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ASYMPTOTIC SU(3) AND K_{13} DECAY FORM FACTORS

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ABSTRACT

The hypothesis of the validity of asymptotic SU(3) symmetry for high energies and large momentum transfers is applied to the K_{13} and π_{13} decay form factors when some of them may satisfy subtracted dispersion relation.

It has been suggested by Gell-Mann [1] that the SU(3) symmetry, even though badly violated due to observed large mass differences within a multiplet, may still be a good symmetry when high energies and momentum transfers are involved. We study in this paper the application of this idea to the K_{l3} and π_{l3} decay form factors when some of the form factors satisfy (once) subtracted dispersion relations. The results following from this idea are in good agreement with experiments.

The K-π form factors in K_{l3} decay are defined by the matrix element:

$$\sqrt{4k_o p_o V^2}\langle \pi^o(k)|V_\mu(0)^3{}_1|K^-(p)\rangle$$

$$= (p+k)_\mu \; F_+(q^2) + (p-k)_\mu \; F_-(q^2) \qquad (1)$$

where $q^2 = (p-k)^2$ and in the exact SU(3) limit $F_-(q^2) = 0$ while $F_+(0) = 1/\sqrt{2}$. The π_{l3} decay form factor is likewise defined by:

$$\sqrt{4k_o p_o V^2}\langle \pi^o(k)|\; V_\mu(0)^2{}_1|\pi^-(p)\rangle = \; f(q^2)\;(p+k)_\mu \qquad (2)$$

where the conserved vector current hypothesis for $V_\mu{}^2{}_1$ implies $f(0) = \sqrt{2}$.

The hypothesis of the validity of SU(3) for large momentum transfers implies that:

$$2F_+(\infty) - f(\infty) = 0$$

and

$$F_-(\infty) = 0 \qquad (3)$$

if $V_\mu{}^1{}_3$ and $V_\mu{}^1{}_2$ belong to the same octet. Thus F_- would satisfy an unsubtracted dispersion relation while F_+ and f may satisfy a subtracted dispersion relation. In fact, recently, in an attempt to explain the $A_{1-} \longrightarrow \rho + \pi$ decay by current algebra technique and to correlate the decay $A_1 \longrightarrow \rho + \pi$, $\rho \longrightarrow \pi + \pi$ and the $\pi^{\pm} - \pi^{0}$ mass difference, it has been suggested [2] that the pion electromagnetic form factor should satisfy a (once) subtracted dispersion relation. It follows from eq. (3) that [3] the form factor F_+ should also satisfy a subtracted dispersion relation. However, the combination $(2\,F_+ - f)$ satisfies an unsubtracted dispersion relation. We obtain thus the relation:

$$(2F_+(0) - f(0)) = (1/\pi) \int_{4m_\pi^2}^{\infty} dq^2 \; \frac{(2 \operatorname{Im} F_+(q^2) - \operatorname{Im} f\,(q^2))}{q^2} \; . \qquad (4)$$

Now, the experiments at low momentum transfer indicate [4] that the variation of the form factors (with momentum transfer) is very well described by assuming that F_+ is dominated by K^* pole while f by the ρ pole. We may thus calculate the right hand side in eq. (4) in pole dominant approximation to obtain:

$$- \frac{2G_{K*}\; G_{K*^- \pi^0 K^-}}{2\, m_{K*}^{\;2}} + \frac{G_\rho\; G_{\rho^- \pi^0 \pi^-}}{2\, m_\rho^{\;2}} = 2\, F_+(0) - \sqrt{2} \qquad (5)$$

where we define:

$$\sqrt{2k_0 V}\; \langle 0 | V_\mu(0)^3{}_1 | K^{*-}(k) \rangle = G_{K*}\; e_\mu^{\;K*}(k)$$

$$\sqrt{4k_0 p_0 V^2}\, \langle K^-(k) |\; j_{\pi^0}(0) | K^{*-}(p) \rangle = G_{K*^-\pi^0 K^-}\; e^{K*} \cdot k \qquad (6)$$

and analogous expressions for other coupling constants.

We can also derive the following relations for K* couplings, assuming that F_+ is once subtracted and is pole dominated:

$$- \frac{G_{K^*} \, G_{K^{*-} \pi^0 K^-}}{2 m_{K^*}^2} \simeq + \lambda \, \frac{m_{K^*}}{m_\pi}^2 \, F_+(0) \; . \qquad (7)$$

Here λ is the parameter defined by

$$F_+(q^2) = F_+(0)(1 - \lambda \, q^2/m_\pi^2) \; . \qquad (8)$$

It is measured experimentally [4] to be +.023.

We obtain from eqn. (5)

$$- \frac{G_\rho \, G_{\rho^- \pi^0 \pi^-}}{2 m_\rho^2} = \sqrt{2} - 2 \left[1 - \lambda \left(\frac{m_{K^*}}{m_\pi} \right)^2 \right] F_+(0) \; . \qquad (9)$$

Using the relation [5] $G_\rho = \sqrt{2} \, m_\rho \, F_\pi$, we find that corresponding to $\sqrt{2} \, F_+(0) = 0.85$, the value obtained by current algebra calculation [3], decay width for ρ comes out to be 119 MeV to be compared with the experimental value 110 - 140 MeV. The corresponding value for $G_{K^*} \, G_{K^{*-} \pi^0 K^-}/2 \, m_{K^*}^2$ is found to be $\simeq$-0.57.

From Weinberg spectral function sum rules [6] we can derive[7]:

$$(G_{K^*}/G_\rho) \simeq (m_{K^*}/m_\rho) \sqrt{2-(F_K/F_\pi)^2} \; . \qquad (10)$$

The equations (7) and (9) then lead to (for [3] $(F_K/F_\pi)^2 = 1.17$ and $\sqrt{2} \, F_+(0) = 0.85$)

$$G_{\rho^- \pi^0 \pi^-} / G_{K^{*-} \pi^0 K^-} = 2.16 \, (m_\rho/m_{K^*}) \qquad (11)$$

which gives a width of 41 MeV for the K* decay, in good agreement with the experimental value of 49 MeV considering the approximations involved in deriving eqn. (10).

We may apply similar arguments for $F_-(q^2)$ to obtain:

$$\xi(0) = F_-(0)/F_+(0) = (m_K{}^2 - m_\pi{}^2)\lambda/m_\pi{}^2 + F_\varkappa G_{K^-\pi^o\varkappa^-}/F_+(0).m_\varkappa^2 \quad (12)$$

where we define:

$$\sqrt{2k_oV}\langle 0|V_\mu(0)_1^3|\varkappa^-(k)\rangle = i\, F_X\, k_\mu$$

$$\sqrt{4\, k_o p_o V^2}\langle K^-(k)|\, j_{\pi^o}(0)|\varkappa^-(p)\rangle = -i\, G_{K^-\pi^o\varkappa^-} \quad (13)$$

for a scalar isospinor strangeness carrying meson $\varkappa$. We can make an estimate of the kappa life-time from eqn. (10). For the interesting case of $m_K < m_\varkappa < m_K + m_\pi$ we find that with [3] $\sqrt{2}\, F_+(0) \sim .85$, $(F_\varkappa/F_\pi)^2 \sim .34$ and $m_\varkappa$ = 570 MeV the lifetime of kappa for decay into $K + 2\gamma$, calculated in a pole model, is 10^{-11}s. if [(4)] $\xi = -0.5$ and it is 2×10^{-12} s. if the value of ξ is + 0.3. For $m_\varkappa$ = 610 MeV the values in the two cases are 1.5×10^{-13} s. and 2.5×10^{-14}s. respectively.

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