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MULTIPLE SCATTERING MEASUREMENTS IN NUCLEAR EMULSIONS EXPOSED TO MOMENTUM ANALYSED PARTICLE BEAMS FROM THE CERN PROTON SYNCHROTRON

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MULTIPLE SCATTERING MEASUREMENTS IN NUCLEAR EMULSIONS EXPOSED TO MOMENTUM ANALYSED PARTICLE BEAMS FROM THE CERN PROTON SYNCHROTRON*

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Abstract. Results of multiple Coulomb scattering measurements on the tracks of high momentum particles in nuclear emulsion are described. It is shown that the method can be used to give reliable momentum estimates, for particles whose tracks are essentially flat in the emulsion, up to 25 GeV/c. Possible deviations from theoretical multiple Coulomb scattering expectations for high cell-sizes are presented.

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1. INTRODUCTION

The aim of the work was to study the reliability of the multiple scattering method for estimating values of the momentum p, of high energy particles in nuclear emulsions. One stack of Ilford G 5 pellicles of size 29 × 14 cm × 600μ was exposed to a negative pion beam of momentum 16.2 ± 0.6 GeV/c, while a second stack of size 19 × 9.5 cm × 500μ was exposed to a proton beam of momentum 24.0 ± 0.4 GeV/c. Great care was taken to align the stacks so that the pellicles were accurately horizontal at the time of exposure. In this way the average track length of the primary particles in both stacks was in excess of 15 cms. The microscope used for the measurements was a Koristka type R 4 which has an X movement of 10 cms. This feature of the microscope enabled us to take measurements over this length on every particle track that was studied. The emulsions were processed by the usual CERN method[1].

The specific objects of study were to investigate:

(a) the effect of different cut-off procedures and scattering constants on the values of the momentum obtained,

(b) the magnitude of spurious scattering effects in the two emulsion stacks, and

(c) scattering measurements in the region of very high cell-sizes (from 12 to 24 mm), where the ratio of signal to noise varied from 15 to 40.

2. EXPERIMENTAL PROCEDURE

A singly charged particle of momentum p and velocity β, undergoes multiple Coulomb scattering in a condensed material. It is convenient to express the magnitude of the scattering in terms of the average second difference $\bar{D}_c$ of y co-ordinate readings taken at regular intervals, t, along the direction of motion of the particle (usually taken as the x axis). The values of $p\beta$ and $\bar{D}_c$ are related by the formula

$$p\beta = \frac{K\, t^{3/2}}{\bar{D}_c \times 0.57 \times 10^3} \qquad (1)$$

where K is the scattering constant for the medium in question, the cell-size t is in units of 100 microns, $\bar{D}_c$ is in microns and $p\beta$ is in GeV/c.

Now $\bar{D}_c$ is the average second difference due to Coulomb scattering only. In practice, the measured second difference, $\bar{D}_{meas}$ contains contributions from effects other than those of Coulomb scattering. We have taken the relation suggested by Dahl-Jensen *et al*[2], viz

$$\bar{D}^2_{meas} = \bar{D}^2_c + \bar{D}^2_\varepsilon + \bar{D}^2_\gamma + \bar{D}^2_{ss} \qquad (2)$$

where:

$\bar{D}_{meas}$ = arithmetic mean of the absolute values of measured second differences;

$\bar{D}_c$ = mean second difference due to multiple Coulomb scattering, obtained from the known momentum of the incident particles;

$\bar{D}_\varepsilon$ = mean second difference due to reading, grain noise and irreproducible part of stage noise;

$\bar{D}_{\gamma}$ = mean second difference due to reproducible part of stage noise (stage profile)*;

$\bar{D}_{ss}$ = mean second difference due to spurious scattering.

We also define:

$$\bar{\delta} = \sqrt{\bar{D}_{\varepsilon}^2 + \bar{D}_{\gamma}^2} = \text{total noise} \qquad (3)$$

and

$$\bar{D} = \sqrt{\bar{D}_{meas}^2 - \bar{\delta}^2} = \sqrt{\bar{D}_c^2 + \bar{D}_{as}^2} \;. \qquad (4)$$

The majority of the experimental results presented in this paper will be given in terms of $\bar{D}$. The statistical errors are given as $\bar{D}/\sqrt{n}$ or $p\beta/\sqrt{n}$, respectively, where n is the number of (independent) measured second differences. To obtain values of momentum, we have used equation (1), with $\bar{D}_c$ replaced by $\bar{D}$ (see equation 4 above), and where $\beta = 1$ for the range of momenta we have investigated.

The tracks were aligned so that they stayed within the eye-piece scale (50μ) over the entire 10 cm movement of the stage. Out of 64 tracks taken in the stack exposed to pions only 3 had to be rejected because they did not fulfil this condition; none of the tracks in the stack exposed in the proton beam had to be rejected. Each track was measured over a distance of 10 cm in a central region of each plate using a Leitz filar eye-piece (12.5 x) and a Koristka objective x55. The filar movement was connected to an automatic device for

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* We have assumed that the individual values of D_{γ} follow a gaussian distribution. This assumption, however, cannot influence our results on account of the high signal to noise ratio.

calculating second differences*. The co-ordinate method of Fowler[3] was employed. In order to obtain a high signal to noise ratio (>4), measurements were taken using basic non-overlapping cell-sizes, t, of 4 and 6 mm.

3. RESULTS

(a) Noise

We have measured the total noise $\bar{\delta}$ using 100μ cell-size across the whole stage movement. It was found to be ~0.2μ and its dependence on cell-size is shown in Fig. 1. We have also measured the total noise with the Bøggild and Scharff[4] diamond line method. We obtained again the value 0.2μ, which shows that reading noise is small in relation to stage plus grain noise. The total noise is negligible in comparison with the signal for the range of cell-sizes employed.

(b) Cut-off procedures (Pion stack)

In order to study the effect of different cut-off procedures and of the choice of scattering constant, we applied to experimental data for t = 4 mm three different cut-off procedures, with the corresponding three different scattering constants:

i) No cut-off, scattering constant K (for $\beta = 1$) given by reference[5] and Fig. 2 of this paper.

ii) Cut-off at $4\bar{D}_{meas}$ with replacement, K values 0.8 units less than

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* This device was designed by Dr. W. M. Gibson and built by Mr. M. A. Roberts in this laboratory.

those of Fig. 2 (according to the recommendations of reference[5]).

iii) Cut-off at $4\bar{D}_{meas}$ without replacement, as given by Voyvodic and Pickup[6].

The distribution of 1454 values of D_{meas} for t = 4 mm is plotted in Figs. 3a and 3b, which show that the distribution is effectively gaussian. The values of $\bar{D}$ and of $p\beta$ obtained are given in Table I. It can be seen that the three procedures give the same results, within statistical errors, as is expected for a large sample. For the results presented in the remainder of this paper we have used the no cut-off procedure. The values of $p\beta$ in Table I are lower than the known beam momentum, presumably due to the presence of spurious scattering.

(c) Presence of Spurious Scattering

The expected magnitude of $\bar{D}_c$ can be calculated from equation (1). Thus any difference between $\bar{D}_{meas}$ and $\bar{D}_c$ at 4 mm cell can be attributed to $\bar{D}_{ss}$, using equation (4).

For the pion stack we find $\bar{D}_{ss} = 0.54 \pm 0.05\mu$ for t = 4 mm and $\bar{D}_{ss} = 0 \pm 0.3\mu$ for t = 8 mm. The average cotangent of the dip angle for the tracks measured was 600.

For the proton stack $\bar{D}_{ss} = 0.23 \pm 0.1\mu$ for t = 4 mm, $\bar{D}_{ss} =$ $= 0.185 \pm 0.35\mu$ for t = 6 mm and $\bar{D}_{ss} = 0.37 \pm 0.6\mu$ for t = 8 mm. For cell-sizes of 6 mm and up to 1 cm the signal obtained is much greater than $\bar{D}_{ss}$ so that a good estimate of the momentum of the track measured is obtained.

(d) Variation of $\bar{D}$ with cell-size

As we have a large sample of values of D_{meas} we have investigated the variation of $\bar{D}$ with cell-size. For the pion stack the results are summarized in Fig. 4. In the figure the full line shows the theoretical variation of $\bar{D}_c$ with t, while the dotted line shows the effect of correcting this curve for the finite size of nucleus[6]. The experimental points seem to fall systematically below the theoretical curves for $t \geqslant 1$ cm. The same effect was found for the tracks in the proton stack as is shown in Fig. 5. In both cases the distribution of the values of D_{meas} is gaussian (Fig. 6 and 7).

If further investigation should confirm the result mentioned in this paragraph, some modification of the statistical theory of multiple Coulomb scattering for high cell-sizes ($t \geqslant 1$ cm) may be necessary.

4. CONCLUSIONS

1) Spurious scattering effects in the two emulsion stacks investigated are small enough so that for cell sizes of 6 mm and greater ($p\beta \geqslant 16$ GeV/c) the contribution to $\bar{D}_{meas}$ from the true multiple Coulomb scattering is much greater than that from spurious scattering and from noise.

2) For large samples of D_{meas}, the three usual alternatives for cut-off procedure are equivalent.

3) Our experimental data suggest that in the region of cell-

sizes greater than 1 cm $\bar{D}$ is smaller than expeted theoretically, even with the correction due to the finite size of the nucleus.

* * *

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5) C. F. Powell, P. H. Fowler, D. H. Perkins, "The Study of Elementary Particles by the Photographic Method", Pergamon Press London, 1959; p. 114.

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* * *

TABLE I

Values of $p\beta$ for pions of 16.2 GeV/c momentum, calculated from measurements taken at 4 mm cell length using three different cut--off procedures, together with the appropriate scattering constants.

	No cut-off	Cut-off $4\bar{D}_{meas}$ with replacement	Cut-off $4\bar{D}_{meas}$ without replacement
$\bar{D}$ (μ)	0.99 ± 0.03	0.98 ± 0.03	0.96 ± 0.03
K used	30.5	29.7	29.0
$p\beta$ (GeV/c)	13.6 ± 0.4	13.4 ± 0.4	13.3 ± 0.4

CAPTIONS TO FIGURES

Fig. 1: Variation of total noise, $\bar{\delta}$, with cell-size for CERN Koristka R4 microscope.

Fig. 2: Theoretical curve of K (without cut-off) vs. cell-size[5].

Fig. 3a: Distribution in the values of D_{meas} for cell-size of 4 mm, π^- beam tracks.

Fig. 3b: Integral plot of $1/N_o \int_{-\infty}^{D_{meas}} N dD_{meas}$ against D_{meas} (4 mm cell) on a Gaussian paper: π^- beam tracks.

Fig. 4: Variation of $\bar{D}$ with cell size: π^- meson tracks. Theoretical curve 1 for $p\beta$ = 16.2 GeV/c, using values of scattering constant K according to reference[5]. Curve 2 takes into account the finite size of nucleus as given by[6].

Fig. 5: Variation of $\bar{D}$ with cell-size, proton tracks. Experimental points:

 for 4 mm basic cell-size,

 for 6 mm basic cell-size.

Theoretical curves 1, 2 show $\bar{D}_c$ (calculated for $p\beta$ = = 24 GeV/c) similarly as in Fig. 4.

Fig. 6: Integral plot of $1/N_o \int_{-\infty}^{D_{meas}} N dD_{meas}$ against D_{meas} for 4 mm cell-size; proton beam tracks.

Fig. 7: Integral plot of $1/N_o \int_{-\infty}^{D_{meas}} N dD_{meas}$ against D_{meas} for 20 mm cell-size; π^- beam tracks.

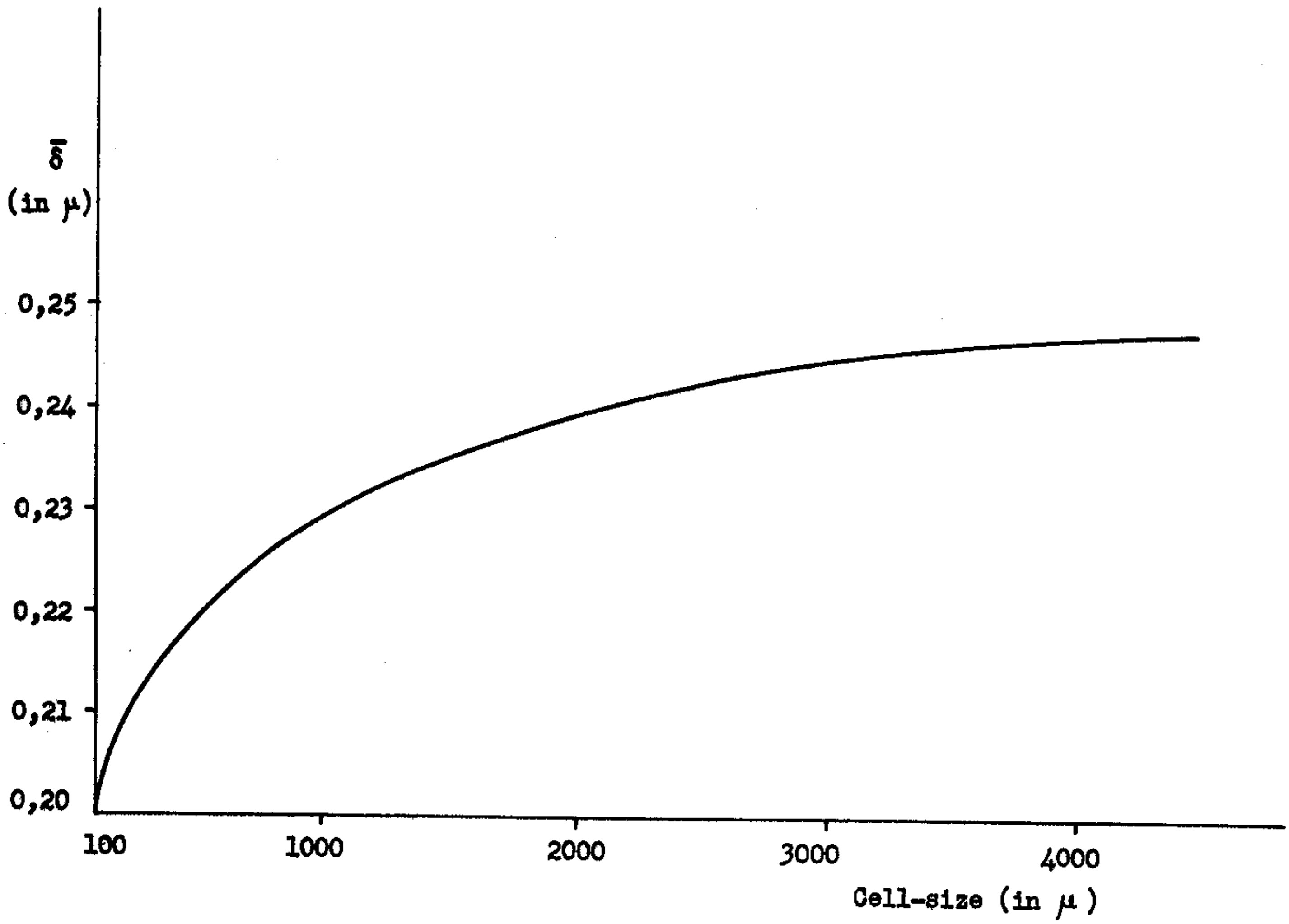

δ̄
(in μ)
0,25
0,24
0,23
0,22
0,21
0,20
100
1000
2000
3000
4000
Cell-size (in μ)

Fig. 1

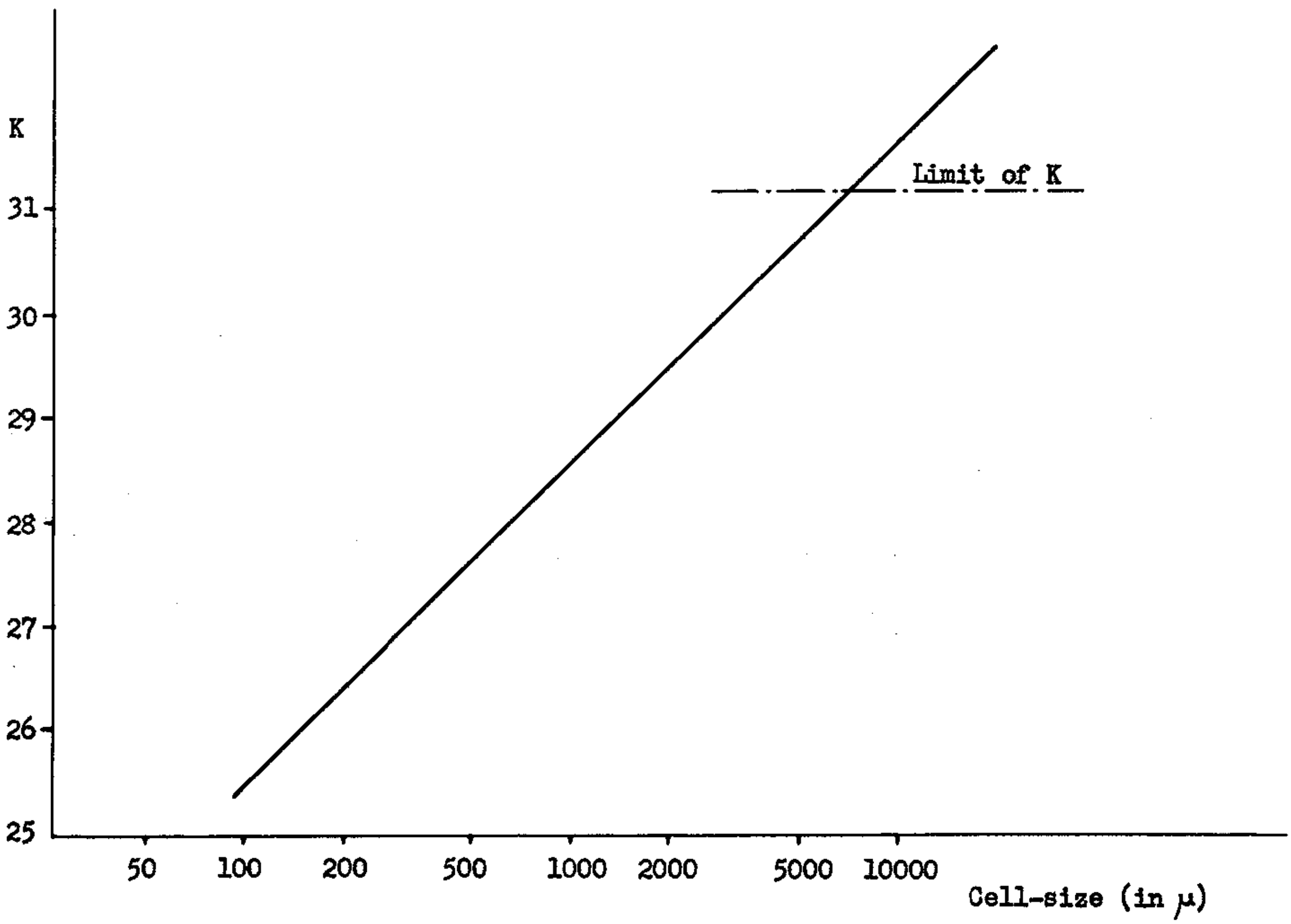
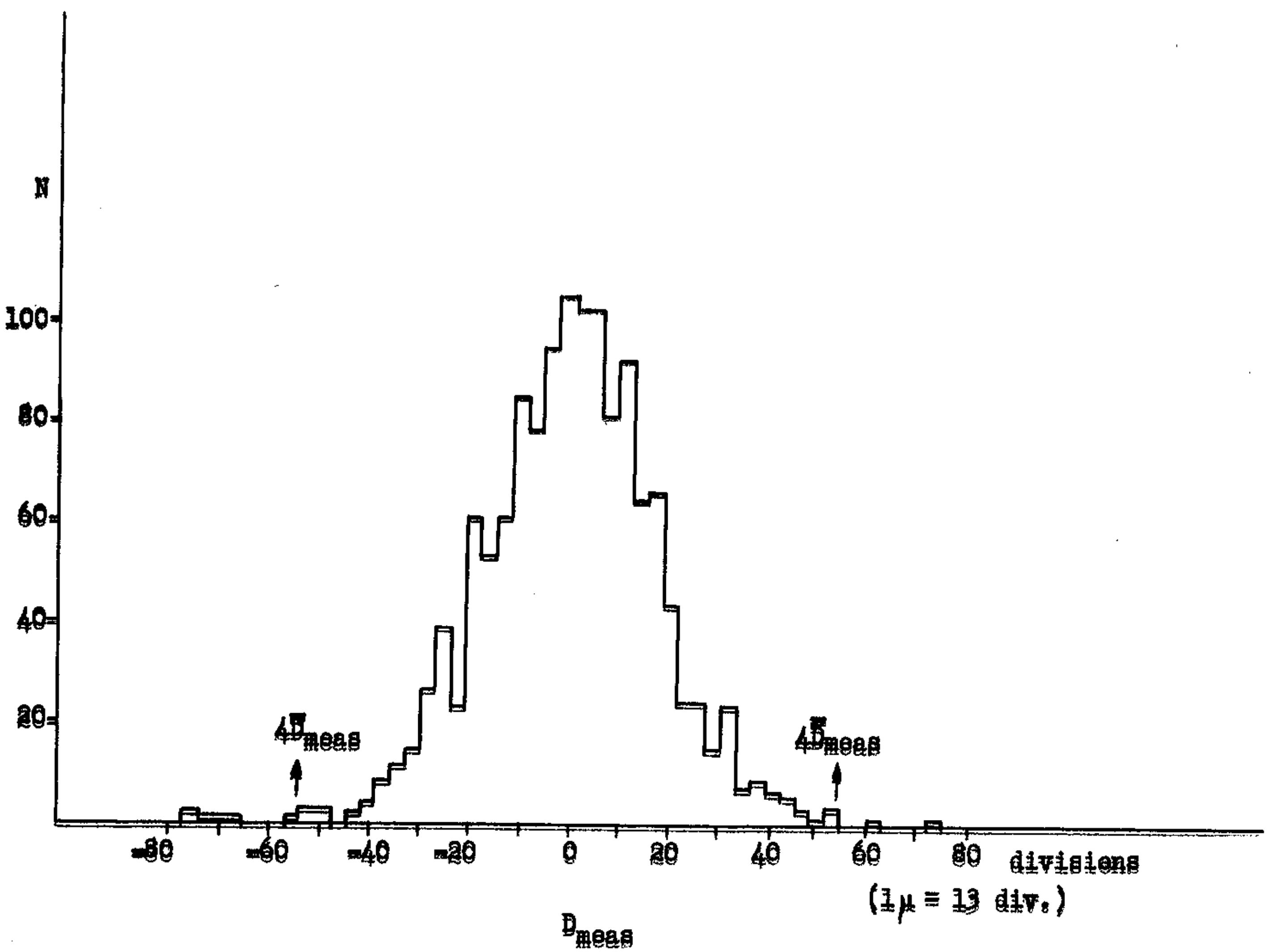

K
31
30
29
28
27
26
25
Limit of K
50
100
200
500
1000
2000
5000
10000
Cell-size (in μ)

Fig. 2



N
100
80
60
40
20
$4\bar{D}_{meas}$
$4\bar{D}_{meas}$
-80
-60
-40
-20
0
20
40
60
80
divisions
(1μ = 13 div.)
D_{meas}

Fig. 3a.

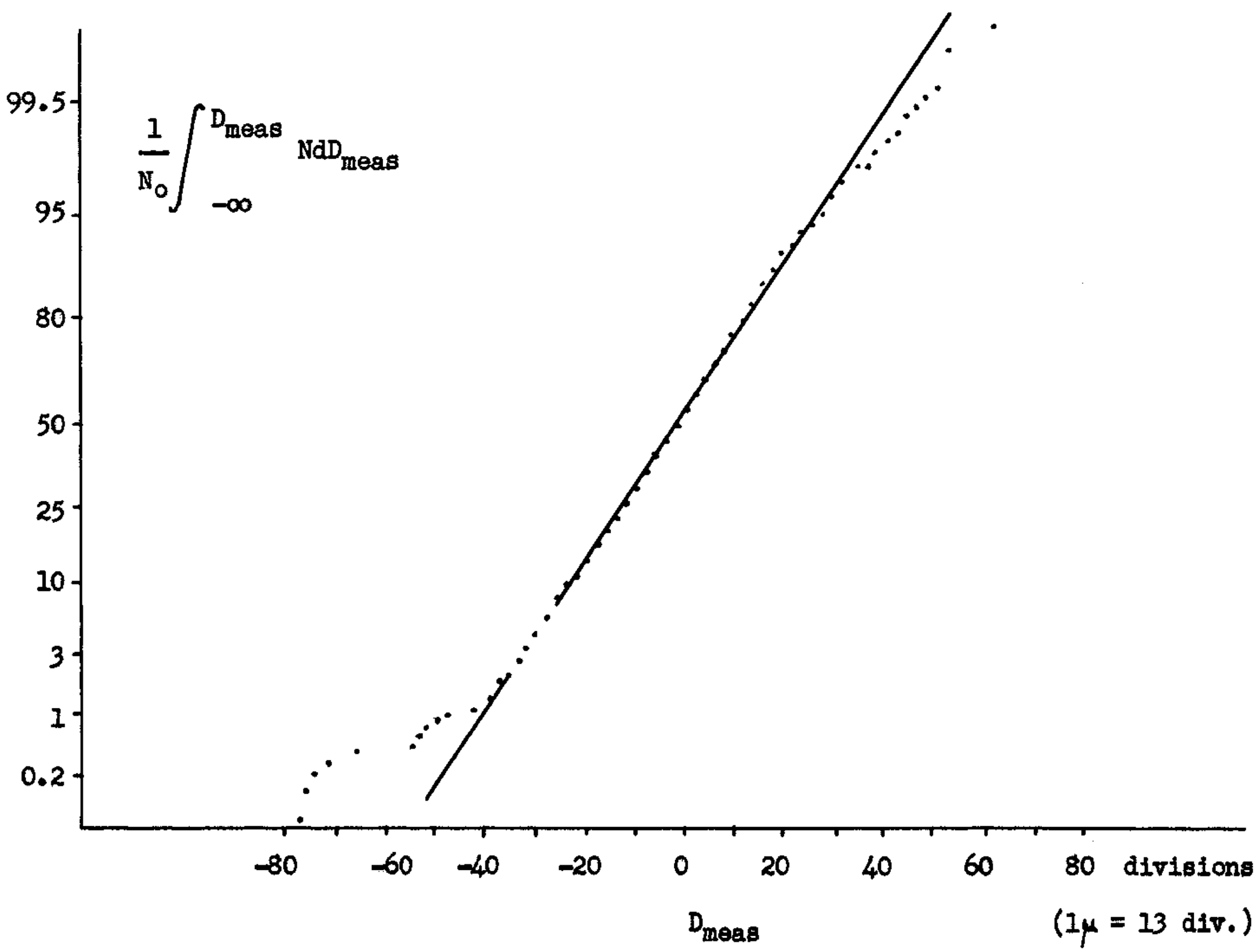
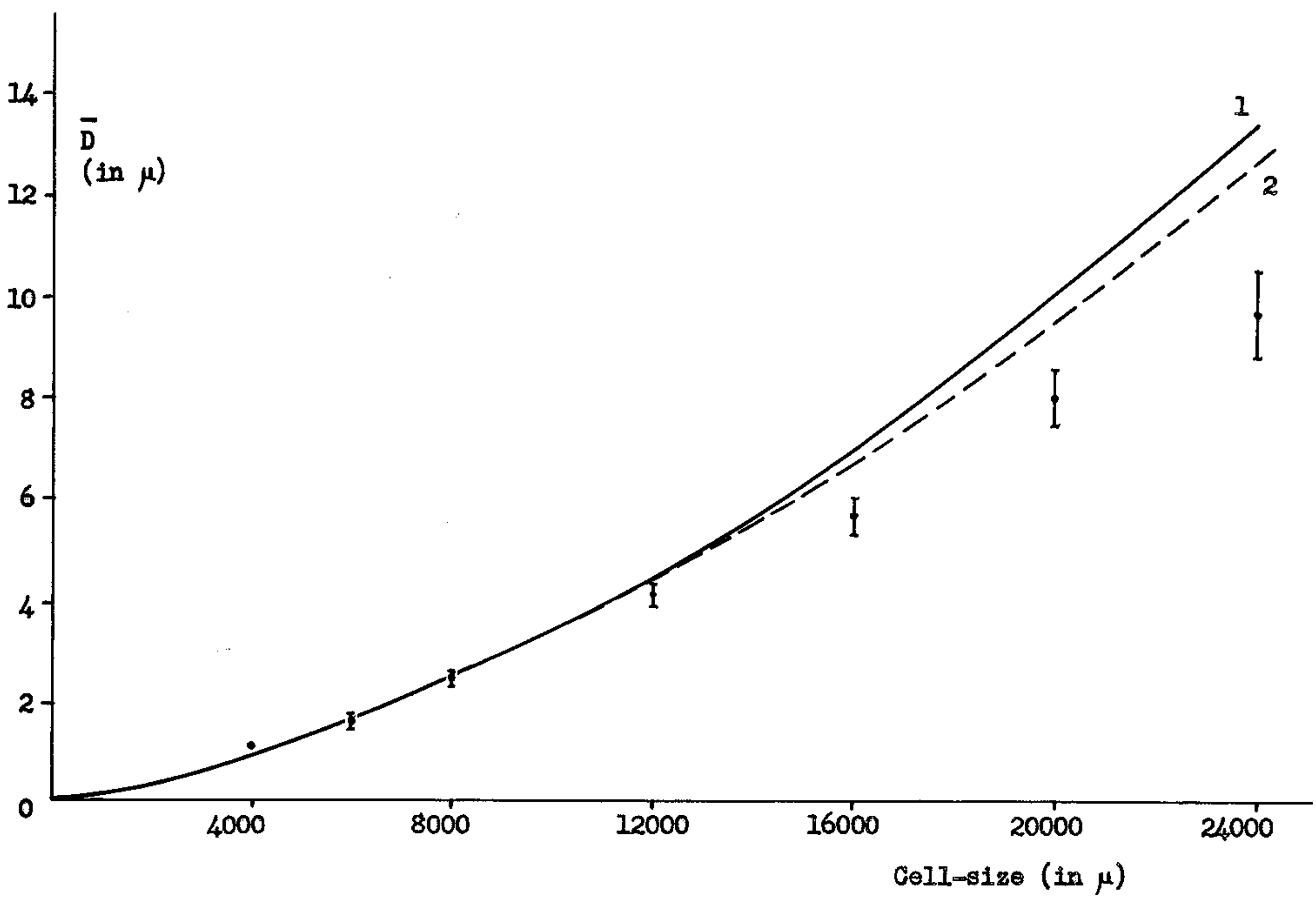
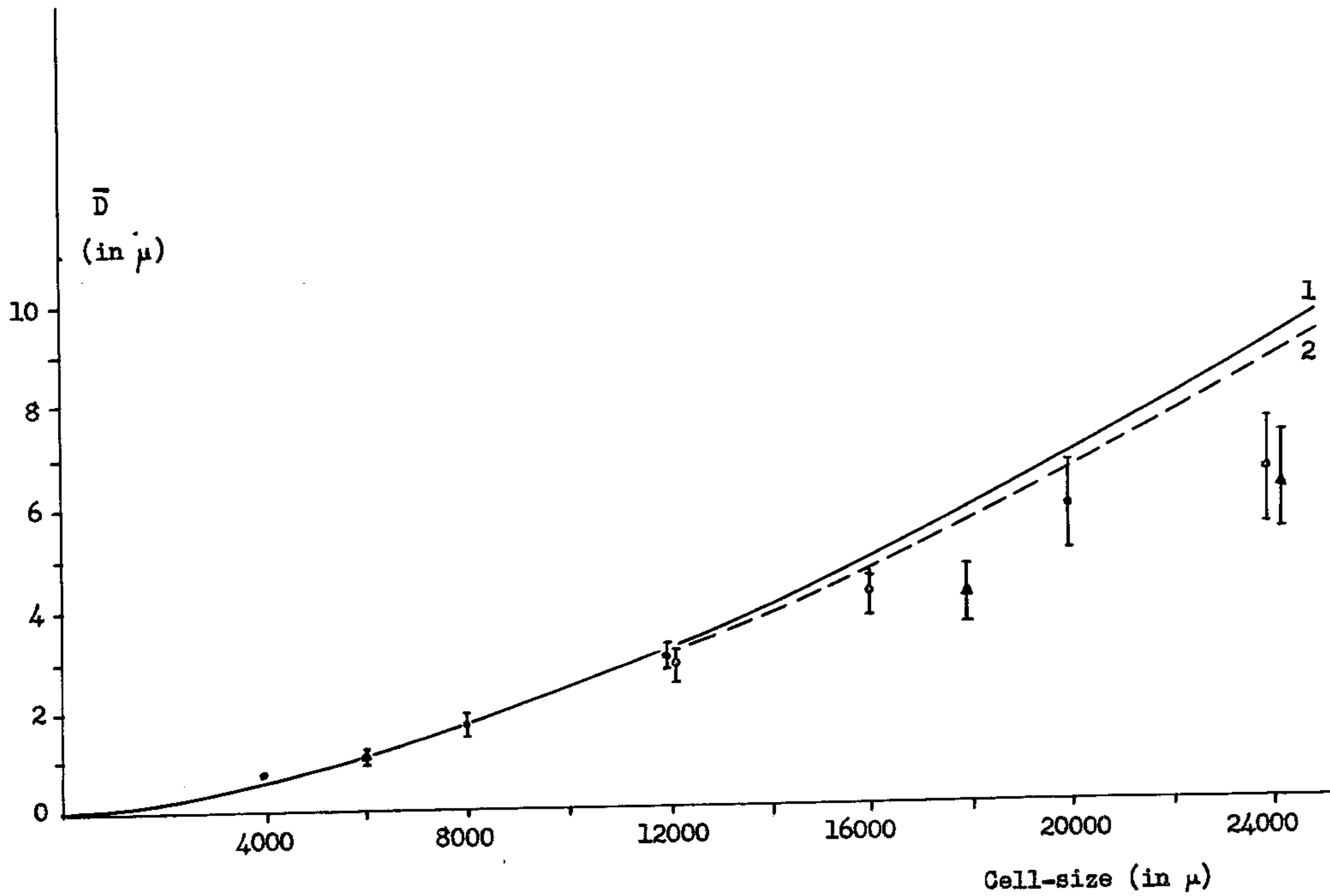

99.5
95
80
50
25
10
3
1
0.2
$\frac{1}{N_o}\int_{-\infty}^{D_{meas}} N dD_{meas}$
-80
-60
-40
-20
0
20
40
60
80 divisions
D_{meas}
(1μ = 13 div.)

Fig. 3b.



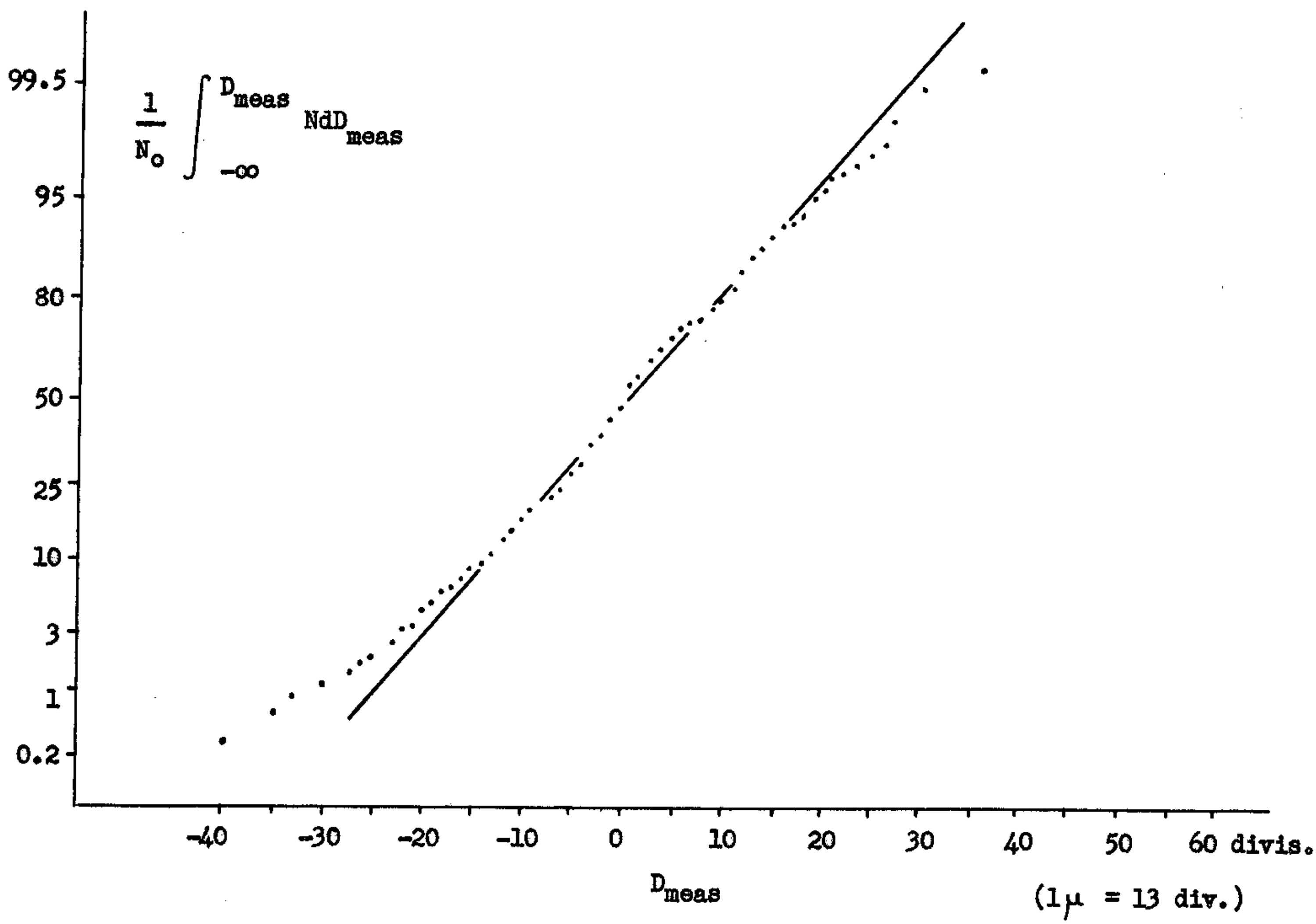
14
12
10
8
6
4
2
0
D
(in μ)
1
2
4000
8000
12000
16000
20000
24000
Cell-size (in μ)

Fig. 4



$\overline{D}$
(in μ)
10
8
6
4
2
0
4000
8000
12000
16000
20000
24000
Cell-size (in μ)
1
2

Fig. 5



99.5
95
80
50
25
10
3
1
0.2
$\frac{1}{N_o}\int_{-\infty}^{D_{meas}} N dD_{meas}$
-40
-30
-20
-10
0
10
20
30
40
50
60 divis.
D_{meas}
(1μ = 13 div.)

Fig. 6

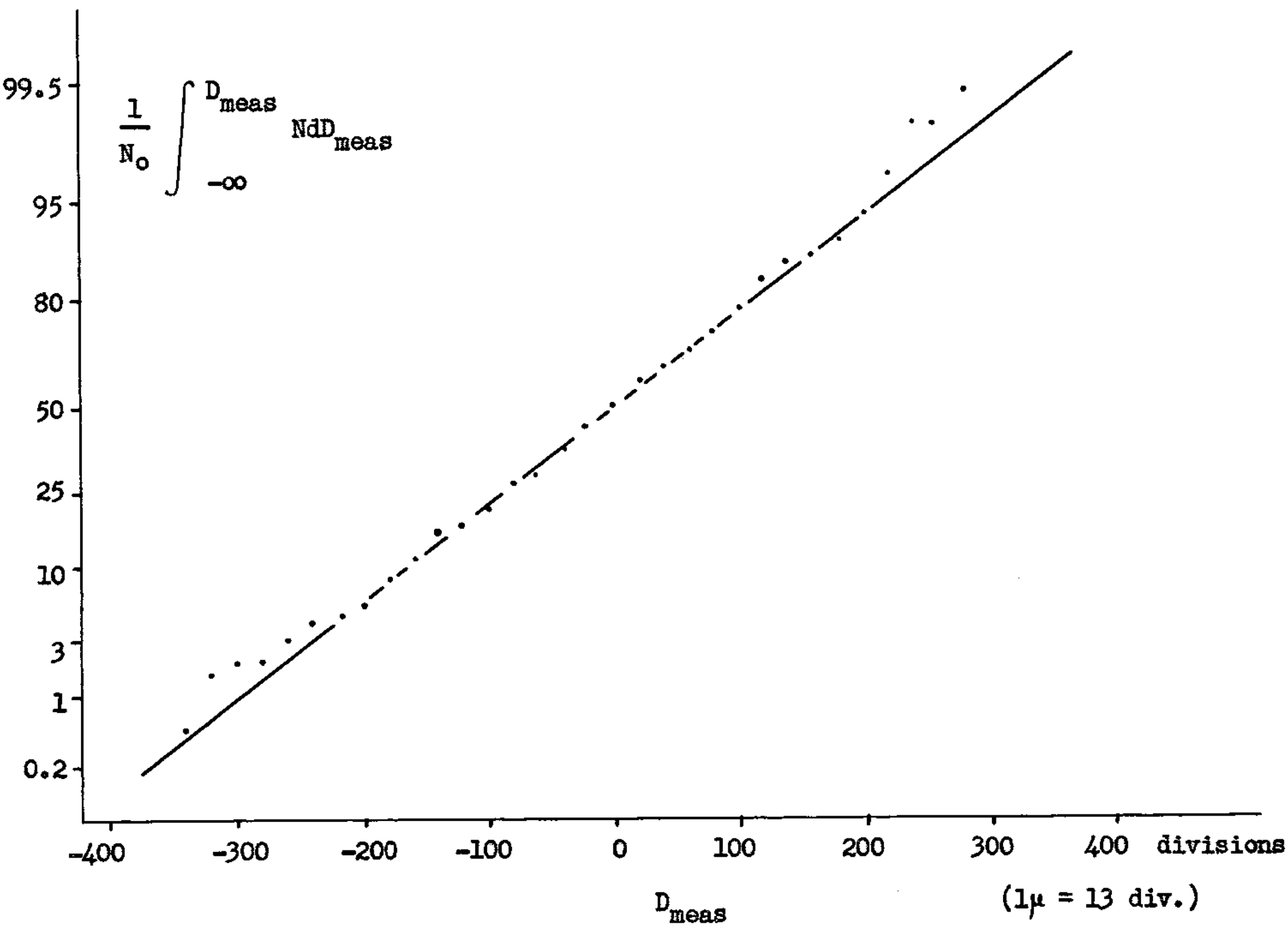

99.5
95
80
50
25
10
3
1
0.2
$\frac{1}{N_o}\int_{-\infty}^{D_{meas}} N dD_{meas}$
-400
-300
-200
-100
0
100
200
300
400 divisions
D_{meas}
(1μ = 13 div.)

Fig. 7