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THE RING FOCUS IN THE SPIRAL ORBIT SPECTROMETER*

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It is shown in this work that the spiral orbit spectrometer has a ring focus near the stable orbit. Taking advantage of this property by means of a cylindrical slit, the characteristics of the spiral orbit spectrometer can be greatly improved. For a particular case studied, the transmission as a function of the line width is given by $T = 0.7W^{1/2}$ for a point source.

1. Introduction

The spiral orbit spectrometer was first built by MIYAMOTO[1], and its theory was developed by IWATA, MIYAMOTO and KOTANI[2]. Spiral orbit spectrometers have been built and used in Japan[3] and in Berkeley[4]. SAKAI[5] has made a study comparing the three types of spectrometers known up to

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now, and in his work he arrives at the relation $T = 0.077W^{1/2}$ between the transmission and the line width for the spiral orbit spectrometer. In this paper we are going to show that there exists a ring focus in the spiral orbit spectrometer, and that by using this property the characteristics of the spiral orbit spectrometer can be improved about a factor of ten from the relation found by SAKAI.

Let us suppose that we have a cylindrically symmetric magnetic field which is radially decreasing, and let us call the median plane a plane perpendicular to the axis of cylindrical symmetry and midway between the pole faces of the magnet that produces such field. We will use a system of cylindrical coordinates with the origin at the intersection of the median plane and the axis of cylindrical symmetry, with the z-axis coinciding with the axis of cylindrical symmetry and pointing upwards. In such a magnet the vertical component of the magnetic field in the median plane can be written as $H_z(r,0) = H_0 f(r)$, where $f(r)$ is a function of r only and it is equal to one when r is equal to 0, and H_0 is the value of the field at the origin.

Once the field is defined in the median plane, it can be computed anywhere else by the relations $\operatorname{div} \vec{H} = 0$ and $\operatorname{curl} \vec{H} = 0$. If the field is decreasing, one finds then that for particles which are moving outside the median plane, the radial component of the magnetic field produces a restoring force that accelerates the particles towards the median plane. This force is the one that produces the ring focus.

If a source that emits monoenergetic electrons of mass m, charge e, and velocity v is placed in the origin, the electrons emitted in the median plane will approach asymptotically to a circle with its center at the origin if for large vlaues of the time t, $\frac{dr}{dt}$ and $\frac{d^2r}{dt^2}$ go to zero. These conditions imply that for a certain value R of r, the following relation holds $\int_0^R rf(r)dr = R^2f(R)$, and that H_o should have a value H_s given by the relation $H_s = mvR / e\int_0^R rf(r)dr$. The first of these two relations is the condition for the existance of a stable orbit, and R is the radius of the stable orbit. The condition of the field being radially decreasing is not sufficient for the existance of the stable orbit.

We can define a dimensionless parameter k by the relation $H_o = kH_s$ and, if one makes a plot of the fraction of the number of electrons which are emitted from a momoenergetic source and received in the detector as a function of k, one obtains a curve called the line profile. The maximum value of this fraction in the line profile is called the transmission T, and the full width at half the maximum is called the line width W. For these and other definitions related to beta-ray spectrometers we follow FRISCH[6].

2. The trajectories and the ring focus

The difficulty of studying the properties of this machine is due to the impossibility of expressing the solutions of the system of differential equations in terms of known or common functions. For this reason we decided to solve them numerically. We chose for f(r) the function $f(r) = 1 - 2r^2/3R^2$ in the region from r equal to zero to r equal to R, where R is a constant length. This function is mathematically simple for the numerical integrations, and it seems that it can be made by suitably shaped pole faces without too great difficulty.

With this field it can be shown that for electrons emitted in the median plane the stable orbit is given by $r = R$ and that $H_s = 3mv/eR$. We made $R = v = 1$ to simplify the equations. The vertical and the radial component of the field at any point are given respectively by $H_z = kH_s(1 - 2r^2/3 + 2z^2/3)$ and $H_r = -4kH_s rz/3$.

The differential equations for the trajectories were left in the following form:

$$\frac{d^2r}{dt^2} = r\frac{d\theta}{dt}\left(\frac{d\theta}{dt} - 3kg\right)$$

$$\frac{d^2z}{dt^2} = -4kzr^2\frac{d\theta}{dt}$$

$$\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = 1 \text{ with } \quad g = 1 - \frac{2r^2}{3} + \frac{2z^2}{3}$$

The trajectories were integrated for various values of the angle β which the emitted electrons form with the median plane at the origin and for several values of k. We calculated with $k = 1.0000$ trajectories with $\sin\beta$ equal to 0.0000, 0.0100, 0.0200, 0.0400 and 0.0600. We also calculated for these same angles and values of k ranging from 0.9950 to 1.0050. The remaining trajectories needed for the problem were found by interpolation. The numeridal integrations were carried out by Milne's method starting with a Taylor's expansion. The interval of t used was 0.0500 or 0.1000 and we carried four decimal places.

In Figure 1 are shown the stable orbit and several trajectories of different values of k and sin β. Notice that for k greater than one all the trajectories come back towards the center of the machine. For k less than one, the trajectories come back towards the center if 1 - k is less than $\sin^2\beta/2$; they approximately coincide with the stable orbit if 1 - k is equal to $\sin^2\beta/2$; and they go beyond the stable orbit if 1 - k is greater than $\sin^2\beta/2$. In Figure 1 is also indicated the position of the ring focus.

In Figure 2 we show z as a function of r; the ring focus is evident from this figure, being located between the values for r of 0.94 to 0.95. By examining closely the region of the ring focus, we found the optimum location of the ring focus and its aperture as a function of the maximum value of sin β that is allowed to pass, and they are shown in Table I.

3. The characteristics of the spectrometer.

In Figure 3 we show schematically the spiral orbit spectrometer using ring focus, the aperture of the slit in the ring focus is narrower than shown. The pole faces should have such shape as to produce the desired field. The source is at the origin. The cylinders A define a slit which gives the maximum value of β that is allowed to pass. The spiral baffles B should extend from A to C. They should have the same shape as the trajectories in the median plane with k equal to 1; and there should be several of them symmetrically placed around the machine. They will serve to separate positrons from electrons. The hollow cylinders C define a slit that corresponds to the ring focus, its radius and aperture being those in Table I. We assume that the width of the detector goes from the ring focus to the stable orbit and that its height covers the maximum value that z can have in this region. This maximum value of z is 0.047 for sinβ equal to 0.0600 and it is approximately proportional to z.

We can see from Figure 1 that many electrons are not received in the detector because they collide with the cylinders C. This situation can be improved by having two identical detectors at 180°, and we have made calculations for this case too. Of course the best case would be if one could have a ring detector, a ring of some detector just slightly wider than the slit in the ring focus and placed around it. Such detector could possibly be made of a scintillator plastic connected with a light pipe to a photomultiplier tube. We also made calculations for the ring detector.

In Figure 4 are shown three line profiles calculated for one detector, two detectors and a ring detector when the maximum value of sinβ is 0.0500. For the other maximum values of sinβ the line profiles have similar shapes. The results of the transmission and line widths obtained are shown in Table II.

4. Conclusions

The relation between the transmission and the line width derived from the results in Table II is $T = 0.7W^{1/2}$ with one detector; $T = 1.0W^{1/2}$ with two detectors; and $T = 1.05W^{1/2}$ with a ring detector. This means that in this respect the spiral orbit spectrometer with ring focus when used with one detector is slightly below the machine of KOFOED-HANSEN[7] and better than all the other machines we know, including those in the review article of FRISCH[6]. When the spiral orbit spectrometer with ring focus is used with two detectors or with a ring detector, it is also better than the machine of KOFOED-HANSEN[7] in the relation between the transmission and the line width.

The maximum transmission that the spiral orbit spectrometer studied can give are 2.1% with one detector; 3.7% with two detectors; and 7.0% with a ring detector. These value depend on the value of β at which the trajectory is tangent to the ring focus, and on the probability of detection when the electron is between the ring focus and the stable orbit.

We have not examined the effect of the source size on the line width, nor we have examined other types of functions f(r) which might possibly improve further the spiral orbit spectrometer with ring focus.

In regard to the possibility of this machine being sucessfully built, we would like to recall that to make a field with the exact f(r) is not necessary, and the machine will work provided its radial dependence fulfills the condition stated in the introduction. However, the cylindrical symmetry of the field is much more critical for the functioning of the machine.

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TABLE I

The optimum radius and aperture of the slit for ring focusing for several values of the maximum sin β allowed to pass. The radius of the stable orbit is unity.

Max. sin β	Radius (r)	Aperture (2z)
0.0300	0.9476	0.0006
0.0400	0.9463	0.0016
0.0500	0.9443	0.0034
0.0600	0.9419	0.0060

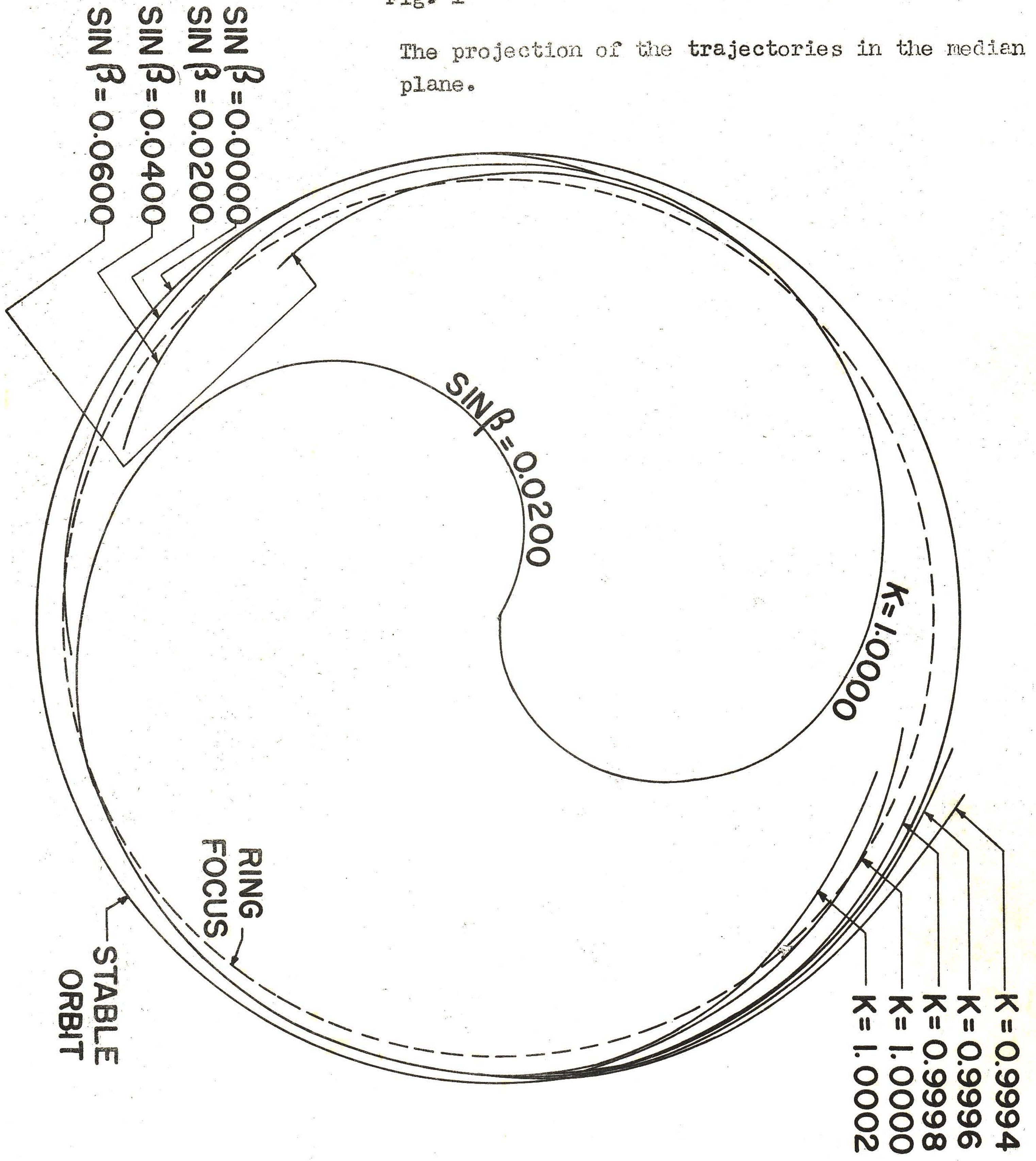
TABLE II

The transmission and the line width for several values of the maximum sin β allowed to pass.

Max. sin β	One detector		Two detectors		Ring detector	
	T	W	T	W	T	W
0.0300	0.0150	0.00033	0.0247	0.00052	0.0300	0.00072
0.0400	0.0174	0.00062	0.0296	0.00078	0.0400	0.00136
0.0500	0.0193	0.00091	0.0334	0.00116	0.0500	0.00232
0.0600	0.0206	0.00111	0.0360	0.00148	0.0600	0.00344

Fig. 1

The projection of the trajectories in the median plane.



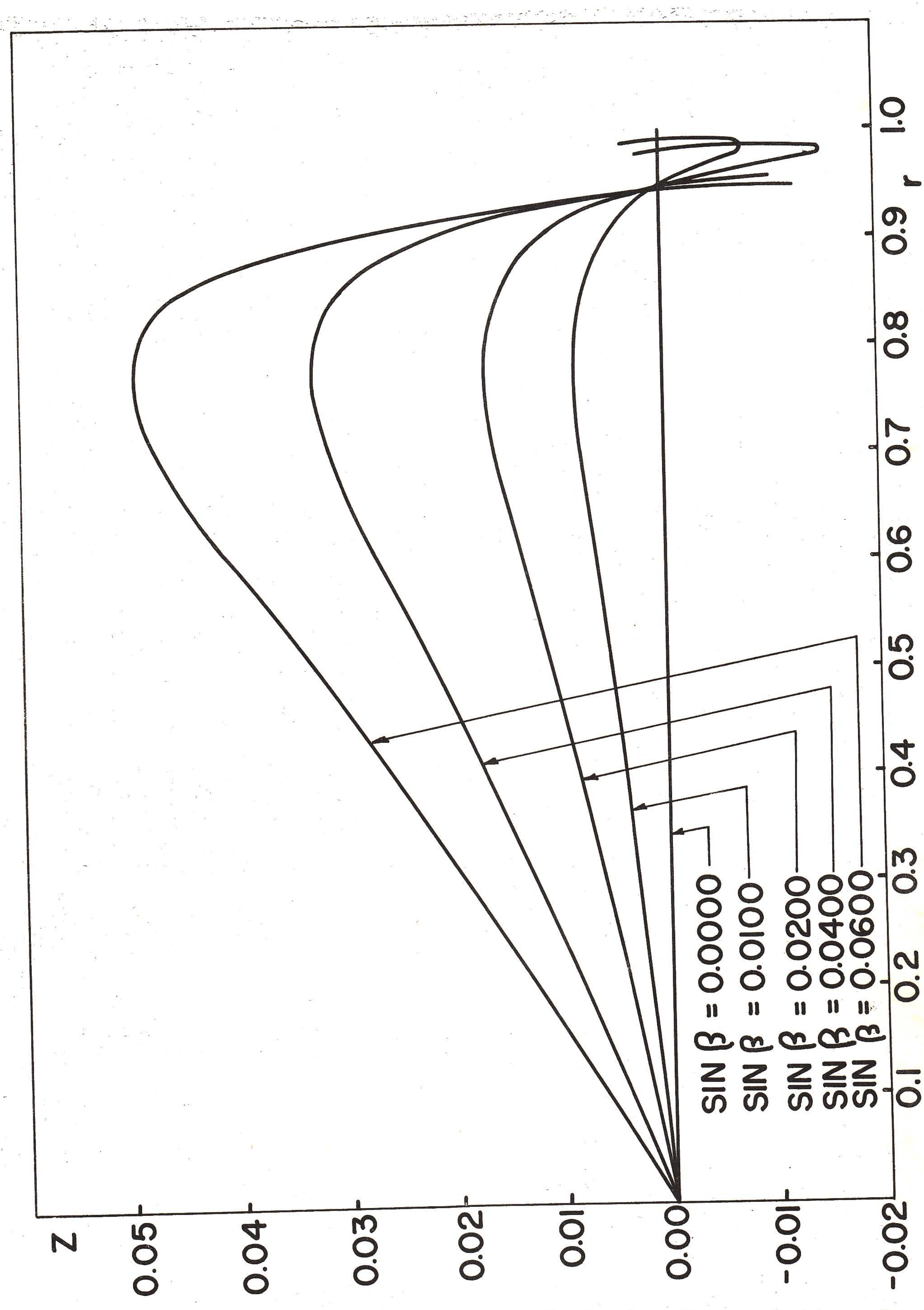


Fig. 2 The projection of the trajectories in a rotating plane containing the cylindrical axis and the electron.

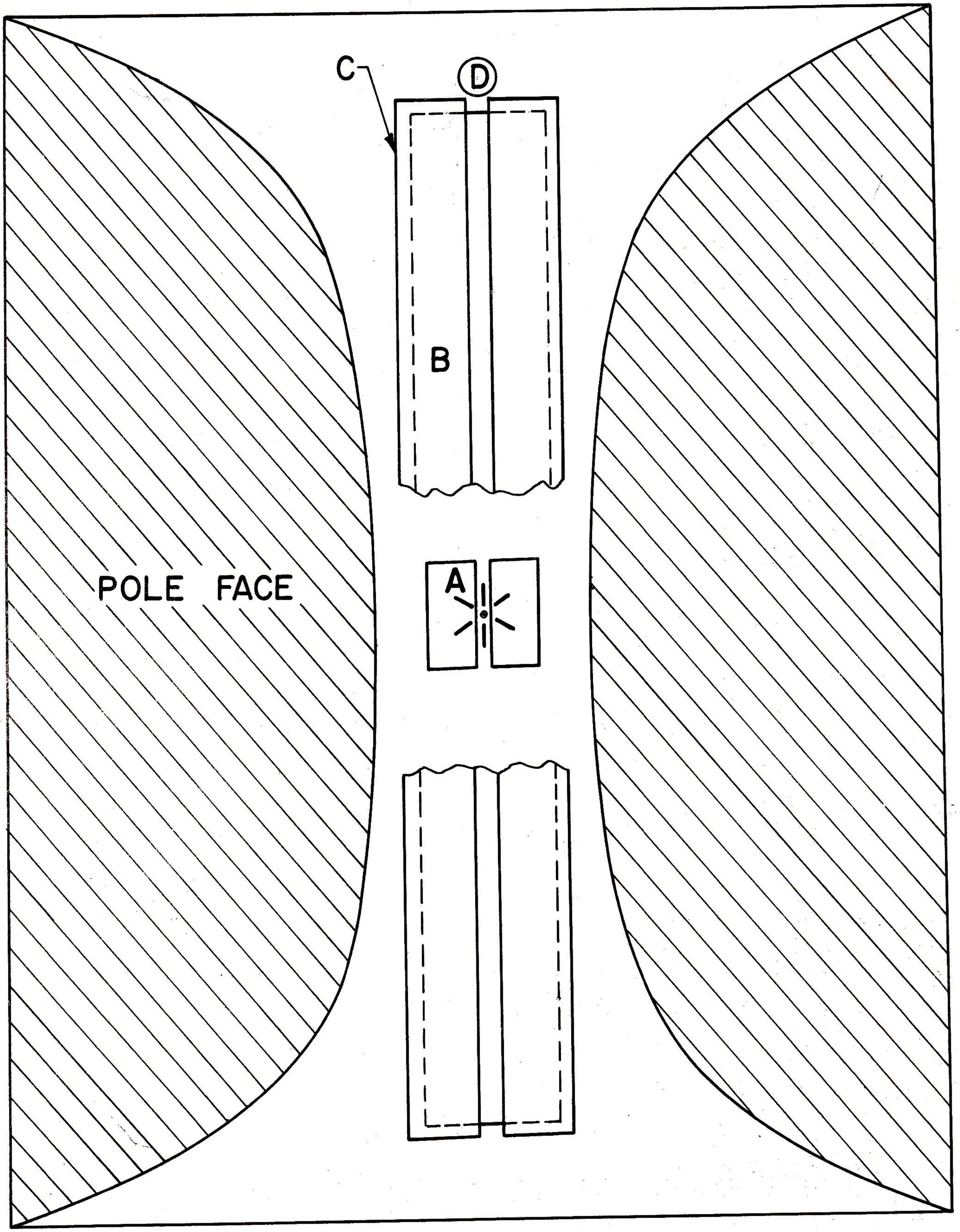


Fig. 3

The spiral orbit spectrometer with ring focus, schematically. For description see text.

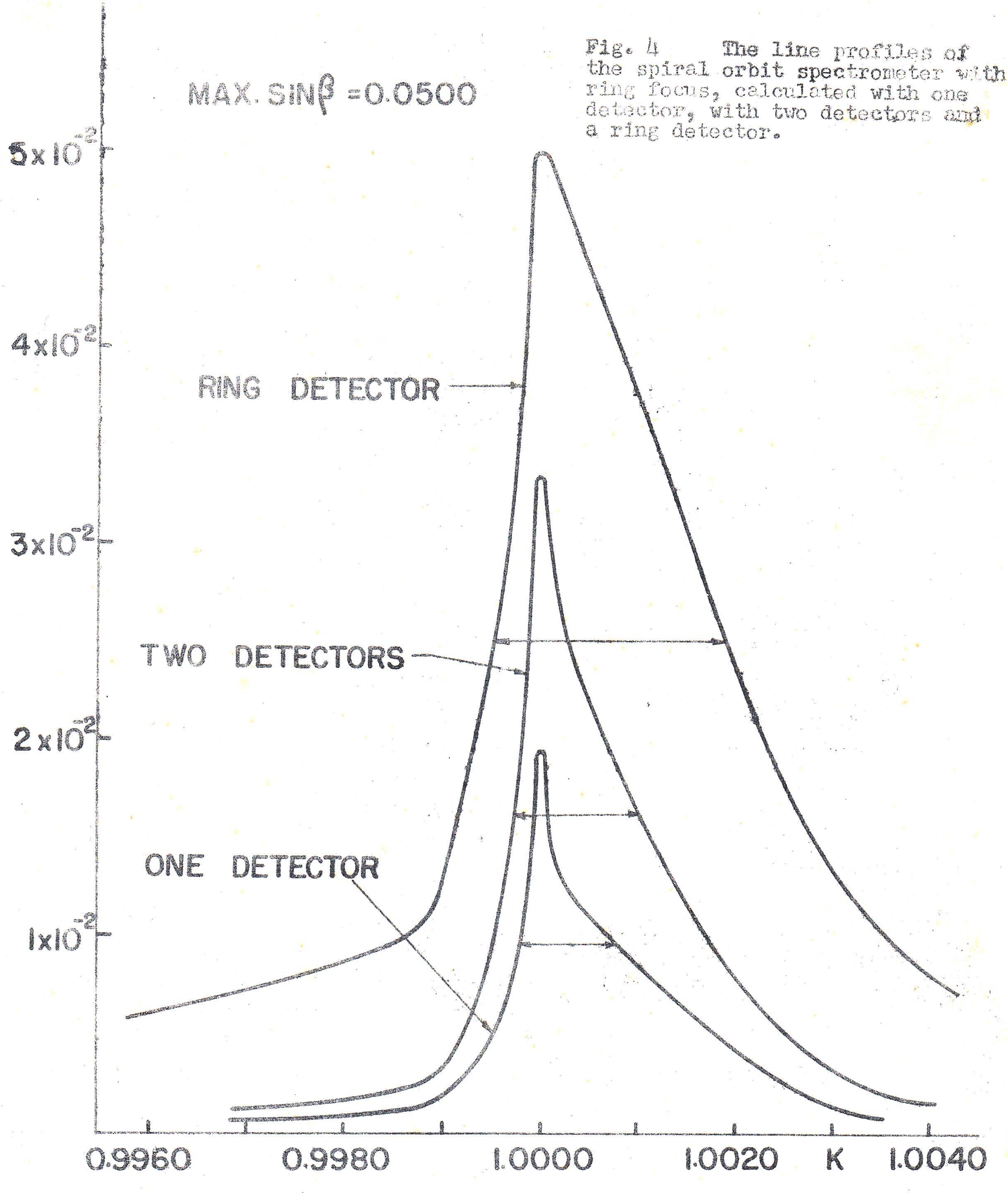


Fig. 4 The line profiles of the spiral orbit spectrometer with ring focus, calculated with one detector, with two detectors and a ring detector.