Bouncing model in brane world theory

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We examine the nonlinear dynamics of a closed Friedmann-Robertson-Walker universe in the framework of brane world formalism with a timelike extra dimension. In this scenario, the Friedmann equations contain additional terms arising from the bulk-brane interaction, which provide a concrete model for nonsingular bounces in the early phase of the Universe. We construct a nonsingular cosmological scenario sourced with dust, radiation, and a cosmological constant. The structure of the phase space shows a nonsingular orbit with two accelerated phases, separated by a smooth transition corresponding to a decelerated expansion. Given observational parameters we connect such phases to a primordial accelerated phase, a soft transition to Friedmann (where the classical regime is valid), and a graceful exit to a de Sitter accelerated phase.

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I. INTRODUCTION

Although general relativity is the most successful theory that presently describes gravitation, it presents some intrinsic crucial problems when we try to construct a cosmological model in accordance with observational data. In cosmology, the ΛCDM model gives us important predictions about the evolution of the Universe and about its current state [1]. However, let us assume that the initial conditions of our Universe were fixed when the early Universe emerged from the semi-Planckian regime and started its classical expansion. Evolving back such initial conditions using the Einstein field equations, we see that our Universe is driven toward an initial singularity where the classical regime is no longer valid [2].

Notwithstanding the cosmic censorship conjecture [3], there is no doubt that general relativity must be properly corrected or even replaced by a completely new theory, let us say a quantum theory of gravity. This demand is in order to solve the issue of the presence of the initial singularity predicted by classical general relativity, either in the formation of a black hole or in the beginning of the Universe. While a full quantum gravity theory remains presently an elusive theoretical problem, quantum gravity corrections near singularities formed by gravitational collapse have been the object of much recent research, from quantum cosmology [4,5] to D-brane theory [6–9]. In the latter scenario, extra dimensions are introduced constituting the bulk space. In the case of spatially homogeneous and isotropic cosmologies, the basic resulting distinction between the two approaches lies in the corrections introduced in the Friedmann Hamiltonian constraint, leading either to modifications in the kinetic energy terms or to extra potential energy terms. In both cases we may have bounces in the scale factor corresponding to the avoidance of a singularity in the models. In this context, the initial conditions from which our Universe has evolved should depend crucially on the adopted version of the theory to describe the dynamics around the singularity.

One of the most important characteristics of our Universe, supported by observational data, is its large scale of homogeneity and isotropy. In fact, the scale of homogeneity and isotropy is empirically well accepted for distances above 100 Mpc. Indeed this is the main reason that makes the geometry of Friedmann-Lemaître-Robertson-Walker (FLRW) a powerful theoretical tool for the construction of a cosmological scenario [1]. However, when we consider a homogeneous and isotropic model filled with baryonic matter, we find several difficulties when we take into account the primordial state of our Universe. Among such difficulties, we can mention the horizon and flatness problems [1].

As a possible solution to these problems emerged the so-called inflationary paradigm [1,10]. Although this fundamental paradigm allows us to solve the horizon and flatness problems, inflationary cosmology does not solve the problem of the initial singularity. Therefore, nonsingular models from a new theory that provide alternative solutions to these problems should be strongly considered.

In this paper we adhere to the brane world scenario, where a timelike noncompact extra dimension is introduced, constituting the bulk space, and all the matter content of our Universe would be trapped on a four-dimensional spacetime embedded in the bulk. At low energies general relativity is recovered [8], but at high energy scales significant changes are introduced into the gravitational dynamics and the singularities can be removed [6].

While spacelike extra dimensions theories have received more attention in the past decades [9], studies involving extra timelike dimensions have been considered [11] despite the fact that propagating tachyonic modes or negative norm states may arise because of timelike extra dimensions. These modes have been regarded as problematic since they might violate causality [12] by considering interactions among usual particles. Issues like the exceedingly small lower bound on the size of timelike extra dimensions [13], the imaginary self-energy of charged fermions induced by tachyonic modes—which seems to cause disappearance of fermions into nothing—and the spontaneous decay of stable particles induced by tachyons with negative energy are major difficulties [12]. Nevertheless, to address the cosmological constant problem in Kaluza-Klein theories [14] or reconcile a solution of the hierarchy problem with the cosmological expansion of the Universe [15], timelike extra dimensions have been considered. On the other hand, it has been shown in Ref. [16] that the appearance of massless ghosts in an effective four-dimensional theory can be avoided by considering topological criteria in Kaluza-Klein theories with extra compactified timelike dimensions. Moreover, avoidance of propagating tachyonic or negative norm states can also be achieved by considering a noncompact timelike extra dimension [7], which is the case in the model of this paper.

We organize the paper as follows. In Sec. II we present a brief review of the modified Einstein field equations in the brane world scenario. In Sec. III, we construct a nonsingular cosmological scenario sourced with dust, radiation, and a cosmological constant. In Sec. IV, we show that given the observational parameters, we can connect such phases to a primordial accelerated expansion, a soft transition to Friedmann (where the classical regime is valid), and a graceful exit to a de Sitter accelerated phase. As our final remarks, we discuss some of its possible imprints in the physics of cosmological perturbations.

II. FIELD EQUATIONS

For the sake of completeness let us give a brief introduction to brane world theory, making explicit the specific assumptions used in obtaining the dynamics of the model. We rely on Refs. [6,9], and our notation basically follows [2]. Let us start with a four-dimensional Lorentzian brane Σ with metric g_{ab}, embedded in a five-dimensional conformally flat bulk $\mathcal{M}$ with metric g_{AB}. Capital Latin indices range from 0 to 4, small Latin indices range from 0 to 3. We regard Σ as a common boundary of two pieces $\mathcal{M}_1$ and $\mathcal{M}_2$ of $\mathcal{M}$, and the metric g_{ab} induced on the brane by the metric of the two pieces should coincide although the extrinsic curvatures of Σ in $\mathcal{M}_1$ and $\mathcal{M}_2$ are allowed to be different. The action for the theory has the general form

$$\begin{aligned} S = \frac{1}{2\kappa_5^2}\Big\{ & \int_{M_1} \sqrt{-\epsilon^{(5)}g}[^{(5)}R - 2\Lambda_5 + 2\kappa_5^2 L_5]d^5x \\ & + \int_{M_2} \sqrt{-\epsilon^{(5)}g}[^{(5)}R - 2\Lambda_5 + 2\kappa_5^2 L_5]d^5x \\ & + 2\epsilon\int_\Sigma \sqrt{-^{(4)}g}K_2 d^4x - 2\epsilon\int_\Sigma \sqrt{-^{(4)}g}K_1 d^4x\Big\} \\ & + \frac{1}{2}\int_\Sigma \sqrt{-^{(4)}g}\Big(\frac{1}{2\kappa_4^2}{}^{(4)}R - 2\sigma\Big)d^4x \\ & + \int_\Sigma \sqrt{-^{(4)}g}L_4(g_{\alpha\beta}, \rho)d^4x. \end{aligned} \tag{1}$$

In the above $^{(5)}R$ is the Ricci scalar of the Lorentzian five-dimensional metric g_{AB} on $\mathcal{M}$, and $^{(4)}R$ is the scalar curvature of the induced metric g_{ab} on Σ. The parameter σ is denoted the brane tension. The unit vector n^A normal to the boundary Σ has norm ϵ. If $\epsilon = -1$ the signature of the bulk space is $(-,-,+,+,+)$, so that the extra dimension is timelike. The quantity $K = K_{ab}g^{ab}$ is the trace of the symmetric tensor of extrinsic curvature $K_{ab} = Y_{,a}{}^C Y_{,b}{}^D \nabla_C n_D$, where $Y^A(x^a)$ are the embedding functions of Σ in $\mathcal{M}$ [17]. While $L_4(g_{ab}, \rho)$ is the Lagrangian density of the perfect fluid [18] (with equation of state $p = \alpha\rho$), whose dynamics is restricted to the brane Σ, L_5 denotes the Lagrangian of matter in the bulk. All integrations over the bulk and the brane are taken with the natural volume elements $\sqrt{-\epsilon^{(5)}g}d^5x$ and $\sqrt{-^{(4)}g}d^4x$, respectively. κ_5 and κ_4 are Einstein constants in five and four dimensions. We use units such that $c = 1$.

Variations that leave the induced metric on Σ intact result in the equations

$$^{(5)}G_{AB} + \Lambda_5{}^{(5)}g_{AB} = \kappa_5^{2(5)}T_{AB}, \tag{2}$$

while considering arbitrary variations of g_{AB} and taking into account (2) we obtain

$$^{(4)}G_{ab} + \epsilon\frac{\kappa_4^2}{\kappa_5^2}(S_{ab}^{(1)} + S_{ab}^{(2)}) = \kappa_4^2(\tau_{ab} - \sigma g_{ab}), \tag{3}$$

where $S_{ab} \equiv K_{ab} - Kg_{ab}$, and τ_{ab} is the energy momentum tensor associated with L_4. In the limit $\kappa_4 \to \infty$ Eq. (3) reduces to the Israel-Darmois junction condition [19]

$$(S_{ab}^{(1)} + S_{ab}^{(2)}) = \epsilon\kappa_5^2(\tau_{ab} - \sigma g_{ab}). \tag{4}$$

We impose the Z_2 symmetry [9] and use the junction conditions (4) to determine the extrinsic curvature on the brane,

$$K_{ab} = -\frac{\epsilon}{2}\kappa_5^2\Big[\Big(\tau_{ab} - \frac{1}{3}\tau g_{ab}\Big) + \frac{\sigma}{3}g_{ab}\Big]. \tag{5}$$

Now using Gauss equation

$$^{(4)}R_{abcd} = {}^{(5)}R_{MNRS}Y^M_{,a}Y^N_{,b}Y^R_{,c}Y^S_{,d} + \epsilon(K_{ac}K_{bd} - K_{ad}K_{bc}) \tag{6}$$

together with Eqs. (2) and (5) we arrive at the induced field equations on the brane

$$\begin{aligned} ^{(4)}G_{ab} = & -\Lambda_4{}^{(4)}g_{ab} + 8\pi G_N\tau_{ab} + \epsilon\kappa_5^4\Pi_{ab} \\ & - \epsilon E_{ab} + \epsilon F_{ab}, \end{aligned} \tag{7}$$

where we define

$$\Lambda_4 := \frac{1}{2}\kappa_5^2\Big(\frac{\Lambda_5}{\kappa_5^2} + \frac{1}{6}\epsilon\kappa_5^2\sigma^2\Big), \tag{8}$$

$$G_N := \epsilon \frac{\kappa_5^4 \sigma}{48\pi}, \tag{9}$$

$$\Pi_{ab} := -\frac{1}{4}\tau_a^c \tau_{bc} + \frac{1}{12}\tau\tau_{ab} + \frac{1}{8}{}^{(4)}g_{ab}\tau^{cd}\tau_{cd} - \frac{1}{24}\tau^{2(4)}g_{ab}, \tag{10}$$

$$F_{ab} := \frac{2}{3}\kappa_5^2\Big\{\epsilon^{(5)}T_{BD}Y^B_{,a}Y^D_{,b} + \Big[{}^{(5)}T_{BD}n^B n^D - \frac{1}{4}\epsilon^{(5)}T\Big]{}^{(4)}g_{ab}\Big\}, \tag{11}$$

E_{ab} is the electric part of the Weyl tensor in the bulk induced in the brane, T_{AB} is the energy momentum in the bulk, and G_N defines the Newton's constant on the brane. For a timelike extra dimension we have that $\epsilon = -1$ in our conventions, implying that $\sigma < 0$ in accordance with observations. We also remark that the effective four-dimensional cosmological constant might be set to zero, or made conveniently small, in the present case of an extra timelike dimension by properly fixing the bulk cosmological constant as $\Lambda_5 \simeq \frac{1}{6}\kappa_5^4\sigma^2$. It is important to note that for a four-dimensional brane embedded in a conformally flat empty bulk we have the absence of the Weyl conformal tensor projection E_{ab} and of F_{ab} in Eq. (8).

Accordingly, Codazzi's equations imply that

$$\nabla_a K - \nabla_b K_a^b = -\frac{1}{2}\epsilon\kappa_5^2\nabla_b\tau_a^b, \tag{12}$$

resulting in

$$\nabla^a E_{ab} = \nabla_b \tau_a^b + \kappa_5^4\nabla^a\Pi_{ab} + \nabla^a F_{ab}, \tag{13}$$

where ∇_a is the covariant derivative with respect to the induced metric g_{ab}. Equations (7) and (13) are the dynamical equations of the gravitational field on the brane.

III. THE MODEL

Let us consider a FLRW geometry on the four-dimensional brane embedded in a five-dimensional de Sitter bulk with a timelike extra dimension ($\epsilon = -1$) [6]. Considering comoving coordinates on the brane, the line element is given by

$$ds^2 = -dt^2 + a^2(t)\Big[\frac{1}{1-kr^2}dr^2 + r^2 d\Omega^2\Big], \tag{14}$$

where $a(t)$ is the scale factor, k is the spatial curvature, and $d\Omega^2$ is the solid angle.

The matter content of the model, restricted to the brane, is given by noninteracting perfect fluids, namely, dust and radiation, with respective equations of state $p_{\rm dust} = 0$, $p_{\rm rad} = \rho_{\rm rad}/3$, and energy momentum tensor $\tau^{ab} := \tau^{ab}{}_{\rm dust} + \tau^{ab}{}_{\rm rad}$ satisfying $\nabla_b\tau^{ab}{}_{\rm dust} = 0 = \nabla_b\tau^{ab}{}_{\rm rad}$. In this situation we have that

$$\Pi_{00} = \frac{1}{12}(\rho_{\rm dust} + \rho_{\rm rad})^2,$$

$$\Pi_{ij} = \Big[\frac{1}{12}(\rho_{\rm dust} + \rho_{\rm rad})^2 + \frac{1}{6}(\rho_{\rm dust} + \rho_{\rm rad})(p_{\rm dust} + p_{\rm rad})\Big]g_{ij}, \tag{15}$$

and Codazzi's equations (12) imply that $\nabla_a \Pi^a_b = 0$, consistent with the contracted Bianchi's identities in (7) and Codazzi's equation (13). The modified Friedmann equations have the first integral

$$H^2 + \frac{k}{a^2} - \frac{\Lambda_4}{3} = \frac{8\pi G_N}{3}(\rho_{\rm dust} + \rho_{\rm rad}) - \frac{4\pi G_N}{3|\sigma|}(\rho_{\rm dust} + \rho_{\rm rad})^2, \tag{16}$$

where $H := \dot{a}/a$ with $\dot{a} \equiv da/dt$. It is worth noting that the bounce is solely engendered because of the presence of a timelike extra dimension that induces the minus sign in the last term of (16). By assuming a spacelike extra dimension, we would obtain a plus sign instead that provides a singular model.

Expressing

$$\rho_{\rm dust} = \frac{E_{\rm dust}}{a^3}, \qquad \rho_{\rm rad} = \frac{E_{\rm rad}}{a^4}, \tag{17}$$

where $E_{\rm dust}$ and $E_{\rm rad}$ are constants of motion, the first integral of motion (16) can be expressed as the Hamiltonian constraint

$$\mathcal{H} = \frac{p_a^2}{2} + V(a) = 0, \tag{18}$$

where

$$V(a) = \frac{k}{2} - \frac{\Lambda_4 a^2}{6} - \frac{8\pi G_N}{6}\Big(\frac{E_{\rm dust}}{a} + \frac{E_{\rm rad}}{a^2}\Big) + \frac{8\pi G_N}{12|\sigma|}\Big(\frac{E_{\rm dust}}{a^2} + \frac{E_{\rm rad}}{a^3}\Big)^2. \tag{19}$$

From (18) we derive the dynamical system

$$\dot{a} = p_a, \qquad \dot{p}_a = -\frac{dV}{da}. \tag{20}$$

It is worth noting that the last term in the potential (19) acts as an infinite potential barrier and is responsible for the avoidance of the singularity $a = 0$. These potential corrections are equivalent to fluids with negative energy densities. This is in accordance with the fact that quantum effects can violate the classical energy conditions and may avoid curvature singularities where classical general relativity breaks down [20]. Such violations tend to occur on short scales and/or at high curvatures, which is the case of the present models.

The behavior of the potential $V(a)$ is illustrated in Fig. 1 for $k = 0.8$, $\Lambda_4 = 1.5$, $E_{\rm rad} = 0.1$, and for $E_{\rm dust} = 0.001$

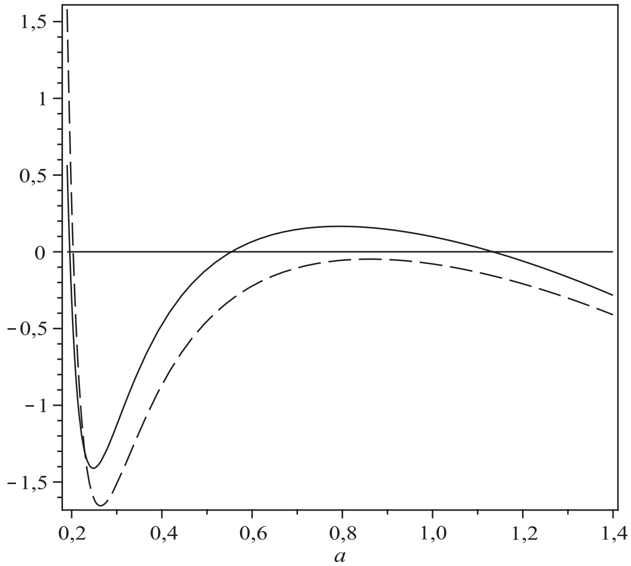

FIG. 1. Behavior of $V(a)$ for parameters $k = 0.8$, $\Lambda_4 = 1.5$, $E_{\rm rad} = 0.1$, and for $E_{\rm dust} = 0.001$ (continuous line) $E_{\rm dust} = 0.09$ (dashed line). The increase of the dust content for a fixed k excludes the presence of perpetually bouncing solutions.

(continuous line), $E_{\rm dust} = 0.09$ (dashed line). We can see that the increase of the dust content for a fixed k excludes the presence of perpetually bouncing solutions by driving the maximum out of the physical space. For a sufficiently large $E_{\rm dust}$ the potential $V(a)$ presents no local maximum or minimum [21].

The critical points in the phase space are stationary solutions of (20), namely, the points of the phase space ($a = a_{\rm crit}$, $p_a = 0$) corresponding to the zeros of the right-hand side of (20). Here, $a_{\rm crit}$ stands for the real positive roots of $dV/da = 0$. By considering the case of closed geometries ($k > 0$), it is not difficult to verify that, depending on the values of the parameters (Λ_4, $|\sigma|$, $E_{\rm rad}$, $E_{\rm dust}$), there are at most two critical points associated with one minimum and one local maximum of $V(a)$. In this case, the minimum of the potential corresponds to a center while the maximum corresponds to a saddle. This configuration allows us to obtain different types of orbits that describe the evolution of universes in this model. In Fig. 2 we illustrate the phase space portrait of the model for $\Lambda_4 = 1.5$, $\sigma = 6000$, $E_{\rm rad} = 0.15$, and $E_{\rm dust} = 0.05$, and for varying k. The value of $E_{\rm dust}$ is sufficiently bounded so that $V(a)$ has a well. The critical points P_1 (center) and P_2 (saddle) correspond to stable and unstable Einstein universes. Typically the model allows for the presence of perpetually bouncing universes (periodic orbits in region I) and one-bounce universes (region II). Region I is bounded by the separatrix $\mathcal{S}$ emerging from the saddle P_2. A separatrix also emerges from P_2 toward the de Sitter attractor at infinity, defining a graceful exit of orbits in region II to an (inflationary) accelerated phase. From now on we will restrict ourselves to the case of closed geometries. In the

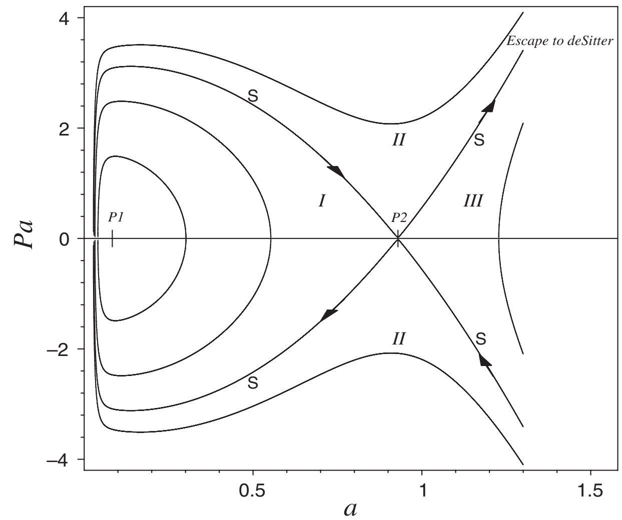

FIG. 2. Phase portrait of the dynamics with the critical point P_1 (center) and P_2 (saddle). Orbits in region II are solutions of one-bounce universes with a graceful exit to an accelerated (inflationary) phase along the separatrix $\mathcal{S}$.

next section we will examine what kind of orbit would be generated when one considers the observational values of (Λ_4, $E_{\rm rad}$, $E_{\rm dust}$).

IV. OBSERVATIONAL COSMOLOGY

As observational cosmology asserts, the domain of homogeneity and isotropy of our present Universe is well accepted for scales around the present horizon, which is given by $a_0 \sim 10^{28}$ cm (here, the subscript 0 denotes the present epoch). In this case, we obtain the following observational parameters:

$$\Lambda_4 \simeq 1.34 \times 10^{-56}\ {\rm cm}^{-2}, \tag{21}$$

$$E_{\rm dust} \simeq 2.6 \times 10^{54}\ {\rm g}, \tag{22}$$

$$E_{\rm rad} \simeq 4 \times 10^{78}\ {\rm g\,cm}, \tag{23}$$

where the Hubble radius is fixed to $H_0 \sim 0.77 \times 10^{-28}\,{\rm cm}^{-1}$.

From Ref. [22], the brane tension has a lower bound that corresponds to $|\sigma|_{\rm min} \sim 10^{22}\ {\rm g\,cm}^{-3}$. That is, a star with the Chandrasekhar mass will not form an event horizon if the brane tension is smaller than $|\sigma|_{\rm min}$. It turns out that this value furnishes us with a curvature scale $l_c \equiv 1/\sqrt{R_b} = (\sqrt{a/\ddot{a}})_b \sim 10^{34} l_P$ at the bounce (where l_P is the Planck length and R_b is the Ricci scalar at the bounce). To guarantee that l_c at the bounce is not smaller than $10^3 l_P$, the brane tension must be less than $10^{85}\ {\rm g\,cm}^{-3}$. Therefore, we have the following physical domain (not spoiling the nucleosynthesis) for the brane tension

$$10^{22}\ {\rm g/cm}^3 \lesssim |\sigma| \lesssim 10^{85}\ {\rm g/cm}^3, \tag{24}$$

where we have set $c = 1$. Feeding the Hamiltonian constraint (18) with $|\sigma|_{\rm min}$ and the parameters (21), we obtain that the spatial curvature is $k \simeq 0.002$ for $|\sigma| \geq |\sigma|_{\rm min}$.

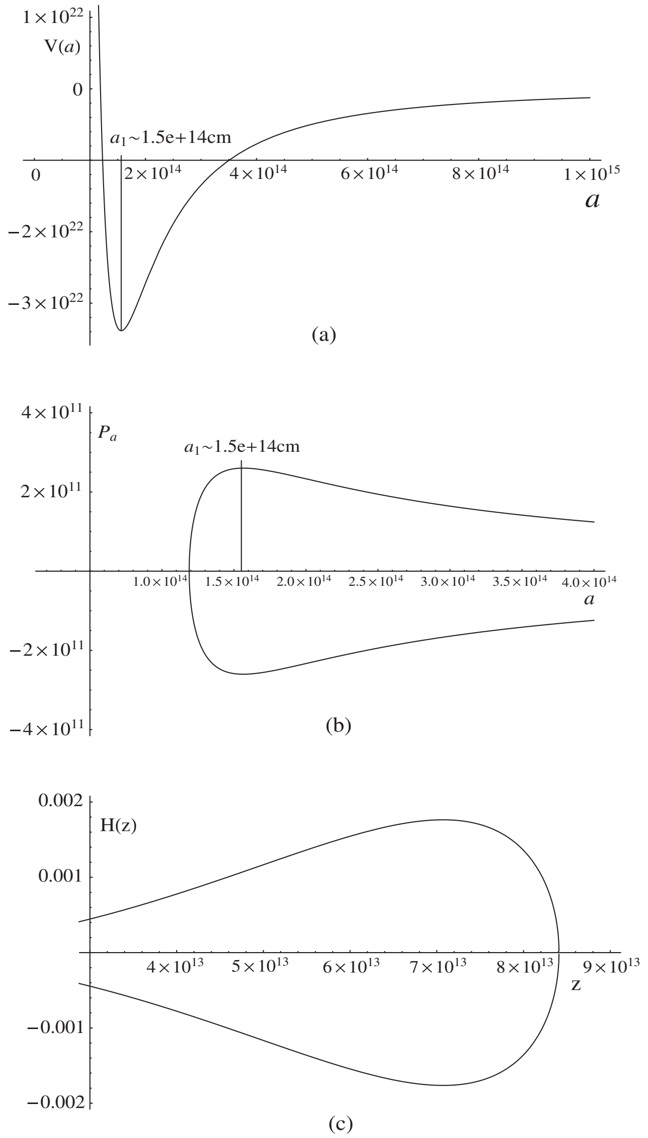

FIG. 3. (a) The potential $V(a)$ and the phase space (b) in the region that encompasses the critical point a_1, considering the observational values (21) and (24). For $|\sigma| \sim |\sigma|_{\min}$, we obtain $a_1 \sim 10^{14}$ cm. Although this primordial accelerated phase does not correspond to usual inflation (0.27 being the number of e-folds), it is important to remark that as our Universe has no beginning of time and the cosmological constant is small, the particle horizon before the bounce was already bigger than the scales of cosmological interest. In (c) we show the behavior of the Hubble factor as a function of redshift in a neighborhood of the bounce given the normalization $a_0 = 1$ and the parameters (30).

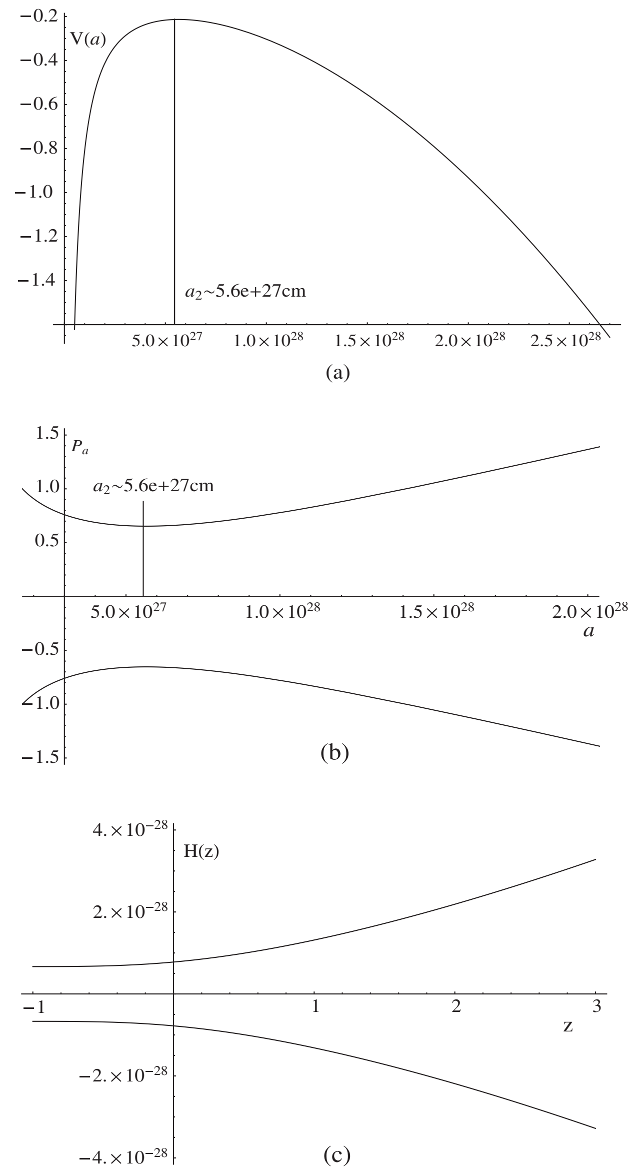

FIG. 4. (a) The potential $V(a)$ and the phase space (b) in the region that encompasses the critical point a_2 and completes Fig. 3. This critical point is of the order of 10^{28} cm, coinciding with the domain of homogeneity and isotropy of our present Universe, regardless of the value of $|\sigma|$. In (c) we show the behavior of the Hubble factor as a function of redshift in a neighborhood of the saddle a_2 given the normalization $a_0 = 1$ and the parameters (30). The domain $0 \geq z > -1$ of the $H > 0$ branch corresponds to the final acceleration phase approaching de Sitter as $z \to -1$, with $H = \text{const}$ at $z = -1$.

Considering the lower bound limit for $|\sigma|$, numerical calculations show that the potential $V(a)$ always has a local minimum at a_1 (corresponding to a center) and a local maximum at a_2 (corresponding to a saddle)—cf. Figs. 3(a) and 4(a). If we increase $|\sigma|$ by 4 orders of magnitude, we obtain a value of a_1 decreased by 1 order of magnitude. On the other hand, the local maximum a_2 is of the order of 10^{28} cm [cf. Fig. 4(a)] for $|\sigma| \geq |\sigma|_{\min}$ (regardless of the value of $|\sigma|$). We exhibit the behavior of $V(a)$ and the phase space (a, p_a) trajectory—for the parameters (21)–(24) and $|\sigma| \sim |\sigma|_{\min}$—in Figs. 3(a), 4(a), 3(b), and 4(b), respectively. We should note that Figs. 3 and 4 display the same potential $V(a)$ and the same universe phase space trajectory in distinct ranges of a, complementing each other.

Given the redshift relation

$$\frac{a(0)}{a(z)} = 1 + z. \tag{25}$$

Equation (16) can be rewritten as

$$H^2 = H_0^2 \Big\{ \Omega_{0\mathrm{dust}}(1+z)^3 + \Omega_{0\mathrm{rad}}(1+z)^4 + \Omega_{0\Lambda} - \Omega_{0k}(1+z)^2 - \frac{3H_0^2}{16\pi G|\sigma|}(1+z)^6 \times [\Omega_{0\mathrm{dust}} + \Omega_{0\mathrm{rad}}(1+z)]^2 \Big\}, \tag{26}$$

where

$$\Omega_{0\mathrm{dust}} \equiv \frac{\rho_{0\mathrm{dust}}}{\rho_0^{cr.}}, \qquad \Omega_{0\mathrm{rad}} \equiv \frac{\rho_{0\mathrm{rad}}}{\rho_0^{cr.}}, \tag{27}$$

$$\Omega_{0\Lambda} \equiv \frac{\rho_\Lambda}{\rho_0^{cr.}} \equiv \frac{\Lambda}{8\pi G}\frac{1}{\rho_0^{cr.}}, \tag{28}$$

$$\Omega_{0k} \equiv \frac{k}{a_0^2 H_0^2}, \tag{29}$$

and $\rho_0^{cr.} \equiv 3H_0^2/8\pi G$. By fixing the normalization $a_0 = 1$, we obtain the following parameters according to the WMAP 7 year results [23]:

$$\Omega_{0\mathrm{dust}} \simeq 0.26, \qquad \Omega_{0\mathrm{rad}} \simeq 10^{-5}, \qquad \Omega_{0\Lambda} \simeq 0.73. \tag{30}$$

Substituting these parameters in (26), we obtain $\Omega_{0k} \simeq 0.004$. In Figs. 3(c) and 4(c) we show the behavior of the Hubble scale factor H with respect to the redshift z.

It is remarkable that considering the interval of 62 orders of magnitude of $|\sigma|$ [cf. (24)], the trajectory in the phase space of the above *observable* universe belongs to region II of the phase space (cf. Fig. 2) corresponding to a one-single-bounce orbit. The part of the trajectory starting from ($a = a_1$, $p_{a_1} < 0$) is an initial acceleration phase that leads the Universe through the bounce and ends in ($a = a_1$, $p_{a_1} > 0$), when the Universe enters in a long and smooth decelerated expansion phase. This primordial bouncing accelerated phase does not correspond to usual inflation, the number of e-folds being 0.27. Note, however, that there is no horizon problem in the model. In fact, before the bounce, because of its cosmological constant dominated contraction from the infinity past until a_2, the particle horizon d_p is given by

$$d_p = \left| \bar{a} \int_\infty^{\bar{a}} \frac{1}{a\dot{a}} da \right| \simeq 10^{28}\ \mathrm{cm}, \tag{31}$$

if $\bar{a} \geq a_2$. Therefore the particle horizon is already of the order of $\Lambda^{-1/2}$, which is constrained by present observations to be of the order of the Hubble radius today. Hence there is no horizon problem for the scales of cosmological interest. The decelerated expansion Friedmann phase ends in the neighborhood of a_2 with a graceful exit to a late de Sitter accelerated phase.

From (8) and (21), we see that the parameter Λ_5 of the model must be adjusted in a very precise way. In fact, Λ_5 must be very close to $8\pi G_N|\sigma|$, which has the minimum value $10^5\ \mathrm{cm}^{-2}$ [see Eq. (24)] and increases as $|\sigma|$ increases, in order to yield the observed value of Λ_4 given in Eq. (21). This is the usual problematic fine-tuning of the cosmological constant, of at least 60 orders of magnitude as we have seen above, which the present model, at least in this first approach, does not solve. It turns out that this is similar to an issue contained in the Randall and Sundrum model [24]. In this scenario the brane is embedded in an anti–de Sitter $(4+1)$ spacetime, and the fine-tuning relation $\Lambda_5 = -\kappa_5^4\sigma^2/6$ has to be satisfied. It was shown in Ref. [25] that the Randall-Sundrum model is unstable under small deviations from this fine-tuning. This is a future investigation we will examine if the same happens in our model.

V. CONCLUSIONS

In the framework of a Brane World formalism with a timelike extra dimension, we have obtained a homogeneous and isotropic bouncing model compatible with all observations at the background level. It starts with a de Sitter contraction from the infinity past, experiences a bounce at very small scales, turning to the usual standard expanding decelerating phases of radiation and matter domination, and has a recent transition to an accelerating expansion. The bounce itself is caused by the appearance of new terms coming from the extra timelike dimension of the bulk in the four-dimensional Friedmann equation, which become important at high curvature scales and avoid the cosmological singularity, inducing a gravitational repulsion owing to the timelike nature of the extra dimension. We have two free parameters: the brane tension σ and the five-dimensional cosmological constant Λ_5. The brane tension can assume a wide variety of values [see Eq. (24)], but Λ_5 must be highly fine-tuned to the value of σ in order to yield an effective four-dimensional cosmological constant compatible with observations. Hence the model solves the singularity problem of the standard cosmological model, together with the horizon and flatness puzzles, but it does not solve the cosmological constant problem.

Our next step will be to perturb the model and investigate the evolution of cosmological perturbations in such a cosmological background. Indeed, our work in progress shows that, if one imposes an unperturbed de Sitter bulk, a numerical treatment of linear hydrodynamical perturbation in the Universe indicates that the bounce has the effect of substantially enhancing the perturbations; nonetheless these perturbations remain bounded with $\delta\rho/\rho \ll 1$ and $\delta p/p \ll 1$. However, a general analysis of cosmological perturbations in this scenario demands also a perturbed de Sitter bulk. In this case the five-dimensional scalar perturbations will induce fluctuations of the Weyl tensor projected on the brane, which will modify the perturbed field equations. This is a technical and conceptually involved problem that will be investigated in future publications.

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[1] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, 2005).
[2] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
[3] R. Penrose, Phys. Rev. Lett. **14**, 57 (1965).
[4] J. Acacio de Barros, N. Pinto-Neto, and M. A. Sagioro-Leal, Phys. Lett. A **241**, 229 (1998); R. Colistete, Jr., J. C. Fabris, and N. Pinto-Neto, Phys. Rev. D **62**, 083507 (2000); F. G. Alvarenga, J. C. Fabris, N. A. Lemos, and G. A. Monerat, Gen. Relativ. Gravit. **34**, 651 (2002); N. Pinto-Neto, F. T. Falciano, R. Pereira, and E. Sergio Santini, Phys. Rev. D **86**, 063504 (2012).
[5] M. Bojowald (Loop Quantum Cosmology Collaboration), Living Rev. Relativity **11**, 4 (2008), http://www.livingreviews.org/lrr-2008-4; M. Bojowald and R. Tavakol, arXiv:0802.4274.
[6] Y. V. Shtanov, arXiv:hep-th/0005193; Y. V. Shtanov, Phys. Lett. B **541**, 177 (2002); Y. Shtanov and V. Sahni, Phys. Lett. B **557**, 1 (2003).
[7] A. Iglesias and Z. Kakushadze, Phys. Lett. B **515**, 477 (2001).
[8] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
[9] R. Maartens, Phys. Rev. D **62**, 084023 (2000); R. Maartens and K. Koyama, Living Rev. Relativity **13**, 5 (2010).
[10] L. F. Abbott and S.-Y. Pi, *Inflationary Cosmology* (World Scientific Publishing, Singapore, 1986).
[11] A. D. Sakharov, Zh. Eksp. Teor. Fiz. **87**, 375 (1984) [Sov. Phys. JETP **60**, 214 (1984)]; J. Barrett *et al.* Int. J. Mod. Phys. A **09**, 1457 (1994).
[12] G. R. Dvali, G. Gabadadze, and G. Senjanovic, in *Many Faces of the Superworld: Yuri Golfand Memorial*, edited by Y. Golfand, M. Shifman, and M. A. Shifman (World Scientific, Singapore, 1999), Vol. 525–532.
[13] F. J. Ynduráin, Phys. Lett. B **256**, 15 (1991).
[14] Y. Aref'eva, B. G. Dragović, and I. V. Volovich, Phys. Lett. B **177**, 357 (1986).
[15] M. Chaichian and A. B. Kobakhidze, Phys. Lett. B **488**, 117 (2000).
[16] Y. Aref'eva and I. V. Volovich, Phys. Lett. **B164**, 287 (1985).
[17] L. P. Eisenhart, *Riemannian Geometry* (Princeton University Press, Princeton, NJ, 1997).
[18] A. H. Taub, Phys. Rev. **94**, 1468 (1954).
[19] W. Israel, Il Nuovo Cimento B **44**, 1 (1966).
[20] N. Pinto-Neto and P. Peter, Phys. Rev. D **78**, 063506 (2008); L. H. Ford, arXiv:gr-qc/0504096.
[21] R. Maier, I. Damião Soares, and E. V. Tonini, Phys. Rev. D **79**, 023522 (2009).
[22] R. Maier and I. Damião Soares, Int. J. Mod. Phys. D **21**, 1250050 (2012).
[23] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[24] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[25] T. Boehm, R. Durrer, and C. van de Bruck, Phys. Rev. D **64**, 063504 (2001).