Chapter 2: Elements of plasma physics

Three theoretical models:

- **Theory of motion of single charged particles** in given magnetic and electric fields; [book: Sec. 2.2]

- **Kinetic theory of a collection of such particles,** describing plasmas *microscopically* by means of particle distribution functions $f_{e,i}(\mathbf{r}, \mathbf{v}, t)$; [book: Sec. 2.3]

- **Fluid theory (magnetohydrodynamics),** describing plasmas in terms of averaged *macroscopic* functions of $\mathbf{r}$ and t . [book: Sec. 2.4]

Within each of these descriptions, we will give an example illustrating the plasma property relevant for our subject, viz. plasma confinement by magnetic fields.

Equation of motion

of charged particle in given electric and magnetic field, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,. \tag{1}$$

- Apply to constant magnetic field $\mathbf{B} = B\mathbf{e}_z$, $\mathbf{E} = 0$:

  (a) projection on $\mathbf{B}$ gives $m\,\dfrac{dv_\parallel}{dt} = 0 \quad \Rightarrow \quad v_\parallel =$ const ,

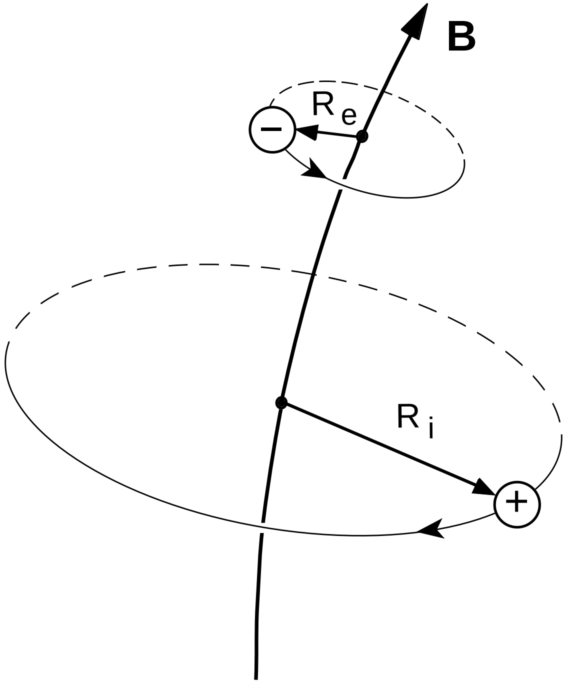
  (b) projection on $\mathbf{v}$ gives $\dfrac{d}{dt}\,(\tfrac{1}{2}mv^2) = 0 \quad \Rightarrow \quad v_\perp =$ const.

- Systematic solution of Eq. (1) with $\mathbf{v} = d\mathbf{r}/dt = (\dot{x}, \dot{y}, \dot{z})$ gives two coupled differential equations for motion in the perpendicular plane:

$$\begin{aligned} \ddot{x} - (qB/m)\,\dot{y} &= 0\,, \\ \ddot{y} + (qB/m)\,\dot{x} &= 0\,. \end{aligned} \tag{2}$$

$\Rightarrow$ periodic motion about a fixed point $x = x_\mathrm{c}$, $y = y_\mathrm{c}$ *(the guiding centre)*.

Cyclotron motion



This yields periodic motion in a magnetic field, with *gyro- (cyclotron) frequency*

$$\Omega \equiv \frac{|q|B}{m} \tag{3}$$

and *cyclotron (gyro-)radius*

$$R \equiv \frac{v_\perp}{\Omega} \approx \frac{\sqrt{2mkT}}{|q|B}\,. \tag{4}$$

$\Rightarrow$ Effectively, charged particles stick to the field lines.

Opposite motion of electrons and ions about guiding centres with quite different gyro-frequencies and radii, since $m_e \ll m_i$:

$$\Omega_e \equiv \frac{eB}{m_e} \gg \Omega_i \equiv \frac{ZeB}{m_i}\,, \qquad R_e \approx \frac{\sqrt{2m_e kT}}{eB} \ll R_i \approx \frac{\sqrt{2m_i kT}}{ZeB}\,. \tag{5}$$

In inhomogeneous fields, these guiding centres drift!

Cyclotron motion (cont'd)

Orders of magnitude

- Typical gyro-frequencies, e.g. for tokamak plasma ($B = 3\,\mathrm{T}$):

$$\Omega_e = 5.3 \times 10^{11}\,\mathrm{rad\,s^{-1}} \quad \text{(frequency of } 84\,\mathrm{GHz}\text{)}\,,$$

$$\Omega_i = 2.9 \times 10^{8}\,\mathrm{rad\,s^{-1}} \quad \text{(frequency of } 46\,\mathrm{MHz}\text{)}\,.$$

- Gyro-radii, with $v_\perp = v_{\mathrm{th}} \equiv \sqrt{2kT/m}$ for $T_e = T_i = 1.16 \times 10^8\,\mathrm{K}$:

$$v_{\mathrm{th},e} = 5.9 \times 10^{7}\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad R_e = 1.1 \times 10^{-4}\,\mathrm{m} \approx 0.1\,\mathrm{mm}\,,$$

$$v_{\mathrm{th},i} = 1.4 \times 10^{6}\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad R_i = 4.9 \times 10^{-3}\,\mathrm{m} \approx 5\,\mathrm{mm}\,.$$

$\Rightarrow$ Tokamak time scales $(\sim 1\,\mathrm{s})$ and dimensions $(\sim 1\,\mathrm{m})$ *justify averaging.*

Since the gyro-frequencies essentially depend on B alone

$\Rightarrow$ excellent diagnostic to *determine the magnetic field strength!*

Relativistic particle motion

- Equation of motion now reads

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \,, \qquad \mathbf{p} = \gamma m \mathbf{v} \quad (\approx m\mathbf{v} \text{ for } v \ll c) \,, \tag{6}$$

  with relativistic momentum $\mathbf{p}$, rest mass m and Lorentz factor $\gamma \equiv (1 - v^2/c^2)^{-1/2}$.

- For motion in constant $\mathbf{B}$,

$$\frac{d\mathbf{p}}{dt} = \frac{q}{\gamma m} \mathbf{p} \times \mathbf{B} \,,$$

  project onto $\mathbf{B}$ and $\mathbf{p}$ $\Rightarrow$ $p_{\|} =$ const and $|\mathbf{p}| =$ const $\Rightarrow$ $v = \mathrm{const}$, $\gamma =$ const .

- *Relativistic gyro-frequency and gyro-radius:*

$$\Omega = \frac{|q|B}{\gamma m} \,, \qquad R = \frac{p_\perp}{|q|B} = \frac{v_\perp}{\Omega} \,. \tag{7}$$

- The ratio $p_\perp/|q| = RB = \gamma m v_\perp/|q|$ depends on particle properties only

  $\Rightarrow$ called *magnetic rigidity* (large for large R, i.e. little deflection by $\mathbf{B}$),

  $\Rightarrow$ useful measure for cosmic ray particle energies.

Drifts

- Single particle motion in constant $\mathbf{E}\ (= E\mathbf{e}_y) \perp$ constant $\mathbf{B}\ (= B\mathbf{e}_z)$.
- Transverse equations of motion:

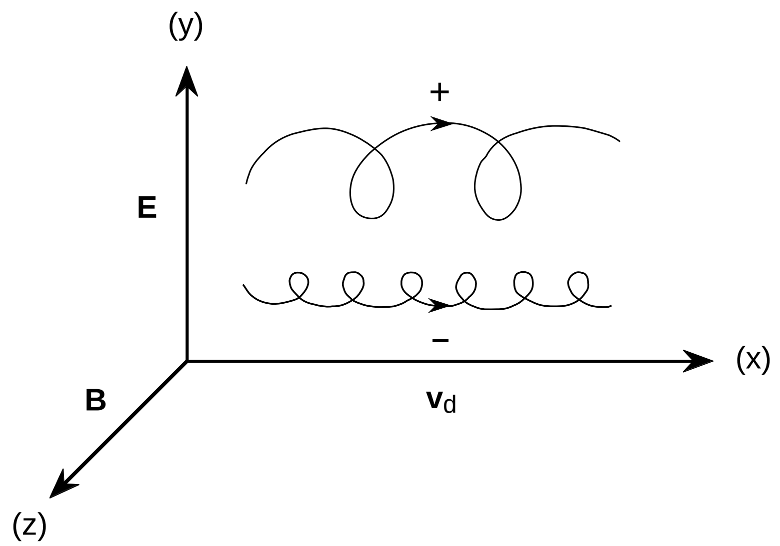
$$\begin{aligned} \ddot{x} - \frac{qB}{m}\dot{y} &= 0\,, \\ \ddot{y} + \frac{qB}{m}(\dot{x} - E/B) &= 0\,, \end{aligned} \tag{8}$$

replacing $\dot{x} \to \dot{x} - E/B \ \Rightarrow$ gyration superposed with constant drift in x-direction.

- Hence, $\perp$ electric field gives $\mathbf{E} \times \mathbf{B}$ drift :

$$\mathbf{v}_\mathrm{d} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}\,, \tag{9}$$

independent of the charge, so that electrons and ions drift in same direction!



Drifts (cont'd)

- Reason: periodic acceleration / deceleration of moving charge in electric field $\mathbf{E}$.
- Lorentz transformation to a frame moving with $\mathbf{v}_\mathrm{d}$ yields:

$$\mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

  $\Rightarrow$ particles move to ensure *vanishing of the electric field in the moving frame !*

- Replace $q\mathbf{E}$ by any other force $\mathbf{F}$:

$$\mathbf{v}_\mathrm{d} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

  $\Rightarrow$ drift velocity now q-dependent: electron and ions drift in opposite directions

  $\Rightarrow$ electric current flow.

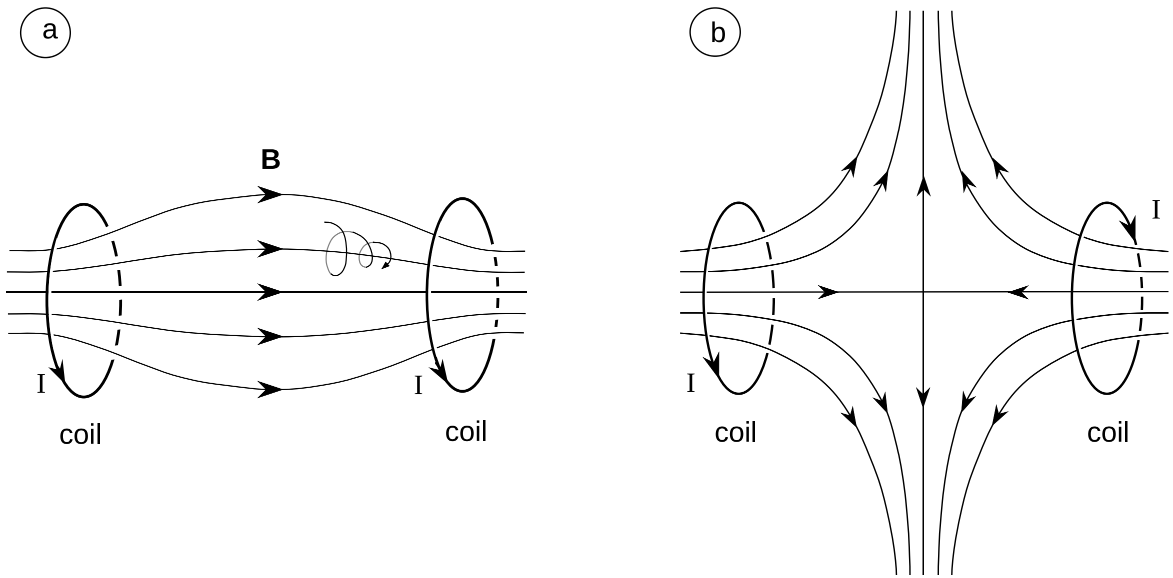
- Other drifts (all due to periodic changes of the gyro-radius)

  $\Rightarrow$ through gradients of the magnetic field: $\mathbf{B} \times \nabla\mathbf{B}$ drift ,

  $\Rightarrow$ through field line curvature (centrifugal force).

Mirror effect

- Particles entering region of higher $|B|$ are reflected back into region of smaller $|B|$ where gyro-radius is larger and $v_\perp$ smaller $\Rightarrow$ (a) mirror, (b) cusp.



- Both confinement schemes have been dropped in thermonuclear fusion research (because of interchange instabilities and leakage through the ends), but the mirror remains important concept to explain *trapping of particles* (e.g. van Allen belts).

- Also, important for the systematic theory of fast periodic particle motion in the slow variation of inhomogeneous magnetic fields $\Rightarrow$ *adiabatic invariants.*

  For example, the reflection of charged particle spiraling into higher field regions of the mirror is described by an adiabatic invariant $\sim v_\perp R$, with $R \sim v_\perp / B$.

Adiabatic invariants

- Allow systematic treatment of periodic motion in inhomogeneous magnetic fields, typically assuming scale gyro-motion $\ll$ scale of inhomogeneities of $\mathbf{B}$.
- Define 'action variables' $J \equiv \oint P\, dQ$ with periodic coordinate Q and generalised momentum $\mathbf{P} \equiv m\mathbf{v} + q\mathbf{A}$, where $\mathbf{A}$ is the vector potential ($\mathbf{B} = \nabla \times \mathbf{A}$).

⇒ *First invariant* (for rapid gyro-motion, the magnetic moment μ is constant):

$$J_1 \equiv \oint \mathbf{P}_\perp \cdot d\boldsymbol{l} = \frac{\pi m v_\perp^2}{\Omega} = \frac{2\pi m}{q}\,\mu\,, \qquad \mu \equiv \pi R^2 I\,. \tag{10}$$

⇒ *Second invariant* (constant for bouncing of particles trapped between mirrors):

$$J_2 \equiv \oint P_\| \, dl \approx \oint m v_z\, dz = \frac{\pi m \hat{v}_z^2}{\omega_\mathrm{b}}\,. \tag{11}$$

⇒ *Third invariant* (constant for slow drift of the guiding centers across the field lines; enclosed flux Ψ_d is also constant):

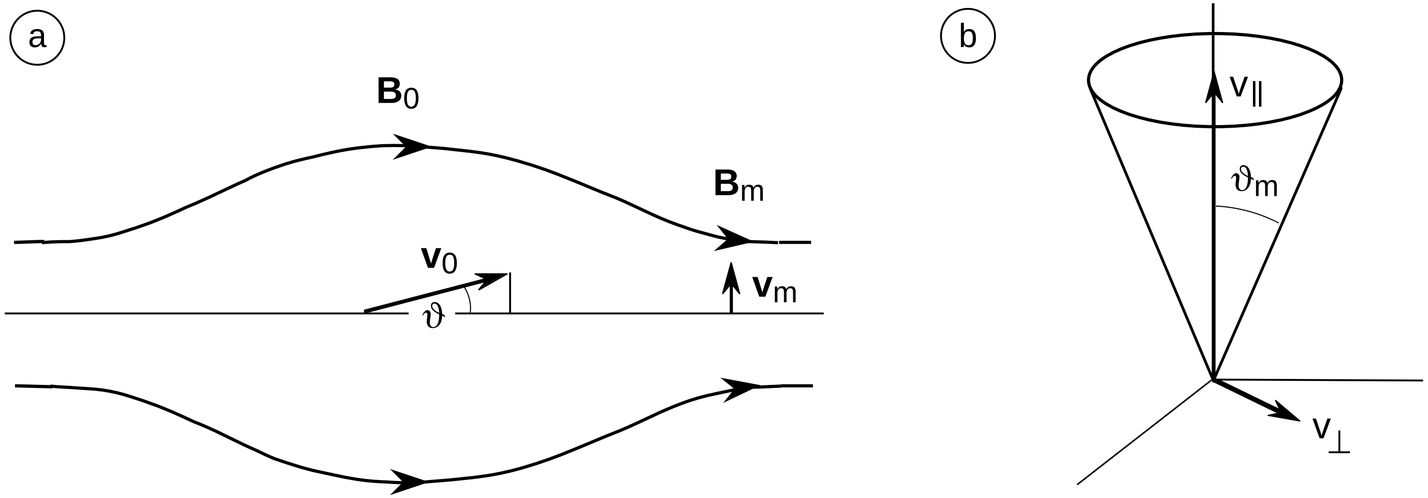
$$J_3 \equiv \oint P_\phi\, r d\phi \approx 2\pi q\, r A_\phi = q\,\Psi_\mathrm{d}\,, \qquad \Psi_\mathrm{d} = 2\pi \int_{r_0}^{r} B_z r\, dr\,. \tag{12}$$

Application to mirror

- Exploit constancy of J_1 $(\sim \mu = \frac{1}{2} m v_\perp^2 / B \,)$ to analyse motion into mirror field:

  $v_\perp$ increases with $B \;\Rightarrow\; v_\parallel$ decreases (energy conservation) $\;\Rightarrow\;$ reflection.

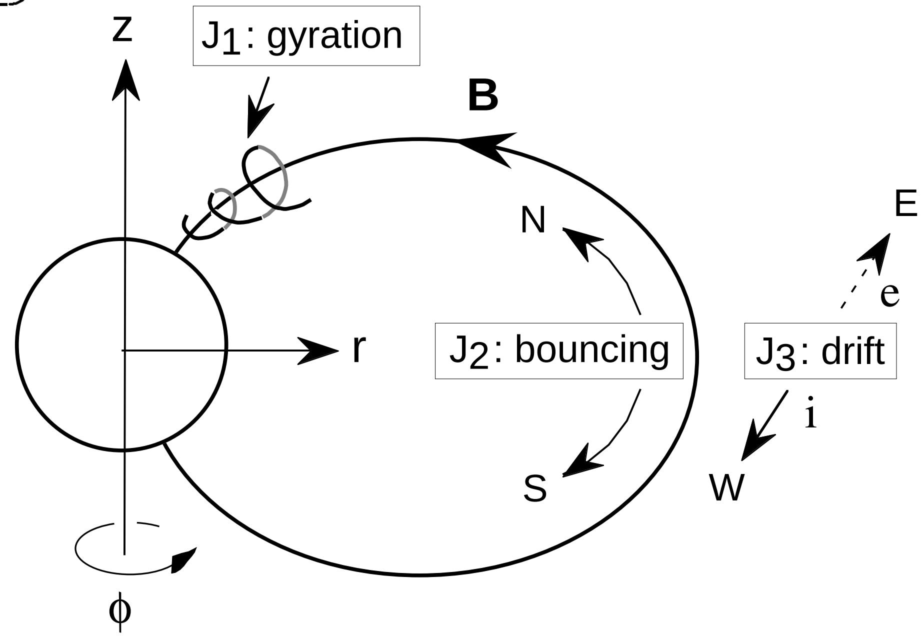
- Not all particles are reflected: particles for small enough $v_\perp / v_\parallel$ are lost.



Transition trapped–untrapped from energy conservation and adiabatic invariant:

$$\left.\begin{array}{r} v_{\parallel,0}^2 + v_{\perp,0}^2 = v_{\perp,m}^2 \\ v_{\perp,0}^2 / B_0 = v_{\perp,m}^2 / B_m \end{array}\right\} \Rightarrow \quad \textit{loss cone:}\;\; \vartheta < \vartheta_m \equiv \mathrm{arctg} \sqrt{\frac{B_0}{B_m - B_0}}\,.$$

Application to magnetosphere



Example: Charged particles in the magnetosphere.

(a) Electrons and ions rapidly gyrate about the magnetic field, conserving J_1;

(b) The guiding centres bounce back and forth between the mirrors on the northern and southern hemisphere on a slower time scale, conserving J_2;

(c) They drift in opposite longitudinal directions on a slower time scale yet, conserving J_3 (magnetic flux inside the drift shell): This invariance is easily invalidated by the fluctuating interaction of the solar wind with the magnetosphere.

Distribution functions

- A plasma consists of a very large number of interacting charged particles $\Rightarrow$ *kinetic plasma theory* derives the equations describing the *collective behavior* of the many charged particles by applying the methods of statistical mechanics.

- The physical information of a plasma consisting of electrons and ions is expressed in terms of *distribution functions* $f_\alpha(\mathbf{r},\mathbf{v},t)$, where $\alpha = \mathrm{e},\mathrm{i}$. They represent the density of particles of type α in the *phase space* of position and velocity coordinates. The probable number of particles α in the 6D volume element centered at $(\mathbf{r},\mathbf{v})$ is given by $f_\alpha(\mathbf{r},\mathbf{v},t)\,d^3r\,d^3v$. The motion of the swarm of phase space points is described by the total time derivative of f_α:

$$\begin{aligned}\frac{\mathrm{d}f_\alpha}{\mathrm{d}t} &\equiv \frac{\partial f_\alpha}{\partial t} + \frac{\partial f_\alpha}{\partial \mathbf{r}}\cdot\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \frac{\partial f_\alpha}{\partial \mathbf{v}}\cdot\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \\ &= \frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}}\,. \qquad (13)\end{aligned}$$

Boltzmann equation

- Interactions (collisions) between the particles determine this time derivative:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = C_\alpha \equiv \left(\frac{\partial f_\alpha}{\partial t} \right)_{\rm coll} . \qquad (14)$$

- Here, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ are the sum of the external fields *and* the averaged internal fields due to the long-range inter-particle interactions. C_α represents *the rate of change of the distribution function due to the short-range inter-particle collisions*. In a plasma, these are the cumulative effect of many small-angle velocity changes effectively resulting in large-angle scattering. The first task of kinetic theory is to justify this distinction between long-range interactions and binary collisions, and to derive expressions for the collision term.

- One such expression is the *Landau collision integral* (1936). Neglect of the collisions (surprisingly often justified!) leads to the *Vlasov equation* (1938).

Completing the system

- *Combine the Boltzmann equation*, determining $f_\alpha(\mathbf{r}, \mathbf{v}, t)$, *with Maxwell's equations*, determining $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. In the latter, charge density $\tau(\mathbf{r}, t)$ and current density $\mathbf{j}(\mathbf{r}, t)$ appear as source terms. They are related to the particle densities $n_\alpha(\mathbf{r}, t)$ and the average velocities $\mathbf{u}_\alpha(\mathbf{r}, t)$:

$$\tau(\mathbf{r}, t) \equiv \sum q_\alpha n_\alpha \,, \qquad n_\alpha(\mathbf{r}, t) \equiv \int f_\alpha(\mathbf{r}, \mathbf{v}, t)\, d^3v \,, \tag{15}$$

$$\mathbf{j}(\mathbf{r}, t) \equiv \sum q_\alpha n_\alpha \mathbf{u}_\alpha \,, \quad \mathbf{u}_\alpha(\mathbf{r}, t) \equiv \frac{1}{n_\alpha(\mathbf{r}, t)} \int \mathbf{v} f_\alpha(\mathbf{r}, \mathbf{v}, t)\, d^3v. \tag{16}$$

  This completes the microscopic equations.

- Solving such kinetic equations in seven dimensions (with the details of the single particle motions entering the collision integrals!) is a formidable problem
  $\Rightarrow$ look for *macroscopic reduction!*

Moment reduction

- Systematic procedure to obtain macroscopic equations, no longer involving velocity space details, is to expand in *finite number of moments of the Boltzmann equation*, by multiplying with powers of $\mathbf{v}$ and integrating over velocity space:

$$\int d^3v \cdots , \quad \int d^3v \, \mathbf{v} \cdots , \quad \int d^3v \, v^2 \cdots \Big|_{\text{truncate}} . \tag{17}$$

- E.g., the *zeroth moment* of the Boltzmann equation contains the terms:

$$\int \frac{\partial f_\alpha}{\partial t} d^3v = \frac{\partial n_\alpha}{\partial t} , \qquad \int \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} d^3v = \nabla \cdot (n_\alpha \mathbf{u}_\alpha) ,$$

$$\int \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} d^3v = 0 , \qquad \int C_\alpha \, d^3v = 0 .$$

Adding them yields the *continuity equation* for particles of species α:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0 . \tag{18}$$

Moment reduction (cont'd)

- The *first moment* of the Boltzmann equation yields the *momentum equation:*

$$\frac{\partial}{\partial t}(n_\alpha m_\alpha \mathbf{u}_\alpha) + \nabla \cdot (n_\alpha m_\alpha \langle \mathbf{v}\mathbf{v} \rangle_\alpha) - q_\alpha n_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) = \int C_{\alpha\beta}\, m_\alpha \mathbf{v}\, d^3 v\,. \quad (19)$$

- The *scalar second moment* of Boltzmann Eq. yields the *energy equation:*

$$\frac{\partial}{\partial t}(n_\alpha \tfrac{1}{2} m_\alpha \langle v^2 \rangle_\alpha) + \nabla \cdot (n_\alpha \tfrac{1}{2} m_\alpha \langle v^2 \mathbf{v} \rangle_\alpha) - q_\alpha n_\alpha \mathbf{E} \cdot \mathbf{u}_\alpha = \int C_{\alpha\beta} \tfrac{1}{2} m_\alpha v^2\, d^3 v\,. \quad (20)$$

- This chain of moment equations can be continued indefinitely. Each moment introduces a new unknown whose temporal evolution is described by the next moment of the Boltzmann equation. The infinite chain must be truncated to be useful. *In fluid theories truncation is just after the above five moments:* continuity (scalar), momentum (vector), and energy equation (scalar).
  How to justify?

Thermal fluctuations

- Split the particle velocity $\mathbf{v}$ in an average part $\mathbf{u}_\alpha$ and a fluctuating part $\tilde{\mathbf{v}}_\alpha$:

$$\tilde{\mathbf{v}}_\alpha \equiv \mathbf{v} - \mathbf{u}_\alpha\,, \quad \text{where } \langle \tilde{\mathbf{v}}_\alpha \rangle = 0\,. \tag{21}$$

This permits the definition of *thermal quantities:*

$$T_\alpha(\mathbf{r},t) \equiv \frac{m_\alpha}{3k}\langle \tilde{v}_\alpha^2 \rangle\,, \qquad p_\alpha \equiv n_\alpha k T_\alpha\,, \qquad \textit{(temperature, pressure)} \tag{22}$$

$$\mathbf{P}_\alpha(\mathbf{r},t) \equiv n_\alpha m_\alpha \langle \tilde{\mathbf{v}}_\alpha \tilde{\mathbf{v}}_\alpha \rangle = p_\alpha \mathbf{I} + \boldsymbol{\pi}_\alpha\,, \qquad \textit{(stress tensor)} \tag{23}$$

$$\mathbf{h}_\alpha(\mathbf{r},t) \equiv \tfrac{1}{2} n_\alpha m_\alpha \langle \tilde{v}_\alpha^2 \tilde{\mathbf{v}}_\alpha \rangle\,, \qquad \textit{(heat flow)} \tag{24}$$

$$\mathbf{R}_\alpha(\mathbf{r},t) \equiv m_\alpha \int C_{\alpha\beta}\, \tilde{\mathbf{v}}_\alpha\, d^3v\,, \qquad \textit{(momentum transfer)} \tag{25}$$

$$Q_\alpha(\mathbf{r},t) \equiv \tfrac{1}{2} m_\alpha \int C_{\alpha\beta}\, \tilde{v}_\alpha^2\, d^3v\,. \qquad \textit{(heat transfer)} \tag{26}$$

Progress by hiding the problems in abbreviations of intricate kinetic processes? Additional information needed about the variables $\boldsymbol{\pi}_\alpha$, $\mathbf{h}_\alpha$, $\mathbf{R}_\alpha$, Q_α to express them in terms of the macroscopic variables n_α, $\mathbf{u}_\alpha$, T_α to close the set!

Maxwell–Boltzmann distribution

- Velocity distribution function for *thermal equilibrium :*

$$f^0_\alpha(\mathbf{r}, \mathbf{v}, t) = n_\alpha \Big(\frac{m_\alpha}{2\pi k T_\alpha}\Big)^{3/2} \exp\Big(-\frac{m_\alpha \tilde{v}^2_\alpha}{2kT_\alpha}\Big) \,. \tag{27}$$

  ⇒ LHS Boltzmann equation (14) vanishes ⇒ $\left(\dfrac{\partial f_\alpha}{\partial t}\right)_{\rm coll} = 0\,.$

  ⇒ solution consistent with definitions of T_α , etc.

- For plasma with two species $\alpha = e,\, i$

  ⇒ each species has Maxwellian velocity distribution,

  ⇒ full equilibrium only when $\mathbf{u}_e = \mathbf{u}_i$ and $T_e = T_i$.

- **Plasma kinetic theory**

  ⇒ deals with deviations from this thermal equilibrium

  ⇒ and the way in which *collisions cause relaxation to thermal equilibrium.*

Closure

- Equations of continuity, momentum, and heat balance then take the form:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0\,, \tag{28}$$

$$n_\alpha m_\alpha \, (\frac{\partial \mathbf{u}_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \mathbf{u}_\alpha) + \nabla \cdot \mathbf{P}_\alpha - n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) = \mathbf{R}_\alpha, \tag{29}$$

$$\tfrac{3}{2} n_\alpha k \, (\frac{\partial T_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla T_\alpha) + \mathbf{P}_\alpha : \nabla \mathbf{u}_\alpha + \nabla \cdot \mathbf{h}_\alpha = Q_\alpha\,. \tag{30}$$

- *The truncated set of moment equations is closed by exploiting the transport coefficients (derived from transport theory)* between the thermal quantities and the gradients of the macroscopic variables. Schematically:

$$\begin{aligned} \boldsymbol{\pi}_\alpha &\sim \mu_\alpha \nabla \mathbf{u}_\alpha & & \textit{(viscosity)}, \\ \mathbf{h}_\alpha &\sim -\kappa_\alpha \nabla (k T_\alpha) & & \textit{(heat conductivity)}, \\ \mathbf{R}_\alpha &\approx -q_\alpha n_\alpha \eta \, \mathbf{j}\,, \qquad \sum Q_\alpha \approx \eta \, |\mathbf{j}|^2 & & \textit{(resistivity)}. \end{aligned} \tag{31}$$

Deriving these coefficients is the second (formidabale) task of kinetic theory.

Collective phenomena: Plasma oscillations

- Extend concepts of quasi-neutrality and Debye length in two steps:

  (a) Perturbations of quasi-neutrality by *plasma oscillations*

  $\Rightarrow$ application of moment equations, 1) neglecting $\mathbf{P}_\alpha$, $\mathbf{h}_\alpha$, $\mathbf{R}_\alpha$, Q_α (cold),

  2) keeping $p_\alpha = n_\alpha k T_\alpha$ (finite pressure).

  (b) Thermal effects on Debye length scale through *Landau damping*

  $\Rightarrow$ application of kinetic equations.

- **Cold plasma oscillations** described by continuity equation (28):

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0 \qquad (\alpha = \text{e, i})\,, \tag{32}$$

simplified ($\mathbf{B} = 0$) momentum equation (29):

$$m_\alpha \Big(\frac{\partial \mathbf{u}_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \mathbf{u}_\alpha\Big) = q_\alpha \, \mathbf{E} \qquad (\alpha = \text{e, i})\,, \tag{33}$$

and $\mathbf{E}$ from Poisson's equation with charge density (15):

$$\nabla \cdot \mathbf{E} = \frac{\tau}{\epsilon_0} = \frac{e}{\epsilon_0}(Z n_i - n_e)\,. \tag{34}$$

(Cold) Plasma oscillations

- Simplify further:

  $m_i \gg m_e$: ions immobile ($\mathbf{u}_i \approx 0$), approx. charge balance ($n_i \approx n_0/Z$), small charge imbalances by slightly displacing the electrons:

$$n_e \approx n_0 + n_1(\mathbf{r}, t)\,, \qquad \mathbf{u}_e \approx \mathbf{u}_1(\mathbf{r}, t)\,. \tag{35}$$

- Yields linearized equations for the electron variables:

$$\begin{aligned} \frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{u}_1 = 0\,, \\ m_e \frac{\partial \mathbf{u}_1}{\partial t} = -e\mathbf{E}_1\,, \\ \nabla \cdot \mathbf{E}_1 = \frac{\tau_1}{\epsilon_0} = -\frac{e}{\epsilon_0} n_1\,. \end{aligned} \tag{36}$$

- May be reduced to a single wave equation for n_1:

$$\frac{\partial^2 n_1}{\partial t^2} = -n_0 \nabla \cdot \frac{\partial \mathbf{u}_1}{\partial t} = \frac{n_0 e}{m_e} \nabla \cdot \mathbf{E}_1 = -\frac{n_0 e^2}{\epsilon_0 m_e} n_1\,. \tag{37}$$

Plasma frequency and Debye length

- Solutions $n_1(\mathbf{r}, t) = \hat{n}_1(\mathbf{r}) \exp(-i\omega t)$ represent electron density oscillations, called *plasma oscillations*, with a characteristic frequency, called *the plasma frequency*:

$$\omega = \pm\omega_{pe} \,, \quad \omega_{pe} \equiv \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}} \,. \tag{38}$$

  For tokamak ($n_0 = 10^{20}\,\mathrm{m}^{-3}$): $\omega_{pe} = 5.7 \times 10^{11}\,\mathrm{rad\,s}^{-1}$ (i.e. $91\,\mathrm{GHz}$) , which is of the same order of magnitude as Ω_e for strong magnetic field ($B \sim 3\,\mathrm{T}$).

- Note: the spatial form of $\hat{n}_1(\mathbf{r})$ is not determined in cold plasma theory. This becomes different for “warm” plasmas, where deviations from charge neutrality due to thermal fluctuations occur in small regions of a size of the order of the *Debye length*

$$\lambda_\mathrm{D} \equiv \sqrt{\frac{\epsilon_0 k_\mathrm{B} T_e}{n_0 e^2}} = \frac{v_{\mathrm{th},e}}{\sqrt{2}\,\omega_{pe}} \,. \tag{39}$$

  For thermonuclear plasma ($\widetilde{T} = 10\,\mathrm{keV}$): $\lambda_\mathrm{D} = 7.4 \times 10^{-5}\,\mathrm{m} \approx 0.07\,\mathrm{mm}$, i.e. of the order of the electron gyro-radius R_e.

(Finite pressure) Plasma oscillations

- **Finite pressure plasma oscillations** described by:

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{u}_1 = 0\,, \tag{40}$$

$$n_0 m_e \frac{\partial \mathbf{u}_1}{\partial t} + \nabla p_1 = -e n_0 \mathbf{E}_1\,, \tag{41}$$

$$\frac{\partial p_1}{\partial t} + \gamma p_0 \nabla \cdot \mathbf{u}_1 = 0\,, \tag{42}$$

$$\nabla \cdot \mathbf{E}_1 = -\frac{e}{\epsilon_0} n_1\,. \tag{43}$$

- Assuming plane waves $n_1(x,t) = \hat{n}_1 \exp i(kx - \omega t)$, and similarly for $\mathbf{u}_1$, p_1, $\mathbf{E}_1$, the gradients $\nabla \to ik\mathbf{e}_x$ and the time derivatives $\partial/\partial t \to -i\omega$, so that Eqs. (40)–(43) become an algebraic system of equations for the amplitudes $\hat{n}_1$, $\hat{\mathbf{u}}_1$, $\hat{p}_1$, and $\hat{\mathbf{E}}_1$. The determinant provides the dispersion equation:

$$\omega^2 = \omega_{pe}^2 (1 + \gamma k^2 \lambda_{\mathrm{D}}^2)\,. \tag{44}$$

  However, this thermal correction of the dependence of ω on k turns out to be incomplete (misses the damping obtained in the proper kinetic derivation).

Collective phenomena: Landau damping

- A more refined analysis of plasma oscillations for “warm” plasmas takes into account velocity space effects, exploiting the Vlasov (or *collisionless* Boltzmann) equation for the perturbations $f_1(\mathbf{r}, \mathbf{v}, t)$ of the electron distribution function. With plane wave solutions $\sim \exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$, one runs into a mathematical problem:

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\cdot\frac{\partial f_1}{\partial \mathbf{r}} = -i(\omega - \mathbf{k}\cdot\mathbf{v})\, f_1 = \frac{e}{m_e}\mathbf{E}_1\cdot\frac{\partial f_0}{\partial \mathbf{v}}\,, \tag{45}$$

  To express f_1 in terms of $\mathbf{E}_1$ one needs to invert the operator $\partial/\partial t + \mathbf{v}\cdot\partial/\partial\mathbf{r}$, which is singular for every $\omega - \mathbf{k}\cdot\mathbf{v} = 0$. Incorporated in a proper treatment of the initial value problem, these singularities were shown by Landau (1946) to give rise to damping of the plasma oscillations. This *Landau damping* is a surprising phenomenon since it occurs in a purely collisionless medium: there is no dissipation!

- An alternative, normal mode, analysis was given by Van Kampen (1955). He showed that the singularities $\omega - \mathbf{k}\cdot\mathbf{v} = 0$ lead to a continuous spectrum of singular modes which constitute a complete set of ‘improper’ eigenmodes for this system. Damping occurs because a package of those modes rapidly looses its spatial phase coherence (*phase mixing*). [Continuous spectra also occur in MHD (as we will see later)!]

'Dispersion equation' (Vlasov)

- 1D Vlasov-Poisson problem:

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial x} = \frac{e}{m_e}\frac{\partial f_0}{\partial v}E_1\,, \qquad \frac{\partial E_1}{\partial x} = -\frac{e}{\epsilon_0}n_1 = -\frac{e}{\epsilon_0}\int_{-\infty}^{\infty} f_1\,dv\,. \tag{46}$$

leads to

$$-\,i(\omega - kv)\,\hat{f}_1 = \frac{e}{m_e}\frac{\partial f_0}{\partial v}\hat{E}_1\,, \qquad ik\hat{E}_1 = -\frac{e}{\epsilon_0}\int_{-\infty}^{\infty}\hat{f}_1\,dv\,. \tag{47}$$

- For $\omega \neq kv$, this would give

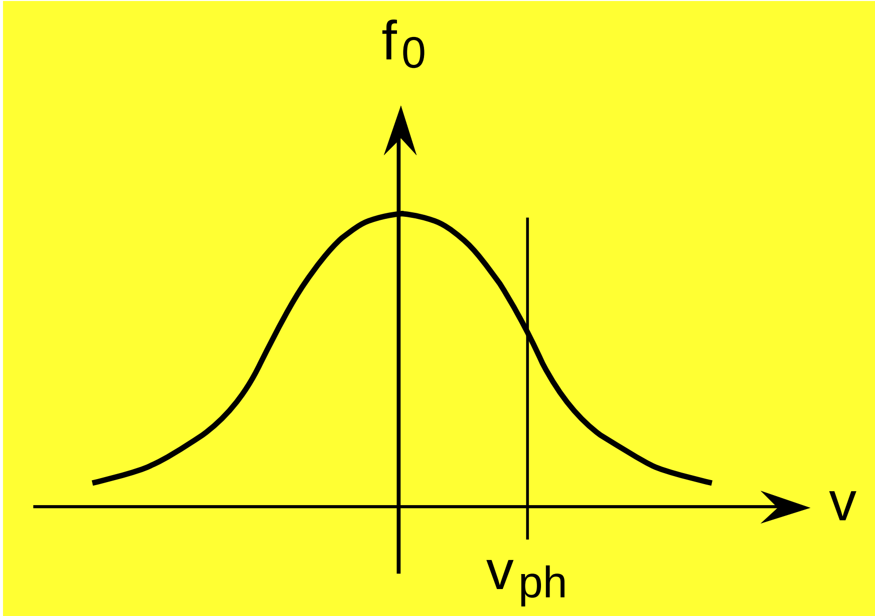
$$\left[1 - \frac{\omega_{pe}^2}{k^2 n_0}\int_{-\infty}^{\infty}\frac{1}{v - \omega/k}\frac{\partial f_0}{\partial v}\,dv\right]\hat{E}_1 = 0\,, \tag{48}$$

where vanishing of the square bracket would provide the *dispersion equation*.

- The singularity $\omega = kv$ was treated in a cavalier manner by Vlasov by exploiting the principal value of the integral for real ω. This reproduced the fluid expression (44) (with $\gamma = 3$). However, there is no justification for this procedure.

Landau's result

- Obviously, assumption $\omega \neq kv$ cannot be justified if frequency ω of the plane waves is real since integration is then right accross the singularity. This singularity occurs for particles with speeds that are resonant with the phase velocity of the waves: $v = v_{\rm ph} \equiv \omega/k$ (vertical line).



- Landau's careful analysis of the singularity revealed that there is an imaginary contribution (the Landau damping) to the frequency of the waves:

$$\omega \approx \omega_{pe}\{1 + \tfrac{3}{2}k^2\lambda_{\rm D}^2 - i\sqrt{\tfrac{\pi}{8}}\,(k\lambda_{\rm D})^{-3}\,\exp\big[-\tfrac{1}{2}(k\lambda_{\rm D})^{-2} - \tfrac{3}{2}\big]\}\,, \qquad (49)$$

- For short wavelengths ($k\lambda_{\rm D} \sim 1$), the damping becomes so strong that wave motion with wavelengths smaller than the Debye length becomes impossible.

From kinetic theory to fluid description

- **(a) Collisionality:** Lowest moments of Boltzmann equation with transport closure gives system of *two-fluid equations* in terms of the ten variables $n_{e,i}$, $\mathbf{u}_{e,i}$, $T_{e,i}$. To establish the two fluids, the electrons and ions must undergo *frequent collisions*:

$$\tau_{\rm H} \gg \tau_i \ \left[\ \gg \tau_e \ \right]. \tag{50}$$

- **(b) Macroscopic scales:** Since the two-fluid equations still involve small length and time scales ($\lambda_{\rm D}$, $R_{e,i}$, ω_{pe}^{-1}, $\Omega_{e,i}^{-1}$), the essential step towards the *MHD* description is to consider *large length and time scales*:

$$\lambda_{\rm MHD} \sim a \gg R_i \,, \qquad \tau_{\rm MHD} \sim a/v_{\rm A} \gg \Omega_i^{-1} \,. \tag{51}$$

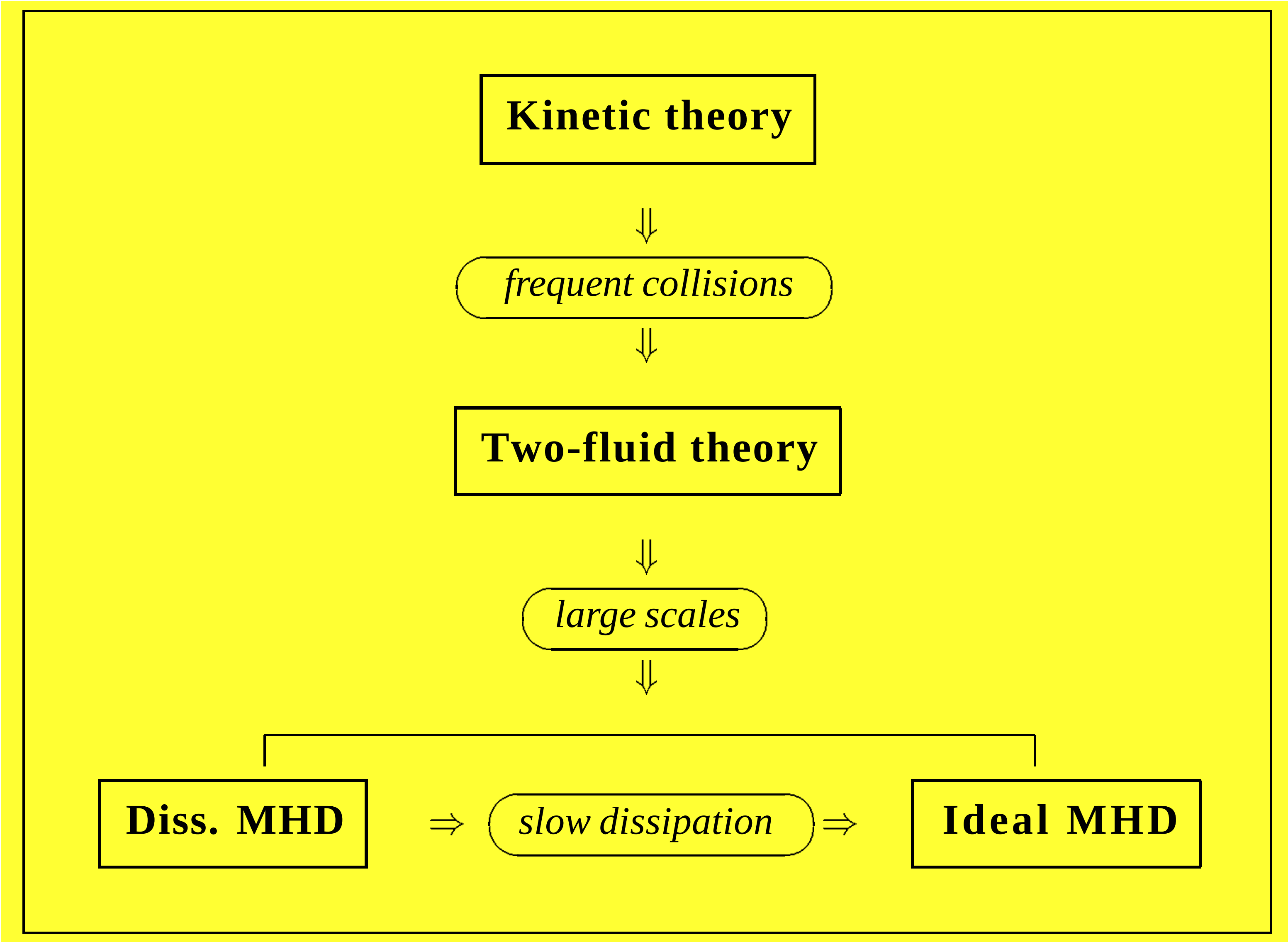
  The larger the magnetic field strength, the more easy these conditions are satisfied. On these scales, the plasma is considered as a *single conducting fluid*.

- **(c) Ideal fluids:** Third step is to consider plasma dynamics on time scales *faster than the slow dissipation* causing the resistive decay of the magnetic field:

$$\tau_{\rm MHD} \ll \tau_{\rm R} \sim a^2/\eta \,. \tag{52}$$

  This condition is well satisfied for the small size of fusion machines, and very easily for the sizes of astrophysical plasmas $\Rightarrow$ model of *ideal MHD*.

In summary:


Kinetic theory
frequent collisions
Two-fluid theory
large scales
Diss. MHD
slow dissipation
Ideal MHD

Resistive two-fluid equations

- Plasma consists of electrons, $q_e = -e$, and one kind of ions, $q_i = Ze$;
- Neglect most of the dissipative terms:

$$\boldsymbol{\pi}_{e,i} \to 0\,, \quad \mathbf{h}_{e,i} \to 0\,; \quad \textit{(neglect of viscosity and heat flow)} \tag{53}$$

- Keep momentum transfer and generated heat associated with resistivity:

$$\mathbf{R}_e = -\mathbf{R}_i \approx e n_e \eta \, \mathbf{j}\,, \qquad Q_e + Q_i = -(\mathbf{u}_e - \mathbf{u}_i) \cdot \mathbf{R}_e \approx \eta |\mathbf{j}|^2\,. \quad \textit{(resistivity)} \tag{54}$$

⇒ *Resistive two-fluid equations* (with $\alpha = e, i$):

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0\,, \tag{55}$$

$$n_\alpha m_\alpha \Big(\frac{\partial \mathbf{u}_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \mathbf{u}_\alpha\Big) + \nabla p_\alpha - n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) = \mathbf{R}_\alpha\,, \tag{56}$$

$$\frac{\partial p_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla p_\alpha + \gamma p_\alpha \nabla \cdot \mathbf{u}_\alpha = (\gamma - 1) Q_\alpha\,. \tag{57}$$

This set is completed by adding Maxwell's equations.

Resistive MHD equations

- Define one-fluid variables that are linear combinations of the two-fluid variables:

$$\rho \equiv n_e m_e + n_i m_i \,, \qquad \textit{(total mass density)} \tag{58}$$

$$\tau \equiv -e\,(n_e - Z n_i)\,, \qquad \textit{(charge density)} \tag{59}$$

$$\mathbf{v} \equiv (n_e m_e \mathbf{u}_e + n_i m_i \mathbf{u}_i)/\rho\,, \qquad \textit{(center of mass velocity)} \tag{60}$$

$$\mathbf{j} \equiv -e\,(n_e \mathbf{u}_e - Z n_i \mathbf{u}_i)\,, \qquad \textit{(current density)} \tag{61}$$

$$p \equiv p_e + p_i\,. \qquad \textit{(pressure)} \tag{62}$$

- Operate on pairs of the two-fluid equations (55)–(57):

$$m_e\,(55)_e + m_i\,(55)_i \;\Rightarrow\; \partial\rho/\partial t\,, \qquad -\,e\,(55)_e + Ze\,(55)_i \;\Rightarrow\; \partial\tau/\partial t\,,$$

$$(56)_e + (56)_i \;\Rightarrow\; \partial\mathbf{v}/\partial t\,, \qquad -\frac{e}{m_e}\,(56)_e + \frac{Ze}{m_i}\,(56)_i \;\Rightarrow\; \partial\mathbf{j}/\partial t\,,$$

$$(57)_e + (57)_i \;\Rightarrow\; \partial p/\partial t\,, \qquad \text{assume } T = T_e = T_i\,.$$

- Evolution expressions for τ and $\mathbf{j}$ disappear by exploiting:

$$|n_e - Z n_i| \ll n_e\,, \qquad \textit{(quasi charge-neutrality)} \tag{63}$$

$$|\mathbf{u}_i - \mathbf{u}_e| \ll v\,, \qquad \textit{(small relative velocity of ions \& electrons)} \tag{64}$$

$$v \ll c\,. \qquad \textit{(non-relativistic speeds)} \tag{65}$$

Resistive MHD equations (cont'd)

Combining one-fluid moment equations thus obtained with pre-Maxwell equations (dropping displacement current and Poisson's equation) results in *resistive MHD equations:*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \,, \qquad \textit{(continuity)} \qquad (66)$$

$$\rho \, (\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) + \nabla p - \mathbf{j} \times \mathbf{B} = 0 \,, \qquad \textit{(momentum)} \qquad (67)$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = (\gamma - 1) \eta |\mathbf{j}|^2 \,, \qquad \textit{(internal energy)} \qquad (68)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \,, \qquad \textit{(Faraday)} \qquad (69)$$

where

$$\mathbf{j} = \mu_0^{-1} \nabla \times \mathbf{B} \,, \qquad \textit{(Ampère)} \qquad (70)$$

$$\mathbf{E}' \equiv \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \, \mathbf{j} \,, \qquad \textit{(Ohm)} \qquad (71)$$

and

$$\nabla \cdot \mathbf{B} = 0 \qquad \textit{(no magnetic monopoles)} \qquad (72)$$

is initial condition on Faraday's law.

Ideal MHD equations

- Substitution of $\mathbf{j}$ and $\mathbf{E}$ in Faraday's law yields *the induction equation*:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \mu_0^{-1} \nabla \times (\eta \nabla \times \mathbf{B}) \,, \tag{73}$$

where the resistive diffusion term is negligible when the *magnetic Reynolds number*

$$R_m \equiv \frac{\mu_0 l_0 v_0}{\eta} \gg 1 \,. \tag{74}$$

- Neglect of resistivity and substitution of $\mathbf{j}$ and $\mathbf{E}$ leads to *the ideal MHD equations:*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \,, \tag{75}$$

$$\rho \, (\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) + \nabla p - \mu_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \,, \tag{76}$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = 0 \,, \tag{77}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \,, \qquad \nabla \cdot \mathbf{B} = 0 \,, \tag{78}$$

which will occupy us for most of this course.

Application: Alfvén waves

- Wave propagation in *homogeneous* plasma with magnetic field in z-direction:

$$\rho_0 = \text{const}\,, \qquad \mathbf{v}_0 = 0\,, \qquad p_0 = \text{const}\,, \qquad \mathbf{B}_0 = B_0\,\mathbf{e}_z\ (\Rightarrow\ \mathbf{j}_0 = 0)\,. \tag{79}$$

- Small perturbations ρ_1, $\mathbf{v}_1$, p_1, $\mathbf{B}_1$ from this state permit to *linearize* Eqs. (75)–(78):

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \mathbf{v}_1\,, \tag{80}$$

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} = -\nabla p_1 + \mu_0^{-1}(\nabla \times \mathbf{B}_1) \times \mathbf{B}_0\,, \tag{81}$$

$$\frac{\partial p_1}{\partial t} = -\gamma p_0 \nabla \cdot \mathbf{v}_1\,, \tag{82}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{v}_1 \times \mathbf{B}_0)\,, \tag{83}$$

producing a complete set of equations for the unknowns ρ_1, $\mathbf{v}_1$, p_1, and $\mathbf{B}_1$.

- Neglecting the pressure, we obtain a wave equation for the velocity $\mathbf{v}_1$:

$$\rho_0 \frac{\partial^2 \mathbf{v}_1}{\partial t^2} = \mu_0^{-1}(\nabla \times \frac{\partial \mathbf{B}_1}{\partial t}) \times \mathbf{B}_0 = \mu_0^{-1}\mathbf{B}_0 \times (\nabla \times (\nabla \times (\mathbf{B}_0 \times \mathbf{v}_1)))\,. \tag{84}$$

Application: Alfvén waves (cont'd)

- Plane wave solutions $\mathbf{v}_1(\mathbf{r},t) = \hat{\mathbf{v}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ (replacing $\partial/\partial t \to -i\omega$, $\nabla \to i\,\mathbf{k}$) yields eigenvalue equation

$$-\rho_0\omega^2\,\hat{\mathbf{v}} = -\mu_0^{-1}B_0^2\,\mathbf{e}_z \times (\mathbf{k}\times(\mathbf{k}\times(\mathbf{e}_z\times\hat{\mathbf{v}})))\,. \quad (85)$$

- $\Rightarrow\ \hat{v}_\parallel = 0$, two remaining components $\hat{\mathbf{v}}_\perp$ oscillate independently. Focus on wave with velocity perpendicular to both $\mathbf{k}$ and $\mathbf{B}_0$. Eigenvalue problem becomes:

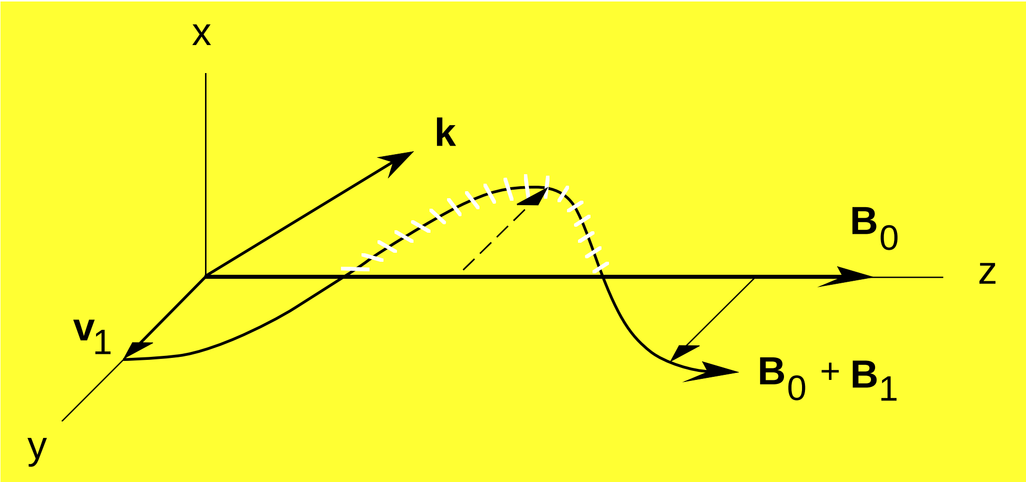
$$(\omega^2 - k_\parallel^2 v_A^2)\,\hat{v}_y = 0\,, \qquad v_A \equiv \frac{B_0}{\sqrt{\mu_0\rho_0}} \quad \textit{(Alfvén velocity)}\,. \quad (86)$$

- Hence, two *Alfvén waves* (1942) (right/left) with frequency

$$\omega = \pm\omega_A\,, \quad \omega_A \equiv k_\parallel v_A\,. \quad (87)$$

Tokamak example:

$$v_A \approx 6\times10^6\,\mathrm{m\,s^{-1}}\,,$$
$$\tau = 2\pi R/v_A \approx 3\,\mu\mathrm{s}\,.$$



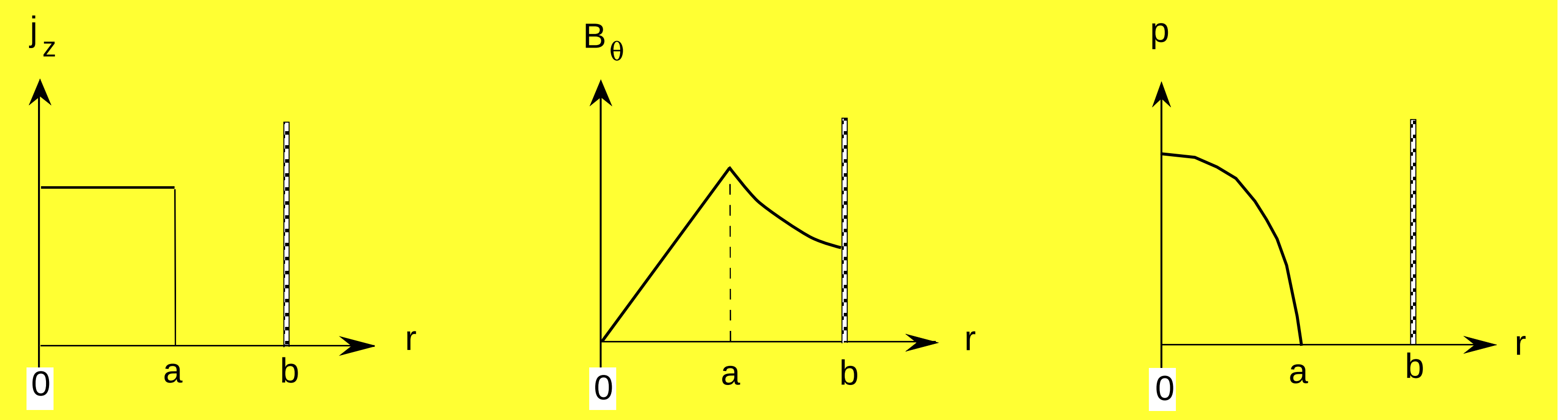
Application: Equilibrium

- Static equilibrium basis of all *magnetic confinement* systems for fusion experiments:

$$\nabla p = \mathbf{j} \times \mathbf{B} , \qquad \mathbf{j} = \mu_0^{-1} \nabla \times \mathbf{B} , \qquad \nabla \cdot \mathbf{B} = 0 . \tag{88}$$

- Example of *z-pinch:*

$$\frac{dp}{dr} = -j_z B_\theta , \quad j_z = \frac{1}{\mu_0 r} \frac{d}{dr}(r B_\theta) \quad \Rightarrow \quad \frac{dp}{dr} = -\frac{B_\theta}{\mu_0 r} \frac{d}{dr}(r B_\theta) . \tag{89}$$



- Numbers:

$$n = 10^{22}\,\mathrm{m}^{-3}, T = 10^8\,\mathrm{K} , a = 0.1\,\mathrm{m} \;\Rightarrow\; p_c = 1.38 \times 10^7\,\mathrm{N\,m}^{-2} \;(= 136\,\mathrm{atm}\,!) ,$$

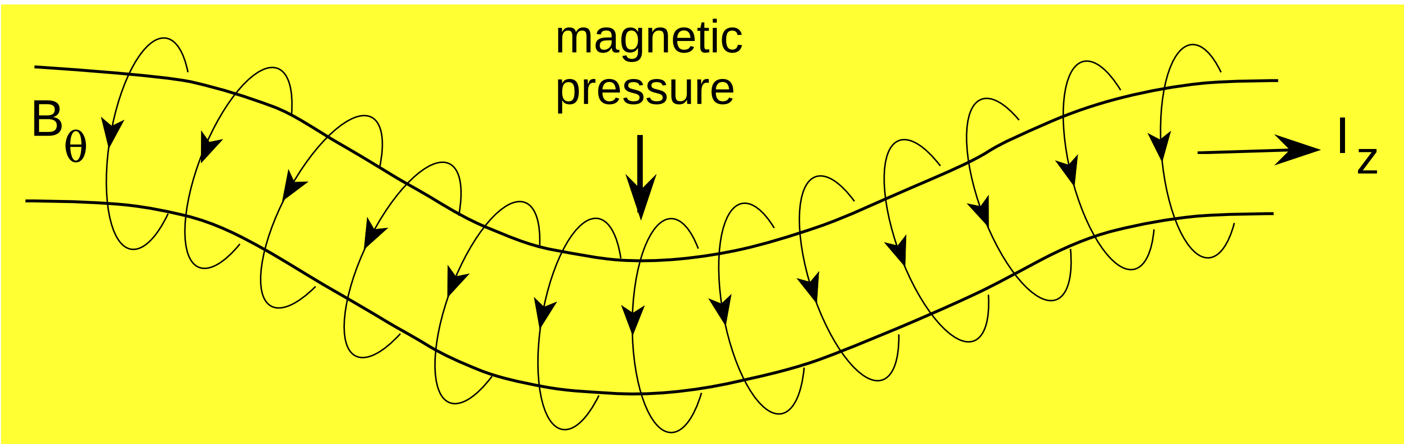
$$I_z = 2\pi a (p_c/\mu_0)^{1/2} = 2.1 \times 10^6\,\mathrm{A} , \quad B_\theta = \mu_0 I_z/(2\pi a) = 4.2\,\mathrm{T}\;(= 42\,\mathbf{kgauss}) .$$

A thermonuclear reactor by just passing a current through a linear plasma column?

Application: instability

- Alfvén waves in a homogeneous plasma with straight magnetic field lines are stable. Field lines of the z-pinch are curved. As a result some "Alfvén" waves have imaginary frequency ($\omega^2 < 0$): exponential growth! These modes are called *kink instabilities* because of the associated helical deformation of the plasma column. For wavelengths $k_z^{-1} \gg a$, the expression for their growth rate reveals cause of the instability, viz. curved magnetic field $B_\theta(a)$ at the plasma edge:

$$\omega^2 \approx -\frac{B_\theta^2(a)}{2\mu_0\rho_0 a^2}\,. \tag{90}$$



- Growth time of the kink instability $\sim 1\,\mu\,\mathrm{s}$ $\Rightarrow$ disastrous!

Cure: tokamak

- Cure: Replace cylinder by torus (tokamak). Since kink modes are long wavelength instabilities, choose parameters such that unstable wavelengths do not fit in the torus. This yields the *Kruskal-Shafranov condition* for external kink mode stability, which puts a limit on the total plasma current:

$$I_z(a) < \frac{2\pi a^2 B_z}{\mu_0 R_0} \,. \tag{91}$$

- In terms of the *'safety factor'* ($\sim$ pitch of the magnetic field lines):

$$q(a) > 1 \,, \qquad q(r) \equiv \frac{r B_z(r)}{R_0 B_\theta(r)} = \frac{2\pi r^2 B_z(r)}{\mu_0 R_0 I_z(r)} \,. \tag{92}$$

- Now, the design of a thermonuclear machine becomes an optimization problem of chosing current distributions that permit both *equilibrium* and *stability*.

Plasma coherence

We introduced the three main theoretical approaches of plasmas (theory of single particle motion, kinetic theory of collections of many particles, and theory of magnetohydrodynamics pertaining to global macroscopic plasma dynamics in complex magnetic fields). Three effects were encountered giving plasmas the *coherence* that is necessary for thermonuclear confinement of laboratory plasmas and which is also characteristic for magnetized plasmas encountered in nature:

- In the single particle picture, we found that *particles of either charge stick to the magnetic field lines by their gyro-motion* which restrains the perpendicular motion.

- In the kinetic description, we found that, because of the large electric fields that occur when electrons and ions are separated, deviations from neutrality can occur only in very small regions (of the size of a Debye length). Over larger regions, *ions and electrons stay together to maintain approximate charge neutrality*.

- In the fluid picture, it was found that *currents in the plasma create their own confining magnetic field* and that *Alfvén waves act to restore magnetic field distortions*. We also encountered the first destructive effect, viz. *the external kink instability*.