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LEPTON NUMBER LAW - ADDITIVE OR MULTIPLICATIVE ?

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A short review of the lepton number conservation law is given in the first part of this report. After a discussion of past experimental tests a tentative analysis is presented in which a distinction can be made between an additive and multiplicative lepton number law.

1. LEPTON NUMBER

In any interaction, fermions can only be created in pairs, by conservation of angular momentum. It was found experimentally, however, that light fermions (= leptons), (as well as heavy fermions (= baryons)), are

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created in pairs separately. The most natural way to explain this was to introduce an additive quantum number for the leptons. The "leptonic charge" or lepton number L, (as well as a baryon number) [1].

Since L is a relative concept, one can <u>define</u> the e^- as the lepton (L = +1). Then, from lepton number conservation in β-decay, the $\bar{\nu}_e$ is the antilepton (L = -1).

With this assignment for the electron, one has two possibilities for the muon: Either one defines the μ^+ or the μ^- as the lepton.

Schematically we have:

TABLE I

	e^-	ν_e	e^+	$\bar{\nu}_e$	μ^-	ν_μ	μ^+	$\bar{\nu}_\mu$	
L	1	1	-1	-1	1	1	-1	-1	(A)
					-1	-1	1	1	(B)

Both sets (A) and (B) are compatible with all known data.

A. *MUON DECAY*

$$\mu^+ \rightarrow e^+ + \text{neutrals} \qquad (1.1)$$

Let us consider the following two possibilities for the decay of the muon:

$$\mu \rightarrow e \; \gamma$$
$$\mu \rightarrow e \; e^- \; e^+ \qquad (1.2)$$

Experimentally, there is no evidence for these processes; the respective branching ratios are [2, 3]

$$\frac{\mu \to e\,\gamma}{\mu \to e\nu\bar{\nu}} < 6 \times 10^{-9} \tag{1.3a}$$

$$\frac{\mu \to e\,e^{+}\,e^{-}}{\mu \to e\,\nu\,\bar{\nu}} < 1.5 \times 10^{-7} \tag{1.3b}$$

From the energy distribution of the electron, we know that at least 2 neutral particles are emitted in addition to the electron:

$$\mu \to e\,A^{o}\,B^{o} \tag{1.4}$$

Since, experimentally, $M_A + M_B < 5.6$ MeV, particles A and B can either be two neutrinos or 2 photons. Since no $e^+ e^-$ - pairs have been observed, the common notion is that A and B are two neutrinos. We will discuss the implications of lepton number conservation for particles A and B later.

B. TWO NEUTRINOS

In high energy neutrino experiments it was shown conclusively that there are definitely 2 distinct neutrinos - the neutrino accompanying the muon being different from that accompanying the electron: [4, 5, 6, 7]

$$\nu_\mu N \not\to \nu_e P \tag{1.5}$$

Hence a lepton number conservation law has to allow leptons to be produced in pairs only, while at the same time forbidding the processes:

$$\nu_\mu N \to \nu_e P$$

$$\mu \to e\gamma \qquad (1.6)$$

$$\mu \to e e^+ e^- .$$

2. POSSIBLE LEPTON CONSERVATION LAWS

Several schemes have been thought of. They all require two distinct neutrinos, ν_μ and ν_e.

A. SCHEME (A)

Lepton number (additive) + Muon number (add)

The easiest way to forbid the unwanted reactions is to make the muon different from the electron by introducing an additional quantum number for the muon, the muon number L_μ [8, 9, 10].

TABLE II

	e^-	ν_e	e^+	$\bar{\nu}_e$	μ^-	ν_μ	μ^+	$\bar{\nu}_\mu$
L	1	1	-1	-1	1	1	-1	-1
L_μ	0	0	0	0	1	1	-1	-1

There would be two separate additive laws. Both L and L_μ would be absolutely conserved.

Examples:

$$\mu^+ \to e^+ \; e^+ \; e^- \qquad\qquad \nu_\mu \; N \to e^- \; P$$

$$L: \; -1 = -1 \; -1 \; +1 \quad \text{OK} \qquad\qquad 1 = 1 \quad \text{OK} \tag{2.1}$$

$$L_\mu: \; -1 \neq 0 + 0 \; +1 \quad \text{X} \qquad\qquad 1 \neq 0 \quad \text{X}$$

Of the two possible ways to assign L for the muon we have chosen (A) since this makes both ν_μ and ν_e leptons. Experimentally, ν_μ and ν_e are lefthanded ($h = -1$). Hence in this scheme we have massless leptons lefthanded and antileptons righthanded. One of the two possible Wey'l equations would suffice to describe nature. For this reason this scheme is normally being used.

We should remark, however, that here we need two 2-component fields for the neutrino; these cannot be united into one 4-component field (Majorano neutrino) since the two fields differ in L_μ.

An alternative way to introduce two additive quantum numbers is to use two lepton numbers completely distinct for e and μ, L_e and L_μ:

TABLE III

	e^-	ν_e	e^+	$\bar{\nu}_e$	μ^-	ν_μ	μ^+	$\bar{\nu}_\mu$
L_e	1	1	-1	-1	0	0	0	0
L_μ	0	0	0	0	1	1	-1	-1

It is easy to see that this scheme is completely equivalent to the one discussed above.

B. *SCHEME (B)*

Lepton number (add) opposite for e^- and μ^-

One can also develop a scheme with just one lepton number, which satisfies all the requirements.[8, 9, 11] For this we define the e^- to be the lepton and the μ^- to be the antilepton.

TABLE IV

	e^-	ν_e	e^+	$\bar{\nu}_e$	μ^-	ν_μ	μ^+	$\bar{\nu}_\mu$	
L'	1	1	-1	-1	-1	-1	1	1	(B)
helicity		-1		1		-1		1	

Examples

$$\left.\begin{array}{llll} & \mu^+ \rightarrow & e^+ & \gamma \\ L': & 1 \neq & -1 & X \end{array} \qquad \begin{array}{llll} \nu_\mu & N \rightarrow e^- & P \\ -1 & \neq \quad 1 & X \end{array}\right\} \qquad (2.2)$$

Now we have (with the conventional names!)

$$\left.\begin{array}{ll} \nu_e & - \text{ lepton lefthanded} \\ \bar{\nu}_e & - \text{ antilepton righthanded} \\ \nu_\mu & - \text{ antilepton lefthanded} \\ \bar{\nu}_\mu & - \text{ lepton righthanded} \end{array}\right\} \qquad (2.3)$$

We have two 2-component fields (with $\psi_\nu = \binom{\phi}{\chi}$): ϕ-space for ν_e and χ-space for $\bar{\nu}_\mu$), which can now be united into one single 4-component field.

$$\left.\begin{aligned} \phi\, \bar{\nu}_\mu\,(x) &= \frac{1}{2}\,(1 - \gamma_5)\,\psi_\nu(x) \\ \phi\, \nu_e\,(x) &= \frac{1}{2}\,(1 + \gamma_5)\,\psi_\nu(x) \end{aligned}\right\} \qquad (2.4)$$

In other words, muon-neutrino and electron-neutrino are the negative and positive chiral parts of one 4-component neutrino field $\psi_\nu(x)$. We see that this scheme is most economical in that it will do with only one lepton number. The neutrinos differ in handedness only.

For all the scheme's economy it is hard to understand why the muon and electron are so different if their neutrinos are distinguished by their handedness only. Also, we have given up the notion of leptons being left-handed and anti-leptons being righthanded.

C. LEPTON NUMBER (ADD) + MUON PARITY (MULT.)

The experimental evidence presently available is insufficient to rule out the possibility that the distinction between (μ^-, ν_μ) and (e^-, ν_e) is given by different values of a multiplicative quantum number, muon-parity P_μ[12].

TABLE V

	e^-	ν_e	e^+	$\bar{\nu}_e$	μ^-	ν_μ	μ^+	$\bar{\nu}_\mu$	all other known pairs
L	1	1	-1	-1	1	1	-1	-1	0
P_μ	1	1	1	1	-1	-1	-1	-1	1

- Examples:

TABLE VI

$\mu^+ \to e^+ e^+ e^-$		$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$		$\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$	
L : $-1 = -1-1+1$	OK	$-1 = -1+1-1$	OK	$-1 = -1-1+1$	OK
P_μ : $-1 \neq (+1)(+1)(+1)$	x	$-1 = (1)(1)(-1)$	OK	$-1 = (1)(1)(-1)$	OK

Perhaps it is unlikely that the lepton number follows a multiplicative law, since all quantum numbers except intrinsic parity C - and G- parity follow an additive law. But there is no really strong theoretical reason favouring an additive or a multiplicative law.

We should also note that the multiplicative law is less restrictive than the additive one as we have seen above.

3. COMPARISON OF ADDITIVE AND MULTIPLICATIVE LAWS

All schemes discussed above imply (a) that leptons are always produced in pairs, (b) that there are two distinct neutrinos and (c) that the unobserved final states in moun decay are forbidden.

There are, however, reactions which are allowed in some schemes but not in others:

TABLE VII

Reaction	Additive (A)	(B)	Multiplicative (C)
(1) $Z \to (Z+2)\ e^- e^- \bar{\nu}_e \bar{\nu}_e$	OK	OK	OK
(2) $Z \to (Z+2)\ e^- e^-$	X	X	X
(3) $\nu_\mu N \to e^- P$	X	X	X
(4) $\mu^+ \to e^+ \gamma$	X	X	X
(5) $\mu^+ \to e^+ e^- e^+$	X	X	X
(6) $\mu^+ \to e^+ \nu_e \nu_\mu$	X	X	X
(7) $\mu^+ \to e^+ \bar{\nu}_e \bar{\nu}_\mu$	X	X	X
(8) $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$	OK	OK	OK
(9) $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$	X	X	OK
(10) $(\mu^+ e^-) \to (\mu^- e^+)$	X	X	OK
(11) $e^- e^- \to \mu^- \mu^-$	X	X	OK

G' Coupling constant for given process

G Coupling constant of β-decay

4. EXPERIMENTAL TESTS (OLD)

A. LEPTON NUMBER CONSERVATION (ANY SCHEME)

The best test comes from measurements of double beta-decay (DBD) (reactions (1) and (2)). DBD is possible both with neutrinos and neutrinoless (if $\bar{\nu}_e \equiv \nu_e$). Since DBD has a lifetime of the order of 10^{21} years it was

e μ

For a multiplicative law, however, $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ also. Then the μ^+ contribution of the ν_e-flux would be depleted and there would be an additional contribution to the $\bar{\nu}_e$-spectrum from $\mu^+ \to e^+ \bar{\nu}$ ν detected indirectly involving mass spectrometric analyses of Tellurium of known age [13, 14]. A relative abundance of Xe^{130} was attributed to DBD of Te^{130}. The lifetime was consistent with DBD accompanied by two neutrinos and the limit for lepton number nonconservation was given as $G'/G < 10^{-4}$.

B. TEST OF SCHEME (B)

Both the process $e^- e^- \to \mu^- \mu^-$ as well as muonium-antimuonium transitions are allowed if a multiplicative law holds. The experimental limits are very poor at present.

$$\begin{aligned} &\text{For } (\mu^+ e^-) \to (\mu^- e^+) \text{ the limit is }^{17}\text{: } G'/G < 5800 \\ &\text{For } (e^- e^- \to \mu^- \mu^-) \quad " \quad " \quad "\ ^{18}\text{: } G'/G < 610 \end{aligned} \qquad (4.1)$$

These upper limits are too high to say anything about the lepton number law. (They only imply that there is no new strong leptonic force).

5. EXPERIMENTAL TEST (NEW)

A. MUON DECAY AS TEST OF SCHEME (C)

As seen from reaction (10) in the above table, muon decay can be used to distinguish between an additive and multiplicative law.

$$\begin{aligned} &\mu^+ \to e^+ \nu_e \bar{\nu}_\mu \quad \text{possible with both add. and mult. law} \\ &\mu^+ \to e^+ \bar{\nu}_e \nu_\mu \quad \text{possible \underline{only} with mult. law.} \end{aligned} \qquad (5.1)$$

The nature of the neutrinos in muon decay can be determined by using the decay neutrinos to induce inverse reactions, i.e. by looking for either μ^- [12] or e^+ [19, 20] produced by neutrinos from μ^+ decay.

In a neutrino run in addition to ν_μ and $\bar{\nu}_\mu$, we have a certain contamination of ν_e and $\bar{\nu}_e$. The ν_e flux is given by contributions from:

a) K^+_{e3}

b) $\mu^+ \rightarrow e^+ \quad \nu_e \; \bar{\nu}_\mu$ in flight (5.2)

c) K^o_{e3}

d) $\overline{K^o_{e3}}$

The $\bar{\nu}_e$ flux is given by contributions from:

a) $\overline{K^o_{e3}}$

b) K^o_{e3} (5.3)

c) K^-_{e3}

If an additive law holds for the lepton number, we have seen that $\mu^+ \rightarrow e^+ \quad \nu_e \; \bar{\nu}_\mu$ only.

For a multiplicative law, however, $\mu^+ \rightarrow e^+ \bar{\nu}_e \; \nu_\mu$ also. Then the μ^+ contribution of the ν_e-flux would be depleted and there would be an additional contribution to the $\bar{\nu}_e$-spectrum from $\mu^+ \rightarrow e^+ \bar{\nu}_e \; \nu_\mu$.

As seen in figure 1, the ν_e-contribution from μ^+ decay in flight is dominant at energies below 2 GeV [21]. That means, the relative magnitude of the ν_e and $\bar{\nu}_e$ flux in this energy region is very sensitive to the branching ratio

$$BR = \frac{\mu^+ \rightarrow e^+ \bar{\nu}_e \; \nu_\mu}{\mu^+ \rightarrow \text{all}} \quad . \tag{5.4}$$

In figure 2 we see the variations of the spectra with different branching ratios.

BR = 0% would mean that the lepton number law is purely additive. For a multiplicative law one would expect BR = 50% [20], if the coupling constant is the same for the two decay modes.

By comparing the number of e^- and e^+ events as a function of energy in a neutrino experiment, one can deduce the relative magnitude of the spectra and hence determine the branching ratio BR experimentally.

B. *EXPERIMENTAL RESULTS*

In the Gargamelle Neutrino Experiment, we have scanned an exposure of $\sim$ 250 000 pictures for electron ($e^{\pm}$) neutrino events. The energy distribution of these is given in Fig. 3.

For $0 < E < 2$ GeV there are 14 e^--events and 1 e^+ - event. The ratio $N_{e^+}/N_{e^-} = 1/14 = 0.07 \pm .09$.

In Fig. 4 we compare this ratio N_{e^+}/N_{e^-} - with the ratio expected from the $\bar{\nu}_e(\nu_e)$-flux assuming BR = 0,50%. We have assumed that

$$\sigma^{\bar{\nu}_e}_{tot} \;/\; \sigma^{\nu_e}_{tot} = \frac{1}{3} \tag{5.5}$$

which was found to be true for ν_μ-N interactions. Since this ratio seems to be a consequence of the hadron-structure, not of the nature of the neutrino, one would expect the same ratio for ν_e - N-interactions.

Taking the ratio of e^+ and e^- events and comparing it with the ratio expected from the spectra has several advantages. One needs to know only the relative flux, which has much smaller errors than the absolute magnitude of it. In addition, corrections for scanning efficiencies drop out if we assume them to be equal for e^+ and e^- events.

6. CONCLUSION

At this time, we do not have sufficient statistics to distinguish with a high confidence level between the laws. The data are consistent with the additive law, the multiplicative cannot be excluded, however.

Analysing in this way the $e^{\pm}$ events from both the complete neutrino and antineutrino runs (and hopefully more freon and propane runs) the statistic should be good enough to clearly distinguish between the two laws.

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