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On the SU(2) Skyrme Model with the Pion Mass Term

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Abstract

We study the SU(2) Skyrme model with the pion mass term, and discuss its physics content. We show that the addition of the pion mass term, through it means a repulsive contribution, makes the baryonic solutions to shrink in space, in a way proportional to the value of the pion mass. Similarly to the simpler case (SU(2) Skyrme model without the pion mass term), the baryonic solutions have a minimum, as a function of the Skyrme parameter, e for the quantum energy. The energies, for all values of e, are consistently larger than those of the simpler model. We show also, at the minimum, the results for the physical properties of the B=1 states. Interestingly, they seem to indicate, for the magnetic moments, a behaviour like the one for charged and neutral point particles, though the eletromagnetic radii are finite.

Key-words: Skyrme model; Chiral solitons; Explicitly broken chiral symmetry.

1 Introduction

The simplest, and most natural, addition to the lagrangian of the SU(2) Skyrme model [1], is well known to be the mass term for the pion [2]. It settles at the classical level the scenery for a situation where the original chiral symmetry is explicitly broken to the isospin (vector) conservation.

The pion mass term contribution is repulsive, and it would, naively, help to spread the mass distribution for the baryons contained in the model as solitons, resulting in increased (eletromagnetic) radii, for instance. The price to be paid should be an expected increase for the masses of the baryons.

It is currently stated, instead, that the introduction of this contribution does not change significantly the values for the physical properties of the nucleon state, which is described by a soliton with baryon number one, and spin and isospin one half. Let us point that this needs a sizeable reduction (by near 20%) in the value of the pion decay constant, f_π, to almost 60% of its experimental value. Nonetheless, it is claimed that the overall picture shows a satisfactory consistency.

We think that several aspects of the Skyrme model including the pion mass term, have not been thoroughly considered. In particular, the physical picture coming from it and its distinction with the simpler version of the model, has received little attention.

Let us recall the main characteristics of the simpler Skyrme model [3]. In order to make the text lighter, we shall use the abbreviation SM_o to refer to the Skyrme model including only in its lagrangian the terms of the non-linear sigma model and Skyrme's stabilizing term, and SM_π will indicate the model as before which has in addition a pion mass term contribution.

The Skyrme model has solutions of integer baryon number B=1,2,3, $\cdots$ and integer and half-integer spins (and isospins) in its hedgehog approximation for the unitary chiral field:

$$U(\vec{x}) = \exp(i\vec{\tau}\cdot\hat{\vec{x}}F(r)) ,$$

where the spherical symmetric function $F(r)$ is the "chiral angle", $\vec{\tau}$ is a vector having as components the three spin Pauli matrices, and $\hat{x}$ is the unit vector in the direction of $\vec{x}$.

For the baryon states, the chiral angle is regular at the origin and at infinity, and the corresponding series expansions contain a classically undetermined dimensional parameter. The baryon solutions are in one to one correspondence with special values of a dimensionless parameter $\phi = F_1/ef_\pi$, where $F_1 \equiv dF(r)/dr|_{r=0}$ and e is the dimensionless Skyrme parameter. Moreover, the regular solutions have a scale invariance, provided e is allowed to vary in a way that compensates the variation of F_1. The pion decay constant (f_π) is taken as fixed parameter that comes from physical information outside the model. As a result, the classical mass for the baryon is undetermined.

The quantum energy that is obtained after introducing (and quantizing) collective coordinates that rotate the static soliton, is a function of the Skyrme parameter, has a (different) minimum for each baryon state, and diverges for $e = 0,\ \infty$. The minimum corresponding to the nucleon curve ($e = 7.67$) corresponds to a narrow object, with mass which corresponds to the observed one for $f_\pi = 0.143$GeV. In order to fit the observed masses for the nucleon (N(939)) and $\Delta(1232)$ [4] one has to move to $e = 5.45$ and fix $f_\pi = 0.129$GeV (70% of its physical value).

An important feature is that the only free, dimensional, quantity at each baryon number trajectory, the pion decay constant, provides a natural unit for all dimensional physical properties. As a result, when changing e from the minimum of the nucleon to the values fitting the masses, the radii increase (a decrease of e brings F_1 to decrease, to keep ϕ at its B=1 value, and since the curve grows towards $e = 0$, f_π must decrease also).

So much for SM_o. In this work we show the results for SM_π.

We consider that the pion decay constant and the pion mass value have to be taken as constant parameters, since they are extracted from a framework foreign to the description of baryons by solitons. At most, they could be taken as a kind of consistency parameters, in the sense of being determined from a large phenomenological analysis within the chiral theory (see in detail what we mean in our discussion [3]).

We will present the regular solutions, at the origin and at the asymptotic region. We will show that taking the pion mass (m_π) as a fixed quantity breaks the scale invariance possible for SM_o. The asymptotic solution displays a larger departure from the one for SM_o, an exponential decay for $m_\pi \neq 0$ and a non-analytic behaviour for $m_\pi \to 0$. The exponential decay brings as a consequence that the baryons are <u>narrower</u> for SM_π than for SM_o, contrary to the expectation that repulsion would make them broader.

Besides, the solutions for the chiral angle miss the freedom for displacements by an integer multiple of π at the extremes of the real axis. This is because (regular) finite energy solutions, with the pion mass term, can only have even multiples of π as asymptotic values.

We apply the arguments by Derrick and Hobart [5], and find, as it should be expected, that the stability of the soliton in SM_π demands an increase in the contribution for the stabilizing term proposed by Skyrme. This is an unwanted feature, since lower order chiral calculations fix the form for the non-linear sigma model term, but the Skyrme term is one of the several of its kind that may result in chiral pertubation theory at the one loop level [6].

Sum rules obtained from the Euler-Lagrange equations of SM_π à la Iwasaki and Ohyama [7] allow to confirm another consequence of the lack of the scale invariance present in SM_0 : the value of ϕ for a given baryon number changes with the value of the Skyrme parameter in a way that determines uniquely F_1 for each value of e. That is, F_1 is now classically determined.

The classical values for the mass of the soliton are larger in SM_π, in agreement with expectation. This indicates that the pion mass term increases the density of the "nuclear matter" for each solution (which, remember, are narrower than in SM_0).

Notwithstanding these changes, the quantum energy obtained after introducing (and quantizing) collective coordinates which rotate the solitons, resembles the one of SM_0 as a function of the Skyrme parameter. Each baryonic state is a single minimum, which is (naturally) higher than for the SM_0 case. This allows to understand why the value of f_π for the fitting of the masses of the nucleon and $\Delta(1232)$ is so much lower.

On evaluating the physical constants for the nucleon, one notices that, at the minimum, the values for the magnetic moments of the two isopin states, as m_π increases, tend to the values for a point Dirac particle (charged and neutral), while preserving a spatial structure. This is an intriguing property, and open new possibilities for the model to represent particle states, which deserve further study.

The text is organized as follows: in section II we present the regular solutions and discuss their features . In section III we face the issue of stability for the chiral solitons of SM_π and discuss the results, including the possibility of looking to an alternate model having no non-linear sigma model term in its lagrangian. In the following we present the sum rules which are obtained from the Euler-Lagrange equations, which are a slight formal modification from the ones for SM_0, but are more cumbersome to use. Section IV is devoted to the discussion of the energy after quantization with collective coordiantes. The physical parameters for the candidate soliton to represent the nucleon are presented and analyzed. Finally, in section V we discuss the results obtained in the previous sections, consider alternative improvements of the model as an approximation to the description of the baryons and other possibilities for it.

2 Classical regular solutions

The static lagrangian of the model is:

$$\mathcal{L} = -\frac{1}{16} f_\pi^2 \mathrm{Tr}\left[L_k L^k\right] - \frac{1}{32e^2}\mathrm{Tr}\left\{[L_k, L_l]^2\right\} - \frac{1}{8} m_\pi^2 f_\pi^2 \mathrm{Tr}(U-2)\,, \tag{1}$$

where

$$L_k = U^\dagger(\vec{x})\frac{\partial U(\vec{x})}{\partial x^k} \tag{2}$$

$$U(\vec{x}) = \exp\left[i\vec{\tau}\cdot\hat{\vec{x}}F(r)\right] \tag{3}$$

$$\hat{x} = \frac{\vec{x}}{\mid\vec{x}\mid}\,, \tag{4}$$

$$r = \mid\vec{x}\mid\,, \tag{5}$$

and $\vec{\tau}$ is a vector with components $(\sigma_1, \sigma_2, \sigma_3)$, the 2 x 2 Pauli matrices.

The corresponding Euler-Lagrange equation gives, for the chiral angle, $F(r)$:

$$\begin{aligned} & r^4\frac{d^2F(r)}{dr^2} + 2r^3\frac{dF(r)}{dr} - r^2\sin[2F(r)] \\ + & \frac{1}{e^2 f_\pi^2}\left[8r^2\sin^2 F(r)\frac{d^2F(r)}{dr^2} - 4r^2\sin[2F(r)]\left[\frac{dF(r)}{dr}\right]^2 - 4\sin^2 F(r)\sin[2F(r)]\right] \\ - & m_\pi^2 r^4 \sin F(r) = 0\,. \end{aligned} \tag{6}$$

This is a convenient form to study the behaviour of the chiral angle at the origin, since the equation has a singularity in this point, coming from the first two terms in the lagrangian.

The behaviour at infinity is best understood by studying the equation after the change of variable: $R = 1/r$. One obtains:

$$\begin{aligned}
& R^2 \frac{d^2K(R)}{dR^2} - \sin\left[2K(R)\right] \\
+ & \frac{1}{e^2 f_\pi^2}\left\{ 8R^4 \sin^2 K(R) \frac{d^2K(R)}{dR^2} + 16R^3 \sin^2 K(R) \frac{dK(R)}{dR} \right. \\
+ & \left. 4R^4 \sin\left[2K(R)\right] \left[\frac{dK(R)}{dR}\right]^2 - 4R^2 \sin^2 K(R) \sin\left[2K(R)\right] \right\} \\
- & \frac{m_\pi^2}{R^2} \sin K(R) = 0 \,.
\end{aligned} \tag{7}$$

Let us first consider the regular solutions at the origin. The regular solutions are not changed at the lowest powers by the pion mass term. A power series solution, regular at the origin, is of the form:

$$F(r) = F_0 + F_1 r + F_3 r^3 + F_5 r^5 + \cdots + F_{2N+1} r^{2N+1} + \cdots \,, \tag{8}$$

where the lower coefficients are:

$$F_0 = n_0 \pi \qquad (n_0 : \text{odd}) \,, \tag{9}$$

$$F_3 = -\frac{4}{5} F_1^3 \cdot \frac{1 + 2\phi^2 - \frac{3\beta^2}{4\phi^2}}{1 + 8\phi^2} \,, \tag{10}$$

$$F_5 = \frac{24}{7} F_1^5 \cdot \frac{N(\phi,\beta)}{D(\phi,\beta)} \,, \tag{11}$$

and

$$N(\phi,\beta) = 1 + \frac{32}{5}\phi^2 + \frac{88}{5}\phi^4 + \frac{448}{5}\phi^6$$

$$+ \quad \beta^2\left(\frac{2}{5} - \frac{7}{8\phi^2} + \frac{104}{7}\phi^2\right) + \beta^4\left(\frac{1}{8\phi^4} - \frac{71}{5\phi^2}\right), \tag{12}$$

$$D(\phi,\beta) \quad = \quad 1 + 24\phi^2 + 192\phi^4 + 512\phi^6 \,. \tag{13}$$

In general, the terms are of the form:

$$F_{2N+1} = C_{2N+1} F_1^{2N+1} R_{2N+1} \,, \tag{14}$$

with C_{2N+1} a numerical coefficient, the same as in the pure non-linear sigma model expansion at the origin. The quantity R_{2N+1} is a rational function of ϕ, which include some contribution from the pion mass term. The dimensional coefficient F_1 is undetermined from the differential equation.

When $\beta = 0$, one could play with the values of F_1 and e, which are not fixed a priori in the model, while keeping fixed the value of ϕ. Since ϕ is a kind of control parameter for the solutions of the model, this is, in principle, allowed. Objections arise from considering e a fixed parameter, like any coupling constant. But since the lagrangian for SM_o (or even SM_π) is a kind of "efective" lagrangian, which should result in some approximate limit of the fundamental theory, QCD, one should by the same expect that for different sectors of the set of internal quantum numbers, the Skyrme coefficient should be modified, casting doubts about taking it as a coupling of the theory, fixed once for ever.

The regular solutions, when F_1 and e are changed arbitrarily, have an interesting scaling invariance. Most quantities in the theory are expressed as integrals, which can be translated entirely into dimensionless variables of integration and scale invariant functions, leaving any dimensional parameter as a coefficient in the integrals.

The inclusion of the pion mass term modifies this. Although the dimensionless parameter β determines ϕ, we are not allowed to play with the value of the mass any longer. The variation of ϕ with e, for fixed pion mass and baryon number, is represented in Fig. 1. The more important deviations are found at small e, in the more classical regime, where β increases with decreasing e. The increase in the value of ϕ indicates that in the region of small e the slope at the origin of the chiral angle is larger than in the $m_\pi = 0$ case, giving a somewhat more tight baryon. In short, for a fixed value of the pion mass different from zero, ϕ varies for a given baryon number, and so does F_1.

Let us now consider the asymptotic solutions. They are of the following form, when written in terms of the dimensionless parameter $\tilde{R} = 1/\tilde{r} = R/ef_\pi$:

$$\begin{aligned} K(\tilde{R}) &= n_\infty \pi + \exp\left[-\frac{\beta}{\tilde{R}}\right] G_1(\tilde{R}) + \exp\left[-\frac{3\beta}{\tilde{R}}\right] G_3(\tilde{R}) + \cdots + \\ &+ \exp\left[-\frac{(2N+1)\beta}{\tilde{R}}\right] G_{2N+1}(\tilde{R}) + \cdots \quad . \end{aligned} \tag{15}$$

The functions $G_k(\tilde{R})$ have power expansions which, for the lower indices, are:

$$G_1(\tilde{R}) = \frac{1}{2} K_2 \tilde{R}^2 (1 + \frac{\beta}{\tilde{R}}) , \tag{16}$$

$$\begin{aligned} G_3(\tilde{R}) &= -K_2^3 \left[\frac{1}{3!}\frac{1}{2^6}\beta^3 \tilde{R}^3 + \frac{1}{4!}\frac{3}{2^5}\beta^2 \tilde{R}^4 + \frac{1}{5!}(30\beta^2 + \frac{285}{2^7}\beta)\tilde{R}^5 \right. \\ &+ \left. \frac{1}{6!}(360\beta^2 + \frac{105}{2^5})\tilde{R}^6 + \frac{1}{7!}(2835\beta + \frac{4725}{2^8}\beta^{-1})\tilde{R}^7 + \cdots \right] . \end{aligned} \tag{17}$$

As we shall see, the integer n_∞ should be even for a regular solution. The second term, the dominant asymptotic contribution, shows already a strong exponential decay, that increases with the pion mass. Its $m_\pi = 0$ limit is rather simple, being just the non-linear sigma model behaviour.

Differently from the Skyrme model, the parameter K_2 is now fixed, at fixed pion mass, for each fixed set of β and e, in complete analogy with what happens with F_1 at the origin.

The limit for negligible pion mass is cumbersome, as wits the inverse powers of β appearing in the functions $G_k(k>1)$. This shows that one should expect a departure from SM$_o$; what is now missing is the freedom to choose any integer multiple of π for the chiral angle at infinity. Before, only the relative difference with respect to the value at the origin mattered.

We have not made the analytic continuation on both regular solutions to an overlap region. We have performed this carefully with the numerical solutions.

Numerically, the behaviour of the solutions looks like an amplified behaviour in SM$_o$. Regular odd baryon number solutions are those that start at the origin with an odd multiple of π and finish, asymptotically, at even multiples of π. These solutions correspond to isolated values of ϕ for given values of m_π (or, better, of β) and e. Between a couple of these values this kind of solutions (starting from odd integer values of π) goes asymptotically to an odd multiple of π, oscillating with increasing wavelength as the control parameter approachs the value corresponding to an allowed baryon state, until they stick to an asymptotic even multiple of π,

$$B = \frac{1}{\pi}[F(r\to\infty) - F(r\to 0)] \qquad (18)$$

$$= n_\infty - n_0 = 1, 3, 5, \cdots \; . \qquad (19)$$

Something analogous is seen starting from even multiples of π at the origin. First, as ϕ increases from zero, the solutions oscillate around the next higher odd multiple of π. The solution has larger amplitude and wavelength as the value of ϕ corresponding to an even baryon number is approached, and, at precisely this value, the solution sticks to the asymptotic, even multiple of π,

$$B = n_\infty - n_0 = 2, 4, 6, \cdots \; . \qquad (20)$$

In fact, we notice that the values of ϕ are, in general, larger, for a given baryon number, than the corresponding value of ϕ for SM_o. This means a larger slope at the origin for the chiral angle, i.e, a more concentrated object. Besides, the exponential decay for the asymptotic region means also that one has a more tight baryon when the pion mass term (which is repulsive!) is added.

As we shall see, the resulting masses for the baryon states are larger than the corresponding ones for the SM_o. This suggests, as a short statement about the effect of the addition of the pion mass term, that it shrinks the baryons and increases the density of the "nuclear matter" described by the soliton.

3 Stability of the classical soliton solution and sum rules

For SM_o, applying the argument advanced by Derrick [5], it is found that the classical mass is inversely proportional to the value of the Skyrme parameter. Provided one keeps ϕ fixed for a given baryon number, the parameter F_1 and e could be changed arbitrarily, and, in principle, the soliton could have an arbitrarily large mass, for an infinitely extended object ($F_1 \to 0$), or could have an infinitesimal mass for a point object.

Now, we are not more able to change parameters at will, taking the pion mass as an external, fixed parameter. The baryon number is not uniquely related to a single value of ϕ. Moreover, in a lagrangian sense, the pion mass contribution is a repulsive one, so the question of the stability is interesting.

Let us begin by writing the classical energy of the soliton:

$$E_{cl} = E_\sigma + E_{Sk} + E_\pi \ , \tag{21}$$

with

$$E_\sigma = \frac{1}{2}\pi f_\pi^2 \int_0^\infty dr \ \left\{ \left[r\frac{dF(r)}{dr} \right]^2 + 2\sin^2 F(r) \right\} \ , \tag{22}$$

being the contribution from the non-linear sigma model term;

$$E_{Sk} = \frac{1}{2}\pi f_\pi^2 \frac{1}{ef_\pi} \int_0^\infty dr \ \left\{ 2\left[\sin F(r)\frac{dF(r)}{dr} \right]^2 + \left[\frac{1}{r}\sin F(r) \right]^2 \right\} \ , \tag{23}$$

the contribution from the stabilizing term proposed by Skyrme; and, finally,

$$E_\pi = \frac{1}{2}\pi f_\pi^2 \cdot 2m_\pi^2 \int_0^\infty dr \ r^2 \left[1 - \cos F(r)\right] \ , \tag{24}$$

the contribution from the pion mass term. Notice that the latter diverges for values of the chiral angle corresponding, asymptotically, to odd multiples of π. The Skyrme term will grow to infinity for an irregular solution at the origin $(F(r \to 0) \sim (2N + 1)\pi/2)$.

Performing a dilatation in the variable of integration

$$r \to \lambda r \quad (\lambda \approx 1) \ , \tag{25}$$

one gets

$$E_\lambda^{cl} = \frac{1}{\lambda} E_\sigma + \lambda E_{Sk} + \frac{1}{\lambda^3} E_\pi \ . \tag{26}$$

For the classical energy to be stable under this change,

$$\left. \frac{dE_\lambda^{cl}}{d\lambda} \right|_{\lambda=1} = 0 \ , \tag{27}$$

one must have

$$E_{Sk} = E_\sigma + 3E_\pi \,, \tag{28}$$

$$\begin{aligned} E^{cl} &= 2E_\sigma + 4E_\pi \\ &= 2(E_{Sk} - E_\pi) \\ &= \frac{2}{3}(E_\sigma + 2E_{Sk}) \,. \end{aligned} \tag{29}$$

The inclusion of the pion mass term changes the relation between the other two terms; for SM_o both should be equal. Now, the Skyrme term must compensate in addition the repulsive contribution from the new term and this is an unwanted result. This will increase the mass of the baryon states, but there is no special reason to have such a large Skyrme term, unbalancing the non-linear sigma model contribution.

The condition of stability, besides, require a positive second derivative near $\lambda = 1$, which gives

$$E_\sigma + 6E_\pi > 0 \,. \tag{30}$$

By the way, from the set of Eqs.(28) and (30), one notices that the pion term and the sigma model term behave alike. It would be interesting to investigate a model where the non-linear sigma model is absent, though it looks quite heretical. The only justification would be that the sigma model contribution is expected to be important for the lowest energy part of the chiral systems with B=0, and, looking for the B $\geq$ 1 sector, one considers a completely different regime.

Notice that for the SM_o, the classical energy was undetermined as a function of the Skyrme parameter at fixed ϕ (or β), because of the scale invariance of the solution for the chiral angle. This is no longer valid after the introduction of the pion mass, the value of the integral changes under a change in the Skyrme parameter.

We may write the integrals in terms of the dimensionless variable $\tilde{r} \equiv e f_\pi r$. In particular, the Euler-Lagrange equations may be handled, in terms of $\tilde{r}$, by the way introduced originally by Iwasaki and Ohyama [7] for the non-linear sigma model. Multiplying Eq.(6) by $\tilde{r}^n dF(\tilde{r})/d\tilde{r}$, and after several integrations by parts, one has

$$\begin{aligned} &\left\{\frac{1}{2}\tilde{r}^{n+4}\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 + 4\tilde{r}^{n+2}\sin^2 F(\tilde{r})\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 - 2\tilde{r}^n \sin^4 F(\tilde{r})\right.\\ &- \left.\tilde{r}^{n+2}\sin^2 F(\tilde{r}) + \beta^2 \tilde{r}^{n+4}\cos F(\tilde{r})\right\}\Big|_{\tilde{r}_1}^{\tilde{r}_2} \\ &+ \int_{\tilde{r}_1}^{\tilde{r}_2} d\tilde{r}\left\{-\frac{1}{2}n\tilde{r}^{n+3}\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 - 4(n+2)\tilde{r}^{n+1}\sin^2 F(\tilde{r})\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2\right.\\ &+ \left.(n+2)\tilde{r}^{2+1}\sin^2 F(\tilde{r}) + 2n\tilde{r}^{n-1}\sin^4 F(\tilde{r}) - \beta^2(n+4)\tilde{r}^{n+3}\cos F(\tilde{r})\right\} = 0 \end{aligned} \tag{31}$$

With the usual assumptions on the values for the chiral angle at the origin and asymptotically [3], we have, for instance, for $n = -2$,

$$\begin{aligned} \sin^2 F(\tilde{r} \to \infty) &= \beta^2 \lim_{\tilde{r}\to\infty}\left[\tilde{r}^2 \cos F(\tilde{r})\right] + \int_0^\infty d\tilde{r}\left\{\tilde{r}\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 - 4\tilde{r}^{-3}\sin^4 F(\tilde{r})\right.\\ &- \left.2\beta^2\tilde{r}\cos F(\tilde{r})\right\} = 0\,. \end{aligned} \tag{32}$$

Whereas for the pure non-linear sigma model ($\beta = 0,\ e \to \infty$) the sum rule is not satisfied, for SM_o ($\beta = 0$), it is acceptable for the irregular solutions ($|\sin F(\tilde{r} \to \infty)| = 1$), and satisfied exactly for the isolated points of ϕ corresponding to baryon states ($\sin F(\tilde{r} \to \infty) = 0$). For the present case, the irregular solutions have also $\sin F(\tilde{r} \to \infty) = 0$, but the right hand side is not zero. The sum rule is satisfied only for the values of ϕ and e, at a given value of m_π, that correspond to the allowed baryon numbers (1,2,3,4,...). As for $\beta = 0$, the sum rule allows a precise value for the parameters (e, ϕ) determining the baryon state.

The sum rule for $n = -4$ is:

$$\begin{aligned}\frac{1}{2}\phi^2 - 2\phi^4 + 2\beta^2 &= \int_0^\infty d\tilde{r}\, \left\{ \frac{8}{\tilde{r}^5}\sin^4 F(\tilde{r}) + \frac{2}{\tilde{r}^3}\sin^2 F(\tilde{r}) \left[1 - 4\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 \right] \right. \\ &- \left. \frac{2}{\tilde{r}}\left[\frac{dF(\tilde{r})}{d\tilde{r}}\right]^2 \right\} \qquad (33)\end{aligned}$$

and provides a consistency check for the parameters of the system.

4 Quantization and some numerical results of interest

The quantization of the system is made appealing to collective coordinates, $A(t)$, which are unitary matrices in SU(2), that rotate the soliton. The states of the quantum hamiltonian for the SU(2) hedgehog should be labelled by the same eigenvalue of the angular momentum, $\vec{J}^2$, and isospin, $\vec{I}^2$. The final results for the energy are like the ones for SM$_o$,

$$E = E^{cl} + \frac{\vec{J}^2}{2\theta}\,, \qquad (34)$$

but E^{cl} is as in Eq.(29), and

$$\theta = \frac{2}{3}\pi f_\pi^2 \int_0^\infty dr\; r^2 \sin^2 F(r) \left\{ 1 + \frac{4}{e^2 f_\pi^2} \left[\left[\frac{dF(r)}{dr}\right]^2 + \frac{\sin^2 F(r)}{r^2} \right] \right\} \qquad (35)$$

has no direct contribution from the pion mass term.

The quantum energy has no explicit dependence on the Skyrme parameter, due to the fact that we have not anymore a scale invariance. Nonetheless, the states for given baryon number, when represented against the Skyrme parameter, display a minimum for some value of e, which in general very much approximates the one for the case $m_\pi = 0$. The masses at the minimum are, however, larger; to obtain a good value for the masses of the nucleon and $\Delta(1232)$ resonance [2] it was necessary to go to even lower values of f_π (~ 0.108GeV) than in the preceding case. In Fig. 2a, b, c, and d we sketch this for the case where B=1, I=J=1/2 and 3/2; B=2, I=J=1; and B=3, I=J=1/2 and 3/2. These curves are drawn for some values of m_π.

We remark that these curves E *vs* e are shown here for the first time for the model. They are interesting in their own. They show, for instance, that for reasonable values of the Skyrme parameter, the state with the quantum numbers of the nucleon is the absolute minimum, which speaks favourably of the model.

The minima represent states whose rotation contributes 1/4 to the total energy [3]. This is, physically, not very good, since the rotation contribution is larger than the energy to emit one pion.

One can divide roughly the domain in Fig. 2 in two regions: the semiclassical one, going from $e = 0$ up to the minimum "nucleon" energy; the results coming from fitting the masses of $N(938)$ and $\Delta(1232)$ are found in this region. The other, where the rotation from collective coordinates dominate, is of no physical interest.

In table I we represent, for the values at the minima, the results for the masses and the parameters of the theory, when the pion mass is varied. The discrepancies for the values of the B=2, 3 states with $\beta = 0$ as compared with those quoted in Ref. [3] follows a more careful integration procedure in the present work, especially at values of $F(r)$ in the vicinity of integer multiples of π.

In table II we compare the results for the nucleon state at the minimum with those coming from adjusting the parameters to fit the masses of the nucleon and the

$\Delta(1232)$ ([2], [4]).

As advanced in preceding section, it is most remarkable that the nucleon state at the minimum shrinks with increasing values for the pion mass, while its mass increases, showing that there is a consequent sizeable increase in the density of "nuclear matter". This goes against what one would expect, the addition of repulsion suggesting that states should stretch.

The shrinking is particularly important for the isovector contributions, the isoscalar quantities being rather indifferent when the pion mass contribution is added.

It might be interesting to comment on the results for the radii resulting from the fitting of the B = 1 states $N(939)$ and $\Delta(1232)$. We called the attention to the fact that the addition of the pion mass term shrinks the particle, and, superficially, the values of the radii are larger from the fit of the masses. The point is that the value of the pion decay constant must be lowered by 20% to attain this result. Since f_π is a kind of natural unit for energies and lengths, it is its reduction to 60% of its experimental value which brings larger radii.

A point to be made regards the values for the magnetic moment at the minimum for the nucleon representative. Curiously, the tendency is towards the values for a pointwise, structureless Dirac particle (g=2 for the charged state, g=0 for the neutral one) though the eletromagnetic radii are finite. This may open possibilities for application of the model outside the realm of strong interaction dynamics.

Another remarkable feature is the increase of both g_A and $g_{\pi NN}$ for the states at the minimum, by 10% and 30% respectively.

5 Discussion of the results and conclusions

In this work we have studied the Skyrme model with the pion mass term added to the lagrangian. We have shown that the resulting theory describes heavier and narrower baryons, the more the pion mass increases. We have been able to understand the known results at the semiclassical level, fitting the masses of the nucleon and $\Delta(1232)$ resonance in a larger framewbrk.

The main shortcoming of the model at this stage is the increasing importance of the Skyrme's stabilizing term in comparison to the non-linear sigma model contribution. In a chiral simmetry theory compatible [6] the stabilizing term is one among several that appear at the one-loop of the effective lagrangian. It would be expected to be a correction to the "tree contribution", but may be we are applying the right argument to the wrong hadron scale.

By the way, this seems to be the principal difficulty to deal with the nucleon in chiral perturbation theory [9,10], since apparently two different scales have to be considered.

In broader terms, contrary to the current opinion, we think that the inclusion of the pion mass term contribution in the lagrangian of the Skyrme model does not help to consider the Skyrme model as the right starting point for the description of baryons in the low energy regime. Maybe the correct addition of π-mesons, as a quantized field, would contribute to improve the description. A "pion cloud", using the old terminology, might contribute to the spreading of charge (and mass). The apparent robustness of the original Skyrme model, specially after quantization through collective coordinates, may indicate that the addition of the pion degrees of freedon will not alter its most salient features.

This work opens also two avenues for eventual progress in the application of the model. One is represented by the possibility of producing a stable classical baryon without the non-linear sigma model term. This may suit a pertubation-like scheme where the current algebra is important for corrections quite neighbouring the mass-shell of a baryon; this may be applicable even for the inclusion of quantized pion degrees of freedom.

The second interesting alternative is the one obtained from the numerical results for the nucleon: maybe the model, without any reference to strong interaction physics as a by-product of the colour gauge theory, may be useful to describe composite particles quite approximate to a Dirac fermion. In this case, the meaning of the coefficients appearing for each term in the lagrangean should be reinterpreted.

In any case, while considering the Skyrme model a good candidate to describe nucleons by chiral solitons is rather premature, it can only benefit from further developments that profit from its features and correct its shortcomings.

Figure Captions

Fig. 1a - Characteristic curves for baryonic number B=1. C1 $\rightarrow m_\pi = 0.0$GeV, C2 $\rightarrow m_\pi = 0.05$GeV, C3 $\rightarrow m_\pi = 0.10$GeV and C4 $\rightarrow m_\pi = 0.139$GeV.

Fig. 1b - Characteristic curves for baryonic number B=2. C1 $\rightarrow m_\pi = 0.0$GeV, C2 $\rightarrow m_\pi = 0.05$GeV, C3 $\rightarrow m_\pi = 0.10$GeV and C4 $\rightarrow m_\pi = 0.139$GeV.

Fig. 1c - Characteristic curves for baryonic number B=3. C1 $\rightarrow m_\pi = 0.0$GeV, C2 $\rightarrow m_\pi = 0.05$GeV, C3 $\rightarrow m_\pi = 0.139$GeV.

Fig. 2a - Quantum energy curves for baryonic number B=1, J=1/2. Curves from botton to top corresponds to: m_π=0.0, 0.05, 0.10 and 0.139GeV.

Fig. 2b - Quantum energy curves for baryonic number B=1. C1 $\rightarrow m_\pi = 0.0$GeV, J=1/2; C2 $\rightarrow m_\pi = 0.0$GeV, J=3/2; C3 $\rightarrow m_\pi = 0.139$GeV, J=1/2; C4 $\rightarrow m_\pi = 0.139$GeV, J=3/2.

Fig. 2c - Quantum energy curves for baryonic number B=2, J=1. Curves from botton to top corresponds to: m_π=0.0, 0.05, 0.10 and 0.139GeV.

Fig. 2d - Quantum energy curves for baryonic number B=3. C1 $\rightarrow m_\pi = 0.0$GeV, J=1/2; C2 $\rightarrow m_\pi = 0.0$GeV, J=3/2; C3 $\rightarrow m_\pi = 0.139$GeV, J=1/2; C4 $\rightarrow m_\pi = 0.139$GeV, J=3/2.

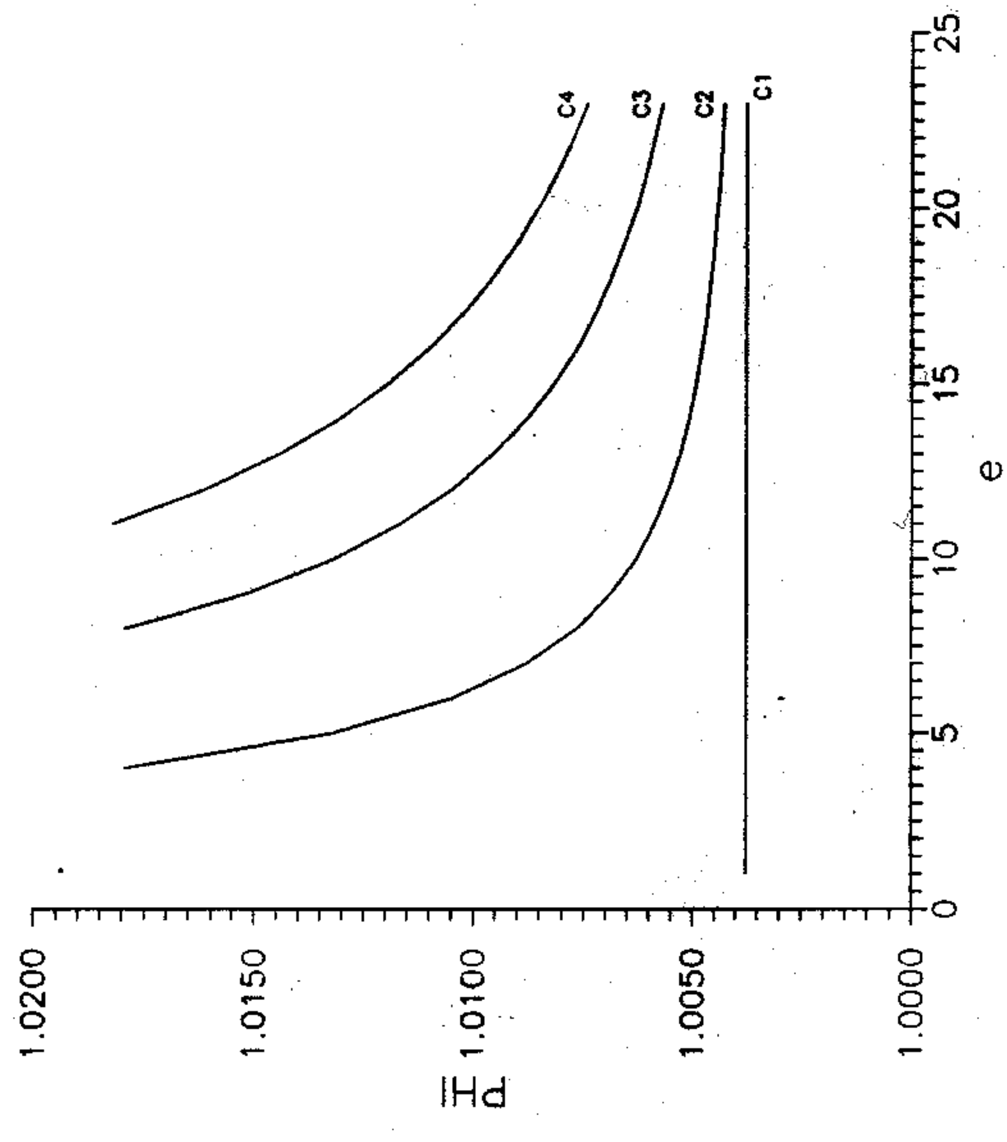

Fig. 1.a

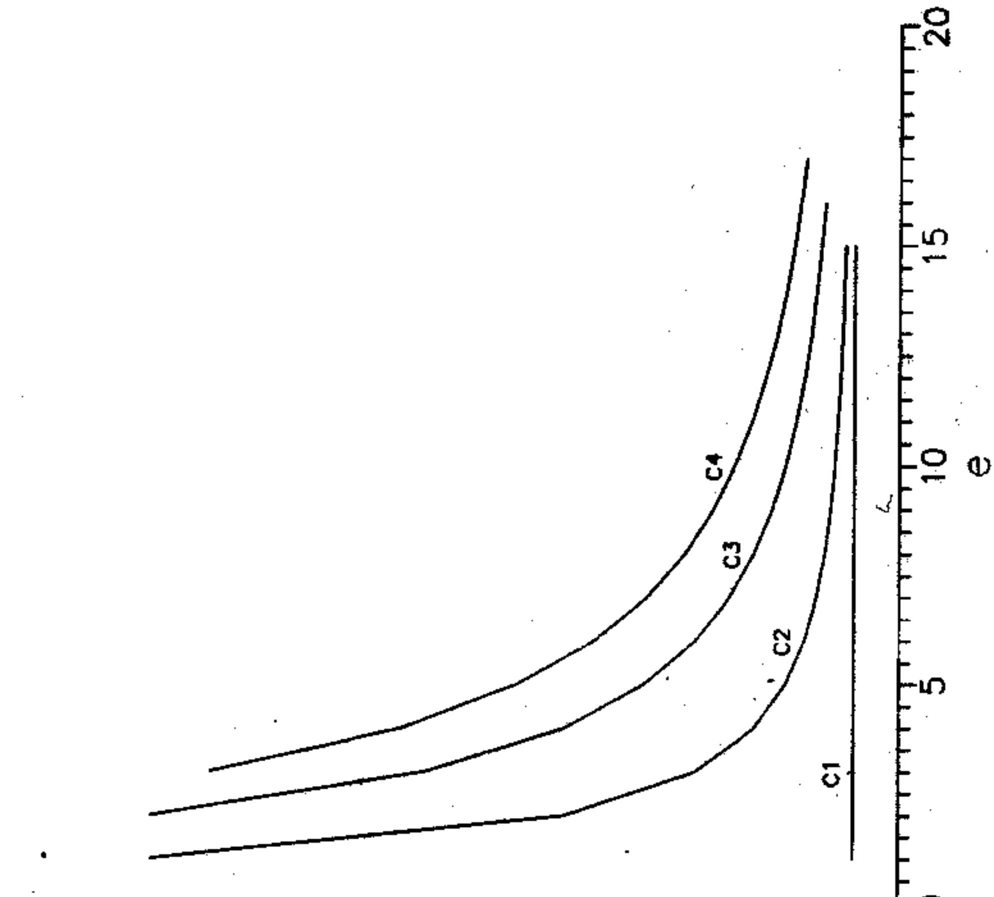

Fig. 1.b

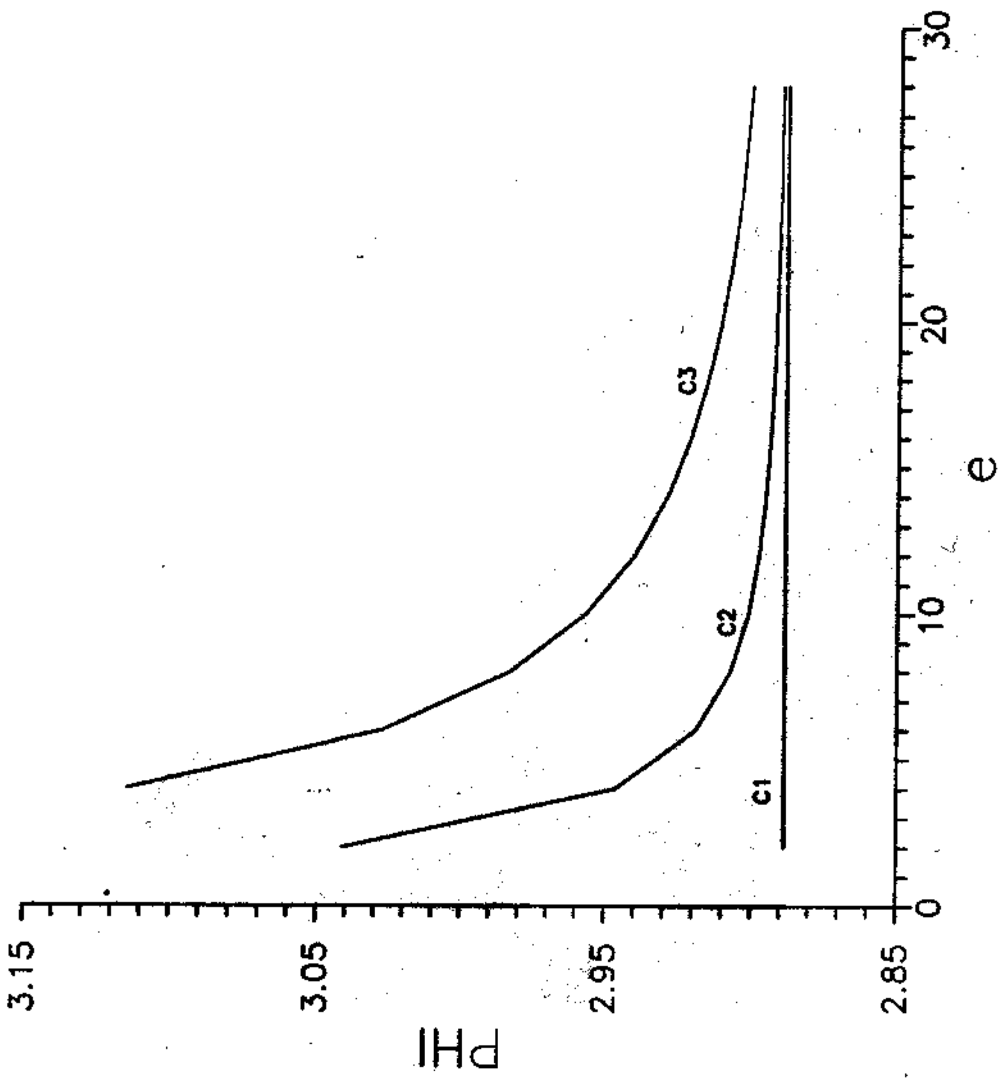

Fig. 1.c

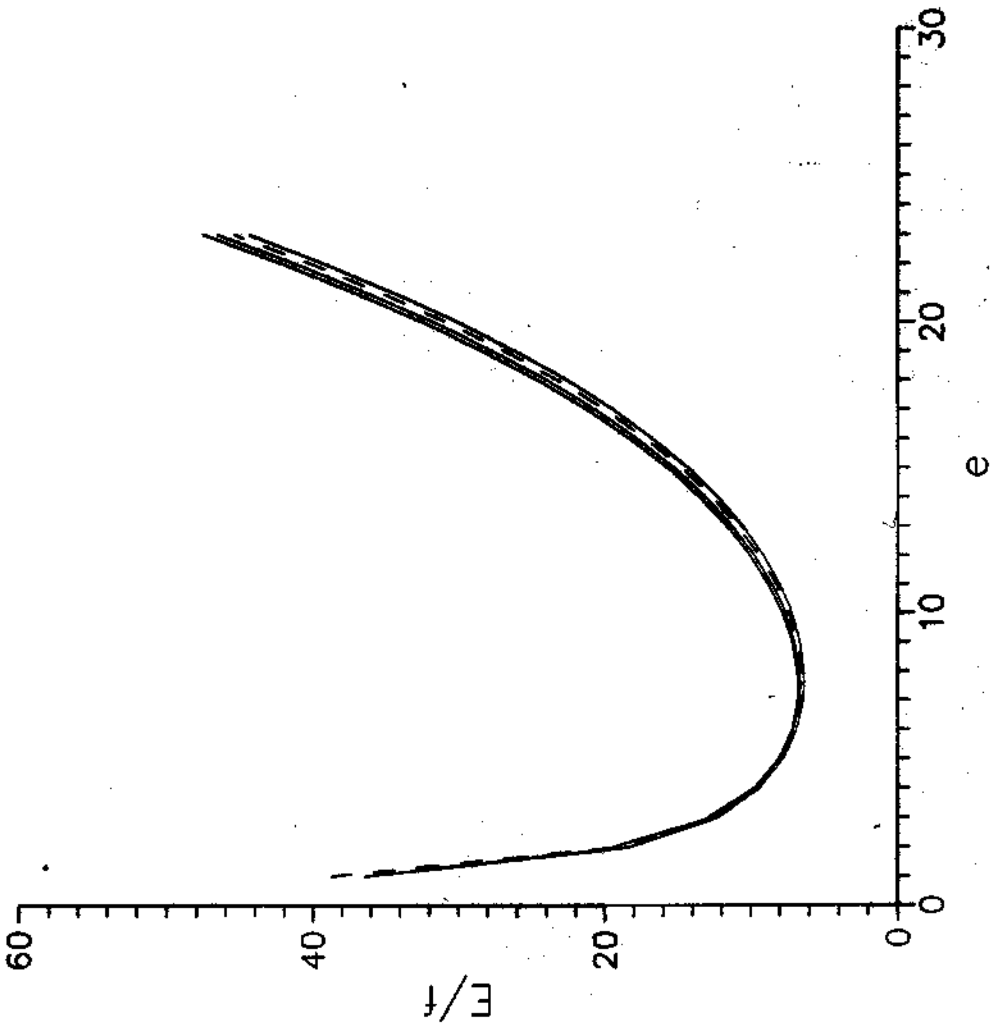

Fig. 2.a

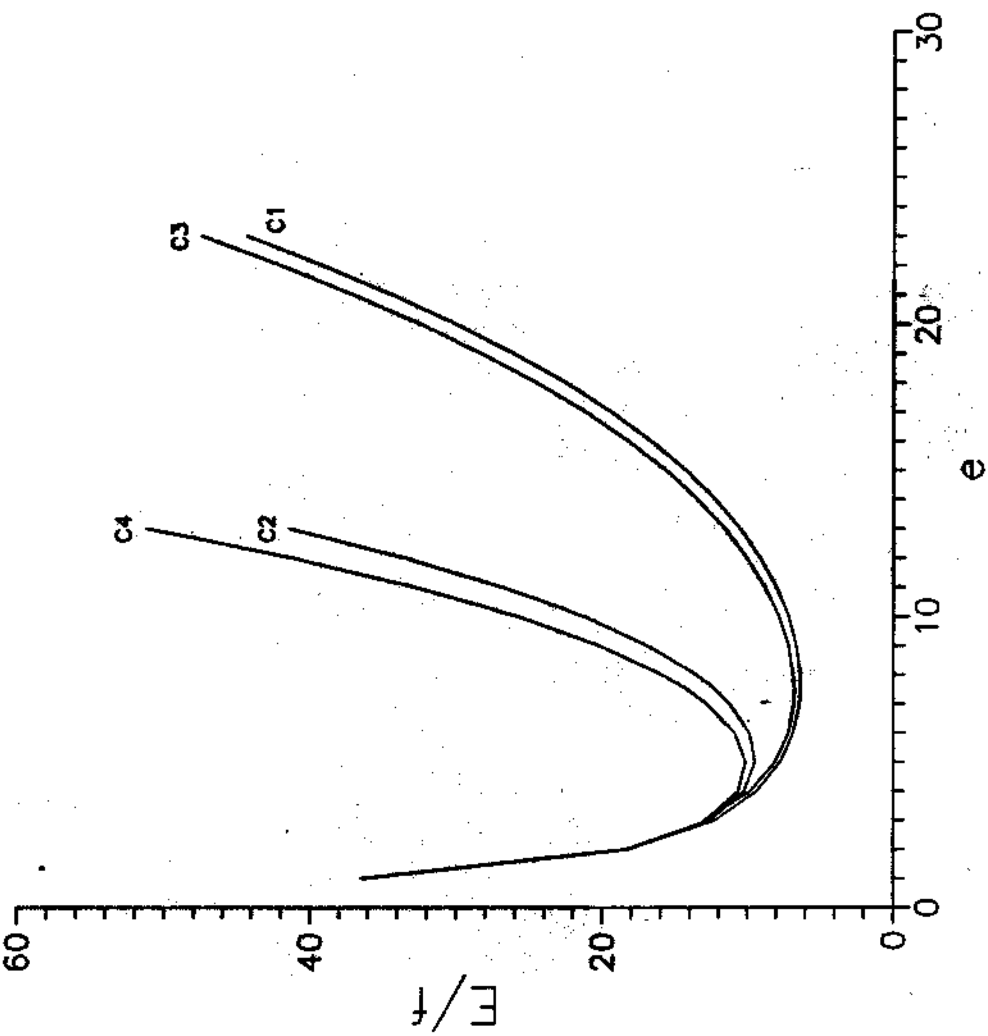

Fig. 2.b

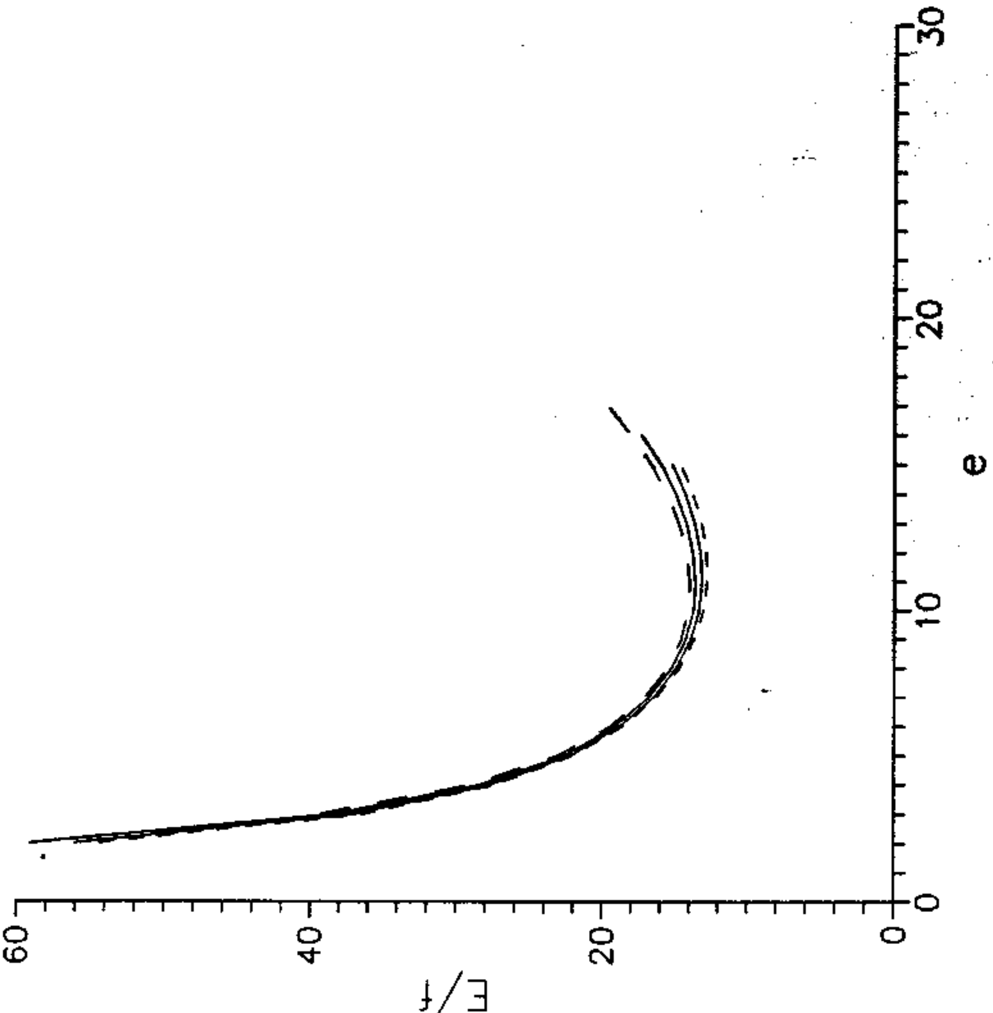

Fig. 2.c

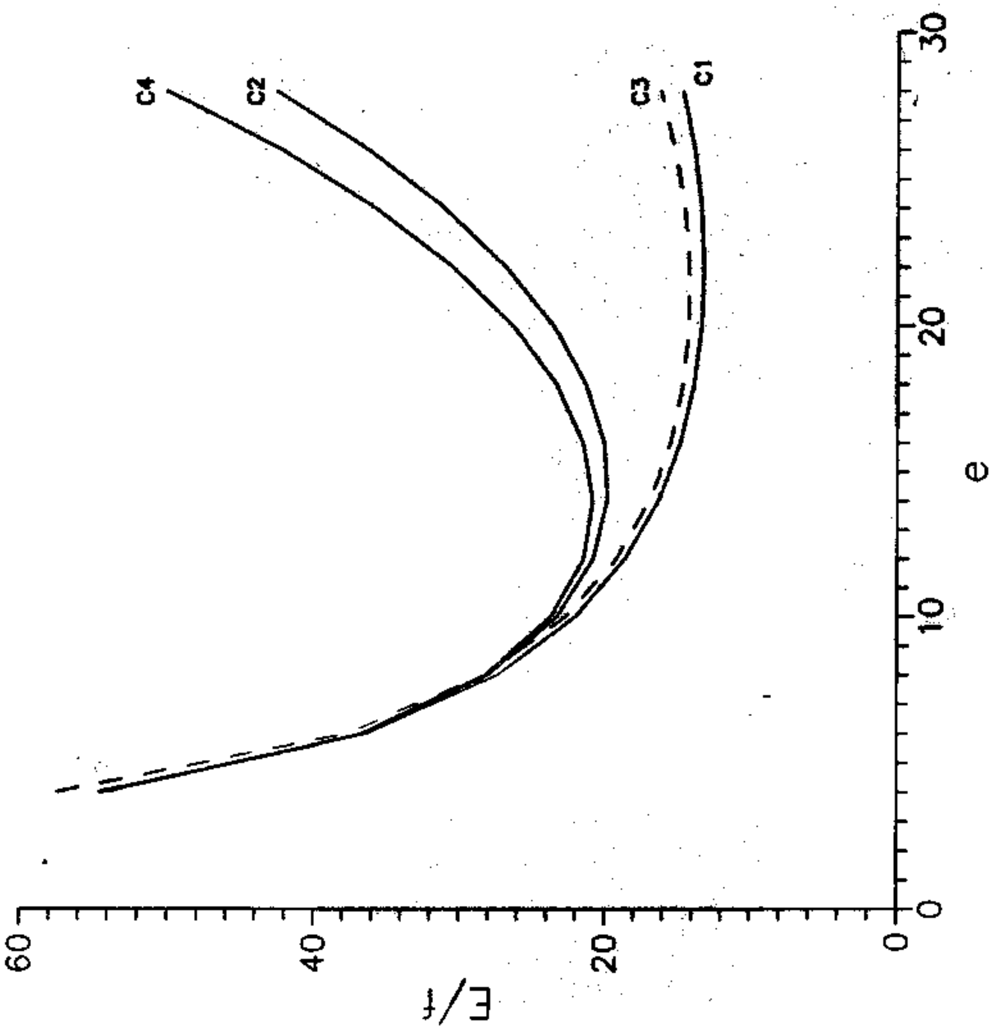

Fig. 2.d

			$f_\pi = 0.186$(GeV)			
m_π(GeV c^{-2})	B	J	e	ϕ	K_2(c^2 GeV^{-2})	M(GeV c^{-2})
0.0	1	1/2	7.67	1.0037...	−17.2771...	1.18
0.05	1	1/2	7.58	1.0080...	−17.1735...	1.21
0.10	1	1/2	7.51	1.0196...	−16.9228...	1.24
0.139	1	1/2	7.45	1.0324...	−16.6735...	1.26
0.0	1	3/2	5.12	1.0037...	−17.2771...	1.76
0.05	1	3/2	5.09	1.0129...	−17.0657...	1.82
0.10	1	3/2	5.06	1.0353...	−16.6194...	1.90
0.139	1	3/2	5.04	1.0583...	−16.2330...	1.96
0.0	2	1	11.33	1.9653...	−51.6729...	2.38
0.05	2	1	11.15	1.9708...	−51.1783...	2.45
0.10	2	1	10.96	1.9853...	−49.9962...	2.53
0.139	2	1	10.82	2.0012...	−48.8500...	2.60
0.0	3	1/2	21.84	2.8886...	−103.8239...	2.47
0.05	3	1/2	21.57	2.8915...	−103.1914...	2.52
0.10	3	1/2	21.26	2.8997...	−101.6226...	2.59
0.139	3	1/2	21.04	2.9090...	−99.9578...	2.65
0.0	3	3/2	14.60	2.8886...	−103.8239...	3.69
0.05	3	3/2	14.35	2.8950...	−102.5250...	3.82
0.10	3	3/2	14.06	2.9118...	−99.4807...	3.98
0.139	3	3/2	13.85	2.9298...	−96.6008...	4.12

Table 1: Results for B=1, 2 and 3 at their minimum quantum energy

Quantity	Ref.[4]	Ref.[2]	This work	This work	Experiment
f_π(GeV)	0.129*	0.108*	0.186	0.186	0.186
M_π(GeV c^{-2})	0.0	0.139	0.0	0.139	0.139
M_N(GeV c^{-2})	input	input	1.18	1.26	0.939
M_Δ(GeV c^{-2})	input	input	2.36	2.62	1.272
M_π(GeV c^{-2})	0.0	input	0	input	0.139
e	5.45*	4.84*	7.67	7.45	– – –
β	0.0	0.264*	0.0	0.10	– – –
$< r^2 >^{1/2}_{E,I=0}$ (fm)	0.59	0.68	0.29	0.29	0.72
$< r^2 >^{1/2}_{E,I=1}$ (fm)	∞	1.05	∞	0.63	0.88
$< r^2 >^{1/2}_{M,I=0}$ (fm)	0.92	0.96	0.45	0.43	0.82
$< r^2 >^{1/2}_{M,I=1}$ (fm)	∞	1.05	∞	0.63	0.80
μ_p	1.87	1.97	0.84	0.87	2.79
μ_n	−1.31	−1.23	−0.16	−0.05	−1.91
$\mid \mu_p/\mu_n \mid$	1.43	1.60	5.32	16.19	1.46
$g_{I=0}$	1.11	1.48	1.36	1.64	1.76
$g_{I=1}$	6.38	6.4	1.99	1.86	9.4
g_A	0.61	0.65	0.31	0.31	1.23
$g_{\pi NN}$	8.9	11.9	3.98	10.26	13.5
$g_{\pi ND}$	13.2	17.9	5.97	15.38	20.3
μ_{ND}	2.3	2.27	0.71	0.66	3.3
F_1(GeV/c)	0.7057	0.5915	1.4319	1.4306	– – –
K_2(c^2 GeV^{-2})	−17.277	−15.810	−17.277	−16.673	– – –

Table 2: Results for the Nucleon Physical Parameters

* Obtained by fitting

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