's curiosity in yet little known ongoing new developments in the conceptual foundations of the more than 80 years old QFT, I will try in the next two sections to take him along through some of its new foundational concepts around "modular localization". Afterwards these concepts are used to obtain a foundational insight into the meaning of integrability/nonintegrability. Far from being a special niche of QFT, it will be clear that this is a central issue of QFT which is closely related to Mandelstam's program of the 60s to find a nonperturbative access to particle theory in which the S-matrix and its properties plays an important role not only as its observational "crown", but also in its computational setting. In particular integrability sheds new light on its recently discovered "semilocal" property of representing a *relative modular invariant* for wedge-localization [18] which in turn leads to a deeper conceptional insight into the crossing property of particle physics. In this way it does not only become clear that Veneziano's [39] dual model was based on the wrong crossing, but it also becomes clear that the particle crossing is much more foundational than it appeared in the setting of LSZ scattering theory.

Fortunately all these self-defeating positions of conceptual points made more than 40 years ago are not simply stupid mistakes, such that after their corrections one can pass to buisiness as usual. Since they have been made on one of the conceptually most intricate points of local quantum physics, there is the unique historical chance to obtain new

[7]The dimensional regularization was first used by Wilson [60] (and rewritten in therms of the Callen-Symanzik setting [61]) for the purpose of calculation of critical indices which are intimately related to properties of the beta-function.

insights and to formulate new aims in particle physics. The new ideas about dynamic and kinematic integrability in the present work are closely related to a correct conceptual understanding of the particle physics' crossing identity. The particle crossing was the conceptual anchor in Mandelstam's first attempt at nonperturbative constructions in particle physics based on the use of the S-matrix; hence the present work may also be seen as revisiting that important cross road of the 60s in order in order to (hopefully) take the right turn this time.

The critical remarks about string theory find their natural explanation that the repair of conceptual error which led to it is very close to the presentation of integrability and related ideas for a new nonperturbative setting of QFT which could establish existence of nontrivial models and pave the way to their control through new nonperturbative approximation ideas. So despite all critical remarks, the aim underlying this paper is upbeat.

2 The modular localization approach of QFT

There are two avenues to modular localization, a mathematical and a more physical-conceptual. The mathematical access starts from the Tomita-Takesaki modular theory of operator algebras and makes contact with QFT by applying this theory to the algebraic formulation of QFT in terms of spacetime-indexed nets of operator subalgebras [16]. An important step was the recognition by Bisognano and Wichmann [33][47] that the abstract modular group $\Delta^{it}_{\mathcal{A}(\mathcal{O})}$ and the modular reflection J acquire a geometric meaning in case of the wedge-localized operator subalgebra $\mathcal{A}(W)$ whereas the compactly localized algebras and their modular data are constructed from intersections of these algebras [46]. The history of modular theory began in the middle of the 60s when, at a conference in Baton Rouge [5] (US) mathematicians interested in operator algebras (Kadison, Tomita, Takesaki,..) met physicists (Borchers, Haag, Hugenholz, Winnink,..) working on an intrinsic formulation of statistical quantum mechanics of open systems, avoiding quantization boxes and taking volume $\rightarrow \infty$ limits. In that context an older computational trick used by Kubo, Martin, and Schwinger took on a fundamental conceptual significance and in this way the KMS property became part of the joint mathematics/physics heritage. Whereas the box-quantized thermal Gibbs states always stay in the realm of the standard (type I_∞) algebras $B(H)$ of all bounded operator, the thermodynamic limit[8] converts this into the same "monad" (hyperfinite type of III_1 factor algebra in Connes_Haagerup classification) algebra as they occur through localization in QFT. However the localization [33] aspect only appeared 10 years after the study of thermal properties of open systems and only then it became clear that localization, thermalization, and the generation of vacuum polarization clouds are inexorably intertwined.

We begin with the physical conceptual setting which starts from Wigner's classification of irreducible positive energy representations of the Poincaré group. In addition to the Born localization associated with the non-covariant (frame -dependent) position operator

[8]The tensor factorization of type I_∞ "thermofield theory" breaks down and the algebra changes its type.

which after adjustment to the relativistic invariant wave function is called *Newton-Wigner localization*, there is a radically different "modular localization". Whereas the Born localization is extrinsic[9] and connects directly with the probability interpretation of wave functions, the modular localization is intrinsic, i.e. it only uses concepts of Wigner's representation theory. For matters of notational simplicity we use the case of a scalar massive particle.

It has been realized, first in a special context in [11] and then in a general mathematical rigorous setting in [34] (see also [35][36]), that there exists a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. The starting point is an irreducible (m>0, s=0) one-particle representation of the Poincaré ´group on a Hilbert space H[10] of wave functions with the inner product

$$(\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0} \tag{2}$$

For other (higher spin, m=0) representations the relation between the momentum space wave function on the mass shell (or light cone) and the covariant wave functions is more involved as a consequence of the presence of intertwiners $u(p,s)$ between the manifestly unitary and the covariant form of the representation. Selecting a wedge region W, that is a Poincaré transform of the standard wedge $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$, one notices that the unitary wedge-preserving boost $U(\Lambda_W(\chi = -2\pi t)) = \Delta_W^{it}$ commutes with the antiunitary reflection J_W on the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The distinguished role of the wedge region is that it produces a commuting pair of (boost, antiunitary reflection). This has the unusual and perhaps unexpected consequence that the unbounded and antiunitary operator

$$\begin{aligned} S_W &:= J_W \Delta_W^{\frac{1}{2}}, \quad S_W^2 \subset 1 \\ since \ \ J_W\Delta_W^{\frac{1}{2}}J_W &= \Delta_W^{-\frac{1}{2}} \end{aligned} \tag{3}$$

which is intrinsically defined in terms of Wigner representation data, is *involutive on its dense domain* and has a unique closure (unchanged notation) with $ranS_W = domS_W$.

The involutivity means that the S_W-operator has ± 1 eigenspaces; since it is antilinear, the +space multiplied with i changes the sign and becomes the - space; hence it suffices to introduce a notation for just one eigenspace

$$\begin{aligned} K(W) &= \{domain\ of\ \Delta_W^{\frac{1}{2}},\ S_W\psi = \psi\} \\ J_W K(W) &= ZK(W)' = K(W'),\ duality \\ \overline{K(W) + iK(W)} &= H,\ K(W) \cap iK(W) = 0 \end{aligned} \tag{4}$$

[9]Born localization entered the already existing QM through Born's famous probabilistic interpretation of (the Born approximation of) the scattering amplitude leading to the notion of cross sections which was later extended to the position operator and its associated wave functions. The old (Bohr-Sommerfeld) QM as well as Heisenberg's new version did not yet deal with probabilistic concepts and (starting from Einstein up to this date) probability remained a point of contention among philosophers of science.

[10]The construction actually works for arbitrary unitary positive energy representations, not only irreducible ones.

wherw Z in this formula is a statistics factor which depends on the Wigner spin s of the representation $Z = e^{i\pi s}$.

It is important to be aware that we are dealing here with real (closed) subspaces K of the complex one-particle Wigner representation space H_1. An alternative is to directly deal with complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the *graph norm* in terms of the positive operator Δ, the dense complex subspace becomes a Hilbert space H_Δ in the graph norm. The second and third line require some more explanation. The upper dash on regions denotes the causal disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on H.

The two properties in the third line are the defining relations of what is called the *standardness property* of a real subspace[11]; any standard K space permits to define an abstract s-operator

$$S(\psi + i\varphi) = \psi - i\varphi, \quad \psi, \varphi \in K \tag{5}$$
$$S = J\Delta^{\frac{1}{2}}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group Δ^{it} and an antiunitary reflection which generally have however no geometric significance. The domain of the Tomita S-operator is the same as the domain of $\Delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that this domain is determined solely in terms of Wigner group representation theory.

The observations up to this point seem to be somewhat remote from physics, but there is an additional fact which converts these formal observations into a foundational properties of QFT. This is the fact that the domain of S_W is precisely the dense subspace of those Wigner wave functions which are localized in W. In field theoretic terms this is the one-particle projection of the space of the dense Reeh-Schlieder domain[12], a notion which was known to physicists before the concept of modular localization. The K spaces result from the one-particle projection of the Hermitian part of the W-smeared fields. Their symplectic complement is identical to the real subspace obtained from applying J_W to K_W

$$K'_W := \{\chi|\ Im(\chi, \varphi) = 0, all\ \varphi \in K_W\} = J_W K_W$$

It is easy to obtain a net of K-spaces by $U(a, \Lambda)$-transforming the K-space for the distinguished W_0. A bit more tricky is the construction of sharper localized subspaces via intersections

$$K(\mathcal{O}) = \bigcap_{W \supset \mathcal{O}} K(W) \tag{6}$$

where $\mathcal{O}$ denotes a causally complete smaller region (noncompact spacelike cone, compact double cone). Intersection may not be standard, in fact they may be zero in which case

[11]According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

[12]The dense space of states generated by applying smeared fields $A(f) = \int A(x)f(x)dx$ with $suppf \subset W$ to the vacuum [16].

the theory allows localization in W (it always does) but not in $\mathcal{O}$. Such a theory is still causal but not local in the sense that its associated free fields are pointlike. One can show that the intersection for *spacelike cones* $\mathcal{O} = \mathcal{C}$ for all positive energy is always standard [34]. A standard subspace is uniquely affiliated with a Tomita s-involution (5).

Unlike the Newton-Wigner position operators and their eigenspaces, these spaces are frame independent (covariantly defined) and for two causally separated regions $\mathcal{O}_1$ and $\mathcal{O}_2$ the simplectic inner product vanishes

$$\begin{aligned} &Im(\psi_1, \psi_2) = 0, \ \psi_i \in K(\mathcal{O}_i) \\ &[\Phi(\psi_1), \Phi(\psi_2)] = 0 \end{aligned} \tag{7}$$

where the $\Phi(\psi)$ are the so-called Segal operators [56]. Hence the symplectic inner product of modular localized one-particle wave functions is nothing else as the free field commutator function: the modular localization preempts the algebraic structure of free fields without the use of any quantization formalism. Naturally this would have been of great interest to Wigner, but the modular localization concepts were only available more than half a century after Wigner's pathbreaking work on the classification of particle spaces.

Note that the relativistic DPI (direct particle interaction, see below) setting [18] also starts from Wigner particles but it ignores the presence of this modular localization structure and follows the logic of multiparticle representation theory of the Poincaré group instead of looking for operators which create modular localized states from the vacuum.

There are three classes of irreducible positive energy representation, the family of massive representations $(m > 0, s)$ with half-integer spin s and the family of massless representation which consists really of two subfamilies with quite different properties namely the $(m = 0, h = half - integer)$ class, often called the neutrino-photon class, and the rather large class of $(m = 0, \kappa > 0)$ massless infinite helicity representations parametrized by a continuous Casimir parameter κ [36].

For the first two classes the the standardness property also holds for double cone intersections of wedges $\mathcal{O} = \mathcal{D} = \cap_{W \supset \mathcal{D}} W$ for arbitrarily small $\mathcal{D}$ i.e. the associated intersection of K-spaces maintains the standardness property. But this is definitely not the case for the infinite helicity family for which the localization spaces for compact spacetime regions turn out to be trivial[13]. Passing from localized subspaces $\mathcal{K}$ in the representation theoretical setting to singular covariant generating wave functions (the first quantized analogs of generating fields) one can show that the $\mathcal{D}$ localization leads to pointlike singular generators (state-valued distributions) whereas the spacelike cone localization $\mathcal{C}$ is associated with semiinfinite spacelike stringlike singular generators [36]. Their second quantized counterparts are pointlike or stringlike covariant fields. It is remarkable that one does not need to introduce generators which are localized on hypersurfaces (branes).

Although the observation that the third Wigner representation class is not pointlike generated was made many decades ago, the statement that it is semiinfinite string-generated, and that one does not have to use generators which localize on larger dimensional submanifolds, is of a more recent vintage [34][36].

But what is the physical significance of the frame independent modular localization as compared to the frame-dependent probabilistic Born-Newton-Wigner localization which is

[13]It is quite easy to prove the standardness for spacelike cone localization (leading to singular stringlike generating fields) just from the positive energy property which is shared by all three families [34].

used in the dissipation of particle wave packets and the formulation of particle scattering[14] ?

The best way to picture modular localized particle wave functions from a field theoretic viewpoint is to say that such states form dense subspaces which correspond in field theoretic terms to the projections of the dense subspaces obtained by applying the $\mathcal{O}$ localized subalgebra $\mathcal{A}(\mathcal{O})$ to the vacuum (the Reeh-Schlieder domain [16] corresponding to $\mathcal{O}$) and projecting the result into the one particle space, in more precise mathematical terms

$$P_1 H(\mathcal{O}) = K(\mathcal{O}) + iK(\mathcal{O}) \equiv H_1(\mathcal{O}) \tag{8}$$
$$H(\mathcal{O}) = domS(\mathcal{O}) \supset \mathcal{A}(\mathcal{O})\Omega,\ S(\mathcal{O})A\Omega = A^*\Omega,\ A \in \mathcal{A}(\mathcal{O})$$

where $S(\mathcal{O})$ is the modular operator which is determined by $\mathcal{A}(\mathcal{O})$ and in whose domain $\mathcal{A}(\mathcal{O})\Omega$ is densely embedded[15].

More important is the opposite use, namely the construction of QFT in a completely intrinsic manner, i.e. without any parallelism to a less fundamental classical theory (quantization). Part of this step has been taken in the beginning of the first part of Weinberg's book [44]. In the following this will be extended from the viewpoint of modular localization and in section 4 the beginnings of an intrinsic approach to interacting systems will be presented.

There is a very subtle aspect of modular localization which one encounters in the second ($m = 0, s \geq 1$) representation class of massless finite helicity representations (the photon-graviton class). Whereas in the massive case the relation of the physical spin s with the formal spin in the spinorial fields $\Psi^{(A,\dot{B})}$ follows the naive angular momentum composition rules [44]

$$\left|A - \dot{B}\right| \leq s \leq \left|A + \dot{B}\right|,\ m > 0 \tag{9}$$

$$s = \left|A - \dot{B}\right|,\ m = 0 \tag{10}$$

The zero mass finite helicity family in the second line has a significantly reduced number of spinorial descriptions. But why is there, in contradistinction to classical field theory no covariant s=1 vector-potential A_μ or no $g_{\mu\nu}$ in case of s=2 ? Why are the admissible covariant generators of the Wigner representation in this case limited to field strengths (for s=2 the linearized Riemann tensor)?

The short answer is that all these missing covariant generators exist as *covariant string-localized fields*, whereas the above method resulting in (10) was (as the quanntization approach) only aimed at pointlike generating (wave function-valued distributions) wave functions of the Wigner unitary representation (Hilbert-) space; there are simply no pointlike generators since their existence would contradict the positivity properties of a Hilbert space! The full range of spinorial possibilities (9) returns as soon as we allow

[14] Actually Born introduces it together with the cross section in the Born approximation; the relation with the quantum mechanical position operator and Schrödinger wave functions came later.

[15] At the time of the discovery of the density of the spaces $\mathcal{A}(\mathcal{O})\Omega$ the modular theory was not yet known. There is no change of content if one uses the same terminology for their modular extension $domS(\mathcal{O})$.

generating wave functions $\Psi^{(A,\dot{B})}(x,e)$ if $s \neq \left|A-\dot{B}\right|$ which are localized on semiinfinite spacelike strings:

$$\begin{aligned} U(\Lambda)\Psi^{(A,\dot{B})}(x,e)U^{*}(\Lambda) &= D^{(A,\dot{B})}(\Lambda^{-1})\Psi^{(A,\dot{B})}(\Lambda x,\Lambda e) \\ \left[\Psi^{(A,\dot{B})}(x,e),\Psi^{(A',\dot{B}')}(x',e')\right]_{\pm} &= 0,\ x+\mathbb{R}_{+}e >< x'+\mathbb{R}_{+}e' \end{aligned} \tag{11}$$

Here the unit vector e is the spacelike direction of the semiinfinite string and the last line expresses the spacelike fermionic/bosonic spacelike commutation. The best known illustration is the $(m=0, s=1)$ vectorpotential representation; in this case it is well-known that although a generating pointlike field strength exists, there is no *pointlike* vectorpotential acting in a Hilbert space. The clash which potentials create between localization and the Hilbert space (which has no counterpart in the classical setting) can be resolved in two opposite ways. The conventional description coming from quantization violates the Hilbert space setting through the presence of ghosts; this is from a conceptual point of view the more radical one because it sacrifies the most cherished principle of QT in order to maintain the less foundational formal rules of quantization. In the BRST formalism the invariance under BRST transformation is an essential tool in order to return to a Hilbert space setting; but the prize to be paid is that physical charge-carrying matter fields remain outside the perturbative formalism dealing with correlation functions of charge-carrying fields[16].

The modular localization approach offers a different option in the form of stringlike covariant vector potentials $A_{\mu}(x,e)$. This has the advantage that the physical origin of the semiinfinite spacelike string localization of the charge carriers[17] becomes manifest [85]. But since every formulation of perturbation theory is (directly or indirectly) based on causal locality of covariant fields, one faces a new problem: an extension of the Epstein-Glaser approach to causal locality situation of spacelike strings (for partial results see [45]).

In the case $(m=0, s=2)$ the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor (it is often referred to as the *linearized* Riemann tensor). In this case the string-localized covariant potential is of the form $g_{\mu\nu}(x,e)$ i.e. resembles the metric tensor of general relativity [85].

Even in case of massive free theories, where the representation theoretical approach of Wigner does not require to go beyond pointlike localization, covariant stringlike localized fields exist. Their attractive property is that they improve the short distance behavior e.g. a massive pointlike vector-potential of *sdd=2* passes to a string localized vector potential of *sdd=1*. In this way the increase of the sdd of pointlike fields with spin s can be traded against string localized fields of spin independent dimension with sdd=1.

[16]They can only indirectly be recovered in the form of recipes for photon-inclusive charged particle scattering cross sections.

[17]The on-shell manifestations are well known since the time of the famous Bloch-Nordsiek paper. In nonabelian gauge theories this problem is more series: non of the correlation functions exists (in any gauge). The infrared divergencies are not there as long as one keeps the string directions e generic charge [85].

This observation would suggest the possibility of an enormous potential enlargement of perturbatively accessible higher spin interaction in the sense of power counting.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [69]. The modular localization approach preempts the spin-statistics connection already in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the symplectic complement with the causal complement the generalized duality relation

$$K(\mathcal{O}') = ZK(\mathcal{O})' \tag{12}$$

where the square of the twist operator $Z = e^{\pi is}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase $= Z^2$ [69]. (Beware that the same letter isused in section 4 for the generator center of the center of the conformal group)

The fact that (apart from Wigner's "infinite spin") one never has to go beyond string localization[18] in order to obtain generating fields for a QFT is remarkable in view of the many attempts to introduce more extended (quasiclassical "branes") objects into particle theory.

It is helpful to be again reminded that modular localization, unlike BNW localization, cannot be connected with probabilities and projectors. It is rather related to aspects of causal localization. As will be seen in the **fourth** section modular localization is also an important tool in the non-perturbative construction of interacting models by representation-theoretical ideas.

3 Algebraic aspects of modular theory

A net of real subspaces $K(\mathcal{O}) \subset H_1$ for a finite spin (helicity) Wigner representation can be "second quantized"[19] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the bosonic or fermionic Fock space $H = Fock(H_1)$ built over the one-particle Wigner space H_1. For integer spin/helicity values the modular localization in Wigner space implies the identification of the symplectic complement with the geometric complement in the sense of relativistic causality, i.e. $K(\mathcal{O})' = K(\mathcal{O}')$ (spatial Haag duality in H_1). The Weyl functor takes this spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: for each Wigner wave function $\varphi \in K(\mathcal{O}) \in H_1$ the associated (unitary) Weyl operator is defined as

$$Weyl(\varphi) := exp\, i\{a^*(\varphi) + a(\varphi)\} \in B(H) \tag{13}$$
$$\mathcal{A}(\mathcal{O}) := \{Weyl(\varphi)|\varphi \in K(\mathcal{O})\}'', \;\; \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$$

[18]Note that for $(m = 0, s \geq 1)$ only the field strength are pointlike generators whereas potentials are string-localized. The use of these potentials in interactions leads however to the necessity to use string-localized generating charged fields.

[19]The terminology 2^{nd} quantization is a misdemeanor since one is dealing with a rigorously defined functor within QT which has little in common with the artful use of that parallellism to classical theory called "quantization". In Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

where $a^*(\varphi)$ and $a(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function φ, and the double prime means the double commutant. This leads to a functorial relation between localization subspaces of the one-particle space and the local subalgebras often referred to as second quantization. Defining the algebra in terms of the double commutant makes it a von Neumann algebra i.e. a weakly closed operator algebra.

This functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization and commutes with the improvement of localization through intersections $\cap$ according to $K(\mathcal{O}) = \cap_{W\supset\mathcal{O}}K(W),\ \mathcal{A}(\mathcal{O}) = \cap_{W\supset\mathcal{O}}\mathcal{A}(W)$. This property of the functorial relation can be conveniently expressed in the commuting diagram

$$\begin{array}{ccc} \{K(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow\cap & & \downarrow\cap \\ K(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{14}$$

Here the vertical arrows denote the tightening of localization by intersection whereas the horizontal ones denote the action of the Weyl functor. This commuting diagram expresses the functorial relation between particles and fields in the absence of interactions. In the interacting case the loss of the diagram and the unsolved particle-field problems are synonymous. It is also the reason why, in contrast to QM, the existence problem of interacting QFTs remains unsolved. The wedge regions continue to play a distinguished role in attempts to construct interacting models (for modular constructions in d=1+1 see below).

The case of half-integer spin representations is analogous [35], apart from the fact that there is a mismatch between the causal and symplectic complements which must be taken care of by a *twist operator* $\mathcal{Z}$ and as a result one has to use the CAR functor instead of the Weyl functor. The case of d=1+2 Bargman-Wigner representation theory permits anomalous spin connected with braid group statistics. This is the only known case for which a "free plekton field" cannot be attained by the functorial second quantization; even the "freest" plektons are not free in the technical sense since plektonic statistics inexorably comes with vacuum polarization independent of the presence of interactions [29].

In case of the large family of irreducible zero mass "infinite spin" representations in which the lightlike little group is faithfully represented, the finitely localized K-spaces are trivial $K(\mathcal{O}) = \{0\}$ and the *most tightly localized nontrivial spaces are of the form* $K(\mathcal{C})$ for $\mathcal{C}$ an arbitrarily narrow *spacelike cone.* As a double cone contracts to its core which is a point, the core of a spacelike cone is a *spacelike semiinfinite string.* The above functorial construction works the same way for the Wigner infinite spin representation, except that in that case there are no nontrivial algebras which have a smaller localization than $\mathcal{A}(\mathcal{C})$ and hence there is no field which is sharper localized than a semiinfinite string.

As stated before, stringlike generators are also available in the pointlike case and they have an improved short distance behavior. In fact in certain cases ("massive QED", massive Yang-Mills) their use could be that of a "catalyzer" for getting over the power-counting hurdle with the string-localization so that after having achieved this one may return to pointlike fields with short distance dimensions bigger than 1. For the construction of stringlike generators it is important to use modular localization properties rather than covariance [36]. The Euler-Lagrange aspects plays no direct role in this construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization and also because most of the spinorial higher spin representations (9) cannot be characterized in terms of unconstrained Euler-Lagrange equations anyhow. The modular localization is the more general method of implementing causal propagation than that from hyperbolic equations of motions.

A basis of local covariant field coordinatizations is then defined by Wick composites of the free fields. The case which deviates furthest from classical behavior is the pure stringlike infinite spin case which relates a *continuous* family of free fields with one irreducible infinite spin representation. Its non-classical aspects, in particular the absence of a Lagrangian, is the reason why the spacetime description in terms of semiinfinite string fields has been discovered only recently rather than at the time of Wigner's representation theoretical approach.

Using the standard notation Γ for the second quantization functor which maps real localized (one-particle) subspaces into localized von Neumann algebras, and extending this functor in a natural way to include the images of the $K(\mathcal{O})$-associated operators $s_{\mathcal{O}}, \delta_{\mathcal{O}}, j_{\mathcal{O}}$ (here we use small letters for the one-particle operators) which are denoted by $S_{\mathcal{O}}, \Delta_{\mathcal{O}}, J_{\mathcal{O}},$ one arrives at the Tomita Takesaki theory of the interaction-free local algebra $(\mathcal{A}_0(\mathcal{O}), \Omega)$ in standard position[20]

$$\begin{aligned} H_{Fock} &= \Gamma(H_1) = e^{H_1}, \ \left(e^h, e^k\right) = e^{(h,k)} \\ \Delta_{\mathcal{O}} &= \Gamma(\delta_{\mathcal{O}}), \ J_{\mathcal{O}} = \Gamma(j_{\mathcal{O}}), \ S_{\mathcal{O}} = \Gamma(s_{\mathcal{O}}) \\ S_{\mathcal{O}} A\Omega &= A^*\Omega, \ A \in \mathcal{A}(\mathcal{O}), \ S_{\mathcal{O}} = J_{\mathcal{O}} \Delta_{\mathcal{O}}^{\frac{1}{2}} \end{aligned} \tag{15}$$

With this we arrived at the core statement of the Tomita-Takesaki theorem in the absence of interactions which is a statement about the two modular objects Δ_0^{it} and J_0 on the algebra

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O})) &\equiv \Delta_{\mathcal{O}}^{it} \mathcal{A}(\mathcal{O}) \Delta_{\mathcal{O}}^{-it} = \mathcal{A}(\mathcal{O}) \\ J_{\mathcal{O}} \mathcal{A}(\mathcal{O}) J_{\mathcal{O}} &= \mathcal{A}(\mathcal{O})' = Z\mathcal{A}(\mathcal{O}')Z^* \end{aligned} \tag{16}$$

In words: the reflection $J_{\mathcal{O}}$ maps an algebra (in standard position) into its von Neumann commutant and the unitary group $\Delta_{\mathcal{O}}^{it}$ defines a one-parametric automorphism-group σ_t of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem holds in complete mathematical generality for "standard pairs" $(\mathcal{A}, \Omega)$. The free fields (20) and their Wick composites are "coordinatizing" singular generators of this $\mathcal{O}$-indexed net of operator algebras in the sense that the smeared fields $A(f)$ with $suppf \subset \mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$ and in a certain sense generate $\mathcal{A}(\mathcal{O})$. In the classifications of von Neumann algebras these local algebras are of a very different type as their global counterpart. The latter is of the same type as quantum mechanical algebras namely an algebra of

[20]The functor second quantization functor Γ preserves the standardness i.e. maps the spatial one-particle standardness into its algebraic counterpart.

all bounded operators on a Hilbert space $B(H)$. The local subalgebras are Murray-von Neumann factor algebras of a completely different type; in physics one only encounters them in local subalgebras of QFT or in thermally represented global algebras. We will refer to such an algebra as a "monads" and refrain here from explaining the terminology "hyperfinite type III_1 factor algebra which would force us to present the Connes-Haagerup classification of von Neumann factors [46].

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic complement passes via the functorial operation to the operator algebra commutant and the spatial one-particle automorphism goes into its algebraic counterpart. The definition of the Tomita involution S through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega = A^*\Omega$ and the action of the two modular objects Δ, J (15) is part of the general setting of the modular Tomita-Takesaki theory of abstract operator algebras in "standard position"; standardness is the mathematical terminology for the physicists Reeh-Schlieder property i.e. the existence[21] of a vector $\Omega \in H$ with respect to which the algebra acts cyclic and has no "annihilators" of Ω. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras is more involved.

The domain of the unbounded Tomita involution S turns out to be "kinematical" in the sense that the dense set which features in the Reeh-Schlieder theorem [16] is determined in terms of the representation of the connected part of the Poincaré group i.e. the particle/spin spectrum[22]. In other words the Reeh-Schlieder domains in an interacting theory with asymptotic completeness are identical to those of the incoming or outgoing free field theory.

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for an interacting standard pair $(\mathcal{A}(W), \Omega)$ the antiunitary involution J depends on the interaction, whereas Δ^{it} continues to be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [11] time that J is related with its free counterpart $J_{W,in}$ through the scattering matrix

$$J_W = S_{scat}\, J_{W,in} \tag{17}$$

This modular role of the scattering matrix as a *relative modular invariant* between an interacting theory and its free counterpart comes as a surprise because usually one thinks of S_{scat} as a fully nonlocal (global) object obtained via LSZ scattering theory. But it is precisely this new semilocal aspect which opens the way for an inverse scattering construction. If one only looks at the dense localization of states which from the domain *domS* (the closure of the Reeh-Schlieder theorem in the graph norm), one misses the dynamics which is fully contained in the modular J which "reshuffles" the states within

[21]In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in H.

[22]For a wedge W the domain of S_W is determined in terms of the domain of the "analytic continuation" $\Delta_W^{\frac{1}{2}}$ of the wedge-associated Lorentz-boost subgroup $\Lambda_W(\chi)$, and for subwedge localization regions $\mathcal{O}$ the dense domain is obtained in terms of intersections of wedge domains.

$domS = ranS$. The properties of J are essentially determined by the relation of localized operators A to their Hermitian adjoints A^*[23].

The physically relevant facts emerging from modular theory can be condensed into the following statements:

- *The domain of the unbounded operators $S(\mathcal{O})$ is fixed in terms of intersections of the wedge localized algebras $\mathcal{A}(\mathcal{O}) = \cap_{W\supset\mathcal{O}}\mathcal{A}(W)$ domains associated to S_W and $domS_W$ is determined by the representation of the Poincaré group (and hence by the particle content alone) and therefore of a kinematical nature. These dense domains change with $\mathcal{O}$ i.e. the dense set of localized states has a bundle structure.*

- *The complex domains $DomS_{\mathcal{O}} = K(\mathcal{O}) + iK(\mathcal{O})$ decompose into real subspaces $K(\mathcal{O}) = \overline{\mathcal{A}(\mathcal{O})^{sa}\Omega}$. This decomposition contains dynamical information which in case $\mathcal{O} = W$ includes the S_{scat}-matrix (17). In the next section arguments will be presented which suggests that with the help of a new emulation formalism (the extension of the Wigner representation approach to the realm of interactions) the S_{scat}-matrix fixes $\mathcal{A}(W)$ uniquely*

Modular localization determines the *holistic aspect of QFT* [32] which places this theory into a sharp contrast with QM, even with relativistic QM in the form of the *direct particle theory* (DPI) [18] since the causal localization is absent. A one-dimensional chain or string of oscillators can without any change be embedded into a higher dimensional QM; this is because quantum mechanical localization has no intrinsic meaning; the position operator may be part of the living space or of an imagined internal space. An embedding of a lower-dimensional into a higher dimensional QFT which respects the principles of modular localization is not possible, or to phrase it the other way around: the restriction of a QFT to a lower dimensional submanifold "remembers" that it is the restriction of a more complete theory as a result of its holistic nature, it does not comply with the physical properties of QFT in that lower dimensional spacetime.

In particular it is not possible to embed a one-dimensional chiral theory into a higher dimensional QFT in form of a spacetime string. It is however a mathematically interesting question to ask whether fields can have *internal symmetry* indices on which noncompact group can act. Here by internal symmetry indices we mean indices which are not tensor/spinor indices referring to the tensor/spinor calculus of the living spacetime of the object under consideration. Classically this is possible, the requirements of classical localization are independent of the nature of the classical *field space* (the classical target space). But for "standard QFTs" (those with mass gaps) the modular quantum localization only permits compact internal symmetry groups (the result of the DHR superselection analysis [65]). The prerequisite for a noncompact target space is a continuous set of superselection sector. The only such situations occur in chiral conformal QFTs, more precisely in

[23] According to a theorem of Alain Connes the existence of operator algebras in standard position can be inferred from the spatial part of the theory if the real subspace K permit a decompositions into a natural positive cone with suitable substructure. Although this construction has been highly useful in Connes classification of von Neumann factors, it has not yet been possible to relate this to physical concepts.

the (lesser understood) non-rational chiral models. The best studied model is that of an n-component abelian current $j_k(x)$. The formally defined exponential "sigma field"

$$\Psi(x,\vec{q}) = e^{i\vec{q}\vec{\Phi}(x)} = e^{i\vec{q}\vec{\Phi}_0 + \vec{q}\vec{\Phi}_{osc.}(x)}, \partial\Phi_k(x) = j_k(x) \tag{18}$$

can indeed support the action of a Poincaré group and this action can even be unitary and fulfill the positive energy restriction. Here the zero mode $\vec{\Phi}_0$ is an n-component number which plays the role of the target localization point i.e. on which the Poincaré group acts as on a localization point of a pointlike field. Having established that the space which the chiral sigma field generates from the vacuum carries a reducible superstring representation of the Poincaré group all properties attributed to string theory follow except that the localization is point- and not string-like. Since the stringlike objects of the Wigner infinite spin representation do not appear as irreducible components the states of this representation have pointlike generating wave-function-valued distributions and the "second quantization" leads to infinite component pointlike fields. The oscillator contributions from $\vec{\Phi}_{osc.}(x)$ act in a quantum mechanical inner space which should be pictured "on top" of a localization point (similar to spin components) without extending the localization "sideways".

What facilitated getting into the "embedding trap" is the $\Phi_k(x) = X_k(x)$ notation [37] by which string theorists want to suggest that the current potentials are really covariant spacetime coordinate operators which by their dependence on the "source parameter" x trace out a stringlike embedded conformal object in the "target (alias inner symmetry) space". This is patently wrong; since Wigner's failure [43] to introduca a covariant localization operator we know that covariance can only be obtained within the field theoretic modular localization whereas for particles the only localization is the quantum mechanical Born localization through the frame-dependent position operator. To strengthen this point further string theorists point to an analogy of the Nambu-Goto Lagrange description with that of a classical relativistic particle (see first section).

The use of the multicomponent current model for obtaining a representation of the Poincaré group on a target space of a non rational chiral model is not its only use. As mentioned in the introduction a much deeper (in the author's opinion) use consists in the so-called *maximal local extensions.* Here the idea is to use those superselection sectors which are generated by fields of integer scale dimensions (commuting for lightlke separations) for an extension of the observable algebra [25]. The maximal ways of doing this can be classified in terms of even integral lattices [26][27]. It turns out that the case of selfdual integral even lattices The associated conformal net of algebras on a circle have the remarkable property that they satisfy Haag duality for algebras of disjoint intervals precisely of the even integral lattice is selfdual which is in a curious way related to the finite number of sporadic group leaving selfdual even integral lattices invariant. In contrast to the quantum mechanical use in the dual model where oscillators contribute to the internal space of a point-localized string wave function space, the relation of localization and its Haag duality concept to selfdual integral lattices (Leech lattices) and sporadic groups (moonshine group) is really deep.

The only somewhat curious aspect which distinguishes the superstring construction from standard constructions is that the space generated by the chiral sigma fields is irreducible i.e. there are operators which intertwine the irreducible Wigner components.

It is this requirement of a *dynamical* reducible Wigner representation which is extremely selective[24] and leads together with the energy positivity to one 10-dim. realization (apart from a finite number of "M-theoretic" variations). The problem starts with the claim that this particular representation in 10 spacetime dimensions is the one from which we should extract our living space by dimensional reduction. Whereas the sequence $foundational \to unicity$ (theory of everything) may have some philosophical support, its inverse the string theoretic credo: $rareness \to foundational$ seems to be more a matter of faith than of rational science. Incorrect understandings of the intrinsic (modular) localization as mentioned at the end of the introduction (1) are probably the cause of this conceptual derailment.

There is another path which leads more directly to dual models and its superstring (m,s) spectrum which has been presented by Mack [40]. It also uses conformal QFT but does not restrict ab inicio to chiral models. Conformal QFTs have a property in that they do not only admit asymptotically convergent Wilson-Zimmermann short distance expansions but possess also globally convergent operator expansions

$$A(x)B(y) = \sum_k \int dz F_{A,B;C_k}(x,y;z)C_k(z) \tag{19}$$

where the sum goes over a basis of (composite) fields with certain properties. Applied to a 4-point function and using spacelike commutativity this leads to 3 different complete pairings. A suitably defined Mellin-transform leads to 3 representations of the Mellin transformed meromorphic function $T(s,t,u)$ whose first order poles are (up to a parameter of a mass which enters through the definition of the Mellin transform) precisely given by the scale dimensions of the $C_k(z)$. This is the conceptual origin of the dual model which was first discovered by Veneziano [39] in a pedestrian tour de force. There are as many dual models in this general sense as there are conformal QFTs. Demanding that the "would be particles" have momenta on which a unitary and positive energy representation of the Poincare group acts requires the scale dimensions to be quadratic in the charges carried by the C's which then leads back to the sigma model associated with a n=10 component (vector and spinor indices) superstring and its M-theoretic variation. For details we reer to Mack's paper [40]. The reason why we mentioned these results at all is to emphasize that this conformal field crossing leading to the dual model has nothing to do with the particle crossing property which is inexorably related to the idea of emulation in the next section. It is precisely at this point where Mandelstam's important attempt at a construcive use of S-matrix ideas went wrong and why the present work may be seen as a synthesis of S_{scat}-matrix ideas and LQP both linked by the property of S_{scat} of representing a relative modular invariant of wedge localization).

There remains the question whether one can associate at all a quantum theory with the Nambu-Goto Lagrangian in it original square root form. On this subject Pohlmeyer [42] has given a rather detailed answer. He first exhibited an infinite number of global classical conservation laws which shows that we are dealing with an integrable system.

[24]The "dynamical" means that it is not a trivial direct sum of free fields over one point, but there are operators (from the chiral theory) which intertwine between the levels of the infinite mass/spin tower. Such fields where looked for in the 60s by Barut, Kleinert and others (Chapter 9, appendix I [41]). The later 10 dimensional superstring field is the only known solution.

The Poisson brackets of these "charges" can be computed and quantized according to the usual wisdom of how Poisson brackets can be translated into commutation relation. This leads to an integrable theory which does not seem to be an integrable QFT in the sense of the present work (there seems to be no way to get localization). The content seems to be unrelated to that associated to the canonical quantization of the string theorists [42].

The holistic nature of QFT presents itself most forcefully in the possibility of characterizing a quantum field theory by the positioning of a finite number of copies of an abstract monad (interpreted as wedge-localized monads) acting in a common Hilbert space. A "modular inclusion" of one monad into another defines a chiral QFT, for a 3-dimensional theory one needs 3 modular positioned monads and placing 6 monads into a specific modular position leads to a model of d=1+3 QFT. The abstract positioning of determines not only the abstract algebraic substrate of a QFT but also the Minkowski spacetime and the action of the Poincaré group on it. The interpretation of a modular inclusion of two monads is context dependent; if there are no other monads present then it defines a chiral theory; if there are other monads around then the monads describe wedge algebras in the position of lightlike inclusions. It is this intrinsic relation of the abstract algebraic positioning of monads in a Hilbert space to the concrete localization of quantum matter in spacetime that deserves the terminology "holistic"; it forbids embedding of a lower into a higher dimensional QFT and it places severe restrictions on "dimensional reduction" in QFT. Quantization is not a boundless game, the holistic nature of QFT shows its limits. The problem is that one cannot see these limitations on the level of Lagrangian quantization; it would be visible if one tries to "curl up" extra dimensions in explicitely computed correlation functions or if one uses structural arguments which reveal the holistic nature of QFT[25].

It is interesting to look at the difficulties which our ancestors encountered with these holistic aspects.

From the time of the "Einstein-Jordan conundrum" [32] through Jordans subsequent discovery of QFT, Heisenberg's discovery of vacuum polarization, Unruh's Gedanken-experiment, the Hawking radiation and the problem of the origin of the cosmological constant, in all those cases the holistic nature of QFT asserts itself.

4 "Emulation" as an adaptation of Wigner's representation theoretical idea to the presence of interactions

Wigner was the first who found an intrinsic way to describe relativistic particles in terms of his classification of unitary positive energy representations which by covariantization [44] lead to spaces of causally propagating wave functions (obeying linear hyperbolic differential equation) without reference to quantization. The transition to free quantum fields in Wigner-Fock space is a functorial construction for which it is better to avoid the

[25] If the model has sufficient analyticity properties which allow real/imaginary time Wick-rotations, one can "curl up" a time component by taking the high temperature limit in a KMS state und create a new time direction by Wick rotation.

terminology "second quantization"[26].

Wigner's theory clearly showed that there can be no covariant position operator $x^{\mu}_{op}(\tau)$ which arises through quantization of the covariant relativistic classical world line $x^{\mu}(\tau)$ which fulfills the Euler-Lagrange equation of $L = \int \sqrt{ds^2}$; but his attempt to find a hint of covariant causal localization in his representation theoretic setting ended in failure, the Born-Newton-Wigner position operator which is an adaptation of Born's quantum mechanical probabilistic concept based on the projectors of the selfadjoint position operator adapted to the covariant inner product of relativistic wave functions is frame dependent (noncovariant). As we know now, he had no chance to find the intrinsic natural entrance which only was discovered more than 50 years later thanks to the new concept of modular localization [12][34][36]. This explains perhaps why Wigner, besides Jordan and Dirac one of the founders of QFT, never participated in the renormalized QED surge. Modular localization is not directly related to probabilities but rather through the thermal statistical mechanics aspects of modular localized ensembles.

He certainly would have embraced modular localization and the functorial completely intrinsic access to interaction-free QFT. This section addresses the problem of its extension to incorporate interactions. The first step consists in a foundational understanding of the particle-field problem which leads to the breakdown of the functorial relation. This is most appropriately achieved by referring to a recent sharpened version [56] of an old theorem (from the times of before modular-localization) which has been established by Jost and the present author in the early 60s [55].

Theorem 1 *(Mund's algebraic extension [56] of the J-S theorem) A Poincaré-covariant QFT in d≥1+3 fulfilling the mass-gap hypothesis and containing a sufficiently large set of "temperate" wedge-like localized vacuum polarization-free one-particle generators (PFGs) is unitarily equivalent to a free field theory.*

The terminology requires some explanation. Here instead of "QFT" it would have been more appropriate to use the expression "AQFT" (algebraic QFT) or "LQP" (local quantum physics) in order to indicate that one is not confined to a quantization approach. But on the other hand this could have created the wrong impression that we are talking about a different theory, whereas the real aim is just a conceptual and methodological extension of the theory beyond the classical bonds to Lagrangian or functional quantization and the ensuing diverging renormalization series. In other words QFT in the present context refers to a fundamental theory of matter which, by being more fundamental than classical theory, should not be forced to dance according to the tune of a supposed parallelisms with a less fundamental theory.

The mass gap hypothesis in the theorem is necessary in order to derive LSZ scattering theory which relates scattering amplitudes with spacetime-dependent (time-ordered) correlation functions and attributes an asymptotic role to particles in their relation to fields which gives, in the case of a "complete particle interpretation" the Hilbert space the structure of a Wigner-Fock space. "PFG" in the theorem stands for localized interacting operators which, similar to smeared free fields, create particle states (in the

[26]Ed Nelson hit the misleading ring of this terminology on its head when he said: "quantization is an art **(Laut Schrader sagte Nelson glaube ich "miracle")**, but second quantization is a functor".

present case one-particle states) without admixture of vacuum polarization but are not themselves free free field operators. The localization region in the case at hand is a wedge. Larger noncompact regions have the full Minkowski spacetime as its causal completion **Stimmt das??**; their $B(H)$ type algebras are generated by free incoming/outgoing creation/annihilation operators and lead to a trivial realization of the above theorem. Its nontrivial aspect results from the tension between the particle structure and causal localization. Its intuitive content is that larger localization regions faciltate the absence of vacuum polarization and hence favor the reconciliation of particle states with causal localization principle of QFT.

Temperateness of these generators means that, like Wightman fields, they have a translation-invariant domain; the translations can then be used to pass from the PFGs to operator-valued tempered distributions [48]. As we will see later, it is this requirement which is very restrictive; the theorem would break down if we only require covariance under those transformations which leave the wedge-localization region invariant (which only allows half-sided and transverse translations besides the wedge preserving boosts). This is the world of nonintegrable theories associated with nontemperate PFGs. The main motivation behind the above theorem is to understand and explore the precise frontier between theories with and without interactions in terms of localization properties in order to find intrinsic methods to classify models of QFT and rigorous ways to demonstrate their existence and mathematically controlled approximations. Its proof uses recent progress about wedge-localization [48]. The theorem destroys the functorial connection between modular localization of particle states and of algebras (14) already on the level of wedges as soon as interactions are present. What remains however in the presence of interactions is the existence of PFG operators with non-temperate (–> non translational invariant) domain properties which are associated with wedge-localized algebras which (as shown below) amounts to a replacement of a functorial relation by the much weaker concept of "emulation".

It turns out that wedge-localized temperate PFGs lead to QFTs with $S_{scat} = 1$[27] in $d > 1 + 1$, and to purely elastic S-matrices S_{scat}^{el} d=1+1 which includes all the integrable factorizing models. This leads to the following foundational definition of integrability in QFT.

Definition 2 *A model of QFT is called (dynamically) integrable if it possesses temperate wedge-localized PFGs. (This will be further specified below, see Definition 4.) Apart from theories with S=1 and free fields in d* $> 1+1$*, this leaves only the known family of factoring models in d=1+1.*

It will be shown that all models which were already known to be integrable in the naive sense of having sufficiently many conserved currents or being solvable within the bootstrap-formfactor program (belonging to the class of factorizing models), are also integrable in this more general and also more abstract way. This definition complies with the *holistic nature* of QFT which places localization and its inexorable companions namely vacuum polarization and thermal manifestations into the center of attention and

[27] It is conceivable that $S_{scat} = 1$ by the use of more powerful mathematical arguments leads to the freeness of the theory as in the case of spacelike-localized cones (strings).

hence places limits on the use of mathematical fact about (differential) geometry. The holistic property of QFT leaves no place for the existence of an integrable subtheory embedded in form of a QFT subalgebra localized on a two-dimensional subspace This is *very different from QM* where localization is not an intrinsic concept and integrability permits realizations in every spatial dimension. This holistic aspect also manifests itself in the connection between the KMS condition for wedge-localized algebras and the crossing properties of formfactors [49]. As will be shown in the sequel, the implementation of the particle crossing property is precisely where Mandelstam's nonperturbative S-matrix based project failed. A new program with similar physical motivation but a different conceptual and mathematical setting which saves the old idea of a unique relation between an S-matrix and a full model of local quantum physics may be viewed as its replacement. As in all previous considerations we assume asymptotic completeness i.e. that the one-particle states and their multi-particle counterparts span the Hilbert space in form of a Wigner-Fock space.

We start **with** our derivation of the particle crossing property whose proof will lead us to the new notion of "emulation" and a foundational understanding of integrability versus nonintegrability within the setiing of modular localization. It states that there exists an identity (the crossing identity) which relates the n-particle vacuum polarization formfactor to an analytic continuation onto the backward mass shell (involving anti-particles) of the connected part of the $k,\ (n-k)$ formfactor (the matrixelement of a local operator between in-out states)

$$\begin{aligned} &\langle 0\,|B|\,p_1,..p_n\rangle^{in} = \ {}^{out}\langle -\bar{p}_{k+1}..,-\bar{p}_n\,|B|\,p_1,..p_k\rangle^{in}_{con} \\ &B\in\mathcal{A}(\mathcal{O}),\ \mathcal{O}\subseteq W,\ \bar{p}=antiparticle\ of\ p \end{aligned} \tag{20}$$

The modular localization theory adds an important intermediate step to this statement: the particle content of the left hand side can be reformulated into a field theoretic expectation of wedge localized operators which in turn can be rewritten with the help of the KMS identity (associated with the thermal manifestation of wedge localization) into another field theoretic expectation. In a second step the latter can be re-expressed in the setting of particles where it turns out to represent an analytically continued formfactor. In other words, the particle reformulation does not lead directly to an on-shell physical quantity, but at least to one which is related to such a a quantity by an analytic continuation which stays on the complex mass shell (i.e. in the particle setting). In many textbooks one finds a formal derivation of particle crossing based on the LSZ scattering formalism. In this derivation the form of the disconnected terms by which the full right hand formfactor deviates from its connected part by the omission of contraction terms consisting of products of invariant $2p_{0i}\delta(p_i-p_j)$ delta functions multiplied with lower formfactors (see [13]) is not correct. These terms have been obtained from the *incorrect assumption* that the LSZ reduction formula also holds for overlapping wave functions; its derivation from the rigorous Haag-Ruelle scattering theory is however restricted to nonoverlapping situations [53] which prevent to get on top of multiparticle threshold singularities[28]. The use of the

[28]Outside these thresholds the Haag-Ruelle approximants approach their asymptotic values faster than any inverse power $1/t^n$, whereas the hitting of a threshold singularity convertes this into power law [52] which is insufficient for the LSZ derivation.

thermal KMS[29] setting shows that the contraction terms are much more involved: instead of being products of delta functions with lower formfactors they involve the interaction (through the explicit appearance of scattering amplitudes) in an essential way.

An alternative derivation of particle crossing from Einstein causality which is essentially restricted to the elastic amplitude has been given in [58][54] under certain analyticity assumptions. This method, although completely correct in its analytic details, does not reveal much about the conceptual setting of crossing, in particular its relation to the thermal aspects of wedge localization which it shares with correct setting for the thermal manifestation of the Unruh effect [57] [32] and with more general situations of black hole Hawking radiation in which the fleeting causal horizon is replaced by an observer-independent event horizon defined in terms of the spacetime metric of curved spacetime.

There is a conceptual complication resulting from the fact that the crossing relation involves the in/out algebras in addition to the interacting algebra. These are different algebras acting in the same Wigner-Fock space and sharing the same $\mathcal{P}$-representation. But the KMS relation involves only one algebra; fortunately this problem can be solved by "emulating" opererators from the free algebra $\mathcal{A}_{ln}(W)$ within the interacting algebra $\mathcal{A}(W)$. Before explaining this procedure it is helpful to recall first the extraction of the particle content from the field theoretic KMS relation in the absence of interactions.

Let B be a wedge-smeared (test function with support in W) composite of a free field $A(x)$ i.e. a Wick-ordered polynomial, and let $A^{(1)} \equiv: A(g_1)..A(g_k)$: and $A^{(2)} \equiv: A(h_1)..A(h_l)$: be Wick-ordered products smeared with W-supported test functions. The KMS relation[30] for the expectation value of a product of B with $A^{(1)}$ and A^2 is the cyclic relation results from the restriction of the global vacuum to the localized wedge algebras

$$\begin{aligned} &\left\langle BA^{(1)}A^{(2)}\right\rangle \overset{KMS}{=} \left\langle A^{(2)}\Delta\ BA^{(1)}\right\rangle, \quad \Delta^{it} = U(\Lambda_W(-2\pi t)) \\ &\curvearrowright \langle 0\,|B|\,p_1,.p_k,q_{k+1}..,q_{k+l}\rangle = \langle -\bar{q}_1,..,-\bar{q}_l\,|B|\,p_1,...,p_k\rangle_{conn} \end{aligned} \tag{21}$$

where the second line results from converting its field content into particle states after retaining only the totally Wick-ordered contribution in the product of the two A. The anti-particle momenta on the backward mass shell which appear on the right hand side are defined by analytic continuation and the subscript *conn* indicates omission of disconnected contribution which contain $2p_0\delta(\vec{p}-\vec{q})$ delta functions and lower particle matrixelements of B; this omission corresponds to the removal of Wick contraction terms. Throwing the $A^{(2)}\Delta$ term onto the left side vacuum and using the fact that the star-conjugation can be expressed in terms of the Tomita modular operator S as well as the commutation relation $\Delta J\Delta^{\frac{1}{2}} = \Delta^{\frac{1}{2}}J$ (important for $q\rightarrow -\bar{q}$) [59], one obtains agreement with the crossing relation (20) since in and out coalesce in the absence of interactions. For $0 < Imt < \pi$ the expectation values are analytic functions, but on the distribution-valued boundaries one finds delta function contributions on both sides which come from poles.

Now we come to the more subtle case with interactions. We use the notation $\mathcal{A}(W)$ for the wedge-localized subalgebra of the interacting system and correspondingly $\mathcal{A}_{in}(W) =$

[29]The replacement of the thermal Gibbs representation, which for open systems (in the thermodynamic limit) ceases to make mathematical sense [51], by the Kubo-Martin-Schwinger analytic boundary formulation.

[30]The global vacuum expections restricted to the wedge algebra becomes a KMS state.

free algebra. A stands for an operator affiliated with the interacting algebra $\mathcal{A}(W)$, in symbols $A\,\eta\,\mathcal{A}(W)$ and likewise $A_{in}\,\eta\,\mathcal{A}_{in}(W)$ for the associated free situation. Since modular localization (i.e. the Tomita S-operator) depends on the algebra, the S-operators for the two wedge algebras are different operators in the same Hilbert space. But since the representation of the Poincaré group is shared between the free and the interacting net of operator algebras, this implies that the two algebras share the modular unitary for the wedge region which is the Lorentz boost.

The idea behind emulation, which creates bijectively related operators in the $\mathcal{A}(W)$-algebra from those in the corresponding $\mathcal{A}_{in}(W)$ algebra, is based on the following lemma (extending the special case of one-particle states in [48])

Lemma 3 *Any state* $|\psi\rangle \in dom \curlyvee S_{\mathcal{A}(W)} = domS_{\mathcal{A}_{in}(W)} = dom\Delta^{\frac{1}{2}}$ *can be generated from the vacuum by two uniquely determined affiliated operators in each of the two algebras*

$$\begin{gathered} |\psi\rangle = A\,|0\rangle = B\,|0\rangle\,,\;\; A\,\eta\,\mathcal{A}_{in}(W),\;\; B\,\eta\,\mathcal{A}(W) \\ AA'\,|0\rangle = A'\,|\psi\rangle\,,\;\; A' \in \mathcal{A}_{in}'(W),\;\; BB'\,|0\rangle = B'\,|\psi\rangle\,,\;\; B' \in \mathcal{A}'(W) \end{gathered} \tag{22}$$

Here the last line defines the operators on a dense set (the Reeh-Schlieder property of the commutant algebras) and hence also the closures of the operators A, B (for which we will maintain the same notation). As will be seen in the sequel, this lemma has some powerful consequences.

This bijection between operators affiliated with different W-localized algebras sharing the same modular unitary Δ^{it} is not an algebraic operator equivalence. Rather the emulation is based on the fact that the infinitely many possibilities of chosing operators for creating a prescribed vecor state becomes unique if the operator belongs to a localized algebra and the state belongs to the domain of the Tomita S associated with that algebra[31]. So strictly speaking what we call somewhat sloppily *emulation* of operators is a bijection of operators between different algebras which is defined through states from the dense *domS*. Different from an isomorphism, it does not respect the star-operation since $A^*\,|0\rangle = S_{\mathcal{A}_{in}(W)}\,|\psi\rangle$ whereas $B^*\,|0\rangle = S_{\mathcal{A}(W)}\,|\psi\rangle \equiv S_{scatt}A^*\,|0\rangle$ is a different state, i.e. the star operation does not commute with the operation of emulation.

Applying this to the situation at hand and denoting the emulated operator by attaching a subscript $\mathcal{A}(W)$ we write

$$\begin{aligned} &A \equiv: A_{in}(f_1)...A_{in}(f_n):,\quad A \longrightarrow A_{\mathcal{A}(W)}\,\eta\,\mathcal{A}(W),\;\; suppf \subset W \\ &if\quad A_{\mathcal{A}(W)}\,|0\rangle = A\,|0\rangle = \left|\check{f}_1,..\check{f}_n\right\rangle^{in},\quad f \rightarrow\; \check{f},\; mass\ shell\ projection \end{aligned} \tag{23}$$

where $\check{f}$ is the particle wavefunction associated to the testfunction f. *Here and in the following we restricted our setting to the presence of just one kind of particle*; in the presence of different types of particles (bound states) there is a separate emulation of

[31] This property is closely related to the Reeh-Schlieder property which is sometimes imprecisely referred to as the "state-operator relation".

each type. The KMS relation, from which the particle crossing is to be derived, reads now [49]

$$\left\langle B(A_{in}^{(1)})_{\mathcal{A}(W)}(A_{in}^{(2)})_{\mathcal{A}(W)}\right\rangle = \left\langle (A_{in}^{(2)})_{\mathcal{A}(W)}\Delta B(A_{in}^{1})_{\mathcal{A}(W)}\right\rangle \tag{24}$$

$$\Delta(A_{in}^{(2)})_{\mathcal{A}(W)}^{*}\left|0\right\rangle = \Delta^{\frac{1}{2}}J_0A_{out}^{(2)}\left|0\right\rangle \tag{25}$$

where $A_{in}^{(1,2)}$ is now a Wick product of W-smeared incoming fields as in (21); its outgoing charge-conjugate counterpart appears on the right hand side by throwing the $(A_{in}^{(2)})_{\mathcal{A}(W)}\Delta$ operator onto the bra-vacuum.

Whenever an emulated k-fold Wickproduct acts directly on the vacuum one can forget the emulation subindex and reconvert it into a particle state; the remaining nontrivial problem is to convert the emulated Wick-product in the middle of the rleft hand side acting on the k-particle state

$$\left\langle 0|B(A_{in}^{(1)})_{\mathcal{A}(W)}|p_1,..p_k\right\rangle^{in} =? \tag{26}$$

$$what\ is\ particle\ content\ of\ (A_{in}^{(1)})_{\mathcal{A}(W)}A_{in}^{(2)}\left|0\right\rangle$$

into particle states. Imagining a decomposition of the unknown state by writing the not explicitly known operator $(A_{in}^{(1)})_{\mathcal{B}(W)}$ in the second line into a possibly infinite series of Wick-ordered products of free fields in the spirit of GLZ expansions [50] with yet unknown coefficient functions, we can Wick-expand the product $(A_{in}^{(1)})_{\mathcal{A}(W)}A_{in}^{(2)}\left|0\right\rangle$. The totally Wick-ordered term is known since the two operators commute inside Wick-ordering and the $\mathcal{A}(W)$-subscript can be omitted. This leads to the expected n-particle contribution i.e. the left hand side in (20).

On the other hand the formal application of the LSZ reduction formalism to this n-particle-vacuum matrixelement would lead to the well-known reduction formula which contains beside the analytic n-particle term disconnected terms consisting of products of delta function contraction terms with B-formfactors with a lesser number of particles depending on the remaining variables (see A1 in [13]). This contribution is precisely what one formally expects by rewriting the lefthand side as an onshell restriction of Fourier transformed time-ordered functions and contraction terms from coinciding momenta. Before we show that the delta function contact structure (however multiplied with different explicitly interaction dependent terms) is also the structure which follows from rewriting (24) in terms of particle states, we specify our notion of integrability in QFT in terms of properties of emulation**:**

Definition 4 *A QFT with a complete particle interpretation is called integrable if it permits a "temperate" emulation of wedge-localized Wick products of incoming free fields i.e. if the domain of the unbounded emulated operators is translation invariant and their Fourier transform can be defined. This is a generalization of PFGs (= one-particle emulats) in [48]. Theories violating this property as a result of the nonexistence of such temperate domains are nonintegrable.*

As a result of the modular localization nature of the emulation construction the domains of the emulats are always invariant under wedge preserving symmetries, but in certain cases they also have a larger invariance which includes translations; in these cases the PFGs and more generally emulated products of wedge-smeared free fields permit a Fourier analysis in case they are polynomial bounded [48].

In [48] it was also shown that the algebraic structure of integrable models is very restrictive. In d>1+1 their scattering matrix is necessarily trivial $S_{scat} = 1$, see also Theorem 1, and in d=1+1 only models with elastic S-matrices are integrable. This leaves only the elastic scattering functions[32] and their associated factorizing models resulting from the solution of the d=1+1 bootstrap formfactor program are all integrable. The absence of inelastic thresholds leads to meromorphic formfactors in the on-shell θ rapidity parametrization $p = m(ch\theta, sh\theta)$. This permits to encode the Bose statistics (a similar construction works for fermions) into the θ-ordering writing formally

$$\left|\theta_{i_1}, ...\theta_{i_n}\right\rangle := \left|\theta_1, ...\theta_n\right\rangle^{in}, \ if\ \theta_{i_1} > ... > \theta_{i_n} \tag{27}$$

which allows to associate other orderings obtained by analytic continuation inside formfactors (i. e. via the other ways to approach the real boundary)[33] with different states generated by the application of operators which are different from the incoming fields. In the setting of [13] the analytic change of this ordering (taking boundary values in different order of $Im\theta \to 0$) inside formfactors can be generated from transpositions in terms of the multiplication with scattering functions. The result is a crossing formula with contraction terms which differ from those of the LSZ reduction formula by additional dynamical factors involving matrixelements of the S-matrix. For our purpose it suffices to illustrate this in the simplest case of a 4-particle formfactor (formula (3.14) in [13])

$$\begin{aligned} &\left\langle\theta_1\left|B\right|\theta_2, \theta_3, \theta_4\right\rangle^{in} = \left\langle 0\left|B\right|\theta_1 + i\pi, \theta_2, \theta_3, \theta_4\right\rangle^{in} + \left\langle\theta_1|\theta_2\right\rangle\left\langle 0\left|B\right|\theta_3, \theta_4\right\rangle^{in} + \\ &+ \left\langle\theta_1|\theta_3\right\rangle S_{scat}(\theta_1 - \theta_2)\left\langle 0\left|B\right|\theta_2, \theta_4\right\rangle^{in} + \\ &+ \left\langle\theta_1|\theta_4\right\rangle S_{scat}(\theta_1 - \theta_2) S(\theta_3 - \theta_1)\left\langle 0\left|B\right|\theta_2, \theta_4\right\rangle^{in} \end{aligned} \tag{28}$$

The $\theta_1 + i\pi$ the vacuum polarization term stands for the analytic continuation. Note that $p(\theta + i\pi) = -p(\theta)$. As explained in [13], the S-matrix factors arise from the re-ordering which is necessary to obtain the identification with the in-state (27).

Note that the last two contraction terms contain dynamic-dependent S-matrix factors which are different from those of the contraction terms in the books. As mentioned before the reason is that the standard contraction terms correspond to coalescing momenta contributions at the onset of multi-particle thresholds where the strong Haag-Ruelle approximation of scattering states for large times (faster than any inverse power) breaks down [52][53]. As those authors emphasize, this limits the derivation of the LSZ reduction formula to non-overlapping wave packets. The presentation based on the analytic exchange of $\theta's$ can be equivalently described in terms of a wedge-localized operator formalism [12][14] with Z-F commutation relations.

[32]In the multicomponent case of elastic scattering (which allows for backward scattering) the scattering functions are matrix-valued and fulfill the Yang-Baxter relation [13].

[33]In higher spacetime dimensions the momenta in the rapidity parametrization have a transverse component and the θ´-ordering has to be replaced by the velocity ordering with respect to the chosen wedge [48].

The appearance of these dynamical modifications has its analog in the formula for the action of the emulated operators on particle states e.g.

$$(A_{in}(f))_{\mathcal{A}(W)} \left|\theta_1, \theta_2\right\rangle^{in} = \int d\theta \check{f}(\theta) \left|\theta, \theta_1, \theta_2\right\rangle + \check{f}(\theta_1) \left|\theta_2\right\rangle + \tag{29}$$
$$+ S_{scat}(\theta_1 - \theta_2)\check{f}(\theta_2) \left|\theta_1\right\rangle, \quad p = m(ch\theta, sh\theta), \ \ \left|p\right\rangle \equiv \left|\theta\right\rangle$$

and two-incoming field emulations on a one-particle state (a consequence of (28))

$$(: A_{in}(f_1)A_{in}(f_2) :)_{\mathcal{A}(W)} \left|\theta_3\right\rangle = \int d\theta_1 d\theta_2 \check{f}_1(\theta_1)\check{f}_2(\theta_2) \left|\theta_1, \theta_2, \theta_3\right\rangle \tag{30}$$
$$+ \int d\theta_1 \check{f}_1(\theta_1)\check{f}_2(\theta_3)S(\theta_2 - \theta_3) \left|\theta_1\right\rangle + \int d\theta_2 \check{f}_1(\theta_3)\check{f}_2(\theta_2)S(\theta_1 - \theta_3) \left|\theta_2\right\rangle$$

The last two terms are the wave function-smeared contact terms which result from applying the reordering. Note that the three-θ state still needs analytic re-ordering in order to rewrite it in terms of a 3-particle in state. These formulas have also an easy algebraic derivation in the operator setting of the reordering with the help of the transpositions in terms of the scattering function [12][14] which we will return to below. The higher particle formulas contain products of two particle S-matrices.

The emulation concept leads to a sharp division into two kinds of situations, integrable d=1+1 QFT models for which the emulated operators have translation-invariant domains permitting Fourier analysis which results in temperate generators of $\mathcal{A}(W)$, and non-temperate PFGs whose domains are only invariant under those restricted transformations which leave W invariant. Unlike ordinary (Wightman) fields, emulated operators do not directly carry physical properties, but as the result of their localization covering the whole noncompact spacetime wedge region which implies that their vacuum polarization properties are especially benign and mathematically more susceptable than interacting pointlike fields. Although one cannot use them directly for the extraction of physical properties their generated wedge algebras can be sharpened by intersections in order to arrive at spacelike cone or double cone localized algebras. It is well known that interactions increase the conceptual distance between particles and local fields and it is helpful to think that wedge localization offers the best compromise of particles with fields.

The construction of the bijection of operators between the incoming (free) wedge algebra with those of the interacting algebra depends on two properties: the equality of the domains of the two Tomita S-operators and the existence of a scattering matrix which enters the definition of $J = S_{scat}J_{in}$ and which is indispensible for checking the wedge commutativity

$$[\mathcal{A}(W'), \mathcal{A}(W)] = 0, \ \mathcal{A}(W') = \mathcal{A}(W)' \equiv J\mathcal{A}(W)J \tag{31}$$

where as usual the dash on the region stands for the causal disjoint and on the algebra for its commutant. The scattering matrix is the only *dynamical* object which enters the construction. It is surprising that integrable QFT exist at all since the temperateness requirement of a translation invariant domain is not natural from the *domS* point of view.

There is an additional more physical attribute of nontemperateness. A QFT associated to an S-matrix with inelastic multiparticle thresholds does not lead to formfactors which are meromorphic in the rapidity variables; the presence of cuts prevents the encoding of analytic $\theta-$exchange into a $\theta-$ordering. The problem is similar to that which occurs in a Wightman theory with plektonic fields. Wheres the Bargman-Hall-Wightman domain for fields with permutation group statistics is "schlicht", it contains cuts in the case of d=1+2 braid group statistics. Hence it is not possible to encode the different ways of approaching the boundary into the the order in which the imaginary parts are taken to zero; one also needs to specify the paths which are used in this process [29]. Here we will not discuss how this can be achieved in the presence of the threshold cuts in formfactors. Rather we will present a conjecture about the nature of contraction terms in the nonintegrable case which reproduces the above formulas if specialized to the integrable case. The nonintegrable analog to formula (29) reads

$$\begin{aligned}(A_{in}(f))_{\mathcal{B}(W)}\left|\theta_1,\theta_2\right\rangle = ((A_{in}(f))\left|\theta_1,\theta_2\right\rangle)_{conn} + \check{f}(\theta_1)\left|\theta_2\right\rangle + \\ \int\int d\theta_1' d\theta_2' S_{scat}(\theta_1',\theta_2|\theta_1,\theta_2')\check{f}(\theta_2')\left|\theta_1\right\rangle + ...\end{aligned} \tag{32}$$

For easier comparison the two-particle contribution was seperated from the remaining inelastic terms whose involved structure will be explained below. Whereas it is true that if the S-matrix for the incoming θ are sufficiently close together the inelastic contributions are absent, this cannot be used inside θ- integrals.

This leaves the problem of the analog of products of elastic S-matrices as in (28). To get an idea about what to do, we first rewrite them in terms of the full 3-particle S-matrix

$$S(\theta_1-\theta_2)S(\theta_3-\theta_1) = S^{-1}(\theta_2-\theta_3)S^{(3)}(\theta_1,\theta_2,\theta_3;\theta_1,\theta_2,\theta_3) \tag{33}$$

in words, the product can be written in terms of a 3-particle factorizing S-matrix in which one of the particles, θ_1, passes through two particles which have no direct interactions (implemented by the multiplication with the inverse two-particle S-matrix). In case of matrix-valued scattering functions we have to use the Yang-Baxter relation in order to compensate the inverse two-particle S-matrix and be left with the left hand side of (33). In this form it is easy to guess (a rather unique looking) expression for the general case:

$$\begin{aligned}S((r+s\text{-}inert)\ p's \rightarrow (r+any)\ p's) \equiv \\ \sum_n S_{full}^{-1}(n\text{-}p's \rightarrow any\ p's)S_{full}(p_1 + k\text{-}inert\ p's \rightarrow p_1 + n\text{-}p's)\end{aligned} \tag{34}$$

where the imposition of particle momentum conservation + Yang-Baxter algebra rules allows to return to the previous formula by specialization to the integrable case.

In this form the idea of a momentum preserving "grazing shot" with p onto an "inert swarm" activates the latter while maintaining the velocity of the "bullet".

Such a guess taken serious for the general non-integrable case would allow to write the expressions which multiply the contractions in terms of infinite sums involving S-matrix elements. The full crossing including the contact terms would then realize an shell version of Murphy's law in particle theory[34] i.e. a particular formfactor would communicate with all other formfactors.

But how is one able to prove such a conjecture. In principle its proof is simple; one "only" has to verify that the PFG behind these KMS properties is "wedge local" i.e.

$$\left\langle \psi \left| \left[J\left(A'_{in}\right)_{\mathcal{A}(W)} J, \left(A_{in}\right)_{\mathcal{A}(W)} \right] \right| \varphi \right\rangle = 0 \tag{35}$$
$$J = S_{scat} J_{in}, \quad A_{in}, A'_{in}\, \eta\, \mathcal{A}_{in}(W)$$

on the dense set of states mentioned before. But this is easier said than done.

There is another important message here. In an operator formulation of crossing in the nonintegrable case it is not possible to encode the operator structure into the permutation group linking the transposition to the scattering function. The analytic prerequisite for doing this was the *use of θ as a uniformization variable*, which breaks down in the presence of inelastic thresholds.

Here it is helpful to look at a similar problem in Wightman's theory when in d=1+2 the permutation group statistics has to be replaced by the more general braid group statistics. In that case there are cuts in the analytic Bargman-Hall-Wightman domain, and the possible ways of reaching the boundary (i.e. the ordering in which the imaginary parts pass to zero) has to be taken into account for the operator interpretation on the physical boundary. This process cannot be encoded into the operator content on the physical boundary; one also must specify the order of paths (crossing cuts). This leaves an infinite number of possibilities instead of the n! permutation group orderings. In fact in the plektonic case these possibilities are parametrized on the boundary by *words in the braid group*. We conjecture that a similar phenomenon may occur in the setting of emulation; this could significantly simplify the emulation formalism for nonintegrable theories.

In the distant future one could expect that this S-matrix-based setting may lead to an existence proof for an associated local net and to controllable approximation techniques for quantities of physical interests. This then would set the same kind of conceptual closure on QFT and make it akin to any other area of theoretical physics.

In the case of integrable models the encoding into operators based on the representation of the analytic transposition in terms of the scattering function leads to the Zamolodechikov-Faddeev structure for the creation/annihilation components of the one-particle PFG emulates

$$\left(A_{in}(\check{f})\right)_{\mathcal{A}(W)} = \int_{\partial_\pm strip} \check{f}(\theta) Z(\theta) d\theta, \tag{36}$$
$$Z(\blacksquare)Z(\blacksquare\prime) = S(\blacksquare - \blacksquare\prime)Z(\blacksquare\prime)Z(\blacksquare);\ Z^*(\theta) \equiv Z(\theta - i\pi)$$

Originally these operators were introduced as a mnemonic device to keep track of combinatorial algebraic structure, but in the 90s it was realized that they represent much

[34] "Anything which can couple (according to the rules of superselected charges) actually does couple". QM is (even in its relativistic form [18]) is par excellence the theory which remains outside the range of Murphy's law.

more than that; they admit a *spacetime interpretation* as generators of wedge localized algebras. Integrable models did not only lead to existence proofs, but they also permit to compute formfactors of pointlike fields [13]. Exact computability can of course not be expected in the nonintegrable situation, but it is not unrealistic that besides being able to assure the existence of models one may be able to find controllable approximations based on the above ideas, which replace the uncontrollable perturbative series. In the integrable (temperate PFGs) case the wedge localization can be directly proven from these algebraic commutation relations [12][14].

The algebraic construction of factorizing models also reveals that the collection of viable QFTs (insofar "viable" makes sense in d=1+1) is much larger than those which can be associated with a Lagrangian; this corresponds to the fact that there are by far more crossing-analytic, Poincaré invariant and unitary elastic scattering functions than local Lagrangian interactions. For each such S-matrix one can construct the formfactors of a local QFT [62]. We cannot tell nature to use only those models which have a Lagrangian name; the best we can hope for in a Lagrangian setting is that of the Lagrangian models comes close to what nature presents us.

The present approach shows in a clear form that the crossing on which Veneziano constructed his dual model has nothing to do with the crossing in the sense of particle physics in this article. Rather what was called crossing in the dual model referred to a "field crossing" in conformal 4-pointfunctions. It also arises from a crossing property but one which results from a conformal 4-point correlation function. More accurately it results from the Mellin transformation of global (converging) conformal operator expansions. The scale dimensions of the composite fields define the pole positions and since there are 3 pairings to which one can apply operator expansions this leads to a crossing identity [40] which according to its derivation is not related to particle physics. In fact conformal fields cannot be related with interacting particles since their LSZ limits vanish (see next section). This implies that also on pure theoretical grounds (forgetting phenomenology) the dual model and string theory was never part of particle physics.

It also cannot be based on a source-target interpretation of chiral sigma models. In the introduction we showed that a lower dimensional QFT can never be embedded into a noncompact target space, in fact d>1+1 QFT do not possess noncompact target spaces and for nonrational conformal QFT, for which this is possible, the oscillatory degrees of freedom are not arranged in form of a string-localized extension in spacetime but rather as inner degrees of freedom sitting "*over a spacetime localization point*" including the mass-spin tower.

The insuffient conceptual understanding which led to the beginning of string theory perpetuated itself in globalized communities which are unable to provide arguments which spot its fault lines, even less to dispose of it in a scientific setting. Even its opponents have to rely on sociological criticism which have a high entertainment value and prolong ST's lifetime. In fact since some opponents built their reputation on sociological arguments and the lack of observability and not on conceptual physical arguments based on particle theory, they in fact have little desire to lead this development to a closure and in this way contribute to its prolongation. There is however a consolation in that particle theory has never experienced an error whose solution presents such a new and potentially deep insight into the foundations of particle theory; after its resolution particle theory will not be the same.

Inspite of the present critique, *the step of Mandelstam to place the S-matrix into a computational approach was not in vain*, even though his later closing of ranks with the dual model setting ended in a blind allay. Taking the original idea and combining it with Haag's idea of local quantum physics one obtains a powerful new tool in nonperturbative particle physics.

5 Kinematic integrability

There exists a different notion of "kinematic" integrability which is not directly related to the dynamics of a model but rather refers to a discrete combinatorial structure of its countable superselected charge sectors which the DHR superselection theory [16] uniquely associates to a local (neutral and invariant under inner symmetries) observable algebra. More explicitely it refers to the structure of the set of higher equivalence classes of localizable representations of the observable net $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset R^4}$ given in its vacuum representation. In the case of massive theories with Bose/Fermi statistics this structure turns out to have the form of a tracial state on the algebra of the infinite permulation group which can be shown to be the dual of a compact "internal symmetry" group commuting with the Poincaré group [16]. The internal symmetry group acts on a larger "field algebra" which contains the observable algebra as a fixed point algebra under the action of the symmetry group [65]. This construction explains the spacetime origin (only localizable representations frature in this construction) of Heisenberg's isospin urform of inner symmetries and in this way demystifies the "inner" aspect. Modulo some natural conventions the entire structure is already preempted in the local net structure (the relative positioning) of the observable algebra[35]. This is a situation which is consistent but not properly understood in the Lagrangian quantization setting; inner symmetry is a quantum concept since it classifies inequivalent representations, there is no classical reason (mechanics, Maxwell fields) for its introduction. In low dimensions ($d \leq 1+2$) where in generic cases the infinite permutation group has to be replaced by the infinite braid group, the strict separation between inner and spacetime symmetries breaks down.

As mentioned before, in the standard case of compactly localizable representations or representations with mass-gaps in $d \geq 1+3$ this additional structure can be identified[36] with a Markov trace on the infinite permutation group P_∞; it is then used in order to enlarge the observable algebra to the "field algebra" [16] which contains the former as the *vacuum sector* together with all other charge-carrying inequivalent representation classes. By construction the field algebra cannot be extended further because all localizable representation classes have already been incorporated. The result confirms the central role of modular localization in QFT.

The result implies that the fields with inner symmetry indices transforming according to a noncompact group (not related to the tensorial/spinorial spacetime symmetries of the "living space") are forbidden as a result of the causal localization which also determines the

[35] There is a strong analogy to Mark Kac's famous dictum: to hear the shape of a drum.

[36] **Meines Wissens ist diese Darstellung der S_∞ doch nur ein Teil der ganzen SA-Struktur. Kann man denn mit ihre Hilfe wirklich das ganze Gruppendual konstruieren?**

localization of charge carriers and the Hilbert space structure, although classically such fields are allowed. *This sets limits to quantization* whose nonobservance would violate the holistic nature of QFT. Wheras in the previous section the Lagrangian quantization setting turned out to be too narrow; most of the large family of integrable models have no known classical counterpart. On the other hand the absence of $d \geq 1+2$ noncompact inner symmetries prevent a quantum source–>target embedding. Although the presence of a continuous superselection structure which only occurs in "nonrational" chiral theories makes inner symmetry ("target") spaces on which noncompact groups can act possible, the resulting target representation are pointlike and not stringlike. This is in particular the case of a 10 component sigma model which is used to obtain the positive energy superstring representation of the Poincaré group, i.e. the chiral theory envisaged as living on a lightlike line (after second quantization) does not become embedded into target space but rather represents an infinite component dynamical field. Already by using the known fact that there are no covariant position operators $X_\mu(\tau)$ one realizes that there is no spacetime dimension in which this is possible; it simply contradicts the causal localization principle of QFT. The degrees of freedom of the infinite chiral oscillators (apart from the zero mode) enrich the inner space over a "target point" and never not go into a stringlike material extension in the sense of "target localization". The string-localization in string theory is an illusion whose origin is the confusion of the intrinsic notion of localization of QFT with the Born's imposed (probabilistic) localization in QM.

To avoid any misunderstanding, within the class of non-rational chiral sigma model target spaces (the quantized counterpart of the classical field space and the generalization of internal symmetry space) *it is possible* to realize noncompact target space symmetries in the associated representation and "their second quantization". But why should anybody get carried away by this observation and erect a foundational gravity-including theory on top of it ? Admittedly it is somewhat surprising that the imposition of unitarity and positive energy on the target structure of a sigma model [59] leads to a an almost unique (apart from a finite number finite "M-theoretic modifications) solution called (misleadingly) the superstring representation. But why infer from this "almost uniqueness" that it should be interpreted as the start of a TOE ? Usually one believes that foundational explanations should be rather unique and not the other way around. Let us not get carried away and interpret M-theoretic properties of 10 parametric Poincaré target symmetry carrying chiral sigma models as foundational observations about our living spacetime, in particular in view of the fact that the misunderstanding of localization is closely related to that which led to the confusion about the particle physics crossing explained in the previous section.

If the terminology "kinematic integrability" would only refer to the fact that the tracial states on the infinite permutation group allow an explicit exact construction of intertwining "charge transporters" of superselected charges and encode inner symmetries, it would not be very interesting; one does not want to shoot sparrows with cannons. There are however two more interesting important cases which remained outside this standard DHR setting and for which this terminology has a nontrivial content. One is local gauge theory involving zero mass potentials. In this case the gauge-invariant observable algebra (generated by currents, field strength and suitable pointlike charge-neutral composites of matter fields) would be the natural candidate for the observable algebra. But such algebras are expected to have continuously many superselection-sectors [16] and the way to extend the observable algebras so that they only admit a countable number has not yet been clearly understood[37]. In particular it has not yet been possible to formulate the expected existence of bilocals with a "gauge bridge" and the status of their asymptotic limiting behavior and its possible relation to the issue of confined and invisible quantum matter. This is not surprising because even a complete understanding of charged "infraparticles" in relativistic abelian gauge theories has not yet been achieved.

There is also a family of massless theories whose conceptual complexity is somewhere between the standard DHR theory and its gauge theoretic extension. These are the conformal theories, which we will treat in the rest of this section. Conformal invariant QFTs began to attract the interest of a few individuals (already in the early sixties) against the resistance of the majority of particle physicists. The reasons for this initially negative attitude by particle physicists were three-fold

1. A conformal field with canonical short distance behavior is inevitably a free field.

2. A conformal QFT cannot be perturbatively constructed directly from free massless fields and the perturbative behavior of massive renormalizable d=1+3 models, contrary to some models in d=1+1, is not "soft" which would allow to take a massless limit within the perturbative Lagrangian setting.

3. The LSZ scattering limits of interacting conformal fields vanish[38]; how can one extract physical observables?

These critical observations were originally (at the beginnings of the 60s) in the form of suspicions and "gut feelings", but they later took the form of mathematical theorems[39]. The proof for two of the statements is actually quite simple, the first follows from the fact that the canonicity of scale dimension requires the two-point function to be that of a massless free field, which in turn implies the freeness of the field itself. The third is a consequence of the fact that the increase of the short distance dimension above its smallest possible value allowed by positivity (that for a free field) automatically lessens the singularity at the place of the zero mass shell $p^2 = 0$ which in turn is too weak to compensate the dissipating behavior of wave packets in order to arrive at a nontrivial LSZ limit. One would be reluctant to mention these evident facts if there was no recent flurry of papers (in connection with conformal gauge theories and the AdS-CFT correspondence) which used the terminology scattering "amplitudes" as if there was some infrared magic which allowed nontrivial zero mass shell restrictions of of correlation functions exist. Whereas one can construct conformal tree graphs, there are no scattering amplitudes of which they are the tree approximation. The Bloch-Nordsiek-Frautschi-Yennie-Suura

[37]However see forthcoming work by Buchholz, Doplicher and Roberts where the DHR superselection theory is extended to include QED.

[38]The Hilbert space positivity forces the Källén-Lehmann spectral measure to have a singularity which is milder than a mass-shell delta function.

[39]In many contemporary articles the fact that the tree-approximation of conformal theory (isomorphic to the classical structure) allow a restriction to a zero mass shell has been used to incorrectly alledge that they can describe quantrum particles in the sense of scattering theory and the S-matrix.

prescription for inclusive cross section cannot be expressed in terms of spacetime correlation functions in the analogy to the LSZ reduction formalism, but at least there is a physical interpretation of the inclusive cross section whereas the physical meaning of the Kinoshita-Lee-Nauenberg "theorem" for QCD (inclusive finiteness after avaraging in addition over colors) without observed gluons remains unclear. None of these statements about the existence of inclusive cross sections holds in conformal QFT, "conformalons" simply do not make sense.

Since conformal theories were expected to appear "in some way" as zero mass limits[40] of particle theories, their structural properties began to attract interest in the 70s; another reason was their expected (nonperturbative) mathematical simplicity as compared to QFTs describing particles: this made them theoretical laboratories for the study of pure field properties. From the viewpoint of the DHR superselection theory, which is based on spacelike locality, these theories are not distinguished from the ones with a particle interpretation since the pointlike nature (no necessity for stringlike generating fields) of generating conformal fields can be proven as a consequence of conformal invariance. What makes the conformal models interesting as a theoretical laboratory is the fact that they allow a new view about the relation between the observable and the superselected charge carrying fields which is in a more direct way based on spacetime properties. It is precisely this simplification which suggests that they may be the first QFTs for which the existence of d=1+3 nontrivial models cannot only be proven but for which their algebras and generating fields can actually be constructed. By definition conformal observables are not only Einstein causal but also "Huygens causal" i.e. their commutators also vanish for timelike separation so that the commutators of the observables live exclusively on the mantle of the light-cone. This leads to a completely new relation between an observable algebra and its (only spacelike localized) field algebra. For defining Huygens observables one does not have to refer to inner symmetries.

An important step in setting up this new relation of a conformal observable algebra and its enlarged field structure is the 1975 conformal decomposition theory [66][67][68]. There were two viewpoints about conformal invariance; one can either say that conformal fields "live" (are univalued) on *the universal covering of the compactified Minkowski spacetime* M_c, or they are distribution-valued sections on M_c. In the first case [68] (which probably goes back to Irving Segal) we encounter infinitely many "heavens" above and "hells" below M_c and there exists a *generator of the center* of the universal conformal covering group $Z \in \widetilde{SO(4,2)}$ (for d=1+3) and Z^n $n = \pm N$ numbering those heavens and hells and $n = 0$ corresponding to the compactification M_c of our living spacetime. The center is a certain *conformal rotation* at the angle 2π which results in the formula $specZ = \left\{e^{i2\pi d_\alpha}\right\}$ where d_α runs over the (anomalous) conformal field dimensions.

There is a strong analogy of this situation to the physics of plektons in d=1+2. In this case the Poincaré group $\mathcal{P}$ has an infinite covering $\widetilde{\mathcal{P}}$, but the spacetime has none. The Wigner-Bargmann representation theory however creates a kind of covering due to the semiinfinite string-like nature of the plektonic wave functions [69]. The anomalous spatial spin corresponds to the anomalous dimension and the plektonic statistics resembles an

[40] Formally such a massless limit forces all multi-particle threshold and their associated cuts to be on top of each other. The difficulty of associating particle physics with conformal QFT is the problem of reconstructing a possible theory decribing massive particles from which it may have resultet in this way.

imagined timelike plektonic exchange, with the statistical phase [16] corresponding to the eigenvalue of Z.

There are special fields for which the dimension is integer[41], in interacting theories these are typically conserved currents (including the energy-momentum tensor) resulting from the "localization" of global symmetries. These fields are not only local, but they fulfill the Huygens property since their commutator vanishes also for timelike separations. We will briefly refer to these observables as "Huygens observables" and they generally form a subalgebra of what one usually calls observables. In terms of group theory this means that these Huygens observables live on the compactification M_c of M and transform according to the conformal group $SO(4,2)$, whereas anomalous dimension fields transform according to the covering $\widetilde{SO(4,2)}$ and live on the covering space $\widetilde{M}_c$. The former fulfill space- and time-like commutativity; as mentioned before their commutators are concentrated on the mantle of the light cone and their correlations can be reduced to multivariable rational analytic functions [70]. Despite their simple appearance, nontrivial d=1+3 Huygens fields have not yet been constructed.

The Huygens observables on M_c are connected to anomalous dimensional fields which together with the Huygens observables create the full conformal field algebra. They are not only enlarging the Hilbert space but they also augment the "living space" $M_c \rightarrow \widetilde{M}_c$. If one wants to stay in M_c, the fields become sections; in particular the anomalous dimensions play the role of generalized superselected charges[42]. An application of the spectral decomposition theory to the generators of the center Z yields

$$A(x) \rightarrow A_{\alpha,\beta}(x) \equiv P_\alpha A(x) P_\beta, \; Z = \sum_\alpha e^{i2\pi d_\alpha} P_\alpha \tag{37}$$
$$A_{\alpha,\beta}(x) B_{\beta,\gamma}(y) = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'}(x,y) B_{\alpha,\beta'}(y) A_{\beta',\gamma}(x)$$

The R-matrices depend discontinuously on spacetime, they are constant but different for time- and space- like separations. For time-like separation the distinction positive/negative timelike is topologically similar to left/right distinction in chiral theories. Indeed the arguments [72] which led to braidgroup representations and exchange algebras in case of chiral theories also apply here; the Artin braids related to statistics which lead to the exchange algebras pass to those which are related to a physically more abstract exchange in time-like direction. But whereas these are the only restrictions in the setting of chiral theories (light-like) or for d=1+2 massive "plektons" (space-like) on the commutation relation of the component fields, the requirement that the components *represent the permutation group for spacelike separations* leads to restrictions which were not there in those cases; they are characteristic for all conformal QFT with $d > 1+1$. The simultaneous fulfillment of the time-like Huygens structure and the spacelike Einstein causality leads to new problems [73] which were solved only partially, to which we will return at the end of this section.

[41]For semiinteger dimension as they already occur for free spinors it is necessary to take the double covering of M_c. These fields fulfill an extended Huygens principle on the double covering.

[42]The analogy works better with squares of charges since the matter-antimatter charge compensation has no counterpart the composition of anomalous dimensions.

The group theoretical cause of the simplification in d=1+1 is the factorization of its conformal group $SO(2,2) = SL(2,R) \times SL(2,R)$ which leaves the 3-parametric Moebius group as the space-time symmetry of a chiral theory on $\mathbb{R}$ or its compactification S^1 (with the possibility to extend it to Diff(S^1)). The group theoretical factorization is followed by a chiral decompostion of the d=1+1 conformal theory into its chiral components. The chiral theories live on a light ray (or its compactification) and have proven to be the most susceptible to classification and construction, in particular in case of rational models (models with a finite number of generating fields). The first illustrative model for the decomposition theory was the exponential Boson field [66]. In this case the analogy of anomalous dimension with superselecting charges takes a very concrete form. In a somewhat formal way of writing

$$j(x) = \partial_x V(x), \quad \langle j(x)j(x')\rangle \sim \frac{1}{(x-x'+i\varepsilon)^2} \tag{38}$$
$$\Psi_q(x) = e^{iqV(x)},\ [Q, \Psi_q(x)] = q\Psi_q(x),\ Q = \int j(x')dx'$$
$$\Psi_q(x) = \sum_{q'} \Psi_q^{q'}(x),\ \Psi_q^{q'}(x) \equiv \Psi_q(x)P_{q'}$$

In the last line the $P_{q'}$ are the projectors onto the subspaces $H_{q'}$ where q' runs over all superselected charge values which occur in the theory (discrete except in non-rational chiral theories). It is now easy to see that the dilation acts quadratically in the charges as[43]

$$U(\lambda)\Psi_q(x)U(\lambda)^* = \lambda^{q^2}\Psi_q(\lambda x) \tag{39}$$

The M_c preserving transformations will not see the indices which result from the projection whereas the generator of the center Z is sensitive to them because of the quadratic relation between charges and anomalous dimensions $Z = e^{2\pi i Q^2}$ which maintains the positivity of the dimensions in the presence of the charge-anticharge symmetry. It is not necessary to work with the double indexed fields since the range projector is fixed by charge conservation.

For a long time the only nontrivial illustration of chiral theories these exponential Bose fields were the only examples. This changed abruptly when Belavin, Polyakov and Zamolodchikov [76] showed that the problem of chiral conformal theories is in an interesting way related with infinite dimensional Lie-algebras which are related to local observable algebras (typically those generated by currents and energy-momentum tensor). They constructed many models with a finite number of generating fields (primary fields) associated with those observable algebras and presented an algorithmus which made it possible to compute 4-point functions as the solution of a finite number of euclidean differential equations (continued back to real time).

This in turn led to a more careful look at the algebraic structure behind the braid group commutation structure [72]. The main step was the reformulation of the DHR superselection theory, in which the permutation group plays the central role, to the braid group [74] of which the permutation group results by simplification in the relation for

[43]The dilation generator D has continuous spectrum whereas the conformal rotation and the generator of the center Z describe the diecrete anomalous dimension spectrum (mod 1 in case if Z).

adjacent generators. The Z-phases are the statistical dimensions in the DHR setting; but whereas the latter only take on the values ± 1 their connection with the R matrices leads to a much more interesting Z-spectrum. The DHR construction of local endomorphisms leads in the simplest case [75] (for which the square of an irreducible endomorphism decomposes into two irreducible ones) to a braid group realization related to the Hecke algebra; for 3 irreducible subendomorphisms one obtains the Birman-Wenzl algebra . All these algebras come with a natural unique state which has the properties of a Markov trace. It turns out that this extended DHR analysis leads also to knot theory and invariants of 3-manifolds; it is a special case of the Vaughn Jones subfactor inclusion theory of type II_1 factors.

All these insights, as deep and interesting as they may be, were not yet of direct help in higher dimensions where the representation theoretical approach turns out to be more demanding and even the structure of Huygens observables is only incompletely understood [70].

An unexpected hint at a mechanism which combines the search for conformal theories with supersymmetry and leads to U(N) central charges came from the observations in [71]. There has been a concerted effort to find perturbative arguments for such supersymmetric conformal models and the most popular candidate has been the N=4 supersymmetric Yang-Mills gauge theory in which a massless Dirac spinor is the supersymmetric partner of the gluon field. In order to be able to assess the present situation of this research, it is helpful to first look at situations which are less controversial.

If the looked-for conformal theory can be viewed as a "smooth" massless limit $m \to 0$ of a massive renormalizable theory one is in the best of all possible situations. In fact the Thirring model is a good illustration for such a situation. Not only was its massless conformal limit explicitely known before its perturbation theory (including its beta function) was worked out, the proof of the vanishing of its Callan-Symanzik beta function was motivated by the smoothness in $m \to 0$ (see [79]) and the vanishing of beta was the perturbative signal of this smoothness. This smoothness seems to be a generic property of all models which are integrable in the sense of the previous section even if they do not contain a coupling parameter as the massive Ising field theory. Contrary to the particle masses which are part of the intrinsic nonperturbative construction, the coupling parameter is a perturbative concept which for a few factorizing models (example: Sine-Gordon) can be abstracted from the mass spectrum from the S-matrix bootstrap.

Short of a perturbative calculation methods for interacting zero mass correlation functions, the only known perturbative argument for a conformal invariant QFT consists in trying to describe it as the $m \to 0$ limit of a massive theory and to show that its Callen-Symanzik beta function vanishes. One of the models for which this was shown to all orders was the massive Thirring model. In this exemplary case one is not only able to demonstrate its low order vanishing, but thanks to a clever combination of the Callen-Symanzik equations with the Ward identities which already proved successful in case of an all order proof for the absence of anomalies [77][78] (in particular for the axial current) one was able to derive a differential equation of the form [79]

$$\beta(g)\partial_g h(g) = 0 \tag{40}$$

where the function $h(g)$ can be expressed in terms of finite normalization parameters which in turn have a well-defined perturbative coupling expansion. For the confirmation of the vanishing of beta to all orders *it is only necessary to show the much easier nonvanishing of h to first order in g.*

This embedding into a massive theory comes into conflict with reality if such a representation as a zero mass limit of a massive QFT is not possible as in the case of the supersymmetric N=4 Yang-Mills theory where the N=4 extension of supersymmetry comes into conflict with the fact that such an enlargement implies conformal invariance. In this case there is no renormalization scheme which permits to define infrared-finite correlation functions in renormalized perturbation theory. There exists however the dimensional regularization prescription which, although not systematically (inductively) applicable to local correlations (and therefore outside the range of the Callen-Symanzik methods) can be used to compute global would be beta functions in lowest order. Since QFT, contrary to QM, depends in an essential way on spacetime dimensions (already the Wigner particle representation theory does!) there is no conceptual justification for such an unguided calculation which is extremely remote from known proofs in integrable models which generically already come with a very smooth behavior for m→ 0[44]. During its more than 40 years of existence there is little conceptual support for such calculations, its fame is buildt on the derivation of asymptotic freedom (and the related Nobel prize) but not on its conceptual transparency.

Recently [85] it became clear that the cause of these infrared divergencies is the semi-infinite string-like localization of covariant gluon fields which, as a result of their self-interaction, are "stronger" string-localized than the observable charge matter fields in QED. Whereas in abelian gauge theories the infrared divergencies are confined to on-shell quantities, in nonabelian gauge theories this happens even off-shell and in the absence of quantum matter. A computational scheme based on renormalizable string-localized potentials $A_\mu(x,e)$ (localized on the spacelike half-lines $x + e\mathbb{R}_+,\ e^2 = -1$) is still in its infancy [85][86], and the situation is still inconclusive. The only known perturbative construcction of d=1+3 interacting conformal QFT starts (in the spirit of footnote 40) from a massive model and checks the vanishing of the beta function. A direct attempt without the intervention of a massive extension in a Hilbert space by dimensional regularization is not reliable.

There is another, this time much more series psychological reason to be careful. The intense interest in this model is not driven by the desire to understand the working of conformal invariance in d=1+3 per se, but rather to contribute to the largest number of papers which were ever written on a rather narrow but extremely popular problem in particle theory during the last 20 years: Maldacena's conjecture that the N=4 SusyYM theory corresponds uniquely to a 5-dim. (gravity related) theory. There exists certainly a $AdS_5 - CFT_4$ correspondence between local algebras; its formulation *in terms of generating pointlike localized quantum fields is however only possible in passing from the higher dimensional to the lower dimensional field* [88], whereas in the inverse direction this can only be done in the algebraic setting of local quantum physics. This correspondence which has been first observed on a group theoretic level in the 60s by Fronsdal extends to local algebras, a fact which underlines the deep relation between positive energy relation of spacetime symmetry groups and locality (spacelike commutativity). But its mathemati-

[44]Many models (e.g. the d=1+1 massive Ising QFT) have no couplings and hence no beta function, but do possess a conformal massless limit.

cally firmly established existence does not mean that starting from a physical theory on one side one obtains a corresponding physical theory on the other side; the study of the free AdS field shows that its CFT counterpart is an (overpopulated, the causal shadow property violating). A similar phenomenon even happens in case on implements a dimensional reduction to a lower dimensional spacetime ("holography onto a brane") where also the spacetime symmetry is reduced. Against naive classical intuition the lower dimenseional QFT is unphysical since it retains all degrees of freedom of the original QFT which formally looks like a QFT on the brane with an unphysical infinite dimensional inner symmetry [40]. This is certainly not the same as the model obtained by first applying a klassical Klein-Kaluza reduction and then applying quantization. Further studies of this property of overpopulation is necessary.

The correct degree of freedom concept showed that this property is quite different in QFT from what one is used to in QM [16]. A simple explicit illustration is provided by applying the correspondence to a 5-dimensional free AdS field. As expected, the resulting 4-dimensional conformal object is mathematically impeccable but unfortunately pathological on one of the two physical side. In fact as far back as 1962 [81] such generalized fields were used to argue that a local algebraic formulation of QFT, based on physical postulates instead of the picture arising from Lagrangian quantization, cannot be solely based on Einstein causality (spacelike commutativity) and Poincaré covariance, but there are also *timelike causal propagation properties* (which formally are fulfilled in every order of Lagrangian perturbation theory). It was therefore satisfying that generalized free fields which always looked suspicious in view of their mass spectrum, are excluded with the help of the time slice requirement. The object coming from the free AdS field is precisely a conformal generalized free field with these unphysical properties. This is the simplest illustration of a causality problem which always occurs; the conformal side may be much more complicated, but it always violates the causal propagation in the indicated way. Of course this does not mean that one cannot play mathematical games as e.g. translating the standard renormalized perturbation theory from the AdS side to the conformal side where it leads to a mathematically consistent non Lagrangian perturbation theory starting from the zero order generalized free field [82]. Mathematically there is no problem.

The problem of perspective candidates for conformal 4-dim. conformal theories and their potential discrete integrable aspects has nothing to do with the sociological phenomenon which led and still leads to such an immense number of publications without tangible result. Here one perhaps should pay attention to the fact that the old pre-electronic vernacular "many people cannot err" has given place to its post-electronic inverse.

Even if it should turn out that there are simply no conformal invariant Lagrangian models in d=1+3 at all, this is by no means the end of the story. Of all the conformal theories in d=1+1 (apart from the rather trivial exponential Bose field) none is associated to a Lagrangian; they rather were constructed by representation theoretical means in the way indicated above. If there is any way of linking a chiral model to a Lagrangion, then such a knowledge comes only very indirectly e.g. through holographic projection on the lightray of one of the few massive factorizing models which admit a Lagrangian name.

For the sake of the argument let us assume that there is really a conformal supersymmetric Yang-Mills model, then the question about the dimensional spectrum of its gauge invariant composites is indeed important, because it would be the first hold into a hitherto hidden area of higher dimensional anomalous dimensions. As in the chiral case, the conformal models can be divided into rational and nonrational ones where rational means that the number of dimensions of (composite) fields modulo integers is finite and the occurrance of nonrational models (continuously many superselection sectors) can presumably be excluded as in higher dimensional massive theories. As will be shown below, the braid group is always a subgroup of the full group following from applying the DHR theory both to the spacelike causality as well as to the timelike Huygens behavior. Hence the spectrum is expected to be similar to that following from the braid group representation encountered in the Hecke algebra representations of certain families of rational chiral models.

As we have shown in the previous section, the notion of dynamical integrability is limited to d=1+1. Partial dynamical integrability on subsystems of higher dimensional models is not possible[45] but fortunately there is the concept of a "kinematical" (and generally discrete) substructure which in the conformal case includes the structure of the spectrum of anomalous dimensions. So attempts to determine that spectrum are reasonable. When we criticise the concrete proposal that the spectrum of anomalous dimensions of composites for the supersymmetric Yang-Mills theory is related to the spectrum of a Heisenberg spin chain [83] it is only because the argument seems to be based on an oscillator representation of the conformal group in which the spectrum of dilations is discete. This cannot be, because in any positive energy unitary representation the dilation and the translations have continuous spectrum.

If one only looks at the timelike structure of conformal theories, the situation would be topologically similar to chiral theory on a light ray, the topological forward/backward lightcone distinction corresponds to the left/right aspect. In both cases one therefore expects the representation theory of the inductive limit of the braid group $\mathbf{B}_\infty$ to be relevant. To be more precise, as mentioned before in connection with the Hecke algebra, the physical braid group matrices arise by dividing out an ideal within the abstract group algebra. The spacelike action of the permutation group $\mathbf{P}_n$ however intertwines with the $\mathbf{B}_n$ in a nontrivial way [73], and the group which is relevant for the higher dimensional conformal QFT is the braid-permutation group $\mathbf{BP}_\infty$ [84]. It is not difficult to write the defining relation between the b_i, t_i $i = 1, 2, ...$ generators but the representation theory has not been developed. The Z spectrum of any 4-dim. conformal model should belong to one of these representations but unlike in the chiral case where the exponential Bose field was available a long time before the later systematic construction of families of chiral models, there exists presently no illustrative nontrivial example; the conformal invariant *generalized free field* (which results from the AdS free field by applying the correspondence) is too far away from physical fields[46] in order to be of much interest. Presently no

[45] This is related to the holistic nature of QFT which makes it impossible to embed a two-dimensional model into a higher dimensional one. Vice versa the restriction of a QFT to a brane (not its quasiclassical image) "feels" that it comes from a higher dimensional theory by retaining the original degrees of freedom. A formulation of the classical Klein-Kaluza dimensional reduction in the form as it has been proposed in the literature is not possible in QFT.

[46] Its abundance of degrees of freedom leads to the before-mentioned pathological timelike causality properties and the absence of reasonable thermodynamic behavior.

representation of BP_∞ of physical relevance is known. To find such representations one should extend the DHR superselection analysis to such a situation.

Finally some remarks about the relation of the present results to the Coleman-Mandula theorem [89] (C-M) are in order. The latter states that under certain assumptions (mainly concerning particle states and the analytic structure of the S-matrix) the full symmetry is a direct product of the Poincaré with internal symmetry. The DR theorem, with somewhat different assumptions, restricts the inner symmetries to compact symmetry groups. Supersymmetry is outside the assumptions of both theorems. In the C-M case this is well known, and the DR theorem simply starts from assumptions which place the bosonic part into the algebra of local observables and the fermionic part into the field algebra which is separated from the former by superselection rules and in this way misses supersymmetry. But that supersymmetry is a symmetry unlike any other is seen in the mechanism of spontaneous breaking in a heat bath. Whereas e.g. the Lorentz symmetry breaking follows the normal pattern of spontaneous symmetry breaking, the loss of supersymmetry is more violent: it "collapses" [90] i.e. the linking together of Bosons and Fermions is less stable than that expected from a "normal" Goldstone spontaneous symmetry breaking.

The 2-dimensional integrable models have been known to be outside the applicability of the C-M theorem since the nature of their infinitely many conservation laws contradict the preconditions of the theorem [47]. For theories involving $(m = 0, s \geq 1)$ representations or for conformal theories, the particle assumptions of C-M are violated. Apart from supersymmetry, this identifies the models outside the range of the C-M theorem precisely with dynamically or kinematically non-integrable models.

6 An epilog

The results about integrability, particle crossing and the modular localization setting appear at first sight far removed from the kind of physics which features in the ongoing LHC measurements. In fact cynics may argue that the associated string-critical consequences are as far off as string theory itself. But this may be a wrong conclusion since the assertion of the causal locality principles could have consequences for the Higgs issue and related problems.

On the one hand it may be helpful to remind readers of some facts which seem to have been lost in the "maelstrom of time". Before the Higgs mechanism degenerated into the presentation of "Gods particle" and its mystic power to generate a mass spectrum by spontaneous symmetry breaking, there was Schwinger's idea of charge screening [91][85][86] which does not break any symmetry (a gauge symmetry is not a spontaneously breakable symmetry!) but rather converts the complex electrically charged field into a real neutal field by forcing the integral over the zero component of the conserved current to vanish (and not to be infinity as in the Goldstone mechanism of spontaneous symmetry breaking). In that case the formfactor has a more analyticity than in QED, which

[47]As the breakdown of the cluster argument (which separates interactions from the identity) for 2-particle elastic scattering shows, integrable theories are close to free fields which violate the C-M assumptions in a trivial way.

allows the alias photon to be a massive object. The total number of degrees of freedom stays the same and this Schwinger screening mechanism is the QFT counterpart of the quantum mechanical Debeye screening which does not change the particle structure but only generates long range effective interaction potentials. Schwinger's bad luck with this useful idea was that he wanted to exemplify it in spinor QED where it has no perturbative realization. If he would have used scalar QED he would have obtained the Higgs model with the same results as Higgs but in a totally different conceptual setting. This is the conceptual historical background why in the old days the mechanism was more correctly called the Schwinger-Higgs screening [91].

But on a more foundational level one could ask to what extent is the Schwinger-Higgs screening the only mechanism for obtaining renormalizable interacting massive vectormesons. This statement does not fall from heaven but is an inevitable technical consequence of using the gauge theoretical indefinite metric (BRST ghosts) setting which requires for its consistent implementation the presence of additional physical degrees of freedom of which the simplest realization is the Higgs field. But in the abelian setting this is not the only mass-generating mechanism, there is also a theory without additional physical degrees of freedom called *massive spinor QED* [77]. It uses some of the indefinite metric ideas borrowed from gauge theory, but in this case they only serve to stay within the power counting limit of renormalization (the short distance dimension 2 of massive vectormesons without the lowering action of ghosts would destroy this property). At this point one may ask the question; is there a way to keep the dimension at d=1 which is required by renormalizability if one does not use ghosts? The answer is positive: semiinfinite string-localized massive $A_\mu(x,e)$ have d=1. The role of the massive string-localized field would be that of a catalyzer which allows to stay below the powercounting barrier so that after having achieved this one may as well return to the pointlike d=2 description of pointlike fields. So the problem is how does one use string-localized fields in a perturbative context. If this turns out to be true I would risk to bet that the resulting point-localized massive theory is identical to that obtained in the old days within a ghost formalism.

In the nonabelian case there is no perturbative massive Yang Mills theory without the Schwinger-Higgs screening in the present gauge formalism. But perhaps there exists a string-localized massive version with pointlike composites (representing $F^2,..$). A massive Yang-Mills theory with the help of a "string-localized catalyzer" in the setting of Callen-Symanzik equation for Epstein-Glaser renormalized correlation functions could also lead to a more credible calculation of the beta function of nonabelian gauge models. None of these ideas have been put to a test.

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