# The angular resolution of the the Pierre Auger Observatory 

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#### Abstract

The Pierre Auger Observatory consists of two independent components, the fluorescence detector and the surface detector, and is acquiring data since 2004. In this work we study the angular resolution for the surface detector component. We develop a model based on the detection technique and some shower parameters. The model is validated by using adjacent surface detector stations and by looking at the flatness of the $\chi 2$ probability distribution. Finally the angular reconstruction accuracy of the surface detector is given as a function of station multiplicity and is compared with the one obtained from the hybrid events, observed simultaneously by both components.


## 1 Introduction

The Pierre Auger Observatory consists of two independent components: the fluorescence detector (FD) and the surface detector (SD) [1]. Whereas the surface detector events have a larger angular resolution than the hybrid events, there is much higher statistics for the former.

The angular resolution for the SD is determined, on an event by event basis, from the zenith $(\theta)$ and azimuth ( $\phi$ ) uncertainties obtained from the geometrical reconstruction, using the relation:

$$
\begin{equation*}
F(\eta)=1 / 2\left(V[\theta]+\sin ^{2}(\theta) V[\phi]\right) \tag{1}
\end{equation*}
$$

[^0]where $\eta$ is the space-angle, and $V[\theta]$ and $V[\phi]$ are the variance of $\theta$ and $\phi$ respectively. If $\theta$ and $\phi / \sin (\theta)$ have Gaussian distribution with variance $\sigma^{2}$, then $F(\eta)=\sigma^{2}$ and $\eta$ has a distribution proportional to $e^{-\eta^{2} / 2 \sigma^{2}} d(\cos (\eta)) d \phi$. Then, if we define the angular resolution $(A R)$ as the angular radius that would contain $68 \%$ of showers coming from a point source, $A R=1.5 \sqrt{F(\eta)}$.

The angular resolution depends strongly on the timing resolution of the water Cherenkov detectors (WCDs) and weakly on the shower front model and the core position uncertainty. The WCDs timing uncertainty is directly modeled from the data (section 2). This model is based on the physics of the shower and the measurement process. It can be adjusted using two pairs of adjacent stations located in the surface array (section 3). The model is validated by studying the $\chi^{2}$ probability distribution for the geometrical reconstruction (section 4). The angular resolution is estimated for the SDonly reconstruction and by comparison with the hybrid data (section 5). We conclude that the angular resolution for events above 10 EeV is better than $1^{\circ}$.

## 2 The Time Variance Model

The angular accuracy of the SD events is driven by the accuracy with which one can measure the arrival time $\left(T_{s}\right)$ of the shower front in each station. The particle arrival time in the shower front can be described as a Poisson process over some interval time $\mathcal{T}$. The first particle arrival time is used as the estimator for the shower front arrival. It is given by $T_{1}{ }^{2}=T_{s}+t_{1}$, where $T_{s}$ is the shower front time and $t_{1}$ has a distribution function [2] given by:

$$
\begin{equation*}
f\left(t_{1}\right)=\frac{1}{\tau} e^{\frac{-t_{1}}{\tau}} \tag{2}
\end{equation*}
$$

Since we estimate the parameter $\mathcal{T}$ from the data itself, the previous distribution is modified to:

$$
\begin{equation*}
f\left(t_{1}\right)=\frac{n-1}{\mathcal{T}}\left(1-\frac{t_{1}}{\mathcal{T}}\right)^{n-2} \tag{3}
\end{equation*}
$$

where $n$ is the number of particles measured during the time $\mathcal{T}$. The variance of $T_{1}$ is given by the variance of $t_{1}$ and is:

$$
\begin{equation*}
V\left[T_{1}\right]=\left(\frac{\mathcal{T}}{n}\right)^{2} \frac{n-1}{n+1} \tag{4}
\end{equation*}
$$

[^1]The variance of the arrival of the first particle in the SD stations, taking into consideration the GPS uncertainty and the resolution of the flash analog-to-digital converters (FADC), can then be written as:

$$
\begin{equation*}
V\left[T_{1}\right]=a^{2}\left(\frac{2 T_{50}}{n}\right)^{2} \frac{n-1}{n+1}+b^{2} \tag{5}
\end{equation*}
$$

where $T_{50}$ is the time interval that contains the first $50 \%$ of the total signal as measured by the photomultiplier FADC traces. The two free parameters $a$ and $b$ can be determined with the adjacent station data. We expect that the parameter $a$ should be close to 1 , while $b$ should be given by the GPS clock accuracy (about 10 ns ) and the FADC trace resolution $25 / \sqrt{12} \mathrm{~ns}$, that is $b \simeq 12 \mathrm{~ns}$.

To calculate the number of particles ( $n$ ) we assume that all particles hit the detector with the same direction than the shower axis, and that the muons are mostly the ones that contribute to the time measurements. Then, we obtain $n$ as the ratio between the total signal $(S)$ in the WCD and the average track length, $T L(\theta)$, of the particles.

The average track length can be computed as the ratio of the detector volume $(V)$ and the area $(A)$ subtended by the arriving particles, and is:

$$
\begin{equation*}
T L(\theta)=\frac{V}{A}=\frac{\pi r^{2} h}{\pi r^{2} \cos (\theta)+2 r h \sin (\theta)} \tag{6}
\end{equation*}
$$

where $\theta$ is the zenith angle, $r=1.8 \mathrm{~m}$ is the detector radius, and $h=1.2 \mathrm{~m}$ is the detector height.

## 3 Testing the model with doublets

Two pairs of adjacent surface detector stations separated by 11 m ("doublets") have been installed in the field of the Auger Observatory. These pairs enable comparison of timing and signal accuracy measurements. We used the data of the doublets to verify the time variance model and also to adjust the constants $a$ and $b$ from it. For each event we computed the time difference as $\Delta T=d T^{(1)}-d T^{(2)}$ where $d T^{(1)}\left(d T^{(2)}\right)$ is the time difference from the first (second) detector of the doublet to the fitted shower front. Doing that, $\Delta T$ does not depend on the shower front shape, since the twin detectors are very close to each other. We used 1693 events (from April/2004 to June/2006) to fit for the two parameters $a$ and $b$, and we obtained:

$$
\begin{aligned}
& a^{2}=0.98 \pm 0.05 \\
& b^{2}=150 \mathrm{~ns}^{2} \pm 18 \mathrm{~ns}^{2}
\end{aligned}
$$



Figure 1: The RMS of the distribution of $\Delta T / \sqrt{V[\Delta T]}$, as a function of the shower zenith angle (top), the average signal in the doublet detectors (middle), and the distance to the shower core (bottom).
which is in good agreement with our expectations $\left(a^{2}=1, b^{2}=144 \mathrm{~ns}^{2}\right)$.

## 4 Validation of the Time Variance Model

If the time variance model describes correctly the measurement uncertainties, the distribution of $\Delta T / \sqrt{V[\Delta T]}$, where $V[\Delta T]=V\left[T_{1}^{(1)}\right]+V\left[T_{1}^{(2)}\right]$, should have unit variance. In figure 1 we show the RMS of the distribution of $\Delta T / \sqrt{V[\Delta T]}$ for the doublets as a function of $\cos (\theta)$ (top), the average signal (middle), and the distance to the core position (bottom). In all the cases, the RMS is almost constant and close to unity, which shows that the time variance model is in very good agreement with the experimental data. In particular, since the time variance model does not explicit depend on the distance of the station to the shower core, the results shown in the bottom panel strengthen our confidence in the validity of the model.

In figure 2 we plot the $\chi^{2}$ probability distribution of the minimizations for all the events with 4 stations or more passing the Auger quality cuts [3] (top), for events with zenith angle smaller than $55^{\circ}$ (middle), and events with zenith angle larger than $55^{\circ}$ (bottom). We only plot probabilities larger than $1 \%$ to avoid the large peak at zero corresponding to badly reconstructed events $(\sim 9 \%)$. This distribution is almost flat as it should be in the ideal


Figure 2: The $\chi^{2}$ probability distribution for all events (top), events with zenith angle smaller than $55^{\circ}$ (middle), and events with zenith angle larger than $55^{\circ}$ (bottom). In the last figure the distribution is plotted with two different scales, the same than the others (full line) for comparison reasons and a zoom (dashed line) to see the details.
case. The flatness is observed both for large and small zenith angles, which means that the model works for all angles without compensating one set from the other. This distribution shows that the variance model properly reproduces the experimental uncertainty.

## 5 Angular Resolution

### 5.1 Surface Detector Only

Considering the quality of the time variance model for the measurement uncertainties we can calculate directly the angular resolution on an event by event basis out of the minimization procedure. In figure 3 left, we show the angular resolution $A R$ (given by $1.5 \sqrt{F(\eta)}$ ) for the geometrical reconstruction as a function of the zenith angle for various station multiplicities (circles: 3 stations, squares: 4 stations, up triangles: 5 stations, down triangles: 6 stations or more).

The angular resolution is about $2.6^{\circ}$ in the worst case of vertical showers with only 3 stations hit. This value improves significantly for 4 and 5


Figure 3: Angular resolution $(A R)$ for the SD as a function of the zenith angle $(\theta)$ extracted from the geometrical reconstruction alone. The $A R$ is plotted for various stations multiplicities (circles: 3 stations, squares: 4 stations, up triangles: 5 stations, and down triangles: 6 or more stations), for the geometrical reconstruction (left) and for the complete one (right), see text.
stations ${ }^{3}$. For 6 or more stations, which corresponds to events with energies above 10 EeV , the angular resolution is in all cases better than $1^{\circ}$. Above $60^{\circ}$, the event multiplicity increases rapidly with zenith angle, and only a few low energy events trigger only 3 stations, hence the poor angular resolution.

In the right panel of figure 3 we show the angular resolution for the complete reconstruction. In all the cases, the angular resolution increases by about $10 \%$, but a "hump" appears around $40^{\circ}$, more visible in the 3 -fold case. This is due to the contribution of the statistical uncertainties on the core position.

All quoted errors are statistical only. We did not, at this stage, investigate possible biases or systematics in the determination of the arrival direction angles.

### 5.2 Comparison with Hybrid events

Finally, in figure 4 we show the space angle between the SD-only and hybrid geometrical reconstructions for showers with different number of stations and different zenith angle ranges. The distributions plotted were fitted with a Gaussian resolution function $d p \propto e^{-\eta^{2} / 2 \sigma^{2}} d(\cos (\eta)) d \phi$, where $\eta$ is the space-angle. The $\sigma$ obtained in the fit is related to the angular resolution by $R A=1.5 \sigma$.

[^2]

Figure 4: Comparison between hybrid and SD-only geometrical reconstructions. Top, for 3 stations with two zenith angle ranges ( $0^{\circ}<\theta<30^{\circ}$ and $30^{\circ}<\theta<50^{\circ}$ ). Middle 4 stations (left) and 5 stations (right) with $30^{\circ}<\theta<50^{\circ}$. Bottom, for 6 stations or more with two zenith angle ranges ( $30^{\circ}<\theta<50^{\circ}$ and $50^{\circ}<\theta<50^{\circ}$ ).

In table 1 we show the $\sigma$ value obtained directly from the SD-only and from the comparison with the hybrid data. The angular accuracy of the hybrid reconstruction $\left(\sigma_{H y b}\right)$ was calculated using equation 1. Then, we calculated the $\sigma_{S D}$ as $\sigma_{\eta}^{2}-\sigma_{H y b}^{2}$, and compared it with $\sigma_{S D-o n l y}$. Here we ignored any possible contribution from systematics that may exist across the two reconstruction methods, and would explain the small differences observed.

## 6 Conclusions

We developed a model to describe the measurement uncertainties of the time arrival of the first particle in the SD stations, based on zenith angle of the shower, and on the integrated signal and rise time measured in the WCD. Absolute predictions of this model or the one adjusted with the doublet data are very similar.

| \# Stations | $\theta$ range | $\sigma_{\eta}$ | $\sigma_{H y b}$ | $\sigma_{S D-\text { only }}$ | $\sigma_{S D}$ from $\sigma_{\eta}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $\left[0^{\circ} ; 30^{\circ}\right]$ | $1.6^{\circ}$ | $0.6^{\circ}$ | $1.8^{\circ}$ | $1.5^{\circ}$ |
| 3 | $\left[30^{\circ} ; 50^{\circ}\right]$ | $1.4^{\circ}$ | $0.6^{\circ}$ | $1.8^{\circ}$ | $1.3^{\circ}$ |
| 4 | $\left[30^{\circ} ; 50^{\circ}\right]$ | $1.3^{\circ}$ | $0.5^{\circ}$ | $1.1^{\circ}$ | $1.2^{\circ}$ |
| 5 | $\left[30^{\circ} ; 50^{\circ}\right]$ | $1.1^{\circ}$ | $0.4^{\circ}$ | $1.0^{\circ}$ | $1.0^{\circ}$ |
| 6 or more | $\left[30^{\circ} ; 50^{\circ}\right]$ | $0.9^{\circ}$ | $0.4^{\circ}$ | $0.6^{\circ}$ | $0.8^{\circ}$ |
| 6 or more | $\left[50^{\circ} ; 70^{\circ}\right]$ | $0.8^{\circ}$ | $0.3^{\circ}$ | $0.4^{\circ}$ | $0.7^{\circ}$ |

Table 1: Results obtained from the Gaussian function fit with and without fixing the core position from the hybrid reconstruction and the SD average for all the cases plotted in figure 4.

Using this model we can obtain an optimal determination of the shower arrival direction and are able to extract the angular resolution of the SD detector on an event by event basis.

The angular resolution of the surface detector was found to be better than $2.7^{\circ}$ for 3 -fold events, better than $1.7^{\circ}$ for 4 -fold and 5 -fold events and better than $1.0^{\circ}$ for higher multiplicity (which corresponds to energies larger than 10 EeV ). These values are compatible with the ones obtained from the comparison with our hybrid data set.

## 7 Acknowledgments

This work was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil, and the Centre National de la Recherche Scientifique, Institut National de Physique Nucléare et Physique des Particules (IN2P3/CNRS), France.

## References

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[2] C. Bonifazi and A. Letessier-Selvon in preparation.
[3] D. Allard et al.:2005, in Proc. of the $29^{\text {th }}$ ICRC, Pune, 7, 287-290


[^0]:    ${ }^{1}$ Pierre Auger Observatory, Av. San Martín Norte 304, (5613) Malargüe, Mendoza, Argentina

[^1]:    ${ }^{2}$ In fact, an unbiased estimator should be $\left.T_{0}=T_{1}-E \hat{[ } t_{1}\right]$, where $\left.E \hat{[ } t_{1}\right]$ is the expectation value.

[^2]:    ${ }^{3}$ For 4 and 5 stations the angular resolution is very similar because in the fitting

