Extending the Spin Projection Operators for Gravity Models with Parity-Breaking in 3-D

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We propose a new basis of spin-operators, specific for the case of planar theories, which allows a Lagrangian decomposition into spin-parity components. The procedure enables us to discuss unitarity and spectral properties of gravity models with parity-breaking in a systematic way.

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I. INTRODUCTION

In the analysis of quantum aspects of any field theory, considerable interest is devoted to the description of the particle spectrum and the relativistic and quantum properties of scattering processes of the theory under investigation. Some of these issues may be understood by means of the analysis of the propagator of the theory. There are various methods for the attainment of propagators, but, particularly in the case of weak field approximation for quantum gravity, which is our interest, algebraic methods have been intensively developed, specially the one based on the spin projection operators (SPO). The SPO has the interesting property of decomposing fields into definite spin-parity sectors and the latter can be expressed in terms of the transverse (θ) and longitudinal (ω) operators as building blocks. The attainment of the propagator by this technique for gravity models, whenever the metric is adopted as the fundamental quantum field, was possible using the basis built up in Ref. [1]. Later Neville [2], and Sezgin and Nieuwenhuizen [3] extended the set of operators in order to provide a complete SPO basis (in four dimensions) for Lagrangians containing a rank-2 tensor and a rank-3 tensor antisymmetric in two indices. With this basis, it was possible to discuss generalized parity-preserving models of gravitation with the vielbein (e^a_μ) and spin connection ($\omega_\mu{}^{ab}$) as fundamental fields.

Motivated by the importance of finding a suitable basis in the task of calculating tensor field propagators, this Letter sets out to propose and discuss a possible extension of the basis of spin operators mentioned above [2, 3] that may prove to be more appropriate for the analysis of propagators of planar models, in special generalised models for 3-D gravity with parity-breaking.

To understand the convenience of the properties satisfied by the basis proposed in [3], let us study a general parity-preserving model:

$$(\mathcal{L})_2 = \sum_{\alpha,\beta} \psi_\alpha O_{\alpha\beta} \psi_\beta, \qquad (1)$$

where $O_{\alpha\beta}$ is some local differential operator and ψ_α carry the 40 components $(\phi_{ab}, \chi_{ab}, \omega_{abc})$, with ϕ, χ being the symmetric and antisymmetric quantum fluctuation pieces of the vielbein respectively. We can systematically analyse the spectrum and unitarity of this model by means of a decomposition in SPO in the momentum space, as described in [3]:

$$(\mathcal{L})_2 = \sum_{\alpha,\beta,ij,J^P} \psi_\alpha a^{\psi\lambda}_{ij}\left(J^P\right) P^{\psi\lambda}_{ij}\left(J^P\right)_{\alpha\beta} \psi_\beta, \qquad (2)$$

where the diagonal operators, $P^{\Psi\Psi}_{ii}\left(J^P\right)$, are projectors in the spin (J) and parity (P) sectors of the field Ψ and the off-diagonal operators ($i \neq j$) implement mappings inside the spin-parity subspace.

This basis of operators is orthonormal and complete in the following sense:

$$\sum_\beta P^{\Sigma\Pi}_{ij}\left(J^P\right)_{\alpha\beta} P^{\Lambda\Xi}_{kl}\left(I^Q\right)_{\beta\gamma} = \delta^{PQ}\delta^{\Pi\Xi}\delta_{jk}\delta_{IJ} P^{\Sigma\Xi}_{il}\left(J^P\right)_{\alpha\gamma}, \qquad (3a)$$

$$\sum_{i,J^P} P_{ii}\left(J^P\right)_{\alpha\beta} = \delta_{\alpha\beta}. \qquad (3b)$$

If the coefficient matrices, $a_{ij}(J^P)$, are invertible, then the propagator saturated with the external sources, S_α, can be written as

$$\Pi = i \sum S^*_\alpha a^{-1\psi\phi}_{ij} P^{\psi\phi}_{ij}\left(J^P\right)_{\alpha\beta} S_\beta. \qquad (4)$$

But, if there are gauge symmetries in the model, the coefficient matrices become degenerate. In this case, as shown in [2], the correct saturated propagator is given by

$$\Pi = i \sum S^*_\alpha A^{\psi\phi}_{ij} P^{\psi\phi}_{ij}\left(J^P\right)_{\alpha\beta} S_\beta, \qquad (5)$$

where the A_{ij} are the inverses of the largest submatrix with nonzero determinant obtained from the a_{ij}. The

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sources, in this case, obey certain constraints. Both, the gauge transformations of the fields and the source constraints, are obtained from the degeneracy structures of the coefficient matrices. They are given, respectively, by:

$$\delta\phi_\alpha = \sum_{J^P,j,\beta,n} V_j^{(R,n)}(J^P)P_{jk}(J^P)_{\alpha\beta}f_\beta(J^P), \quad \text{for any k} \tag{6a}$$

$$\sum_{j,\beta} V_j^{(L,n)}(J^P)P_{kj}(J^P)_{\alpha\beta}S_\beta(J^P) = 0, \quad \text{for any } k \text{ and } J^P \tag{6b}$$

with $f_\beta(J^P)$ being arbitrary functions and $V^{(R,n)}$ and $V^{(L,n)}$ being the right and left null eigenvectors of $a_{ij}(J^P)$. So, they are given by the relations:

$$\sum_j a_{ij}(J^P)V_j^{(R,n)}(J^P) = 0, \tag{7a}$$

$$\sum_j V_j^{(L,n)}(J^P)a_{ji}(J^P) = 0, \tag{7b}$$

We see, by this brief discussion, that with the basis (3a), (3b), the analysis of the particular model we have at hand can be reduced to the task of discussing the coefficient matrices. So, it is interesting to generalise this basis in order to accommodate more general models while keeping the same type of formalism. Even if this procedure may readily be generalised to arbitrary dimensions [4], it may however leave aside important models with parity violation.

The motivation for our quest comes mainly from the Chern-Simons term which appears for Yang-Mills and gravity theories in $(1+2)$-dimensional space-time, that have been extensively discussed in the literature [5]-[9]. Our point is that the operator brought about by the Chern-Simons term in a Maxwell-Chern-Simons model (we shall refer to such an operator as $S_{\mu\nu}$), motivates us to search for operators more fundamental than the ordinary $\theta_{\mu\nu}$- and $\omega_{\mu\nu}$-operators. Indeed, we shall find out two new projection operators, $\rho_{\mu\nu}$ and $\sigma_{\mu\nu}$, in terms of which $\theta_{\mu\nu}$ can be expressed. Our task here consists in building up a whole set of new SPO in 3-D and, with the help of the results presented in this Letter, we shall pave the road for the analysis of the spectral consistency of planar quantum-field theoretic models with vector and tensor fields that may encompass generalised gravity models in 3-D.

II. BUILDING UP THE SPO BASIS

To fix ideas before we go on searching for the new basis, it is instructive to consider a simpler case where the Levi-Civita tensor is present. In 3-D, we can define the Maxwell-Chern-Simons Lagrangian as:

$$\mathcal{L}_{MCS} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mu}{2}\epsilon^{\mu\nu\kappa}A_\mu\partial_\nu A_\kappa. \tag{8}$$

It is easy to convince ourselves that, if one allows to express the wave operators only in terms of the metric tensor and derivatives (powers of momenta in momentum space), the basic elements needed to expand the operator are the θ's and ω's. This is not the case if the Levi-Civita tensor appears in the wave operator. Since ϵ cannot be written in terms of θ's and ω's, we are forced to enlarge the number of building blocks and, this is actually our main point to extend the usual basis of spin-operators as we have already mentioned above.

The Lagrangian (8) can be brought into the form:

$$\mathcal{L}_{MCS} = \frac{1}{2}A^\mu O_{\mu\nu}A^\nu, \tag{9}$$

with $O_{\mu\nu}$ in momentum space, given by:

$$O_{\mu\nu} = \theta_{\mu\nu} + \mu S_{\mu\nu}, \tag{10}$$

where $S_{\mu\kappa} = \epsilon_{\mu\nu\kappa}k^\nu$.

If we wish to obtain the propagator, we must know the algebraic properites between basic operators that we have at hand. We can show that:

$$\begin{gathered}\theta^2 = \theta, \omega^2 = \omega, \theta\omega = \omega\theta = 0, \\ S^2 = -\theta, S\theta = \theta S = S, S\omega = \omega S = 0.\end{gathered} \tag{11}$$

With these relations, we see that the operator S is a transverse one. That is, it is a linear operator that maps an arbitrary vector into another vector that lies in the transverse subspace. But, this vector is independent of the vector that is obtained by the action of the θ-operator. This is possible since the transverse subspace in 3-D is two dimensional. Surely, we can exhaust all possible transverse operators if we define a basis in this transverse subspace. Taking two orthonormal space-like vectors (e_1 and e_2) in the transverse subspace, we may define two operators that project onto the one-dimensional subspace spanned by each one of these vectors and two operators that implement mappings between these two subspaces. Let us define the two projectors by the relation:

$$\theta_{\mu\nu} = \rho_{\mu\nu} + \sigma_{\mu\nu}, \tag{12}$$

with

$$\rho_{\mu\nu} = -(e_1)_\mu(e_1)_\nu, \tag{13a}$$

$$\sigma_{\mu\nu} = -(e_2)_\mu(e_2)_\nu, \tag{13b}$$

where,

$$e_1.e_1 = e_2.e_2 = -1, e_1.e_2 = 0. \quad (14)$$

One can show that the other two operators that accomplish the mappings can be given by:

$$(P_{12})_{\mu\nu} = \rho_{\mu\rho}\sigma_{\nu\sigma}\epsilon^{\rho\sigma\lambda}\frac{k_\lambda}{\sqrt{k^2}}, \quad (15a)$$

$$(P_{21})_{\mu\nu} = \sigma_{\mu\sigma}\rho_{\nu\rho}\epsilon^{\rho\sigma\lambda}\frac{k_\lambda}{\sqrt{k^2}}. \quad (15b)$$

The four operators we have defined satisfy:

$$(P_{11})^2 = P_{11}, \quad (16a)$$
$$(P_{22})^2 = P_{22}, \quad (16b)$$
$$P_{12}P_{21} = P_{11}, \quad (16c)$$
$$P_{21}P_{12} = P_{22}, \quad (16d)$$

with $P_{11} \equiv \rho$ and $P_{22} \equiv \sigma$.

We stress that our interest is on gravity theories in first-order formalism. In this manner, the fundamental fields are the vielbein and spin connection. Another important application of these basis is the possibility of taking advantage from the dual aspect of the fields. So, even if the wave operator does not explicitly contain the Levi-Civita tensor, the latter may indirectly appear. Once the Lagrangian is written in terms of the quantum fluctuations of e^a_μ and $\omega_\mu{}^{ab}$, the ϵ-tensor may come in if the Lagrangian needs to be written in termos of the dual fields of these fluctuations. Using the duality relations, the quantum fluctuations can be written as:

$$\tilde{e}_{\mu\nu} = \phi_{\mu\nu} + \epsilon_{\mu\nu\kappa}\chi^\kappa, \quad (17a)$$
$$\tilde{\omega}_\mu{}^{\nu\kappa} = \epsilon^{\nu\kappa\sigma}\left(\psi_{\mu\sigma} + \epsilon_{\mu\sigma\rho}\lambda^\rho\right), \quad (17b)$$

We have dropped the distinction between greek and latin indices, since we have assumed that the fluctuations are about a Minkowski vacuum. In the relations (17a) and (17b), $\phi_{\mu\nu}$ is the symmetric part of the vielbein fluctuation and χ^κ is the vector dual to the antisymmetric one, $\psi_{\mu\sigma}$ is the symmetric part of the field dual to the spin connection fluctuation and λ^ρ is the vector dual to the antisymmetric part of the dual field. In the sequel, we shall consider these fields as the fundamental ones.

The task of finding a basis of operators that act on the vectors fields χ and λ has already been carried out, since we only need to add the longitudinal operator, ω, to the list (16d). In the work of Ref.[4], the spin projectors for symmetric rank-2 tensor was obtained for arbitrary dimension. These projectors have been written in terms of θ's and ω's. But, as we have seen, θ can be split into two more basic projectors and, with this, we increase the possibilities of construction of wave operators. In the same way, we can also use the relation (12) to split the spin projectors of [4] for D=3 into more basic ones. As an example, let us take one of the projectors and analyse how this works:

$$P^{\phi\phi}(2^+)_{ab;cd} = \frac{1}{2}(\theta_{ac}\theta_{bd} + \theta_{ad}\theta_{bc}) - \frac{1}{2}\theta_{ab}\theta_{cd}. \quad (18)$$

Substituting (12) in the expression (18), we obtain two projectors in terms of ρ and σ, one for each degree of freedom of spin:

$$P^{\phi\phi}_{11}(2^-)_{ab;cd} = \frac{1}{2}(\rho_{ac}\sigma_{bd} + \rho_{ad}\sigma_{bc} + \sigma_{ac}\rho_{bd} + \sigma_{ad}\rho_{bc}), \quad (19a)$$

$$P^{\phi\phi}_{22}(2^+)_{ab;cd} = \frac{1}{2}(\rho_{ad}\rho_{bc} + \sigma_{ad}\sigma_{bc}) - \frac{1}{2}(\rho_{ab}\sigma_{cd} + \sigma_{ab}\rho_{cd}). \quad (19b)$$

The mappings between the degrees of freedom are carried out by:

$$P^{\phi\phi}_{12}(2^{-+})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\rho_{ac}\sigma^h_b\rho^g_d + \rho_{bc}\sigma^h_a\rho^g_d - \sigma_{ad}\rho^g_b\sigma^h_c - \sigma_{bd}\rho^g_a\sigma^h_c)\frac{k^e}{\sqrt{k^2}}, \quad (20a)$$

$$P^{\phi\phi}_{21}(2^{+-})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\rho_{ca}\sigma^h_d\rho^g_b + \rho_{da}\sigma^h_c\rho^g_b - \sigma_{cb}\rho^g_d\sigma^h_a - \sigma_{bd}\rho^g_c\sigma^h_a)\frac{k^e}{\sqrt{k^2}}. \quad (20b)$$

Before we proceed, let us clarify the notation. The notation in (18) is imported from 4-D and it makes strictly physical sense only in 4-D. If the symbols do not lead to wrong physical conclusions, we preserve them in 3-D. But, in terms of ρ and σ, extra care must be taken. Actually, the operators above do not project over the whole spin-2 space, but rather over a sub-sector of the degrees of freedom carried by a spin-2. The most important difference concerns parity. In 4-D, we can fix the parity of an operator by counting the number of field contractions with the θ's present in the given operator. This is so because θ projects a Lorentz index in the 1^--sector. That is, we associate a parity "-" with the subspace projected by θ. This makes sense, since the representation of parity in Minkowsky vector space is given by:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (21)$$

and, for a massive particle in the rest frame, we can assume that the transverse space is the 3-D spatial part of Minkowsky space. So, the parity operation changes the sign of the spatial components of the vector. However, in

3-D, a parity operator distiguishes one particular space direction. For example, we can define it as:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{22}$$

In this form, the transverse operator can be split as the direct sum of two subspaces, each one associated with one parity. By convention, let us choose the subspace projected by σ as the one related to the "-" parity. So, in 3-D, the parity of the operators is given by counting the number of indices contracted by the σ operator. This justifies the prescription we have done to the operators (19a), (19b), (20a) and (20b).

By construction, the operators defined above satisfy the relations below:

$$P^{\phi\phi}(2^{+}) = P_{11}^{\phi\phi}(2^{++}) + P_{22}^{\phi\phi}(2^{--}), \tag{23a}$$
$$(P_{11})^2 = P_{11}, \tag{23b}$$
$$(P_{22})^2 = P_{22}, \tag{23c}$$
$$P_{12}P_{21} = P_{11}, \tag{23d}$$
$$P_{21}P_{12} = P_{22}. \tag{23e}$$

This process of decomposition can be repeated for all operators needed to exhaust all the possiblities of contraction of the fields in the free Lagrangian. Before we write down the explicit form of the operators in our basis, it is worthy to mention that they carry a pair of superscripts ($\psi_1 = \psi$, $\psi_2 = \phi$, $\lambda_1 = \lambda$, $\lambda_2 = \chi$) and a pair of subscripts ($i_1, j_1 = 1, 2$; $i_2, j_2 = 3, 4$; $i_3, j_3 = 5, 6$; $i_4, j_4 = 7, 8$). The operators, in terms of ρ, σ and ω, are finally cast below:

1. $P_{i_1 j_1}^{\psi_{i_1}\psi_{j_1}}(1^{++})_{ab;cd} = \frac{1}{2}(\rho_{ac}\omega_{bd} + \rho_{bc}\omega_{ad} + \rho_{ad}\omega_{bc} + \rho_{bd}\omega_{ac})$

2. $P_{i_2 j_2}^{\psi_{i_2-2}\psi_{j_2-2}}(1^{--})_{ab;cd} = \frac{1}{2}(\sigma_{ac}\omega_{bd} + \sigma_{bc}\omega_{ad} + \sigma_{ad}\omega_{bc} + \sigma_{bd}\omega_{ac})$

3. $P_{i_1 j_2}^{\psi_{i_1}\psi_{j_2-2}}(1^{+-})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\rho_a^g\sigma_c^h\omega_{bd} + \rho_b^g\sigma_c^h\omega_{ad} + \rho_a^g\sigma_d^h\omega_{bc} + \rho_b^g\sigma_d^h\omega_{ac})\frac{k^e}{\sqrt{k^2}}$

4. $P_{i_2 j_1}^{\psi_{i_2-2}\psi_{j_1}}(1^{-+})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\sigma_a^h\rho_c^g\omega_{db} + \sigma_a^h\rho_d^g\omega_{cb} + \sigma_b^h\rho_c^g\omega_{da} + \sigma_b^h\rho_d^g\omega_{ca})\frac{k^e}{\sqrt{k^2}}$

5. $P_{i_3 j_3}^{\lambda_{i_3-4}\lambda_{j_3-4}}(1^{++})_{ab} = \rho_{ab}$

6. $P_{i_4 j_4}^{\lambda_{i_4-6}\lambda_{j_4-6}}(1^{--})_{ab} = \sigma_{ab}$

7. $P_{i_3 j_4}^{\lambda_{i_3-4}\lambda_{j_4-6}}(1^{+-})_{ab} = \rho_a^g\epsilon_{ghe}\frac{k^e}{\sqrt{k^2}}\sigma_b^h$

8. $P_{i_4 j_3}^{\lambda_{i_4-6}\lambda_{j_3-4}}(1^{-+})_{ab} = \sigma_a^h\epsilon_{ghe}\frac{k^e}{\sqrt{k^2}}\rho_b^g$

9. $P_{i_1 j_3}^{\psi_{i_1}\lambda_{j_3-4}}(1^{++})_{ab;c} = \frac{1}{\sqrt{2k^2}}(\rho_{ac}k_b + \rho_{bc}k_a)$

10. $P_{i_3 j_1}^{\lambda_{i_3-4}\psi_{j_1}}(1^{++})_{a;bc} = \frac{1}{\sqrt{2k^2}}(\rho_{ba}k_c + \rho_{ca}k_b)$

11. $P_{i_2 j_4}^{\psi_{i_2-2}\lambda_{j_4-6}}(1^{--})_{ab;c} = \frac{1}{\sqrt{2k^2}}(\sigma_{ac}k_b + \sigma_{bc}k_a)$

12. $P_{i_4 j_2}^{\lambda_{i_4-6}\psi_{j_2-2}}(1^{--})_{a;bc} = \frac{1}{\sqrt{2k^2}}(\sigma_{ba}k_c + \sigma_{ca}k_b)$

13. $P_{i_2 j_3}^{\psi_{i_2-2}\lambda_{j_3-4}}(1^{-+})_{ab;c} = \frac{1}{\sqrt{2}}\epsilon_{ghe}(\sigma_a^h\rho_c^g\omega_{eb} + \sigma_b^h\rho_c^g\omega_{ea})$

14. $P_{i_3 j_2}^{\lambda_{i_3-4}\psi_{j_2-2}}(1^{+-})_{a;bc} = \frac{1}{\sqrt{2}}\epsilon_{ghe}(\sigma_b^h\rho_a^g\omega_{ec} + \sigma_c^h\rho_a^g\omega_{eb})$

15. $P_{i_1 j_4}^{\psi_{i_1}\lambda_{j_4-6}}(1^{+-})_{ab;c} = \frac{1}{\sqrt{2}}\epsilon_{ghe}(\rho_{ag}\sigma_c^h\omega_{be} + \rho_{bg}\sigma_c^h\omega_{ae})$

16. $P_{i_4 j_1}^{\lambda_{i_4-6}\psi_{j_1}}(1^{-+})_{a;bc} = \frac{1}{\sqrt{2}}\epsilon_{ghe}(\rho_{bg}\sigma_a^h\omega_{ce} + \rho_{cg}\sigma_a^h\omega_{be})$

17. $P_{i_1 j_1}^{\psi_{i_1}\psi_{j_1}}(0^{++})_{ab;cd} = \frac{1}{2}\theta_{ab}\theta_{cd}$

18. $P_{i_2 j_2}^{\psi_{i_2-2}\psi_{j_2-2}}(0^{++})_{ab;cd} = \omega_{ab}\omega_{cd}$

19. $P_{i_1 j_2}^{\psi_{i_1}\psi_{j_2-2}}(0^{++})_{ab;cd} = \frac{1}{\sqrt{2}}\theta_{ab}\omega_{cd}$

20. $P_{i_2 j_1}^{\psi_{i_2-2}\psi_{j_1}}(0^{++})_{ab;cd} = \frac{1}{\sqrt{2}}\omega_{ab}\theta_{cd}$

21. $P_{i_3 j_3}^{\lambda_{i_3-4}\lambda_{j_3-4}}(0^{++})_{ab} = \omega_{ab}$

22. $P_{i_1 j_3}^{\psi_{i_1}\lambda_{j_3-4}}(0^{++})_{ab;c} = \frac{1}{\sqrt{2}}\theta_{ab}\frac{k_c}{\sqrt{k^2}}$

23. $P_{i_3 j_1}^{\lambda_{i_3-4}\psi_{j_1}}(0^{++})_{a;bc} = \frac{1}{\sqrt{2}}\theta_{bc}\frac{k_a}{\sqrt{k^2}}$

24. $P_{i_2 j_3}^{\psi_{i_2-2}\lambda_{j_3-4}}(0^{++})_{ab;c} = \omega_{ab}\frac{k_c}{\sqrt{k^2}}$

25. $P_{i_3 j_2}^{\lambda_{i_3-4}\psi_{j_2-2}}(0^{++})_{a;bc} = \omega_{bc}\frac{k_a}{\sqrt{k^2}}$

26. $P_{i_2 j_2}^{\psi_{i_2-2}\psi_{j_2-2}}(2^{--})_{ab;cd} = \frac{1}{2}(\rho_{ac}\sigma_{bd} + \rho_{ad}\sigma_{bc} + \sigma_{ac}\rho_{bd} + \sigma_{ad}\rho_{bc})$

27. $P_{i_1 j_1}^{\psi_{i_1}\psi_{j_1}}(2^{++})_{ab;cd} = \frac{1}{2}(\rho_{ad}\rho_{bc} + \sigma_{ad}\sigma_{bc}) - \frac{1}{2}(\rho_{ab}\sigma_{cd} + \sigma_{ab}\rho_{cd})$

28. $P_{i_2 j_1}^{\psi_{i_2-2}\psi_{j_1}}(2^{-+})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\rho_{ac}\sigma_b^h\rho_d^g + \rho_{bc}\sigma_a^h\rho_d^g - \sigma_{ad}\rho_b^g\sigma_c^h - \sigma_{bd}\rho_a^g\sigma_c^h)\frac{k^e}{\sqrt{k^2}}$

29. $P_{i_1 j_2}^{\psi_{i_1}\psi_{j_2}}(2^{+-})_{ab;cd} = \frac{1}{2}\epsilon_{ghe}(\rho_{ca}\sigma_d^h\rho_b^g + \rho_{da}\sigma_c^h\rho_b^g - \sigma_{cb}\rho_d^g\sigma_a^h - \sigma_{bd}\rho_c^g\sigma_a^h)\frac{k^e}{\sqrt{k^2}}$.

The off-diagonal operators have been obtained in such a way that the following multiplicative rules and completeness relation are fulfilled:

$$\sum_\beta P_{ij}^{\Sigma\Psi}(I^{PQ})_{\alpha\beta}P_{kl}^{\Lambda\Xi}(J^{RS})_{\beta\gamma} = \delta_{jk}\delta^{\Psi\Lambda}\delta^{IJ}\delta^{QR}P_{il}^{\Sigma\Xi}(I^{PS})_{\alpha\gamma}, \tag{24a}$$

$$\sum_{i,I^{PP}} P_{ii}(I^{PP})_{\alpha\beta} = \delta_{\alpha\beta}, \tag{24b}$$

and, as we have claimed at the beggining, this makes possible to analyse generalised parity-violating gravity models in 3-D, by using the same techniques as the ones presented in [3]. There are only slight differences due to the notation and role played by parity. In the present case, the wave operators is written as:

$$O_{\alpha\beta} = \sum_{J,ij} a_{ij}^{\Sigma\Lambda}(J)P_{ij}^{\Sigma\Lambda}(J^{PQ})_{\alpha\beta}, \tag{25}$$

and the saturated propagator, in the case of gauge symmetries, can be cast as below:

$$\Pi = i\sum_{J,ij} S_\alpha^* A_{ij}^{\Sigma\Lambda}(J)P_{ij}^{\Sigma\Lambda}(J^{PQ})_{\alpha\beta}S_\beta, \tag{26}$$

where $A_{ij}(J)$ is the inverse of the largest submatrix of the $a_{ij}(J)$ with the degeneracies extracted. The important fact is that these coefficient matrices accommodate the coefficients of the operators with both parities. Besides these subtle aspects, the rest of the analysis goes along the same paths as it has been carried out with the basis (3a).

A detailed application of the general procedure we develop here is worthwhile. Indeed, the study of a 3-D model for gravity in the presence of dynamical torsion and higher power of the curvature is under progress and the efficacy of the projectors we have presented here becomes manifest in this application. These results shall soon be reported elsewhere [10].

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