

Cosmology without inflation

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We propose a new cosmological paradigm in which our observed expanding phase is originated from an initially large contracting Universe that subsequently experienced a bounce. This category of models, being geodesically complete, is non-singular and horizon-free, and can be made to prevent any relevant scale to ever have been smaller than the Planck length. In this scenario, one can find new ways to solve the standard cosmological puzzles. One can also obtain scale invariant spectra for both scalar and tensor perturbations: this will be the case, for instance, if the contracting Universe is dust-dominated at the time at which large wavelength perturbations get larger than the curvature scale. We present a particular example based on a dust fluid classically contracting model, where a bounce occurs due to quantum effects, in which these features are explicit.

I. INTRODUCTION

With the recent release of Wilkinson microwave anisotropy probe (WMAP) data [1, 2], the inflation paradigm [3, 4, 5, 6, 7] has been set on firmer ground. Apart from solving some of the standard cosmological puzzles (horizon, flatness, isotropy), the simplest models predict an almost scale invariant spectrum of long wavelength scalar perturbations, as observed, with low amplitude tensor perturbations. This successful paradigm suffers, however, from some weakening issues and omissions. The existence of an initial singularity (a point where no physics is possible) in the standard cosmological model is not addressed by inflation [8]. There is no consensus yet as to whether inflation really solves the homogeneity problem [9, 10] as long as one still needs special initial conditions in a relatively large patch to initiate inflation [10, 11, 12]. It seems that we cannot go forward on this problem without a precise knowledge of how the Universe leaves the Planck scale and/or a theory of initial conditions, i.e. without having an unambiguous and complete theory of quantum gravity [10]. Furthermore, some cosmologically relevant wavelengths must, at some early stage, have been trans-Planckian [13, 14, 15, 16]; this can cast doubts on the validity of the cosmological perturbation predictions of inflation. Finally, the usual and simpler models of inflation need a scalar field [17], whose theoretical properties demanded for setting up the inflationary phase are not obviously compatible with those obtained from well-motivated fundamental particle physics theory [18, 19] (a new perspective was, however, suggested [20]). In view of these difficulties, the question can be asked whether the inflationary solution is unique.

Mechanisms that eliminate the initial singularity belong to one of the following scenarios: either they assume a quantum creation of a small but finite Universe

and hence a beginning of time [21, 22], or they are based on an eternal Universe, hence with no beginning of time. This last possibility can itself be divided into two distinct categories: a monotonic time dependence of the scale factor, i.e. an expansion lasting forever, or different phases including contractions and expansions, and therefore bounces. The first situation is realised in the pre-big-bang (PBB) scenario [23, 24]; it requires a long accelerated phase originating from either an asymptotically zero volume flat spacetime or from a finite but small compact region [25, 26] before the usual decelerated expansion of the standard model. As for bouncing models, they can be embedded in many theoretical situations [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39], including classically singular cases [40, 41, 42]. In a string approach, both situations are in practice equivalent due to the presence of the dilaton which allows for a field reparametrization (as opposed to conformal transformation as is usually, and erroneously, stated): the PBB evolution of the Jordan (or string) frame is turned to a bounce in the Einstein frame.

Up to now, there is not a single observation which favors one of these three scenarios (time creation, eternal expansion or bounce) with respect to the others, rendering these three possibilities susceptible to physical investigation, without prior preferences.

Bouncing models differentiates, however, very strongly from the other two scenarios above in one important aspect: initial conditions may not be anymore put in a very small region, perhaps with Planckian size, but in a very large and almost flat Universe. In this framework, the flatness and the homogeneity problems are viewed from a very different perspective. Hence, bouncing models not only solve, by construction, the singularity problem, but they may also possibly solve, as we discuss in this work, other important puzzles of the standard model without the need for an inflationary phase¹.

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¹ Note that, strictly speaking, the bounce itself could be seen as

Note also that a transition from contraction to expansion demands non-standard or non classical physics, and/or non standard matter in order to avoid the singularity in between. If, for having inflation, violation of the strong energy condition is necessary and sufficient, for bouncing models it may not be sufficient, requiring also violation of the null energy condition in most cases (i.e. Friedmann models with nonpositive spatial hypersurface curvature [43]). This suggests that there could possibly be observational implications to which we shall come later. For now, we turn to the way a bounce addresses the usual puzzles, before presenting an actual model in which these features can be readily implemented.

II. COSMOLOGICAL PUZZLES

Bouncing models lead to a new framework for uncovering completely new solutions to the standard cosmological puzzles. Let us list them in what follows.

- **Singularity:** Bouncing universes are, by construction, geodesically complete, and hence singularity free, so this point, not addressed by inflation, is a non-issue here.

- **Horizon:** The size d_H of the horizon is given by the time integral $d_H(t) \equiv a(t) \int_{t_i}^t a^{-1}(\tau) d\tau$, with t_i some initial value. If the dynamics is driven by a perfect fluid with constant equation of state $\omega \equiv p/\rho$, with p and ρ the pressure and energy density, respectively, of the fluid, the scale factor behaves, for flat hypersurfaces, as $a(t) \propto |t|^{2/[3(1+\omega)]}$ [here and in what follows, we assume for simplicity that the bounce takes place at $t = 0$, so that $t < 0$ ($t > 0$) represents the contracting (expanding) phase].

Integrating, we obtain the horizon as

$$d_H = \frac{3(1+\omega)}{1+3\omega} \left\{ |t_i|^{(1+3\omega)/[3(1+\omega)]} - |t|^{2/[3(1+\omega)]} + t \right\}. \quad (1)$$

If $\omega > -\frac{1}{3}$, then clearly, as $t_i \rightarrow -\infty$ (bouncing case), d_H diverges. At any finite time before or after the bounce, the horizon is infinite and remains so for all subsequent times. Note that this solution would cease to be valid if, as seems to be the case now, the Universe had been dominated by some kind of dark energy ($\omega < -\frac{1}{3}$) in the contracting phase. This observation thus appears to require a non symmetric bounce.

- **Flatness:** The problem stems from the classical equation giving the density $\rho(t)$ relative to the critical one $\rho_c(t) = 3H^2(t)/(8\pi G_N)$, with $H \equiv \dot{a}/a$ the Hubble ex-

pansion rate, through

$$\frac{d}{dt} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}, \quad (2)$$

where $\Omega \equiv \rho/\rho_c$. As Ω is close to unity now, implying almost flat spatial sections (the term involving the spatial curvature \mathcal{K} would be negligible in the Friedmann equation), Eq. (2) implies that it must have been arbitrarily closer in the past in the usual big-bang scenario based on decelerated ($\ddot{a} < 0$) expansion ($\dot{a} > 0$) since then $|\Omega - 1|$ is an ever-increasing function of time. To solve this problem, one must have had a long enough period during which $|\Omega - 1|$ decreases. This can be accomplished either through an inflationary expansion phase ($\ddot{a} > 0$ and $\dot{a} > 0$) or through a long decelerated contracting phase ($\ddot{a} < 0$ and $\dot{a} < 0$). In the latter framework, we would say that the Universe is seen to be almost flat now because it has expanded much less than it has contracted before.

- **Homogeneity:** This is perhaps the deepest problem of the standard model. There are essentially two approaches to this issue. The first, exemplified here by the Weyl curvature hypothesis [44, 45] (other examples on this approach have been proposed [21, 22], based on boundary conditions on the wave function of the Universe), is to provide some theory of initial conditions². The second possibility, of which inflation is prototypical, is to invoke a dynamical process which wipes out any preexisting inhomogeneity and anisotropy. In both cases, the outcome should be the outstandingly special Friedmann-Lemaître-Robertson-Walker (FLRW) geometry. It is unquestionable that inflation, providing such a mechanism, significantly alleviates the problem [9, 10, 12], but it is not clear whether it precludes special initial conditions [10, 11] to be imposed. It seems likely that a combination of these two approaches will turn out to be necessary. One expects that whenever (if ever) a consistent theory of quantum gravity is consensually accepted, it will, once applied to cosmology, provide the required initial conditions to homogenize the primordial Universe.

In bouncing models, one may indeed envisage a solution for this problem using a mixture of the above-mentioned two approaches within a complete new perspective. In a very large and dilute Universe, the energy-momentum tensor of matter, and hence the Ricci tensor, should be very small. This requirement, by itself, does not ensure that the geometry is almost flat since Einstein equations do not fix the Weyl tensor. Under the

including an inflationary phase since $\ddot{a} > 0$ near the bounce. However, inflation is usually assumed (as we do in the present work) to be not only a period of acceleration, but one inducing many e-folds of expansion in a very short time. This is clearly not the case during a bounce.

² Based on thermodynamical considerations, the Weyl curvature hypothesis consists in saying that the arrow of time implies the Universe to have an initially very low total entropy. It turns out that, if its gravitational part, the dominant one, depends only on the Weyl tensor, as the conjecture states, then it suffices to argue that the latter should be initially negligible. Note, however, that this particular hypothesis is not sufficient by itself to guarantee homogeneity as long as the conformal factor of the metric may have non-negligible spatial gradients at the beginning.

Weyl curvature hypothesis, however, geometry should be almost flat at that time, which we take to be our initial condition. The question then becomes: do the initial inhomogeneities grow?

Consider first the initial regime where the Universe is very large, rarefied and, as discussed above, almost flat. Then the self-gravitation of any inhomogeneity, even with $\delta\rho/\rho \gtrsim 1$, is negligible, as long as ρ is very small. Note that this would not be true if we were to take initial conditions at a time for which the Universe is small and dense. These original inhomogeneities therefore get dissipated in much the same way as air gets rapidly homogeneous if perturbed (sound waves do not condense). By assumption, neither gravity nor its entropy are relevant in this regime. By the Hamilton theorem, the entropy of matter grows undisturbed for a long time and thus can reach its maximum value.

Let us now go one step forward and assume a matter-(dust-) dominated cosmological contraction. In that case, the dust field velocity evolves as $v \propto a^{-1}$, its number density $n \propto a^{-3}$, and consequently its mean free path reads $\lambda_{\text{MFP}} = (n\sigma)^{-1} \propto a^3$, where σ is the dust cross section (necessarily small for the dust approximation to make sense).

In a very large dust-dominated Universe, the Jeans length is $\lambda_{\text{J}}^{\text{ph}} = c_{\text{s}}[\pi/(G_{\text{N}}\rho)]^{1/2} \propto a^{1/2}$ and can be made larger than any large scale we see today. The dissipation time t_{d} for a given inhomogeneity of wavelength λ smaller than the Jeans length is given by

$$t_{\text{d}} = \frac{\lambda}{v} \left(1 + \frac{\lambda}{\lambda_{\text{MFP}}} \right), \quad (3)$$

and this time ought to be compared with the Hubble time scale. For dust, $a \propto R_{\text{H}}^{2/3}$, where we set $R_{\text{H}} = yR_0$ the Hubble radius at any time and R_0 its present value. Writing $\lambda = xR_0$ and $\lambda_{\text{MFP}} = AR_{\text{H}}^2$, A being a constant, Eq. (3) transforms into

$$t_{\text{d}} \propto xy^{2/3} \left(1 + C \frac{x}{y^2} \right) T_0, \quad (4)$$

where T_0 is the value of the Hubble time today and $C^{-1} = AR_0$. Comparing with the Hubble time $t_{\text{H}} = yT_0$, Eqs. (1) and (4) yield

$$\frac{t_{\text{d}}}{t_{\text{H}}} \propto \frac{x}{y^{1/3}} \left(1 + C \frac{x}{y^2} \right). \quad (5)$$

The dependence of (5) on y obtained by the simple calculation above shows that, for a sufficiently large $R_{\text{H}} = yR_0$, any scale up to the size of our Universe today becomes homogeneous, being dissipated before gravity can play any role. In fact, depending on the amount of time spent in this dust contraction regime, and this time can be fixed arbitrarily large, even infinite if one wishes, the dissipation is so effective that only quantum fluctuations given by the uncertainty principle survive. This provides, as a bonus, unique initial conditions for the perturbations: vacuum fluctuations.

- Dark energy: This problem is not addressed by inflation, and the simplest bouncing cases also remain silent here. However, as discussed above, although dark energy is mostly harmless as inflation proceeds, it may be problematic (see the horizon problem above) for bouncing models. Hence, either dark energy was produced near or after the bounce, or it cannot have dominated in the asymptotic past, as in the transient dark energy example [46]. In this case, one could turn this potential difficulty into a means of reducing the spectral index of perturbations: with a small amount of dark energy in the primordial fluctuation enhancement epoch, the effective equation of state could be made negative, thus implying a slightly red spectrum (see below). This is something to be investigated in more detail in the future.

III. INITIAL CONDITIONS FOR STRUCTURE FORMATION

The main achievement of the inflation paradigm was the realization that, due to the quantum fluctuations of the scalar field and the metric, initial conditions for semi-classical perturbations could be set in a natural way, demanding that at some stage the relevant scales had been in a vacuum quantum state. Implementing this condition then led to the prediction that the spectral index of scalar perturbations is close to 1 [4]. What similar initial conditions can be imposed in bouncing models, if any, and what do they lead to in terms of observations?

Setting vacuum initial conditions is in fact even more natural in a bouncing case. Indeed, the Universe is supposed to be very large in the far past and in fact, for the idea to make any sense at all, much larger than any observable scale today. This means that, for any given scale of interest, there exists a time, sufficiently before the bounce, for which the scale in question is much smaller than the curvature scale. As a result, one can safely work in the tangent Minkowski space. Furthermore, imposing vacuum for the corresponding perturbations at that time is then not only a plausible requirement but also a necessary consequence of the homogeneity solving scenario discussed above, where inhomogeneities are dissipated up to quantum vacuum fluctuations in a huge and slowly contracting Universe.

In the simple quantum cosmology background presented in the following section, stemming from action (8), explicit calculations starting with vacuum initial conditions yields, for the scalar and tensor spectral indices, respectively [47, 48, 49, 50]

$$n_{\text{s}} = 1 + \frac{12\omega}{1 + 3\omega}, \quad (6)$$

and

$$n_{\text{T}} = \frac{12\omega}{1 + 3\omega}. \quad (7)$$

In the dust (pressureless) limit $\omega \rightarrow 0$, one can easily get a scale invariant spectrum for both tensor and scalar

perturbations, in agreement with observations. Furthermore, fitting the amplitude of the perturbations with cosmic microwave background (CMB) data leads to the nice constraint that the curvature scale at the bounce should be greater than roughly a thousand Planck lengths, ensuring that the model is not spoiled by some discrete nature of spacetime such as induced by string effects [47].

The above calculations might erroneously lead one to believe that the model necessarily involves only one fluid, and that it ought to be dust at all times. Clearly, this would ruin the central idea. In fact, it is not mandatory that the fluid dominating the dynamics during the bounce be dust. This is fortunate since densities and temperature increase as the Universe contracts, eventually reaching the point above which particle masses becomes negligible and the Universe becomes radiation-dominated. This would also happen as time goes on if an initial bunch of monopoles and antimonopoles were to annihilate. In any case, a matter to radiation transition is expected.

The reason why it is not necessary that dust dominate also during the bounce is the following: the spectra of the growing and constant modes of the Bardeen potential in the contraction phase are obtained far from the bounce, and they do not change in a transition, say, from matter to radiation domination (although amplitudes may change [51]). The effect of the bounce is essentially to mix these two coefficients (this also happens in other frameworks [52, 53]): the constant mode in the expansion phase is thus very likely (although this is a model-dependent statement [54]) to acquire the scale invariant piece previously built up. This happens whatever the fluid dominating at the bounce [47]. Hence, the bounce may be dominated by any other fluid, such as radiation. In short, providing the perturbations enter the potential during an almost dustlike epoch, one expects the spectrum to be almost scale invariant. We shall generally assume this hypothesis, bearing in mind that it ought to be checked explicitly afterwards [55].

Other bouncing representations have been discussed, among which are purely classical fluids, one of which, whose role is restricted to the bounce itself, is of negative energy [56, 57]. Such a negative energy classical fluid might also be an effective fluid originated from interactions among ordinary fluids in the early Universe [58]. Again, a scale invariant spectrum can be recovered provided the positive energy fluid dominating at the early stage, when the Universe is large, has an equation of state close to vanishing (dust), irrespective of the negative energy fluid which drives the bounce. Finally, and even though they are not mandatory, classical scalar fields can also lead to bouncing models with a scale invariant spectrum [48]. Hence this result is quite robust and not a mere particular feature of a given specific model.

We now turn to our specific case which exemplifies all of the basic requirements for the bounce paradigm we wish to defend as a would-be “challenger” to inflation. We would like to emphasize that, although it possesses all

of the features expected for a data-reproducing bounce, its use merely serves the purpose of exhibiting how it can practically be realized. As for inflation, many other solutions can be found, each with its specificities; present [1, 2] or future [59] observational constraints might, however, hopefully discriminate between the many possibilities.

IV. A QUANTUM COSMOLOGICAL BOUNCE

What could be more simple for cosmology than to use Einstein action sourced by a constant equation of state perfect fluid in 4 dimensions? Amazingly enough, such an overwhelmingly simple framework manages to reproduce all of the cosmic data, as we want to emphasize here. The theory we deal with is thus

$$\mathcal{S} = - \int d^4x \sqrt{-g} \left(\frac{R}{6\ell_{\text{Pl}}^2} + \rho \right), \quad (8)$$

with R the Ricci scalar and ρ the energy density with associated pressure $p = \omega\rho$, assuming ω to be a constant.

We restrict our attention, to begin with, to homogeneous and isotropic models and thus choose to consider the subset of metrics of the FLRW form, namely,

$$ds^2 = g_{\mu\nu}^{(0)} dx^\mu dx^\nu = N^2(\tau) d\tau^2 - a^2(\tau) \gamma_{ij} dx^i dx^j, \quad (9)$$

with $\gamma_{ij} = (1 + \frac{1}{4}\mathcal{K}\mathbf{x}^2)^{-2} \delta_{ij}$ the 3-space metric and $a(\tau)$ the scale factor. Note that we do not assume flat spatial sections, so the spatial curvature \mathcal{K} is free, although normalizable: $\mathcal{K} \in \{0, \pm 1\}$. Finally, the lapse function can be chosen as $N = a^{3\omega}$, so that τ is identified with cosmic time if the fluid is made of dust, and conformal time if it is made of radiation.

Going on to perturbations around such a background, we write the full metric as

$$ds^2 = \left(g_{\mu\nu}^{(0)} + \delta g_{\mu\nu} \right) dx^\mu dx^\nu. \quad (10)$$

This, in principle, provides a full set of quantum observables. Let us consider an arbitrary quantum state $\psi[g_{\mu\nu}(x), \dots]$, where the dots stand for whatever other degree of freedom is present. Consistency of the linear quantum perturbation approach in this case might be asserted, or at least addressed, provided that, for non vanishing values of the background expectation values, the constraint

$$\langle \psi | \delta g_{\mu\nu} | \psi \rangle \equiv \langle \delta g_{\mu\nu} \rangle_\psi \ll \langle g_{\mu\nu}^{(0)} \rangle_\psi \quad (11)$$

holds.

In the perturbed FLRW case in the longitudinal gauge, considering scalar and tensor perturbations only³

$$ds^2 = N^2 (1 + 2\Psi) d\tau^2 - a^2 [(1 - 2\Phi) \gamma_{ij} + h_{ij}] dx^i dx^j, \quad (12)$$

³ Both at classical and quantum levels, scalar, vector and tensor

the constraint (11) amounts to $(\langle\Psi\rangle_\psi, \langle\Phi\rangle_\psi, \langle h_{ij}\rangle_\psi) \ll 1$. Note that the tensor modes are traceless and divergence-free, i.e. $\gamma^{ij}h_{ij} = 0$ and $h^i{}_{;j} = 0$, with the covariant derivative taken with respect to γ_{ij} .

The crucial point concerning this expansion is that it can be shown [60] that the Fourier modes of these perturbations, in the restricted case of a constant equation of state perfect fluid, satisfy equations of motion that are exactly those of the classical theory. In fact, at least in the flat $\mathcal{K} = 0$ situation, they can be obtained without any appeal to the background field equations and therefore can be used straightforwardly in the quantum regime [47, 60, 61, 62]. A consistent Hamiltonian constraint $\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H}$ was obtained, where \mathcal{H}_0 describes the background geometry while $\delta\mathcal{H}$ is the Hamiltonian constraint for the perturbations written in very simple form and suitable for Dirac quantization.

The way to proceed is to go a step forward with respect to the usual approach, where perturbations are quantized and the background remains classical, and use the whole Hamiltonian constraint above to Dirac quantize both the background and the perturbations⁴, making a wavefunction separation into zeroth and second orders as

$$\psi = \psi^{(0)}(a, \tau) \times \psi^{(2)}[a, \Psi(\mathbf{x}), \Phi(\mathbf{x}), h_{ij}(\mathbf{x}), \tau], \quad (13)$$

and solve the zeroth order using a Bohmian approach [64, 65], where actual trajectories can be calculated. In the case of a perfect fluid, the Bohmian quantum trajectory for the scale factor reads [47]

$$a(\tau) = a_0 \left[1 + \left(\frac{\tau}{T_0} \right)^2 \right]^{1/[3(1-\omega)]}, \quad (14)$$

where a_0 , the value of the scale factor at the bounce⁵, and T_0 are arbitrary constants to be eventually determined by observations, and the time parameter τ is related to conformal time η through

$$d\eta = [a(\tau)]^{3\omega-1} d\tau. \quad (15)$$

perturbations decouple, and vectors are rapidly diluted away by the expansion as $\propto a^{-2}$; they are not measurable today, so one merely needs to ensure they never spoiled the linear approximation.

⁴ An attempt in this direction was done [63], which, however, could not be taken much forward due to the complicated form of $\delta\mathcal{H}$ they use.

⁵ The background wave function at the bounce, which is a Gaussian centered at the singular point $a = 0$, gives the probability of having a particular value for a_0 , and it is very low when evaluated at sufficient big values of a_0 that can describe the large Universe in which we live. However, if one takes background wavefunctions at the bounce consisted of Gaussians traveling away from the singular point $a = 0$ with speed parameter u , this problem can be overcome and large Universes can be obtained with reasonable probabilities [66]. This is an example of the fact that the scales of the Universe are not uniquely determined by Planck scale but also on parameters appearing in its quantum state

Note that this solution has no singularities and tends to the classical solution when $\tau \rightarrow \pm\infty$ ⁶. Hence, once an initial condition has been given, $a(\tau)$ can really be understood as a mere function of time. This function is henceforth plugged into the Fourier mode equations for the perturbations, where it serves as a source for “particle production” just as in the usual inflation calculations. This mode equation reads [60]

$$v_k'' + \left(\omega k^2 - \frac{a''}{a} \right) v_k = 0, \quad (16)$$

where v reduces to the Mukhanov-Sasaki variable [51] when the background satisfies the classical Einstein equations and a prime means derivative with respect to conformal time. The potential $V = a''/a$, which yields the scale of curvature of the bouncing quantum background $\ell_c \equiv aV^{-1/2}$, has the same qualitative properties as the potential for perturbations in inflation: it is negligible when $|\eta| \rightarrow \infty$ and has its maximum around $\eta = 0$. As an explicit example, its form for a radiation fluid reads

$$V_{\text{rad}} = \frac{1}{T_0^2 \left(1 + \frac{\eta^2}{T_0^2} \right)^2}, \quad (17)$$

whereas the dust case reads

$$V_{\text{dust}} = \frac{2a_0^2}{9T_0^2} \frac{3 + x^2}{(1 + x^2)^{4/3}}, \quad (18)$$

where we have set $x \equiv \tau/T_0$. In both cases, the potential is vanishing in the limit $|\tau| \rightarrow \infty$, i.e. far from the bounce, and reaches its maximum at the bounce itself. Hence, as in inflation, scales of physical perturbations are much smaller than the curvature scale in the far past (they are above the potential, i.e. $k^2 \gg V$), where they oscillate and can be set in quantum vacuum state. When the bounce approaches, these scales get larger with respect to the curvature scale and eventually enter the potential ($k^2 \ll V$), where they get amplified. Finally, they become smaller again than the curvature scale in the far future (exit from the potential), where they oscillate again, now amplified. Most of the time, i.e. far from the bounce itself, the background scale factor thus obtained is undistinguishable from the solution classical Einstein (Friedmann) equation. As all of the effects discussed in the previous section take place in these regimes, one can consistently assume Einstein gravity throughout the relevant history of the Universe.

⁶ As all quantum trajectories, and hence the mean value of a , have this same functional form, then using a probabilistic interpretation, like the Many Worlds interpretation [67], will presumably give the same forthcoming results: we expect the explicit use of a Bohmian interpretation for quantum mechanics to be of no practical consequence.

In this category of models, the index n_s of Eq. (6) can be tuned as close to unity as one wishes, but from above. This means the spectrum is expected to be slightly blue, as opposed to at least the simplest single field inflationary models in which it is slightly red. The latest WMAP3 [1, 2] observations do not currently favor this prediction but do not rule it out either, especially if n_s is sufficiently close to 1 [68].

The amplitude of the perturbations needs be calculated numerically. The free parameters of the background must then be adjusted in order to fit observational data and theoretical consistency and completeness constraints. On the observational side, one must have a background compatible with the large Universe we see today and perturbations which fit the CMB data. As for the theoretical issues, one must impose that the gauge invariant variables always remain in the linear regime and, at least in principle, that scales of cosmological interest were never smaller than the Planck length in order to avoid any trans-Planckian problem by construction.

Taking into account the constraints on the parameters due to the normalization conditions and the compromise that the model should describe our real Universe in fact leads to imposing that the scale factor at the bounce must be large in Planck units. Once this is done, there is no trans-Planckian problem [13, 14] and no departure from linearity.

Finally, we should like to emphasize a major, possibly observable in the future, difference between inflation and such bouncing models: the so-called consistency [51] relation between the tensor-to-scalar ratio T/S and the spectral index. While a typical inflation prediction is a linear relation, the bounce case, on the other hand, predicts [47] $T/S \propto \sqrt{n_s - 1}$. In the case the scalar index is very close to 1, which is the current best fit with WMAP data [68], then T/S would be very small. Further improved data, notably on B modes in the CMB, will provide a very stringent, and hopefully discriminating, test as they will have the ability to provide a measure of T/S up to values [69] of order 10^{-3} .

V. CONCLUSIONS

The theory of linear quantum perturbations has been successfully applied in the framework of a classical inflationary background: only the perturbations were quantized, leading to a sort of semiclassical approximation to quantum gravity [4]. We have developed a Hamiltonian formalism where not only the perturbations but also the background could be quantized [47, 60, 61, 62]. This led to a picture of quantum perturbations evolving in a nonsingular bouncing background spacetime from a vacuum state yielding spectral indices and amplitudes that can be made to agree with observations provided the dominant fluid in the background when the perturbation scale becomes smaller than the curvature radius is dust. The curvature scale at the bounce can always be

set larger than the Planck length, and hence the calculations are not spoiled by higher order quantum gravity effects. Finally, such a model can be extended to include a radiation-dominated decelerating phase before nucleosynthesis without corrupting its main features properties. This thus provides a simple theoretical framework where only the basic principles of general relativity and quantum mechanics, together with the assumption of the existence of a dustlike fluid (dark matter?), yield what can be argued to be a sensible model. Furthermore, such behaviors can also be obtained in other nonquantum bouncing models [48, 56, 57], indicating that these are not particular properties of the specific models here discussed.

We have also argued that general bounces provide different perspectives on old issues such as flatness and homogeneity. In fact, these problems may be alleviated or solved using simple physical arguments which can be applied only in this context.

There are, however, many open questions left to be addressed and some weak points. Let us list them below.

→ Baryogenesis and dark energy are not addressed, but the latter could actually provide a means of obtaining a redder spectrum.

→ Was radiation always there, or it was produced at the bounce, e.g. through the evaporation of mini black holes or monopolonium bound states? Does its presence alter the amplitude of the perturbations, and if so, how?

→ As primordial perturbations are enhanced at the bounce, similarly one could think that they also might lead to large amounts of particle production. The relic density of these particles needs be evaluated for each model.

→ Although spatial curvature is expected to be negligible during most of the evolution, particularly in the expanding phase, it may be quite important at the bounce itself and modify the amplitude of the perturbations.

All of the properties of bouncing models and their open issues show that they seem to provide a robust alternative to inflation. A less ambitious role, although still very important, should be that they can complement inflation by solving the singularity problem, ease the homogeneity problem and yield appropriate initial conditions for it [52]. In any case, bounce cosmology leads to numerous new, hopefully measurable [70], ideas and effects, yet to be investigated. The tensor-to-scalar prediction is already an example of such an effect rendering the paradigm testable.

As a final remark, we would like to stress that, in contradistinction with models in which time begins, there is no point to asking what the probability is of the appearance of some particular eternal model out of nothing. Contrary to the usual perspectives, one can as well assume existence to be conceptually prior to nonexistence, i.e. existence itself may not be deserving explanation. This is the idea underlying our category of models: the Universe always existed and its “appearance” is thus a non question.

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