

Cosmic acceleration from interaction of ordinary fluids

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Abstract

Cosmological models with two interacting fluids, each satisfying the strong energy condition, are studied in the framework of classical General Relativity. If the interactions are phenomenologically described by a power law in the scale factor, the two initial interacting fluids can be equivalently substituted by two non interacting effective fluids, where one of them may violate the strong energy condition and/or have negative energy density. Analytical solutions of the Friedmann equations of this general setting are obtained and studied. One may have, depending on the scale where the interaction becomes important, non singular universes with early accelerated phase, or singular models with transition from decelerated to accelerated expansion at large scales. Among the first, there are bouncing models where contraction is stopped by the interaction. In the second case, one obtains dark energy expansion rates without dark energy, like Λ CDM or phantomic accelerated expansions without cosmological constant or phantoms, respectively.

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1 Introduction

In general, the fluids describing the matter content of a cosmological model are considered to be non interacting. For instance, in the Standard Cosmological Model, cold dark matter, neutrinos, dark energy and the baryon-photon fluid are usually taken to be decoupled. However, it is well known that in the early Universe many of these fluids were coupled through annihilation and/or scattering processes [1]. Many approaches to describe these primordial interactions have been proposed [2, 3, 4, 5, 6, 7]. Furthermore, interactions between the two dark components have been investigated in order to explain the coincidence problem, and effective phantom acceleration without phantom equation of state [8, 9, 10, 11]. Finally, interactions between two dark matter components, or dark matter self-interaction [12], and interaction between dark matter and baryons [13] have also been investigated. In the last case, the possibility of interactions between baryons and dark matter due to the strong force has been proposed because it can solve some conflicts between numerical simulations on structure formation with observations. One can justify the fact that such strong interactions have not yet been detected because most of the WIMP detectors are ground based [13].

In this report, we explore the consequences of some phenomenological models of interaction between two positive energy fluids with constant equation of state, both satisfying the strong energy condition. We follow the line of research of Refs [10, 11], postponing the physical justification of these phenomenological proposals for future publications in order to concentrate on their physical consequences, and their potentialities to solve cosmological puzzles (e.g. concordance problem, cosmological singularities), with the hope that they may be tested soon [14]. Assuming a power law form in the scale factor for these interactions, similar to what was done in Ref. [11] but not restricting to dark energy-dark matter interactions, we arrive at an effective Friedmann equation containing two effective non interacting perfect fluids where one of them, depending on the interaction, may have arbitrary equation of state, including phantom-like behaviour, and/or negative energy density³. This gives rise to many types of cosmological analytical solutions which we obtain explicitly: bouncing universes, as those studied in Refs. [16, 17, 18], and singular models with late acceleration phases, like the Λ CDM model or others with phantom acceleration leading to a big-rip. In all these cases, the occurrence of interactions between the fluids may lead, even in the framework of classical General Relativity, to accelerating phases in the Universe, without the need to suppose that such fluids violate the strong energy condition.

The paper is organized as follows: in Section II we describe our phenomenological models and we arrive at the effective Friedmann equations, from which we obtain the analytical solutions presented and discussed in Section III. Section IV is devoted to the conclusions.

³Some similar situations were studied in Refs. [15].

2 Cosmology with two interacting fluids

We model the interaction between two fluids satisfying the strong energy condition and with equations of state $p_i = w_i \rho_i$, $i \in \{1, 2\}$, with $w_i = \text{const.} > -1/3$, and $\rho_i > 0$ as follows:

$$\begin{aligned}\dot{\rho}_1 + 3H(1 + w_1)\rho_1 &= -Q, \\ \dot{\rho}_2 + 3H(1 + w_2)\rho_2 &= Q,\end{aligned}\tag{1}$$

where $H = \dot{a}/a$ is the Hubble parameter, Q is the interaction rate, and a dot represents derivative in cosmic time.

We assume, without loss of generality, that the energy density of the second fluid can be written as:

$$\rho_2 = \rho_{2_0} \frac{f}{f_0} \left(\frac{a_0}{a}\right)^{3(1+w_2)},\tag{2}$$

where ρ_{2_0} , f_0 and a_0 are constants, and f is an arbitrary time-dependent function. It follows from (1) and (2) that

$$Q = \rho_2 \frac{\dot{f}}{f} = \rho_{2_0} \frac{\dot{f}}{f_0} \left(\frac{a_0}{a}\right)^{3(1+w_2)}\tag{3}$$

We now assign an ansatz for f ,

$$\frac{f}{f_0} = \left(\frac{a}{a_0}\right)^{-3w_f},\tag{4}$$

where w_f is a constant. Hence

$$\rho_2 = \rho_{2_0} \left(\frac{a}{a_0}\right)^{[-3(1+w_2+w_f)]}.\tag{5}$$

Consequently

$$Q = -3w_f \rho_{2_0} H \left(\frac{a}{a_0}\right)^{-3(1+w_2+w_f)},\tag{6}$$

The solution of the conservation equation for the first fluid

$$\dot{\rho}_1 + 3H(1 + w_1)\rho_1 = 3w_f \rho_{2_0} H \left(\frac{a}{a_0}\right)^{-3(1+w_2+w_f)}\tag{7}$$

is given by

$$\rho_1 = C_1 a^{-3(1+w_1)} + \frac{\rho_{2_0} w_f}{w_1 - w_2 - w_f} \left(\frac{a}{a_0}\right)^{-3(1+w_2+w_f)},\tag{8}$$

with C_1 a constant. The total energy density then reads

$$\rho_T = C_1 a^{-3(1+w_1)} + \frac{\rho_{2_0}(w_1 - w_2)}{w_1 - w_2 - w_f} \left(\frac{a}{a_0}\right)^{-3(1+w_2+w_f)}.\tag{9}$$

Making the definitions,

$$w_+ \equiv w_1, \quad w_- \equiv w_2 + w_f, \quad \rho_+ \equiv C_1, \quad \rho_- = \frac{\rho_{20} a_0^{3(1+w_2+w_f)} (w_+ - w_2)}{w_+ - w_-} \quad (10)$$

the Friedmann equation can be written as:

$$H^2 = l_{pl}^2 \left(\rho_+ a^{-3(1+w_+)} + \rho_- a^{-3(1+w_-)} \right), \quad (11)$$

where $l_{pl}^2 = 8\pi G/3$.

Hence, from two interacting fluids satisfying the strong energy condition, $w_1, w_2 > -1/3$, and positive energy density, $\rho_1, \rho_2 > 0$, interacting via Eq. (1) with Q given in (6), one obtains two effective non interacting fluids characterized by the parameters w_+, w_-, ρ_+, ρ_- in the Friedmann equation (11), where one satisfies the strong energy condition, $w_1 = w_+ > -1/3$, but the other can violate the strong energy condition and/or have negative energy as long as, from definitions (10), and depending on the values of w_f in (6), one may obtain $w_- < -1/3$ and/or $\rho_- < 0$. This fact leads to interesting cosmological solutions where bounces and/or late accelerated phases may happen. We present these possibilities in the following section. Note that if the two fluids have the same equation of state, $w_1 = w_2$ there is no second effective fluid. Also, if $w_+ = w_-$, the quantity ρ_- cannot be defined. Hence we do not consider this singular point in parameter space below.

3 Analytical solutions

By introducing a new coordinate time τ :

$$d\tau = \frac{dt}{a^\beta}, \quad \text{with} \quad \beta = \frac{3}{2}(2w_+ - w_- + 1) \quad (12)$$

we have as solution for the scale factor:

$$a(\tau) = a_b \left(\frac{\tau^2}{\tau_0^2} - \frac{|\rho_-|}{\rho_+} \right)^\alpha, \quad (13)$$

with

$$\alpha = \frac{1}{3(w_- - w_+)} \quad (14)$$

$$a_b = \left(\frac{|\rho_-|}{\rho_+} \right)^\alpha \quad (15)$$

$$\tau_0^2 = \frac{4\alpha^2 |\rho_-|}{l_{pl}^2 \rho_+^2}. \quad (16)$$

With the new coordinate time τ , it is therefore possible to generalize the solution in the conformal time found in [17], and the cosmological models with

dust plus dark energy with arbitrary constant equation of state obtained in [20], to arbitrary values of w_+, w_- .

The Hubble function is

$$\frac{\dot{a}}{a} = \frac{2\alpha\tau}{\tau_0^2 a^\beta \left(\frac{\tau^2}{\tau_0^2} - \frac{|\rho_-|}{\rho_-} \right)}, \quad (17)$$

while cosmic acceleration reads

$$\frac{\ddot{a}}{a} = -\frac{2\alpha}{\tau_0^2 a^{2\beta} \left(\frac{\tau^2}{\tau_0^2} - \frac{|\rho_-|}{\rho_-} \right)^2} \left[(1 + 3w_+) \alpha \frac{\tau^2}{\tau_0^2} + \frac{|\rho_-|}{\rho_-} \right]. \quad (18)$$

One may have transitions from decelerated to accelerated phases, and vice-versa, when

$$\tau^2 = -\frac{|\rho_-|}{\rho_-} \frac{\tau_0^2}{\alpha(1 + 3w_+)}. \quad (19)$$

The asymptotic behaviours occur for $\tau \rightarrow \pm\infty$ and, if $\rho_- > 0$, for $\tau \rightarrow \pm\tau_0$. In the first case one has

$$t \propto |\tau|^{3\alpha(1+w_+)}, \quad a(t) \propto |t|^{2/[3(1+w_)]}, \quad (20)$$

while for the second one obtains,

$$t \propto |\tau \pm \tau_0|^{3\alpha(1+w_-)/2}, \quad a(t) \propto |t|^{2/[3(1+w_-)]}, \quad (21)$$

Finally, looking at the evolution of the interaction term (6), one should expect that its modulus $|Q|$ would decrease when the Universe expands and Hubble time $t_H = 1/H$ increases. which is indeed the case of Eq. (6) unless $w_- < -1 \Rightarrow w_f < -1 - w_2$. However, this extreme situation can be explained using Eqs. (5,8), where it is shown that the energy densities of the two fluids ρ_1 and ρ_2 increase with expansion in that case, making the interaction stronger. We will analyze these situations below in more details using the equation

$$\begin{aligned} & - \frac{a_b^{6(1+w_+)}}{3w_f \rho_{2_0} a_0^{3(1+w_2+w_f)}} \dot{Q} = \\ & - \frac{2\alpha}{\tau_0^2 \left(\frac{\tau^2}{\tau_0^2} - \frac{|\rho_-|}{\rho_-} \right)^{2(1+w_-)/(w_- - w_+)}} \left[3\alpha(3 + 2w_- + w_+) \frac{\tau^2}{\tau_0^2} + \frac{|\rho_-|}{\rho_-} \right]. \end{aligned} \quad (22)$$

We will now discuss in more details the possible cases in terms of the sign of ρ_- . As we are assuming that both original fluids satisfy the strong energy condition, then $w_+ = w_1 > -1/3$.

3.1 $\rho_- < 0$

The scale factor reads:

$$a(\tau) = a_b \left(\frac{\tau^2}{\tau_0^2} + 1 \right)^\alpha, \quad (23)$$

implying that $-\infty < \tau < \infty$, and hence, from Eq. (17), the scale factor has an extremum at $\tau = 0$.

3.1.1 $\alpha > 0$

When $\alpha > 0$, Eq. (18) shows that this extremum is a minimum, and we have a non singular bouncing universe. There are transitions from deceleration to acceleration and vice-versa because there are real roots from Eq. (19) in that case: $\alpha(1 + 3w_+) > 0$. As $w_- > w_+$, the negative energy fluid dominates when the universe is small, avoiding the singularity in the accelerated phase, while the positive energy fluid dominates when the universe is large, which expands decelerately in this regime, as it can be seen from Eq. (20).

Regarding the interaction term Q , one can see from Eq. (22) that for

$$0 < \frac{\tau^2}{\tau_0^2} < \frac{1}{3\alpha(3 + 2w_- + w_+)} \approx 1 \quad (24)$$

Q is increasing while the model expands. This can be understood by noticing that this is the period near the bounce⁴, where the scale factor increases slowly while the Hubble time $t_h = 1/H$ decreases abruptly, largely compensating the slow expansion.

3.1.2 $\alpha < 0$

When $\alpha < 0$, Eq. (18) shows that the extremum is a maximum. When the universe is small, the w_+ fluid dominates and the w_- fluid becomes important only around the maximum. We have a big-bang big-crunch model (remember we are assuming $w_+ > -1/3$, hence $\tau \rightarrow \pm\infty$ implies t finite in this case, see Eq. (20)). There are no transitions from deceleration to acceleration and vice-versa because $\alpha(1 + 3w_+) < 0$ in this case (see Eq. (19)), whatever is the value of w_- .

The interaction term may increase with expansion only for the period shown in Eq. (24), now around the maximum of the scale factor, if $w_- < -(3+w_+)/2 < -4/3$ (remember that $w_- < w_+$ in this case). This can be justified in terms of the slow increasing of a in that period, the increasing of the energy densities of the two fluids ρ_1 and ρ_2 because $w_- < -1$.

3.2 $\rho_- > 0$

In this case the scale factor reads

$$a(\tau) = a_b \left(\frac{\tau^2}{\tau_0^2} - 1 \right)^\alpha, \quad (25)$$

implying that $-\infty < \tau < -\tau_0$, or $\tau_0 < \tau < \infty$. Hence, from Eq. (17), the scale factor has no extremum being either an always contracting or always expanding model. We will concentrate on the always expanding solutions.

The conditions for having transitions from deceleration to acceleration and vice-versa in this case are (see Eq. (19)), besides $\alpha(1 + 3w_+) < 0$, the condition

⁴As $3 + 2w_- + w_+ > 2$ in this case, $[3\alpha(3 + 2w_- + w_+)]^{-1}$ in Eq. (22) can be big only if $0 < \alpha \ll 1$, but then the bounce also lasts very long in τ (see Eq. (19))

that $|\tau| > |\tau_0|$ (see Eq. (13)), which implies that $|\alpha(1 + 3w_+)| < 1$. Hence, in this case, $\alpha > 0$ imposes that $w_+ < -1/3$ and $w_- > -1/3$, and $\alpha < 0$ implies that $w_+ > -1/3$ and $w_- < -1/3$, as it should be if the two effective fluids have positive energy. As we are assuming $w_+ > -1/3$, there are no transitions when $\alpha > 0$.

3.2.1 $\alpha > 0$

When $\alpha > 0$, both effective fluids satisfy the energy conditions and have positive energy. These are the standard cases of two ordinary fluids governing a universe with decelerated expansion from an initial singularity. The interaction term Q always decreases with expansion.

3.2.2 $\alpha < 0$

When $\alpha < 0$, the w_+ fluid dominates when the universe is small, necessarily reaching a singularity (from Eq. (20), $\tau \rightarrow \pm\infty$ implies t finite in this case), and the w_- fluid dominates when it is large. If $w_- < -1/3$ one can have a transition from decelerated to accelerated expansion, and if $w_- < -1$, one gets a big rip.

As pointed above, in this last case one may have increasing $|Q|$ with expansion. Looking at Eq. (22), one can see that, for $w_- \leq -(3 + w_+)/2$, or $w_f \leq -(3 + w_1 + 2w_2)/2$, one has the bizarre situation where $|Q|$ always increases with expansion, even near the initial singularity, when the ordinary $w_+ = w_1$ fluid dominates and expansion takes place in the standard way. It seems that the increasing in ρ_2 is so strong (see Eq. (5)), that it compensates expansion, and the decreasing of ρ_- and H . When $-(3 + w_+)/2 < w_- < -1$ one has the more reasonable situation where $|Q|$ decreases with expansion when the ordinary $w_+ = w_1$ fluid dominates and increases only in the period given by

$$1 < \frac{\tau^2}{\tau_0^2} < -\frac{1}{3\alpha(3 + 2w_- + w_+)}, \quad (26)$$

when we expect that the energy densities of the two fluids ρ_1 and ρ_2 increase with expansion (see Eqs. (5,8)), making the interaction stronger, and Hubble time $t_H = 1/H$ decreases. Note that, depending on the value of w_+ and w_- , the increasing of $|Q|$ may happen after $\rho_- \rho_+$ equilibrium, which takes place at $\tau^2 = 2\tau_0^2$. For instance, if $w_+ = 0$, there is a period when $|Q|$ decreases and the w_- fluid dominates if $-6/5 < w_- < -1$.

For $w_- \geq -1$, $|Q|$ always decreases with expansion.

3.3 Final remarks

For the sake of completeness, let us mention some different models one may obtain if the condition $w_+ > -1/3$ is relaxed. One may have pre-big bang [19] and singular inflationary models without graceful exit when $\rho_- > 0$. When $\rho_- < 0$, there are bouncing models between big rips or between accelerated contraction and expansion, and models which expands from a pre-big bang like

initial state, till a maximum size, and then contracts to a time reversed pre-big bang like final state, with transitions from accelerated to decelerated phases and vice-versa around the maximum. These are not realistic models.

From all these possibilities, the most interesting cases are the non singular solutions and the models with late accelerated expansion. Among the first, there are the bouncing models with $\rho_- < 0$ and $\alpha > 0$. They are non singular models which are connected to a standard expansion phase dominated by a fluid satisfying the strong energy condition when the universe is large. In particular, the bouncing model studied in Ref. [16] can be obtained with the choice $w_1 = 1/3$, $w_2 = 0$ and $w_f = 1$, yielding an interaction term given by $Q = -3KH a^{-6}$, which is strongly suppressed when a is large. One can view this interaction as happening in a temperature where baryons are relativistic but dark matter is not, and they interact through the strong force as suggested in Ref. [13]. This could also happen in a temperature where both are relativistic, but not exactly with the same equation of state parameter. In that case one should have $w_f \approx 2/3$ and $Q \approx -2KH a^{-6}$.

In the second group there are the models with transition from decelerated to accelerated expansion, as in the cases with parameters $\rho_- > 0$, $\alpha < 0$, and $w_- < -1/3$. One possibility would be to have two different dark matter components, or dark matter and baryons, with $0 < w_1 \ll 1$, $0 < w_2 \ll 1$ and $w_1 > w_2$, interacting with $w_f = -1$, yielding an effective Λ CDM model with $w_+ \approx 0$ and $w_- \approx -1$. In this case, one has $Q \approx 3KH$, a mild decreasing with expansion depending only on the Hubble parameter. Note that, as the effective cosmological constant is given by $\Lambda \propto \rho_- \propto (w_1 - w_2)$, its smallness could then be related with the smallness of w_1 and w_2 .

4 Conclusion

We have obtained some interesting cosmological models with flat spatial sections from ordinary fluids which interact with the phenomenological interaction rate (6). Bouncing models, which cannot be obtained from two non interacting fluids within classical General Relativity unless one of them violates the null energy condition [21], may arise in this context even if the two interacting fluids can be modelled by dust and/or relativistic fluids, like dark matter components and baryons, depending on the background temperature. This same combination of fluids, with another choice of the power w_f in Eq. (6), may lead to the Λ CDM expansion rate without the need to introduce any fundamental cosmological constant, whose small observational value is a challenge for particle physics theory.

Of course the relevance of the phenomenological interaction rate (6) in different phases of the history of the real Universe must be justified in micro-physical grounds. We will come back to this problem in forthcoming publications. Also, the evolution of primordial quantum perturbations in such models must be carried out in order to compare the results with CMBR data. For instance, one should investigate if the bouncing models presented here could also lead to scale

invariant spectra of perturbations as it was verified in other bouncing universes [22]. These are subjects beyond the aim of the present report, which is to point out that it is maybe not necessary to evoke the existence of exotic and/or unknown substances to yield the present acceleration of the Universe and/or to avoid the initial singularity within classical General Relativity.

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