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ABSTRACT

Breaking down of $(n,\gamma) \rightleftarrows (\gamma,n)$ equilibrium of the r-process nucleosynthesis is studied. It is shown that the semi-freezing stage of r-process is relatively long, suggesting the necessity of more careful calculation of dynamical freezing process to reproduce the correct r-process abundance of elements.

1 - INTRODUCTION

It is widely accepted that the r-process nucleosynthesis is responsible for the origin of heavy nuclei in the universe[1,2].

Since the work of Burbidge et al, many calculations have been done in order to reproduce the observed abundance of r-process elements. Recent progresses in the calculation of supernova explosion enable us to treat the r-process dynamically[3], and in fact some attempts have been made[4,5,6] to couple the nucleosynthesis process to the time evolution of density and temperature of supernova explosion.

It is customy assumed that during the r-process, the nuclear equilibrium between reactions (n,γ) and (γ,n) is attained so that the isotopic abundance of a given Z-family (Z: atomic number) is calculated from the Saha equation. After a certain time interval, when the density and the temperature decrease sufficiently, the r-process is cut off artificially and some after-freezing effects (post-freezing neutron capture, delayed neutrons, etc.) are considered[7,8,9].

Although the freezing time scale is expected to be very small, the process of non-equilibrium phase might alter the resultant atomic abundance. Especially if we follow the time evolution of supernova explosion and couple it to the r-process calculation, it is very important to include the effect of non-equilibrium process.

In this paper, we investigate carefully the applicability of nuclear equilibrium and derive the condition for temperature and density for which the Saha equation begins to fail.

In the following section, we establish the criteria for nuclear equilibrium, then calculate numerically time scales of $(\gamma,n) \rightleftarrows (n,\gamma)$ process and β-decay of all nuclei in concern. Finally we show graphically the region of density and temperature where non-equilibrium process must be considered.

2 - CONDITION FOR NON-EQUILIBRIUM

2.1 - $(\gamma,n) \rightleftarrows (n,\gamma)$ Process Time Scale

Consider a family of isotopes of a given atomic number Z. When the β-decay rate is very small compared to (γ,n) or (n,γ)

reaction rates isotopic abundances $n(Z,N)$ of this family satisfy the equation

$$\frac{d}{dt} n(Z,N_i) = -(\lambda_{\gamma n}^{i} + \lambda_{n\gamma}^{i})\, n(Z,N_i) + \lambda_{\gamma n}^{i+1}\, n(Z,N_{i+1})$$

$$+ \lambda_{n}^{i-1}\, n(Z,N_{i-1}) \qquad (2.1)$$

where $\lambda_{\gamma n}^{i}$ and $\lambda_{n\gamma}^{i}$ denote rates of (γ,n) and (n,γ) reactions of i-th isotope, respectively. They are given by respective cross sections $\sigma_{\gamma n}$ and $\sigma_{n\gamma}$ as

$$\lambda_{\gamma n}^{i} = \frac{8\pi}{(hc)^3} \int_0^\infty c\, \sigma_{\gamma n}^{i}(E)\, E^2\, e^{-E/kT}\, dE$$

$$\lambda_{n\gamma}^{i} = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} n_n \int_0^\infty (v\, \sigma_{n\gamma}^{i}(E))\, v^2\, e^{-E/kT}\, dv \qquad (2.2)$$

with $v = (\frac{2E}{\mu})^{1/2}$, where μ is the reduced mass for (n,γ) reaction, and n_n the neutron density. For the sake of convenience, we use the matrix notation and rewrite eq. (2.1) as

$$\frac{d}{dt} |n\rangle = -\Lambda |n\rangle \qquad (2.3)$$

where $|n\rangle$ is a colum vector whose components are given by $n(Z,N_i)$, and Λ is the matrix of coefficients of the right hand side of eq. (2.1).

This matrix has an eigenvalue zero which corresponds to the conservation of the total number of nuclei $n = \sum_i n(Z,N_i)$. All other eigenvaluesare positive.

The smallest non-zero eigenvalue gives an estimate of the time-scale for attaining equilibrium distribution.

At the equilibrium isotopic abundances satisfy the following relation

$$\frac{n(Z,N_i)}{n(Z,N_{i-1})} = \frac{\lambda^{i-1}_{n\gamma}}{\lambda^{i}_{\gamma n}} \quad . \qquad (2.4)$$

If $\lambda^{i}_{\gamma n}$ and $\lambda^{i-1}_{n\gamma}$ are related by the detailed balance theorem[1,5],

$$\frac{\lambda^{i}_{\gamma n}}{\lambda^{i-1}_{n\gamma}} = \frac{2}{n_n} \left(\frac{h^2}{2\pi\mu kT}\right)^{-3/2} \frac{W(Z,N_{i-1})}{W(Z,N_i)} \; e^{-S_n/kT} \qquad (2.5)$$

with $S_n = B(Z,N_i) - B(Z,N_{i-1})$, $B(Z,N_i)$ the binding energy of isotope (Z,N_i) and $W(Z,N_i)$ the nuclear partition function, equation (2.5) together with eq. (2.4) immediately leads to Saha equation.

In our case eq. (2.5) is applied more widely than the Saha equation since the thermal equilibrium is attained very rapidly. Thus we can estimate the eigenvalue of the matrix Λ if we know all the (n,γ) reaction rates of a given Z-family. However in order to obtain the required value of the time scale, only several nuclei are included in the diagonalization procedure. As we are interested to know the values of temperature and density for which the nuclear equilibrium begins to fail, it is sufficient to include several nuclei around the peak of the equilibrium isotopic distribution.

The time dependent solution of eq. (2.2) is given by

$$|n\rangle = |n_0\rangle + \sum_i |c_i\rangle \, e^{-\lambda_i t} \qquad (2.6)$$

where $|n_o>$ is the vector of equilibrium distribution, λ_i eigen-value of Λ, $|c_i>$ constant vector which depends on the inicial condition.

Now in the dynamical r-process, there are two factors which intervene the equilibrium distribution: the β-decay of nuclei in concern and the change of density and temperature due to hydrodynamical process. In order to judge whether the Saha equation is a good approximation or not, we may compare the time scale of $(\gamma,n) \rightleftarrows (n,\gamma)$ equilibrium, $\tau_{\gamma n}$, with those of nuclear β-decay, τ_β, and the hydrodynamical process, τ_h.

The β-decay time scale is defined as

$$\tau_\beta^{-1} = \frac{1}{<n_o|n_o>} <n_n|\Lambda_\beta|n_o>$$

when Λ_β is the diagonal matrix whose diagonal elements are the β-decay rates of corresponding isotopes.

If $\tau_{eq} << \tau_\beta$ and $\tau_{eq} << \tau_h$, the Saha equation is a good approximation. For the sake of definiteness, we define the non-equilibrium condition as $\tau_{eq} \leq 0.1\ \tau_\beta$ and $\tau_{eq} \leq 0.1\ \tau_h$. For $\tau_{eq} = 0.1\ \tau_\beta$ or $0.1\ \tau_h$, the relative accuracy of Saha equation is expected to be of the order $e^{-10} \simeq 0.005\%$.

2.2 - Reaction and Decay Rates

In order to estimate the (n,γ) reaction rate we adopted the empirical formula of Schramm and Blake[9]. The comparison of the result with experimental data[10] for kT = 30 KeV is reasonable.

For (γ,n) rates, we used the detailed balance formula eq. (2.5), and calculate $\lambda_{\gamma n}$ from $\lambda_{n\gamma}$ of Schramm and Blake's formula. It may asked that why the empirical formula of $\lambda_{\gamma n}$ based on the giant resonance model is not used in spite of the fact that (γ,n) reactions are more extensively studied and well known than (n,γ) reactions. However at the temperature we are interested, the (γ,n) reaction is suppressed compared to (n,γ) reaction by a factor $e^{-S_n/kT}$, where S_n is the neutron separation energy, so that if we calculate the (n,γ) reaction rate by the (γ,n) reaction rate, errors are enhanced by the factor $e^{+S_n/kT}$ which is very large. Furthermore the giant resonance formula is not so accurate at very lower energy than the resonance energy ($\simeq$ 10 $\sim$ 20 MeV).

The detailed balance relation is necessary to reproduce the Saha equation as an equilibrium limit.

For the β-decay rate, we apply the Gross Theory formula[11], which is expected a good approximation for the nuclei far off the β-stability line[8]. The hydrodynamical time scale is taken from Schramm's value[4], τ_h = 0.056 sec.

3 - RESULTS AND DISCUSSION

Fig. 1 shows the temperature for which $\tau_{eq} = 0.1\ \tau_h$ happens (i.e., the beginning of non equilibrium with respect to the hydrodynamical process) as a function of Z. The two curves correspond to different values of neutron density, which is related to the approximate neutron separation energy S_n of the nuclei at the peak of isotopic abundance as[5]

$$S_n = (34.075 - \log n_n + 3/2 \log T_9)T_9/5.04.$$

The graph for $\tau_{eq} = 0.1\ \tau_{\beta}$ is quite similar to this figure.

It is clear from Fig. 1 that while isotopic families whose distribution covers neutron magic number ($Z \sim 28, 49, 78$) are still in equilibrium, other families already begin to break down the equilibrium. The values of temperature and density for which the first isotopic family begins to break the equilibrium are considerably diferent from those for the last family. To see this more explicitly, in Fig. 2 we plotted 3 contors in $\log n_n$ (neutron density) - T_9 diagram. The contor $h_{3/4}$ shows sets of temperatures and neutron density for which 3/4 of all the families are in equilibrium with respect to the non-equilibrium criterion $\tau_{eq} = 0.1\ \tau_h$, on $h_{1/2}$ 1/2 of families are in equilibrium and on $h_{1/4}$ only 1/4 of families are still in equilibrium.

The contors $\beta_{3/4}$, $\beta_{1/2}$ and $\beta_{1/4}$ are defined analogously with respect to the condition $\tau_{eq} = 0.1\ \tau_{\beta}$.

The region between two contors $h_{3/4}$ and $h_{1/4}$ or $\beta_{3/4}$ and $\beta_{1/4}$ may be considered as a region of intermediate equilibrium in the sense that some families are still in equilibrium but others are not. We found that such a intermediate equilibrium is relatively wide so that the final abundance curve may be altered during this semifreezing stage. It is interesting to note that the two different criteria $\tau_{eq} = 0.1\ \tau_h$ and $\tau_{eq} = 0.1\ \tau_{\beta}$ define almost the similar region of intermediate equilibrium.

Precise calculation of this freezing stage of r-process combined to the process of supernova explosion is in progress.

REFERENCES

1 - Burbidge, E.M.; Burbidge, G.R.; Fowler, W.A.; Hoyle, F. - Revs. Modern Phys., 29 (1967), 547.

2 - Seeger, P.A.; Fowler, W.A.; Clayton, D.D. - Astrophys.J. Suppl. nº 97, 11 (1965), 121.

3 - Cameron, A.G.W.; Delano, M.D.; Truran, J.W. - Proc. Int. Conf. on the Properties of Nuclei far from the Region of Beta-Stability (Leysin, 1970, CERN Report 70-30, vol. 2, Pag. 735).

4 - Schramm, D.N. - Astrophys. J., 185(1973), 293.

5 - Sato, K. - Progr. Theor. Phys., 51 (1973) 726.

6 - Hillebrandt, W.; Takahashi, K.; Kodama, T. - Astron. & Astrophys., 52 (1976), 63.

7 - Blake, J.B.; Schramm, D.N. - Astrophys.Lett.,14(1973),207.
Kodama, T.; Takahashi, K. - Phys.Lett., 43B (1973), 167.

8 - Kodama, T.; Takahashi, K. - Nucl. Phys. A239 (1975), 489.

9 - Blake, J.B.; Schramm, D.N. - Astrophys.J. 179 (1973), 569.

10 - Allen, B.J.; Gibbons, J.H.; Macklin, R.L. - Adv. Nucl.Phys., 4 (1971), 205.

11 - Takahashi, K.; Yamada, M. and Kandoh, T. - Atomic Data and Nucl. Data Tables 12 (1973) 101.

FIGURE CAPTIONS

Fig. 1 - Non-equilibrium temperature defined by $\tau_{\gamma n} = 0.1\ \tau_h$ plotted as functions of Z. Two curves correspond to two different values of neutron density, which is indicated by the neutron separation energy S_n.

Fig. 2 - Contor plot of neutron separation energy S_n (dash--dotted curve) in log n_n - T_9 plane. The attached numbers are values of S_n. The intermediate equilibrium region is specified by curves β and h . (See the text).

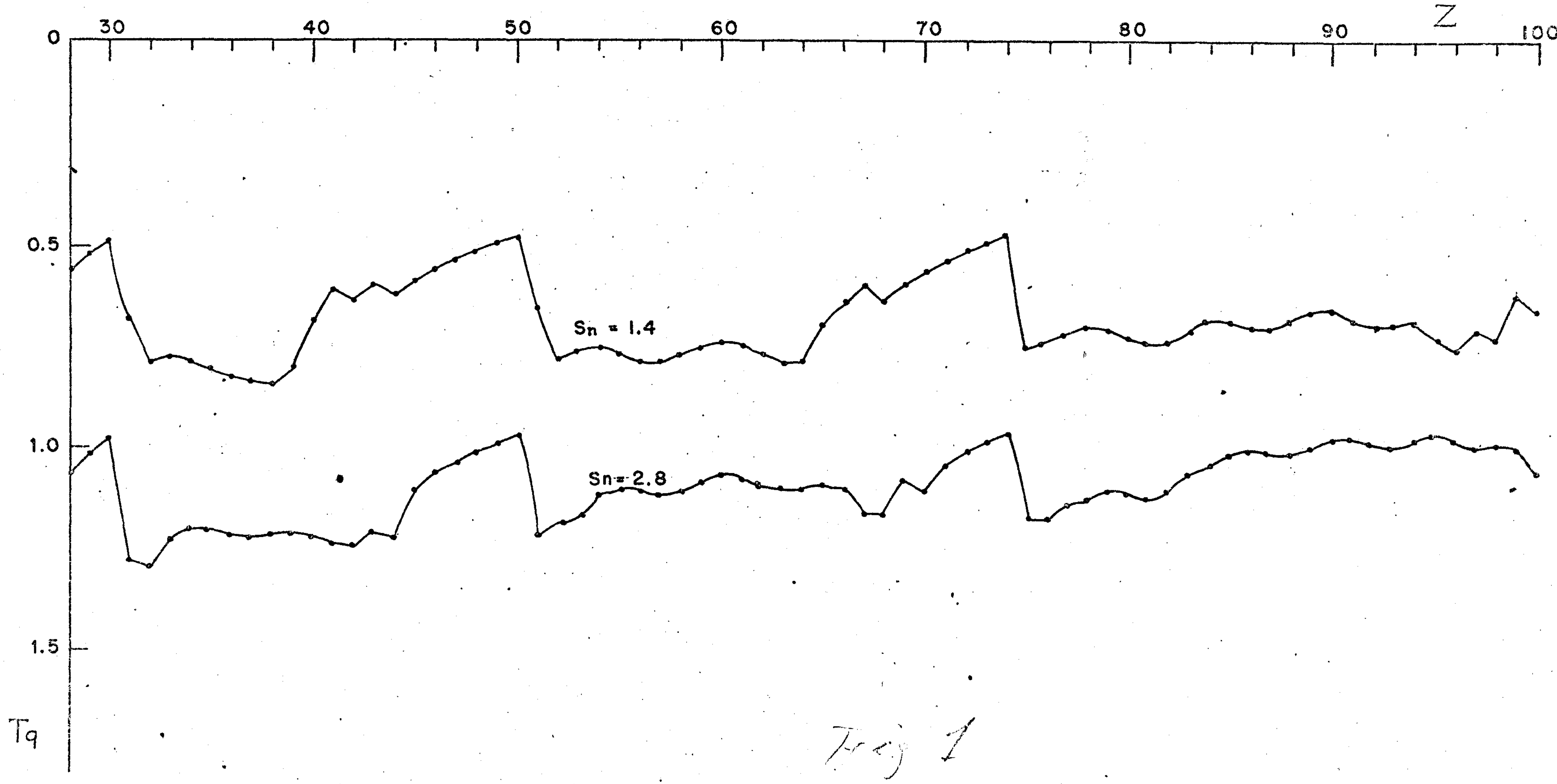

Fig 1

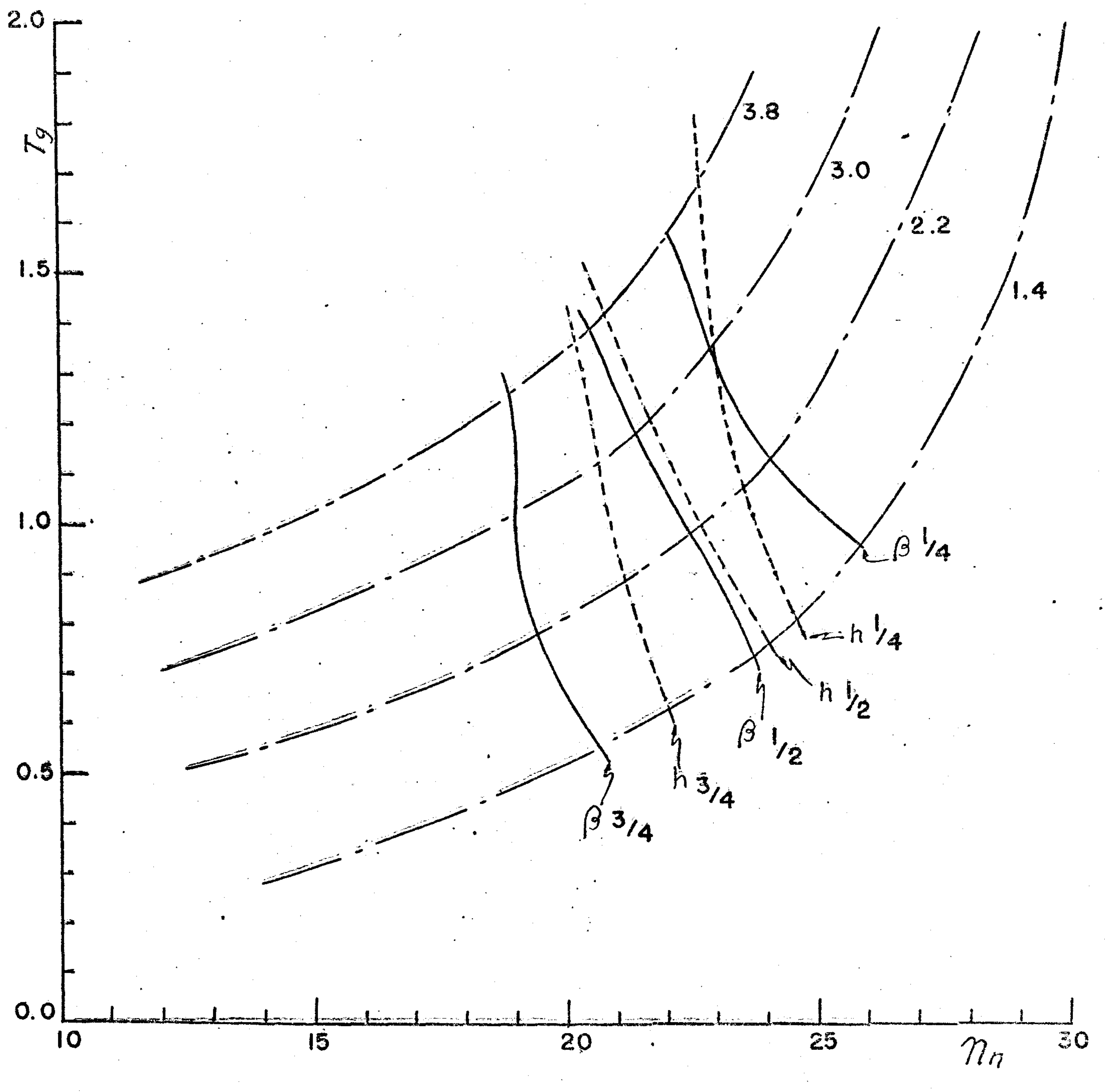
2.0
1.5
1.0
0.5
0.0
T_g
3.8
3.0
2.2
1.4
β 1/4
h 1/4
h 1/2
β 1/2
h 3/4
β 3/4
10
15
20
25
30
n_n

Fig 2