ELASTIC SCATTERING OF α - PARTICLES*

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Porter's explanation [1] of the elastic scattering of α - particles is based upon a nuclear model in which the protons are more concentrated close to the center than the neutrons as suggested by Johnson and Teller [2]. Indeed, the distorsion the coulomb orbits due to the distribution of protons in the nucleus, may be neglected if the α - particles do not penetrate deeply into this distribution. The nuclear absorption would be due to the presence of the external neutron layer.

If we assume a uniform distribution of protons in sphere of radius $R = 1.2 \times 10^{-13} A^{1/3}$ cm, then α -particles,

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22 Mev energy colliding with nuclei of Ag^{108} can penetrate only slightly into the charged sphere. However, at higher energies, such as 48 Mev,[3] the α-particles may penetrate deeply into this sphere and are able to overcome the coulomb barrier. Therefore we must expect essential deviations from the pure hyperbolic paths. Thus, we cannot use, at these energies, the coulomb potential as in Porter's work. But when the uniformly charged sphere potential is used to calculate the classical trajectories, we get into a serious difficulty as the scattering angle has a maximum value in complete disagreement with the experimental results.

The scattering angle, as given by Hamilton-Jacobi theory is:

$$\theta = \omega + 2\cos^{-1}\left[\left(1+\frac{\alpha}{2}\,\mathrm{ctg}^2\frac{\omega}{2}\right)\sin\frac{\omega}{2}\right] - \cos^{-1}\left\{\frac{\frac{\alpha}{2}\,\mathrm{ctg}^2\frac{\omega}{2} - \left(\frac{1}{\alpha}-\frac{3}{2}\right)}{\sqrt{\frac{\alpha}{2}\,\mathrm{ctg}^2\frac{\omega}{2} + \left(\frac{1}{\alpha}-\frac{3}{2}\right)^2}}\right\}, \qquad (1)$$

where $\alpha = \frac{ZZ' e^2}{RE}$ and ω is the corresponding scattering angle in a pure coulomb field. In Fig. 1 θ is plotted against ω and it is shown that θ_{max} is about 42°; experimental data give a finite scattering cross-section until far beyond this angle. In order to avoid this difficulty, it should be necessary to take a much smaller radius for the distribution of protons, than is indicated in any other experiment.

On the other hand, when we try to improve Porter's calculation of the hyperbolic orbits in a pure coulomb field, we are led again to unsatisfactory results. We have assumed an uniform distribution of matter in a sphere of radius $R = R_{Ag} + R_{\alpha} = 8.4 \times 10^{-13}$ cm; therefore, the ratio

$$\frac{\sigma}{\sigma_c} = \exp\left(-\int\frac{\rho \, ds}{\rho_o \, l_o}\right),$$

becomes

$$\frac{\sigma}{\sigma_c} = e^{-s/l} \qquad (2)$$

and we used for s an approximate formula:

$$S = \frac{1}{3}\left(v_M + 2\,v_m\right) T, \qquad (3)$$

where T is the time spent by the α-particles inside the sphere, v_M is the velocity at the entrance into the nucleus and $\bar{v}_m$ that at its nearest distance from the center. (This formula would be exact if $v = v_m + \text{const.}\, t^2$ which is a reasonably good approximation.) Hamilton-Jacobi theory gives for T :

$$T = \sqrt{\frac{m}{2E}} \cdot \frac{Z Z' e^2}{E}\left\{\left(\Delta^2 - \csc^2\frac{\omega}{2}\right)^{1/2} + \right.$$

$$\left. + \ln\left(\left[\Delta + \left(\Delta^2 - \csc^2\frac{\omega}{2}\right)^{1/2}\right]\sin\frac{\omega}{2}\right)\right\}, \qquad (4)$$

where $\Delta = \frac{2\,RE}{Z\,Z' E^2} - 1$.

Fig. 2 shows the experimental values of $\frac{\sigma}{\sigma_c}$ plotted against s varying either the energy[4] or the scattering angle[5]. According to (2) both curves should be coincident straight lines. Moreover, it can be shown that, at constant energy, $\int \rho \, ds$ shows a maximum, at a certain angle, whatever the distribution of matter is, provided ρ is a decreasing function of r. At this angle $\frac{\sigma}{\sigma_c}$ has a minimum. With a uniform distribution, this minimum occurs at an angle where a decreasing behaviour has been observed experimentally.

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(2) M. H. Johnson and E. Teller, Phys. Rev. 93, 357 (1954)

(3) R. E. Ellis and Larry Scheeter, Phys. Rev. 101, 636 (1956)

(4) Wall, Rees, and Ford, Phys. Rev. 99, 825 (1955)

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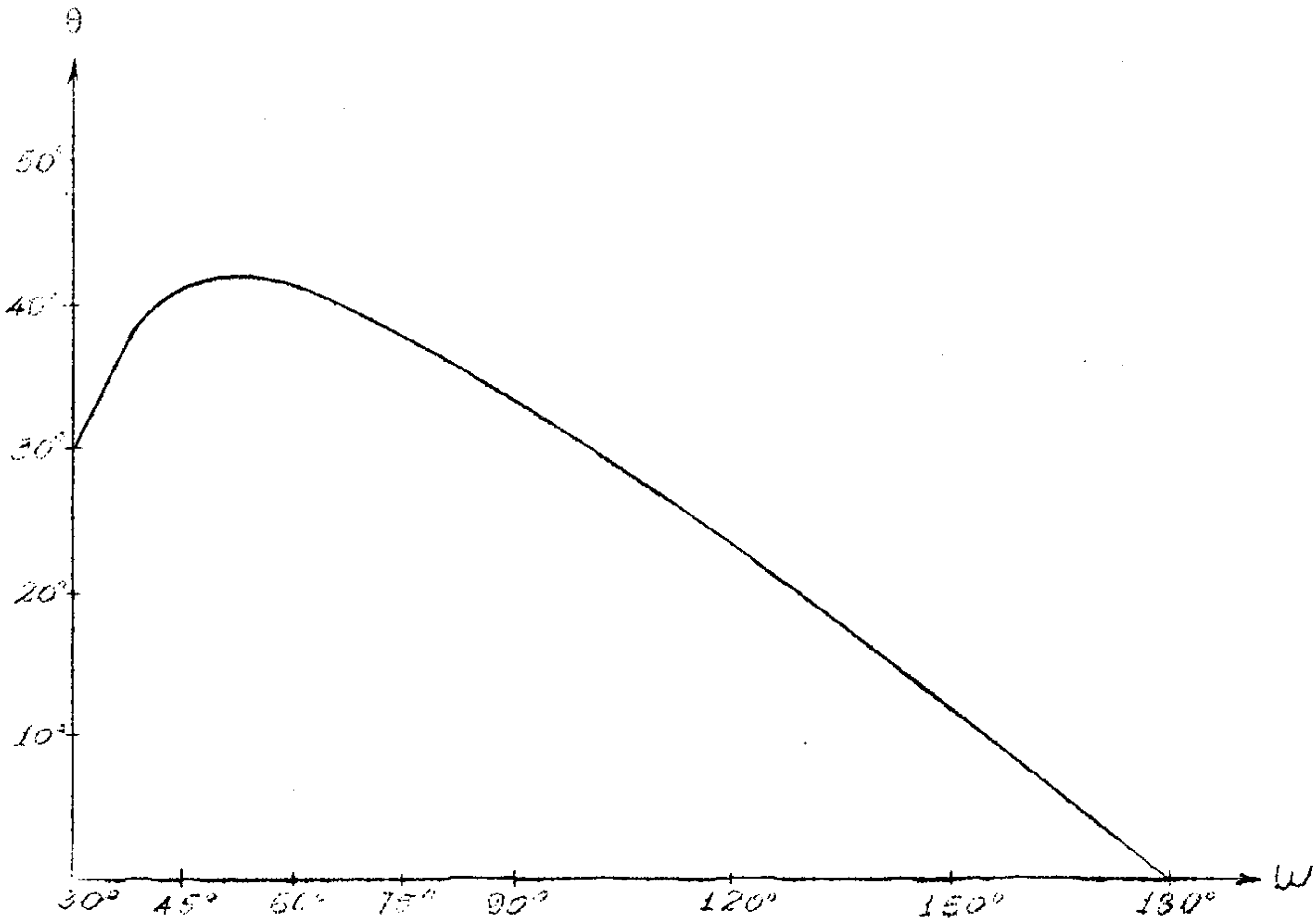

Fig. 1

Theoretical scattering angle of 48 Mev α - particles by Ag^{108}. The curve shows the scattering angle for a uniform distribution of charge with radius of $1.2 \times 10^{-13} A^{1/3}$ cm. as function of the coulomb scattering angle.

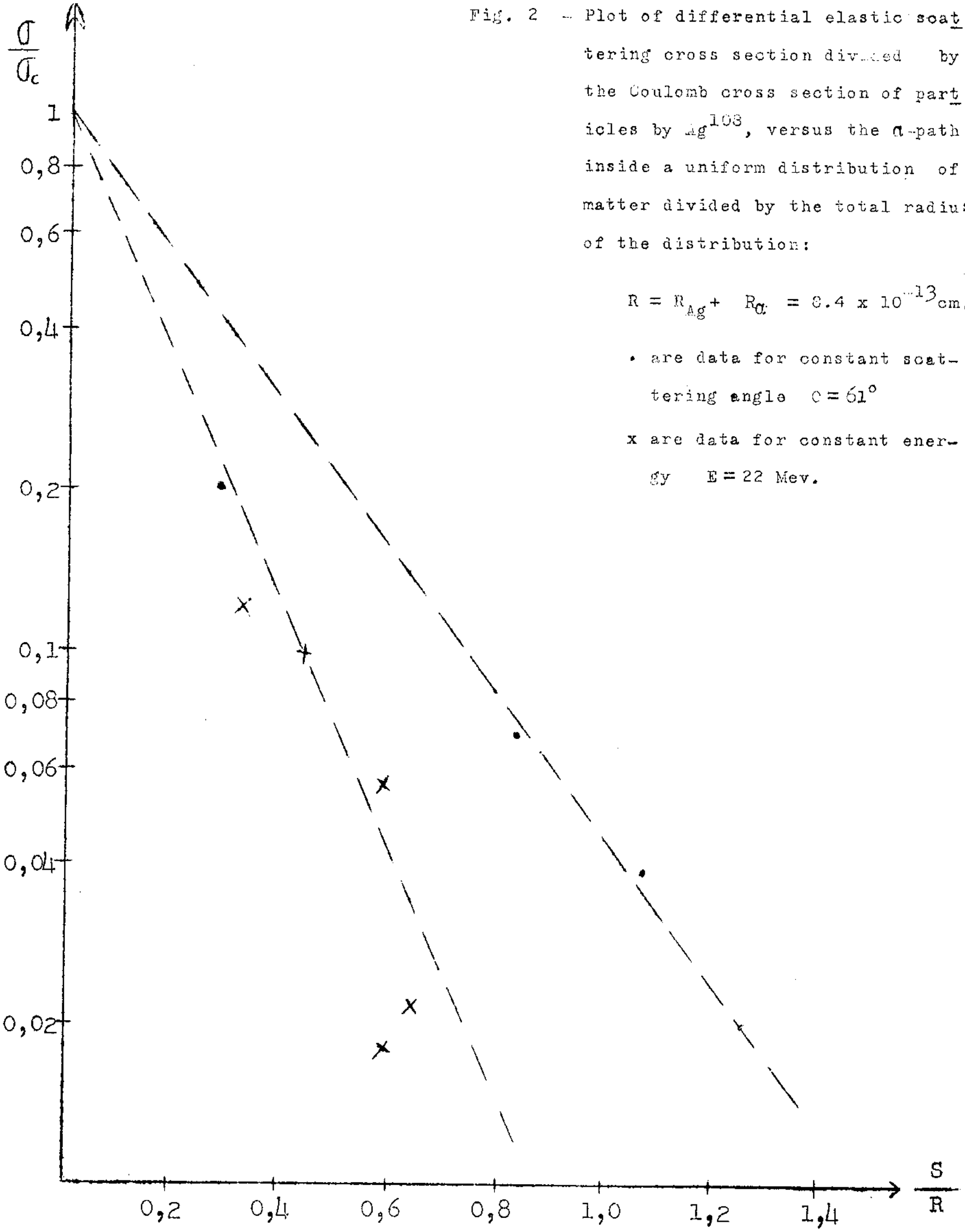

Fig. 2 – Plot of differential elastic scattering cross section divided by the Coulomb cross section of particles by Ag^{108}, versus the α-path inside a uniform distribution of matter divided by the total radius of the distribution:

$$R = R_{Ag} + R_{\alpha} = 8.4 \times 10^{-13} cm.$$

• are data for constant scattering angle $\theta = 61^{\circ}$

x are data for constant energy $E = 22$ Mev.