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By

S. W. MACDOWELL

CENTRO BRASILEIRO DE PESQUISAS FISICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

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S. W. MACDOWELL§

Centro Brasileiro de Pesquisas Físicas

Rio de Janeiro, D.F.

ABSTRACT

The relativistic density matrix formalism developed by Stapp is used to calculate the polarization vector of muons from $K_{\mu 3}$-decay and the depolarizing effects due to high energy atomic collisions.

I. The polarization vector of muons produced in $K_{\mu 3}$-decay is derived in a straighforward way using the relativistic density matrix formalism developed by Stapp[1]. The relativistic density matrix in momentum space may be written in the form:

$$\rho(p) = \frac{1 \mp \gamma_5 \not{a}}{2} \Lambda_{\pm} \gamma_0 \lambda(p), \tag{1}$$

where $\Lambda_{\pm} = (\not{p} \pm m)/2m$ is the projection operator for particle and anti-particle states respectively and a_μ is a four-vector such that $p.a = 0$ and $0 \leqslant -a^2 \leqslant$. The probability of finding particles with momentum within the interval $(\underline{p}, \underline{p} + d\,\underline{p})$ is $(m/E)\lambda(p)\, d^3 p$. The expectation value of an operator $O(p)$ is given by:

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(§) Now at the Department of Mathematical Physics, University of Birmingham, England.

$$\langle O \rangle = \int \mathrm{Tr.}\ (\rho\, O)\ \frac{m}{E}\, d^3 p. \tag{2}$$

The "polarization vector" for particles of given momentum is the expectation value of the spin-matrix $\underline{\sigma}$ in the centre of mass system. One can see that:

$$\underline{P} = \underline{a} - a_o\, \underline{p}\, /(E + m). \tag{3}$$

For the decay process $K^{\pm}_{\mu 3} \longrightarrow \mu^{\pm} + \nu + \pi^{o}$, assuming a primary Fermi interaction[2], one has:

$$\rho\ (p)\ \frac{m}{E}\, d^3 p = \Lambda_{\pm}\ \bar{Q}\ \not{p}_{\nu}\ Q\ \Lambda_{\pm}\ \gamma_o\ (2\pi)^4\ \delta(p_f - p_i)\, d\,\tau\ , \tag{4}$$

where p_f and p_i are the total momenta in the initial and final states respectively, $d\tau$ is an element of phase space volume and $Q = \frac{1}{2}(1 - \gamma_5)\ Q\ (g) + \frac{1}{2}(1 + \gamma_5)\ Q\ (f)$, where

$$Q(g) = \frac{1}{M}\, g_S - \frac{1}{M^2}(g_V\ \not{p}_K + g_{V'}\ \not{p}_{\pi}) + \frac{1}{M^3}\ g_T\, \sigma_{\mu\nu}\ p_K^{\mu}\ \ p_{\pi}^{\nu}\ . \tag{5}$$

Re-writing (4) so as to bring it into the form given by (1), one finds:

$$\underline{P} = (\underline{A}_g - \underline{A}_f)\ /\ (\Delta_g + \Delta_f), \tag{6}$$

where:

$$\underline{A} = -\Big\{ \Big[\ |X_S|^2\ p.p_{\nu} - |X_V|^2(2EE_{\nu} - p.\ p_{\nu})\Big]\ \underline{p}/\,E - \Big[(|X_S|^2 + |X_V|^2)m\ + + 2\,\mathrm{Re}(X_S X_V^{*})E\Big]\ (\underline{p}_{\nu N} + \frac{m}{E}\ \underline{p}_{\nu L}) - 2\ \mathrm{Im.}\ (X_S X_V^{*})\ \underline{p}\ \times\ \underline{p}_{\nu}\Big\} \tag{7}$$

and

$$\Delta = |X_S|^2\ p.p_{\nu} + |X_V|^2(2E\ E_{\nu} - p.\ p_{\nu}) + 2\,\mathrm{Re}\ (X_S\ X_V^{*})\ m\ E_{\nu}\ ; \tag{8}$$

where $\underline{p}_{\nu N}$ and $\underline{p}_{\nu L}$ stand for components of $\underline{p}_{\nu}$ normal and parallel to $\underline{p}$, and

$$X_S = \frac{1}{M}\left(g_S - \frac{m}{M} g_{V'} + \frac{E - E_\nu}{M} g_T\right); \quad X_V = \frac{1}{M}\left(g_V + g_{V'} - \frac{m}{M} g_T\right); \tag{9}$$

with entirely analogous expressions with the g's replaced by the f's. Analogous expressions have been obtained by Charap[3], using spin projection operators[4].

The two-component theory[5] corresponds to setting either all g's or all f's equal to zero. In this case for a given final configuration of the momenta the polarization is complete ($p^2 = 1$). Therefore the analysis of correlated measurements in the decay chain $K_{\mu 3} - \mu - e$ is a very promising approach to the investigation of the decay mechanism. On the basis of the law of conservation of leptons, the alternative $g \approx 0$ should be excluded.

As shown in a previous paper[6], all parameters must be real for invariance of the interactions under time reversal. Direct evidence of the violation of this principle would be provided by observation of a polarization normal to the plane of decay. However, this results from interference between S, V, T (or P, A, T) couplings and could not occur if the Fermi interaction were a (V+ A) combination. Correlated polarization provides an independent way of determining the best set of parameters g's and f's. In the analysis of the pion energy dependence of these parameters it supplies information additional to that from angular correlation experiments at fixed pion energies.

II. The expressions for normal and longitudinal polarization must be corrected by factors which take into account the depolarization. This may easily be done in the case when the depolarizing factors are identical and independent of the primary muon energy. The main cause of depolarization, which could depend on the spin orientation and the muon energy is atomic collisions at high energies.

Let M be the matrix element for collision, $\underline{p}_1$ and $\underline{p}_2$ the initial and final momenta, ρ_1 and ρ_2 the density matrix in the initial and final states. One can write:

$$\rho_2 \gamma_o = \Lambda_{2+} \bar{M} \rho_1 \gamma_o M \Lambda_{2+} . \tag{10}$$

Transforming the density matrices to the rest system of the particle, the above relation becomes:

$$\rho_2^o = \frac{\gamma_o + 1}{2} L_2 \bar{M} L_1 \rho_1^o L_1 M \bar{L}_2 \frac{\gamma_o + 1}{2} , \tag{11}$$

where $L = (\gamma_o \not{p} + m)/[2m (E+m)]^{1/2}$ is a Lorentz translation of velocity $- \underline{p}/E$. Since one can write:

$$\frac{\gamma_o + 1}{2} L_1 M \bar{L}_2 \frac{\gamma_o + 1}{2} = (\alpha + \underline{\sigma}.\beta) \frac{\gamma_o + 1}{2} \tag{12}$$

it follows that:

$$\rho_2^o = (\alpha^* + \underline{\sigma} . \underline{\beta}^*) \frac{1 + \underline{\sigma} \, \underline{p}_1}{2} (\alpha + \underline{\sigma} . \underline{\beta}) \frac{\gamma_o + 1}{2} = \frac{1}{2} \Big\{ (\alpha \alpha^* + \underline{\beta} . \underline{\beta}^*)(1 + \underline{\sigma} . \underline{p}_2) +$$

$$+ \underline{p}_1 . [- i \underline{\beta} \times \underline{\beta}^* + 2 \mathrm{Re} \, \alpha \underline{\beta}^*] + \underline{\sigma} . [-i \underline{\beta} \times \underline{\beta}^* + 2 \, \mathrm{Re}(\alpha \underline{\beta}^* + i \alpha \underline{\beta}^* \times \underline{p}_1 - \tag{13}$$

$$- \underline{\beta} . \underline{\beta}^* \underline{p}_1 + \underline{\beta} . \underline{p}_1 \underline{\beta}^*)] \Big\} \times \frac{\gamma_o + 1}{2} .$$

Supposing that the particles are slowing down in a medium with n scattering centres per unit volume, the average depolarization per unit path is the product of nx(cross section) by the average depolarization per collision. All the terms in (13) of the form $\propto \underline{\beta}^*$ and $\underline{\beta} \times \underline{\beta}^*$ vanish on the average; one then obtains:

$$- \frac{d\,\underline{P}}{d\,x} = 2n\,\frac{m}{p} \int (\underline{\beta} \times \underline{P}) \times \underline{\beta}^*\, d\tau \Big]_{Av.} , \qquad (14)$$

where the result of integration over phase space is averaged over the incident directions. The normal and longitudinal depolarization are then given by

$$- \frac{1}{P_N} \frac{d\,P_N}{d\,x} = n\,\frac{m}{p} \int \left[\underline{\beta}\cdot\underline{\beta}^* + |\underline{\beta}\cdot \underline{\xi}_1|^2 + \frac{1}{2} (\underline{\beta}\cdot\underline{\beta}^* - 3|\underline{\beta}\cdot\underline{\xi}_1|^2) \langle \sin^2\theta \rangle\right] d\tau \qquad (15,a)$$

$$- \frac{1}{P_L} \frac{d\,P_L}{d\,x} = 2n\,\frac{m}{p} \int \left[\underline{\beta}\cdot\underline{\beta}^* - |\underline{\beta}\cdot \underline{\xi}_1|^2 - \frac{1}{2} (\underline{\beta}\cdot\underline{\beta}^* - 3|\underline{\beta}\cdot\underline{\xi}_1|^2) \langle \sin^2\theta \rangle\right] d\tau \qquad (15,b)$$

where $\underline{\xi}_1 = \underline{p}_1/p_1$ and θ is the deviation of $\underline{\xi}_1$, from the initial direction after the particle has travelled a distance x. At high energies the mean square deviation is small and the term in θ is negligible.

For elastic scattering neglecting the nuclear recoil and using first Born approximation, one obtains:

$$\frac{2}{P_N} \frac{dP_N}{dx} = - \frac{1}{P_L} \frac{dP_L}{d\,x} = 4\pi\, n\, \frac{Z^2\, e^4}{(E+m)^2}\, \ln \left(\frac{\sin\,\theta\,\max/2}{\sin\,\theta\,\min/2}\right) , \qquad (16)$$

where θ min, estimated by means of the Thomas-Fermi statistical model, is given by $\sin(\theta \min/2) = Z^{1/3}\, m_e e^2/p$ and $\sin(\theta \max/2) \leqslant 1/r_n p$ where $r_n = r_o\, A^{1/3}$ is the nuclear radius. The value of θ max may also be fixed by the experimental conditions. For inelastic scattering one can suppose the muon to be interacting directly with one electron, which may be considered as free and initially at rest. The results in first order perturbation theory are:

$$-\frac{1}{P_N}\frac{dP_N}{dx}=\pi n\frac{Z\ e^4}{(E+m)^2}\left\{\ln\left(\frac{2\ m_e\ p^2}{Z\ I\ m^2}\right)+\frac{1}{m^2}\left[p^2+mE+(m+E)^2\right]\right\} \qquad (17,\ a)$$

$$-\frac{1}{P_L}\frac{dP_L}{dx}=2\pi n\frac{Z\ e^4}{(E+m)^2}\left\{\ln\frac{2\ m_e\ p^2}{Z\ I\ \ m^2}+\frac{1}{m^2}\left[p^2+m\ E\right]\right\}. \qquad (17,\ b)$$

Dividing these results by the energy less and integrating one obtains the depolarizing factor for these processes. For muons with initial kinetic energy ~ 130 Mev, slowed down to ~ 0.5 Mev in carbon, the depolarization due to atomic collisions is ~ 1%. Appreciable depolarization occurs only at very low energies due to capture processes. Therefore one can expect that the depolarization will be almost isotropic and independent of the primary muon energy.

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2. This matrix is actually only the diagonal part with respect to the momentum variables of the density matrix in momentum representation.
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