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Is There a Super-Selection Rule in Quantum Cosmology?

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Abstract—A certain approach to solving the Wheeler-DeWitt equation in quantum cosmology, based on a type of super-selection rule by which negative frequency solutions are discarded, is discussed. In a preliminary analysis: we recall well-known results in relativistic quantum field theory, showing that adopting this approach of super-selection by discarding a sector of the frequencies does not lead to acceptable results. In the area of quantum cosmology, a qualitatively similar result is obtained: we show that by discarding solutions with negative frequencies, which is usually done in order to demonstrate "strong" results on the resolution of the singularity, important physical processes are lost, namely, the existence of cyclic solutions, which, under certain reasonable assumptions, can be interpreted as processes of creation-annihilation at the Planck scale that are typical of any relativistic quantum field theory.

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1. INTRODUCTION

The singularity problem in quantum cosmology has been addressed since the beginning of this area of research [1]. A long discussion on the possibility of resolve the singularity by means of quantum effects took place since then.In the context of the Wheeler-DeWitt (WDW) approach [1, 2]; models have been obtained where the singularity persists in the quantum regime (see, e.g., [2–7]), and also models in which the singularity is avoided due to quantum effects (see [8–13]). That is to say, there has been no absolute consensus on whether in the quantum regime the singularity is maintained as a strong result in the WDW approach. Furthermore, there is no general agreement on the necessary criteria for quantum avoidance of singularities (for a clarifying analysis of the various criteria see [13], and for the important problem of preservation of unitarity in the course of the evolution see [14, 15]).

In the framework of Loop Quantum Cosmology (LQC), a series of results have been obtained, where the big-bang singularity is avoided and substituted by a bounce coming from solutions of the equation of differences which is the equation that takes place instead of the WDW equation (see [16]).[1] At the same time, it has been claimed that the Wheeler-DeWitt approach to quantum cosmology does not solve the singularity problem of classical cosmology (see, e.g., for example [17–21]). This general assertion was already widely criticized in [23]. There it was shown, firstly, that the assertion is not precise because to address this question it is necessary to specify not only the quantum interpretation adopted but also the quantization scheme chosen. On the second place, it was demonstrated that quantum bounces occur when one considers the Bohm-de Broglie interpretation in any of the two different usual quantization schemes: the Schrödinger-like quantization, which essentially takes the square-root of the resulting Klein-Gordon equation through the restriction to positive frequencies, and their associated Newton-Wigner states, or the induced Klein-Gordon quantization that allows both positive and negative frequencies together. We refer the reader, interested in this study, to the last cited reference.

The aim of this letter is to analyze and to discuss the quantization scheme which involves the restriction to a single sector of frequencies (say, discarding the negative frequency solutions[2]) which is made invoking a kind of super-selection rule [21]. As a preliminary study, we analyze what happens in a basic quantum field theory (QFT), i.e. a quantum free scalar field, when we discard the negative-frequency

[1] LQC has an important advantage over the Wheeler-DeWitt quantum cosmology because it has its foundations in the theory of Loop Quantum Gravity, where there are less conceptual problems than in the case of the canonical quantization that leads to the WDW equation.

[2] The problem of negative frequencies in quantum cosmology is known since the early works on the subject, see, e.g., [3].

solutions. As is well known, the fundamental Lorentz symmetry is lost, or, in other words, we are led to a violation of causality. We then study the WDW approach to quantum cosmology. At the beginning, we briefly outline the arguments of [21] for a super-selection rule, making some discussion, and after that we develop the central part of this letter: we analyze the behavior of the Bohmian trajectories obtained from the solutions of the WDW equation for a Friedmann-Lemâitre-Robertson-Walker (FLRW) model with flat spatial sections, assuming the content of matter of the universe as given by a free, massless, minimally coupled scalar field, while negative frequencies are incorporated in the positive-frequency initial solution. This is implemented using a superposition of solutions modulated by two Gaussians symmetrically located around $k = 0$. The Gaussian width is varied from an initial value representing an almost non-overlapped configuration (the Gaussians are completely disjoint), which indicates a positive-frequency solution, then going through several increasing values representing a greater partial overlap, indicating a greater weight of negative frequencies in the integral, until we overcome a certain "threshold" (see below) from which the phenomena qualitatively different begin to occur. We were able to show that when the negative-frequency solutions are incorporated beyond a certain value, fundamental phenomena appear: the existence of cyclic universe solutions which could be interpreted as a processes of creation-destruction of universes at Planck scale, provided the scalar field is assumed to play the role of time. These phenomena are usual in any QFT, and we see that when the negative-frequency solutions are not considered in quantum cosmology, theses processes are lost, or, in other words, no cyclic universe solutions is present.

This paper is organized as follows: in Section 2, as a preliminary study, we analyze the case of a QFT, then, in Section 3, the case of quantum cosmology is studied. In Section 4 the BdB quantum cosmology is discussed and our results are obtained, in Section 5 we present our conclusions.

2. DISCARDING NEGATIVE-FREQUENCY SOLUTIONS IN KLEIN–GORDON

This section contains well-known results in quantum field theory. But we wish to clarify the problem to be studied in the next section, using a model already known, which is a scalar field satisfying the Klein-Gordon equation, and recreating the type of problem that can occur when the frequencies of a sector (say, negative) are discarded from the general solution. The idea is to confront it qualitatively with the model of quantum cosmology discussed in the next section. The model of a massless scalar field, which would be the most appropriate for comparison with a quantum cosmological model of the next section, is subject to the same analysis and satisfies the same results recreated here since it can be obtained without problems by taking the limit of zero mass from a massive field considered here (always with zero spin) [22], Chapter 5.9.

We consider a free scalar field ψ satisfying the usual Klein-Gordon equation

$$\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + m^2\psi = 0 \ . \tag{1}$$

The general solution can be written as a sum of two terms:

$$\psi = \int_{-\infty}^{\infty} dp\tilde{\psi}_{+}(p)e^{\frac{i}{\hbar}(px-Et)} + \int_{-\infty}^{\infty} dp\tilde{\psi}_{-}(k)e^{\frac{i}{\hbar}(px+Et)} \tag{2}$$

the first term being the "positive-frequency solution" and the second one the "negative-frequency solution".

As we know, the energy E satisfies (for a given momentum p) the condition $E^2 = p^2 + m^2$, i.e., it can have two values, $\pm\sqrt(p^2 + m^2)$. In principle, only positive values of E can have a physical significance of free particle energy. But the negative values cannot be simply omitted: the general solution of the wave equation can be obtained only by superposing all its independent particular solutions, the negative solutions being reinterpreted in the second quantization formalism. We have a complete set of commuting observables given by the energy and the momentum.[3]:

$$\hat{E} \equiv i\hbar\frac{\partial}{\partial t}, \tag{3}$$

$$\hat{p} \equiv -i\hbar\frac{\partial}{\partial x}. \tag{4}$$

As is known, these are "even" operators, which means that they transform positive-frequency solutions to positive-frequency ones and negative-frequency solutions to negative-frequency ones. This feature does not allow us, in any way, to dispense one of the sectors (say, negative). There is no selection rule that allows us to dispense with one of the sectors. If we did that, above all, there will be no room for antiparticles, which means the absence of the rich processes of creation and annihilation. Second, it would not be possible to satisfy completely

[3]Indeed, we must also know the helicity, which is zero, but we can ignore it without affecting our analysis.

the Lorentz symmetry, more precisely, invariance under the four-dimensional inversion (which is a four-rotation with determinant $+1$) will be violated, since its fulfillment make necessary the simultaneous presence in Eq. (2) of terms having both signs of E in the exponents: these signs are changed by the substitution $t \to -t$ (see [24], Section 11). With this, a violation of CPT invariance would be arbitrarily introduced (see [24], section 13).

Another way to see the problem is to analyze the process consisting in a particle propagating from space-time point x to space-time point y. The field $\psi^*(x)$ creates a particle at x, and $\psi(y)$ destroys a particle at y. The amplitude of this process is given by

$$\langle 0|\psi(y)\psi^*(x)|0\rangle \tag{5}$$

and, at spacelike separations (i.e. out of the light cone), it must vanish because no signal can propagate faster than light. Now, as we know $\psi(y)|0\rangle = 0$, because the vacuum is annihilated by ψ, then if the commutator $[\psi(y), \psi^*(x)]$ vanishes for spacelike separated regions, i.e,

$$[\psi(y), \psi^*(x)] = 0 \quad \text{for} \quad (x-y)^2 < 0, \tag{6}$$

then the amplitude (5) will indeed vanish at spacelike separations. In other words, (6) is a sufficient condition for the amplitude (5) to vanish at spacelike separations. In computing the commutator in (6), we can use the plane wave expansion for the field operators ψ and ψ^*. If we assume that this expansion involves a sum over plane waves with only positive frequencies (as in the case of nonrelativistic free fields), then it is not mathematically possible to adjust the coefficients of those expansions in such a way that they verify (6), unless they commute identically in all the space-time. It is necessary to allow negative-frequency plane waves in the field expansions in order to satisfy (6), i.e commutation at spacelike separations but not everywhere. Thus discarding the negative frequencies leads to a violation of causality [22, 25].

3. DISCARDING NEGATIVE FREQUENCY SOLUTIONS IN QUANTUM COSMOLOGY

We have pointed out in the introduction that in several papers the WDW approach to quantum cosmology has been criticized because they show that it is not possible to solve the Big Bang singularity. We have already noted that this strong statement has been criticized in [23]. But there is an argument of “superselection rule” used to arrive at that statement, which we want to discuss now. We are going to analyze the validity of the procedure which involves working with a single sector of frequencies. Here we outline the argument of [21]: That reference studied the WDW limit of Loop Quantum Cosmology (LQC) by working in the regime where effects of quantum discrete geometry can be neglected. The WDW equation obtained has the same form as the Klein-Gordon equation in static space-time:

$$\frac{\partial^2 \Psi}{\partial \phi^2} + \Theta \Psi = 0, \tag{7}$$

where the field ϕ plays the role of time, and Θ is the spatial Laplacian given by Eq. (3.4) of [21]:

$$\Theta \equiv -\frac{16\pi G}{3B(\mu)} \frac{\partial}{\partial \mu} \sqrt{\mu} \frac{\partial}{\partial \mu}, \tag{8}$$

where μ is the spatial coordinate and $B(\mu)$ is an eigenvalue of the operator $\widehat{|\mu|^{-3/2}}$, within a multiplicative constant. A general solution is obtained as a superposition of positive and negative frequencies:

$$\begin{aligned}\Psi(\mu, \phi) &= \int_{-\infty}^{+\infty} dk \tilde{\Psi}_+ e_k(\mu) e^{i\omega\phi} \\ &+ \int_{-\infty}^{+\infty} dk \tilde{\Psi}_- \bar{e}_k(\mu) e^{-i\omega\phi} \\ &= \Psi_+(\mu, \phi) + \Psi_-(\mu, \phi),\end{aligned} \tag{9}$$

where $e_k(\mu)$ are the eigenvectors of Θ with eigenvalues ω^2. A complete set of Dirac observables is given by

$$\hat{p}_\phi \Psi(\mu, \phi) \equiv -i\hbar \frac{\partial \Psi}{\partial \phi}, \tag{10}$$

$$\begin{aligned}\widehat{|\mu|_{\phi_0}} \Psi(\mu, \phi) &\equiv e^{i\sqrt{\Theta}(\phi-\phi_0)} |\mu| \Psi_+(\mu, \phi_0) \\ &+ e^{-i\sqrt{\Theta}(\phi-\phi_0)} |\mu| \Psi_-(\mu, \phi_0),\end{aligned} \tag{11}$$

and we quote verbatim from [21]: “both these operators preserve the positive and negative frequency subspaces. Since they constitute a complete family of Dirac observables, we have *superselection*. In quantum theory we can restrict ourselves to one superselected sector. We focus on the positive frequency sector, and from now on, drop the suffix $+$.”

The restriction to the positive-frequency sector through a type of “superselection rule” allows one to build a “physical” Hilbert space which is the space of wave functions of positive frequency with finite norm (the norm given by equation (3.15) of [21]). This may be correct from a mathematical point of view, especially because it allows for showing that the WDW evolution does not resolve the singularity (section III B of [21]).

However, in addition to the fact that the issue of singularity resolution, as presented in [21], seems to depend on the inclusion or not inclusion of negative frequencies, it does not seem like a good way to follow as this procedure causing a loss of important physical characteristics associated with the existence of negative-frequency solutions, as we are going to show in section 4.

4. THE BOHM-DE BROGLIE THEORY APPLIED TO QUANTUM COSMOLOGY

The Bohm-De Broglie quantum theory (see [26]) can be consistently implemented in quantum cosmology (see [27]). Considering homogeneous minisuperspace models, which have a finite number of degrees of freedom, the general form of the associated Wheeler-De Witt equation reads

$$-\frac{1}{2}f_{\rho\sigma}(q_\mu)\frac{\partial\Psi(q)}{\partial q_\rho \partial q_\sigma}+U(q_\mu)\Psi(q)=0, \qquad (12)$$

where $f_{\rho\sigma}(q_\mu)$ is the DeWitt minisuperspace metric of the model, whose inverse is denoted by $f^{\rho\sigma}(q_\mu)$. By writing the wave function in its polar form, $\Psi = R\,e^{iS}$, the complex equation (12) decouples into two real equations:

$$\frac{1}{2}f_{\rho\sigma}(q_\mu)\frac{\partial S}{\partial q_\rho}\frac{\partial S}{\partial q_\sigma}+U(q_\mu)+Q(q_\mu)=0, \qquad (13)$$

$$f_{\rho\sigma}(q_\mu)\frac{\partial}{\partial q_\rho}\left(R^2\frac{\partial S}{\partial q_\sigma}\right)=0, \qquad (14)$$

where

$$Q(q_\mu) := -\frac{1}{2R}f_{\rho\sigma}\frac{\partial^2 R}{\partial q_\rho \partial q_\sigma} \qquad (15)$$

is called the quantum potential. The Bohm-De Broglie interpretation applied to quantum cosmology states that the trajectories $q_\mu(t)$ are real, independently of any observations. Equation (13) represents their Hamilton-Jacobi equation, which is the classical one added with a quantum potential term Eq. (15) responsible for the quantum effects. This suggests to define

$$\pi^\rho = \frac{\partial S}{\partial q_\rho}, \qquad (16)$$

where the momenta are related to the velocities in the usual way:

$$\pi^\rho = f^{\rho\sigma}\frac{1}{N}\frac{\partial q_\sigma}{\partial t}, \qquad (17)$$

N being the lapse function. To obtain quantum trajectories, we have to solve the following system of first-order differential equations, called the guidance relations:

$$\frac{\partial S(q_\rho)}{\partial q_\rho} = f^{\rho\sigma}\frac{1}{N}\dot{q}_\sigma. \qquad (18)$$

The above equations (18) are invariant under time re-parametrization. Therefore, even at the quantum level, different time gauge choices of $N(t)$ yield the same space-time geometry for a given non-classical solution $q_\alpha(t)$. Indeed, there is no problem of time in the de Broglie-Bohm interpretation for minisuperspace quantum cosmological models [28]. However, this is no longer true when one considers full superspace (see [29, 30]). Notwithstanding, even with the problem of time in the superspace, the theory can be consistently formulated (see [31]).

Let us then apply this interpretation to our minisuperspace model, which is given by a spatially flat Friedmann (FLRW) universe with a massless free scalar field. The Wheeler-DeWitt equation reads[4]

$$-\frac{\partial^2\Psi}{\partial\alpha^2}+\frac{\partial^2\Psi}{\partial\phi^2}=0, \qquad (19)$$

where ϕ is the scalar field, and $\alpha \equiv \log a$, and a is the scale factor. Comparing Eq. (19) with Eq. (12), we obtain from Eqs. (13) and (14):

$$-\left(\frac{\partial S}{\partial\alpha}\right)^2+\left(\frac{\partial S}{\partial\phi}\right)^2+Q(q_\mu)=0, \qquad (20)$$

$$\frac{\partial}{\partial\phi}\left(R^2\frac{\partial S}{\partial\phi}\right)-\frac{\partial}{\partial\alpha}\left(R^2\frac{\partial S}{\partial\alpha}\right)=0, \qquad (21)$$

where the quantum potential reads

$$Q(\alpha,\phi) := \frac{1}{R}\left[\frac{\partial^2 R}{\partial\alpha^2}-\frac{\partial^2 R}{\partial\phi^2}\right]. \qquad (22)$$

The guidance relations (18) are

$$\frac{\partial S}{\partial\alpha} = -\frac{e^{3\alpha}\dot{\alpha}}{N}, \qquad (23)$$

$$\frac{\partial S}{\partial\phi} = \frac{e^{3\alpha}\dot{\phi}}{N}. \qquad (24)$$

We can write Eq. (19) in null coordinates:

$$v_l := \frac{1}{\sqrt{2}}(\alpha+\phi) \qquad \alpha := \frac{1}{\sqrt{2}}(v_l+v_r)$$
$$v_r := \frac{1}{\sqrt{2}}(\alpha-\phi) \qquad \phi := \frac{1}{\sqrt{2}}(v_l-v_r), \qquad (25)$$

yielding

$$\left(-\frac{\partial^2}{\partial v_l \partial v_r}\right)\Psi\left(v_l,v_r\right)=0. \qquad (26)$$

The general solution is

$$\Psi(u,v) = F(v_l)+G(v_r), \qquad (27)$$

[4] It is the same model studied in [23], Sec.II and IV.

where F and G are arbitrary functions. Using the method of separation of variables, one can write these solutions as Fourier transforms given by

$$\Psi(v_l, v_r) = \int_{-\infty}^{\infty} dk U(k)\, e^{ikv_l} + \int_{-\infty}^{\infty} dk V(k)\, e^{ikv_r}, \tag{28}$$

U and V also being two arbitrary functions, or, because for our purpose it is better to work in the original coordinates α and ϕ, by

$$\Psi(\alpha, \phi) = \int_{-\infty}^{\infty} dk U(k)\, e^{ik(\alpha+\phi)/\sqrt{2}} + \int_{-\infty}^{\infty} dk V(k)\, e^{ik(\alpha-\phi)/\sqrt{2}}. \tag{29}$$

For our numerical analysis we take the arbitrary functions $U(k)$ and $V(k)$ as the Gaussians

$$U(k) = e^{-(k-d)^2/\sigma^2}, \tag{30}$$

$$V(k) = e^{-(k+d)^2/\sigma^2}, \tag{31}$$

then we have

$$\Psi(\alpha, \phi) = \int_{-\infty}^{\infty} dk e^{-(k-d)^2/\sigma^2}\, e^{ik(\alpha+\phi)/\sqrt{2}} + \int_{-\infty}^{\infty} dk e^{-(k+d)^2/\sigma^2}\, e^{ik(\alpha-\phi)/\sqrt{2}}. \tag{32}$$

After integration and within a normalization[5] constant, we have

$$\Psi(\alpha, \phi) = |\sigma|\sqrt{\pi} \exp\left[i\frac{d\phi}{\sqrt{2}} - \frac{\sigma^2(\alpha^2+\phi^2)}{8}\right] \times \left\{ \exp\left[\frac{id\alpha}{\sqrt{2}} - \frac{\sigma^2\alpha\phi}{4}\right] + \exp\left[-\frac{id\alpha}{\sqrt{2}} + \frac{\sigma^2\alpha\phi}{4}\right]\right\}. \tag{33}$$

To obtain the quantum trajectories, it is necessary to calculate the phase S of the above wave function and to substitute it into the guidance equations. We will work in the gauge $N = 1$.

[5]In our study, the normalization of the wave function is irrelevant because we are going to extract information only from its phase.

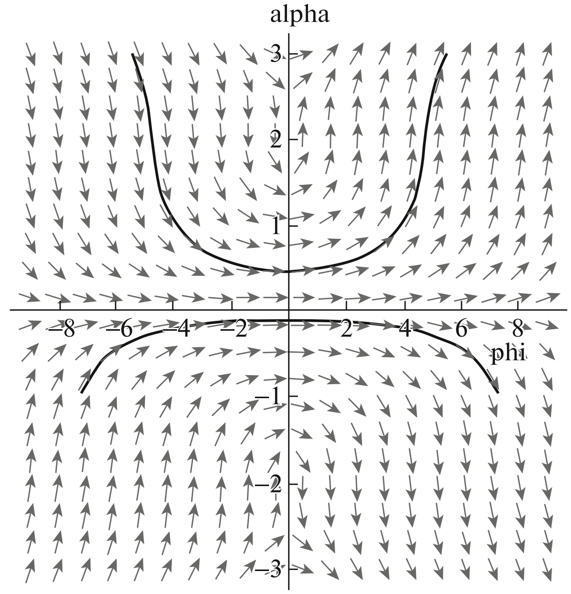

Fig. 1. The field plot (α versus ϕ)) shows the family of trajectories for Bohmian guidance equations (35), (36) associated to a wave functional with positive frequencies only. Two of them that describe their general behavior are depicted in solid lines: the first one represents a bouncing universe while the second one corresponds to a universe which begins and ends in singular states (a "Big Bang–Big Crunch" universe).

Computing the phase, we have:

$$S = \frac{d\phi}{\sqrt{2}} + \arctan\left[\tanh\left(\frac{\sigma^2\alpha\phi}{4}\right)\tan\left(\frac{d\alpha}{\sqrt{2}}\right)\right], \tag{34}$$

which, after substitution into (23) and (24), yields a planar system given by

$$\dot{\alpha} = \frac{\phi\sigma^2\sin(\sqrt{2}d\alpha) + 2\sqrt{2}d\sinh\left(\frac{\sigma^2\alpha\phi}{2}\right)}{e^{3\alpha}4\left[\cos(\sqrt{2}d\alpha) + \cosh\left(\frac{\sigma^2\alpha\phi}{2}\right)\right]}, \tag{35}$$

$$\dot{\phi} = \frac{2\sqrt{2}d\cosh\left(\frac{\sigma^2\alpha\phi}{2}\right) + 2\sqrt{2}d\cos(\sqrt{2}d\alpha)}{e^{3\alpha}4\left[\cos(\sqrt{2}d\alpha) + \cosh\left(\frac{\sigma^2\alpha\phi}{2}\right)\right]} - \frac{\alpha\sigma^2\sin(\sqrt{2}d\alpha)}{e^{3\alpha}4\left[\cos(\sqrt{2}d\alpha) + \cosh\left(\frac{\sigma^2\alpha\phi}{2}\right)\right]}. \tag{36}$$

Equations (35),(36) give the direction of the geometrical tangents to the trajectories which solve this planar system. By plotting the tangent direction field, it is possible to obtain the trajectories (Fig. 1). The line $\alpha = 0$ divides the configuration space into two

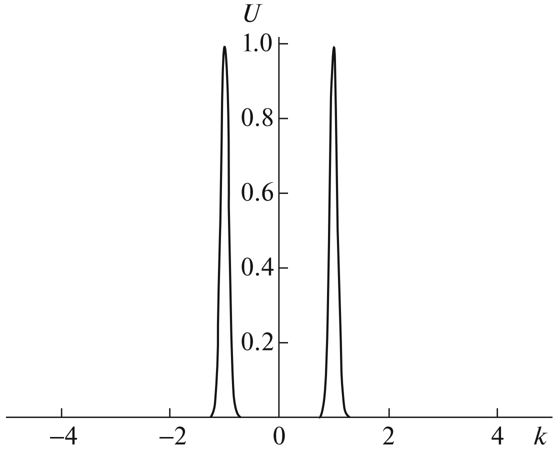

Fig. 2. $U(k)$ and $V(k)$ are two sharply peaked Gaussians centered at $k = d$ and $k = -d$, respectively. This gives a positive-frequency solution, equation (37).

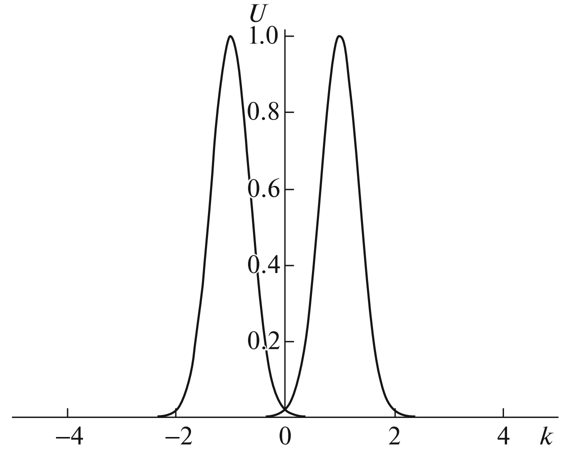

Fig. 3. $U(k)$ and $V(k)$ can no longer be considered as almost disjoint ones but will begin to overlap. Negative frequencies will begin to have an appreciable weight in the integral.

symmetric regions, and the line $\phi = 0$ contains all singular points, which are nodes and centers. The nodes arise when the denominator in the above equations, proportional to the norm of the wave function, is zero. No trajectory passes through these points. They happen when $\phi = 0$ and $\alpha = (2n+1)\dfrac{\pi}{\sqrt{2}d}$, n an integer, with periodicity $\sqrt{2}\pi/d$. The center points appear when the numerators are zero. They are given by $\phi = 0$ and

$$\alpha = \frac{2\sqrt{2}d}{\sigma^2}\cot\left(\frac{\sqrt{2}}{2}d\alpha\right).$$

In [23], the Bohmian trajectories corresponding to the solutions of the WDW equation of positive frequency, were obtained, such as shown in Fig/ 1. Here it is possible to distinguish two kinds of trajectories. The upper half of the figure contains trajectories describing bouncing universes, while the lower half corresponds to universes that begin and end in singular states (“Big Bang—Big Crunch” universes).

4.1. Positive and Negative Frequencies

If we allow both negative and positive frequencies in the solution, it is possible to observe the occurrence of cyclic universes which are shown by oscillatory trajectories in ϕ, as we will see. In this case, if one wishes to interpret ϕ as time, this corresponds to creation and annihilation of expanding and contracting universes that exist for a very short duration.[6]

[6] At this point we must note that a continuity equation for the ensemble of trajectories with a certain distribution function of initial conditions, is absent. For a discussion of this point, see [23], section IV.

To get an idea of how the inclusion of negative frequencies works in the behavior of solutions, we gradually introduce them.

We start choosing the values of σ and d in (32) with $\sigma \ll 1$ and $d \geq 1$. In this case, the functions $U(k)$ and $V(k)$ are two sharply peaked Gaussians centered at $k = d$ and $k = -d$, respectively (Fig. 2). Then the wave function (32) can be written very approximately as

$$\Psi(\alpha,\phi) \approx \int\limits_0^\infty dk e^{-(k-d)^2/\sigma^2}\, e^{ik\frac{1}{\sqrt{2}}(\alpha+\phi)} + \int\limits_{-\infty}^0 dk e^{-(k+d)^2/\sigma^2}\, e^{ik\frac{1}{\sqrt{2}}(\alpha-\phi)}, \quad (37)$$

which means a positive-frequency solution (note that ϕ, i.e., the “time”, appears in the exponential only with a positive sign).

If we sufficiently increase the parameter σ (“width”), the two Gaussians can no longer be considered almost disjoint but will begin to overlap (Fig. 3). This means that the approximation (37) is no longer valid, and negative frequencies will begin to be included with a greater weight in the integral. As we continue, to increase the parameter σ more and more negative frequencies will have a greater weight in the solution.

We have solved the Bohm guidance equations (35), (36) and obtained Bohmian trajectories for several increasing values or the parameter σ (the parameter d was kept constant) and for the same initial conditions. The results are depicted in Figs. 4–8.

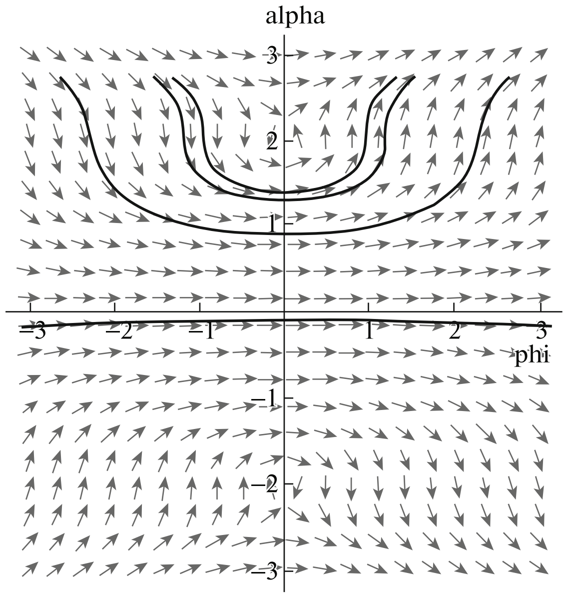

Fig. 4. The field plot (α versus ϕ) shows a family of trajectories for the Bohm guidance equations (35) and (36), associated to a wave functional with only positive-frequency solutions, $\sigma = 0.5$ and $d = 1$.

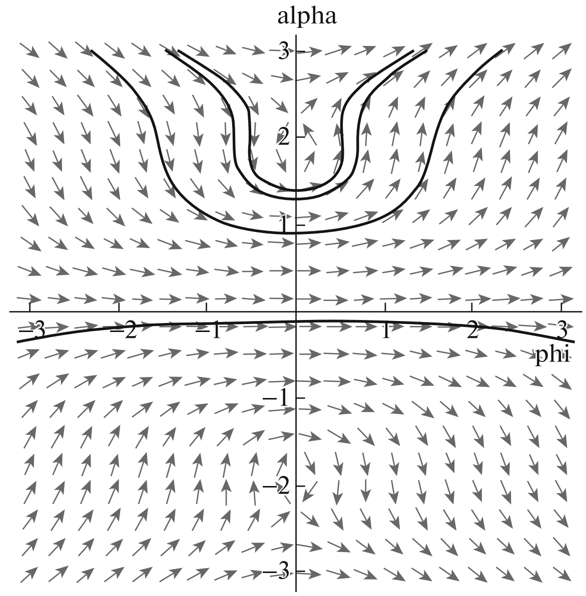

Fig. 5. The same as in Fig. 4 but with a bit of negative frequencies, which begin to weigh on the integral. $\sigma = 0.7$ and $d = 1$.

In Fig. 4 we have practically only positive-frequency solutions, in Fig. 5 negative-frequency solutions begin to weigh on the integral, in Fig. 6 there is a little more negative-frequency weight in the integral . We add more and more negative frequencies (this is because each Gaussian has an increasingly significant tail on the semi-axis opposite to the one containing its center) until we see that, at a certain point, a trajectory becomes cyclic (Fig. 7). Here there seems to be a threshold for this particular trajectory, which will be discussed below. At the end, in Fig. 8 positive and negative frequencies appear, in some sense, alike, as would be in a general solution to the WDW equation. In this last figure we observe the occurrence of cyclic universes, where, as we said, we can interpret that as processes of creation and destruction of universes if we accept ϕ fulfilling the role of a time. This situation of creation and annihilation of universes is a typical feature of relativistic quantum field theory. After all, the Wheeler-DeWitt equation already represents a second-quantized field theory. As such, it is expected that creation-annihilation processes occur naturally. These fundamental processes are lost along the demonstrations using the "superselection rule", then, in our humble understanding, we think that such a "rule" does not exist.

4.2. A threshold for the Emergence of Cyclic Universes?

We can ask whether there is a threshold of contribution of negative frequencies, above which a certain

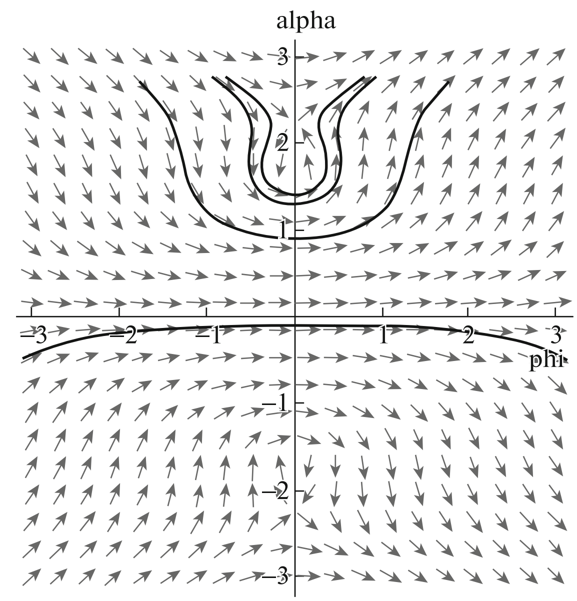

Fig. 6. The same as in Figs. 4 and 5 but with a larger contribution of negative frequencies, $\sigma = 0.8$ and $d = 1$.

trajectory becomes cyclic, i.e. a threshold for the emergence of processes of creation-annihilation[7]. As a partial answer, we have found numerically that, for example, the trajectory whose initial conditions are $\alpha(0) = 1.4$, $\phi(0) = 0$ becomes cyclic when $\sigma \approx 0.9$ for $d = 1$ (Fig. 7). This can be characterized by the area enclosed under each Gaussian, between $k = 0$

[7] I thank Prof. Nelson Pinto-Neto from CBPF, for asking along this line.

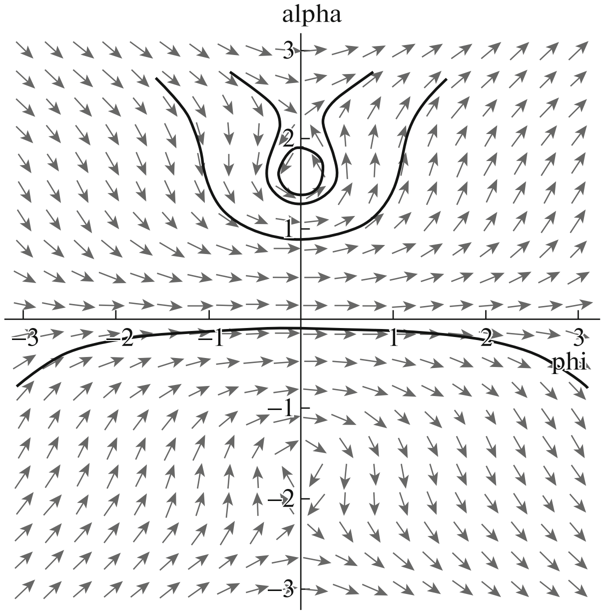

Fig. 7. The same as in Figs. 4–6 but with such a contribution of negative frequencies that a cyclic universe is formed (the trajectory passing through $\alpha = 1.4$, $\phi = 0$); $\sigma = 0.9$ and $d = 1$.

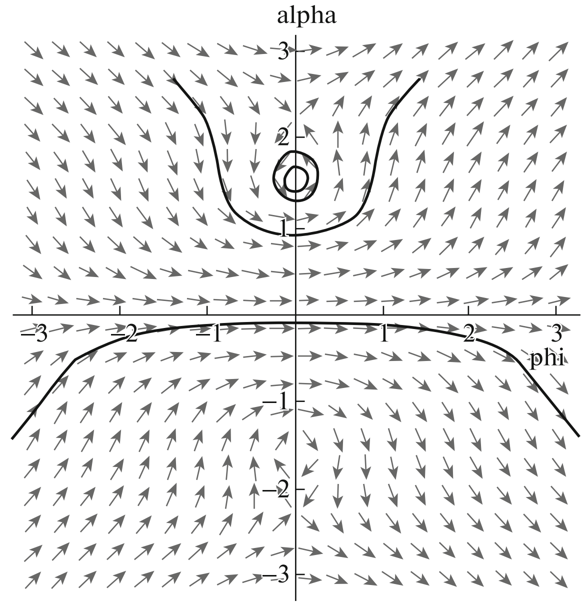

Fig. 8. The same as in Figs. 4–7 but with such a weight of negative frequencies that more cyclic universes emerge (note the trajectory passing through $\alpha = 1.3$, $\phi = 0$); $\sigma = 1$ and $d = 1$.

and $-\infty$ for the Gaussian centered at d and between $k = 0$ and $+\infty$ for the Gaussian centered at $-d$, the area that we call T (Fig. 9):[8]: the threshold occurs for $T \approx \sqrt{\pi}\sigma(1 - \text{Erf }(d/\sigma)) \approx 0.185$ (≈ 5.8 percent of the total area of the Gaussians). There is another trajectory, with initial conditions $\alpha(0) = 1.3$, $\phi(0) = 0$, that becomes cyclic for $\sigma \approx 1$ (Fig. 8), which means a threshold $T \approx \sqrt{\pi}\sigma(1 - \text{Erf }(d/\sigma)) \approx 0.279$ (≈ 7.9 percent of the total area of the Gaussians). We see that the value of the threshold T strongly depends on the initial conditions, i.e., it is different for each of this type of trajectory.

At the point where a trajectory closes, we have $d\alpha/d\phi = 0$ (the trajectory is instantaneously horizontal). This derivative is obtained by dividing equations (35) and (36), leading to (with $\overline{d} = \sqrt{2}d$)

$$\frac{d\alpha}{d\phi} = \frac{\phi\sigma^2 \sin(\overline{d}\alpha) + 2\overline{d}\sinh\dfrac{\sigma^2\alpha\phi}{2}}{2\overline{d}\cosh\dfrac{\sigma^2\alpha\phi}{2} + 2\overline{d}\cos(\overline{d}\alpha) - \alpha\sigma^2\sin(\overline{d}\alpha)}. \tag{38}$$

In this way, knowing that at that point is $\phi = 0$, it is observed that the numerator and then the derivative is zero, independently of the value of σ. This shows that there is no generic value of σ for which the trajectory closes, this will depend on the dynamics, as we will outline in the next section. It is clear that of course not every trajectory becomes a cyclic universe by allowing all negative frequencies. It seems it would be possible to determine the set of trajectories that can become cycling. Note that, for example, in the case $\sigma = 0.9$ fixed, there may be more cyclic trajectories than indicated in Fig. 7, say, all trajectories interior with respect to the one shown in the figure. The same occurs for fixed $\sigma = 1$.

As we said, the lower half of the graphs contains trajectories that describe universes of the Big-Bang / Big-Crunch type but now we see that, due to the perfect symmetry with respect to the horizontal axis, there are also cyclic trajectories, which correspond to cyclic universes of size even smaller than the Planck scale. However, all of them never reach a macroscopic size.

4.3. Qualitative Physical Explanation

The transition from open to closed trajectories can be qualitatively explained if we consider the energy balance of the system.

An equation that represents that balance can be obtained from Eq. (20), which is the quantum version of the Einstein-Hamilton-Jacobi equation (see [32, 33]), and for our model is given by

$$-\left(\frac{\partial S(\phi,\alpha)}{\partial\alpha}\right)^2$$

[8] In other words, it is a sum of the areas of the tails along the semi-axis which is opposite to the one containing the center of each Gaussian.

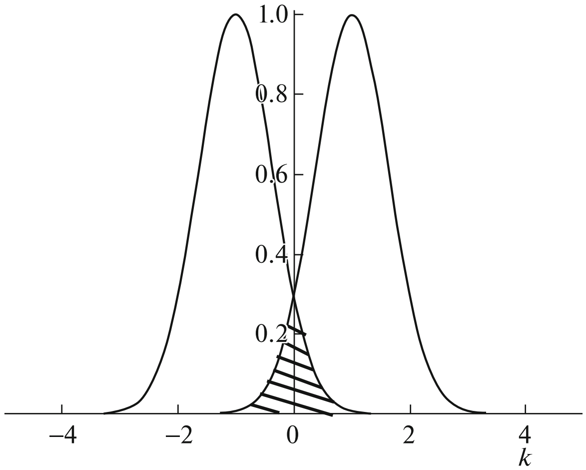

Fig. 9. A given trajectory, potentially cyclic, becomes effectively cyclic when the shaded area (which represents the contribution of negative-frequency solutions) exceeds a threshold given by $T \approx \sqrt{\pi}\sigma(1 - \mathrm{Erf}\,(d/\sigma))$.

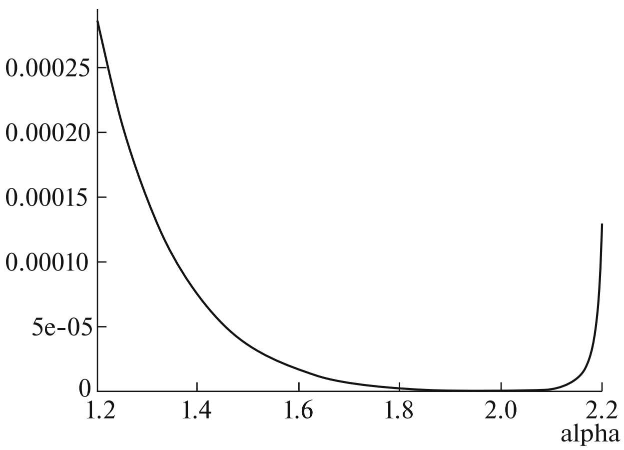

Fig. 10. The minimum of the effective potential for $\sigma = 0.5$ is at $\alpha = 1.9768$. The pole is fixed at $\alpha = 2.22$.

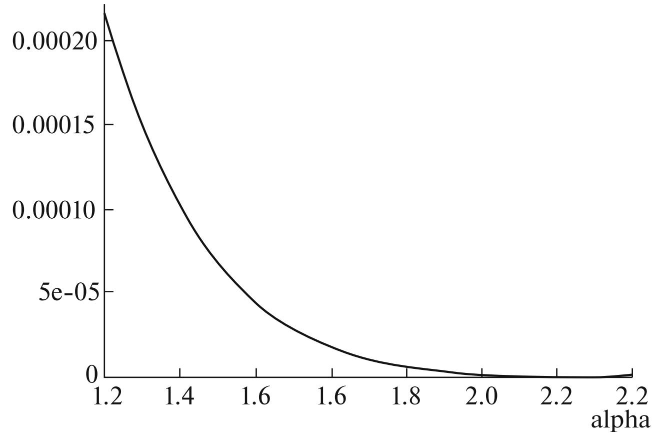

Fig. 11. The minimum of the effective potential for $\sigma = 0.7$ is at $\alpha = 1.795$, i.e, it has moved away from the node which remains fixed at $\alpha = 2.22$.

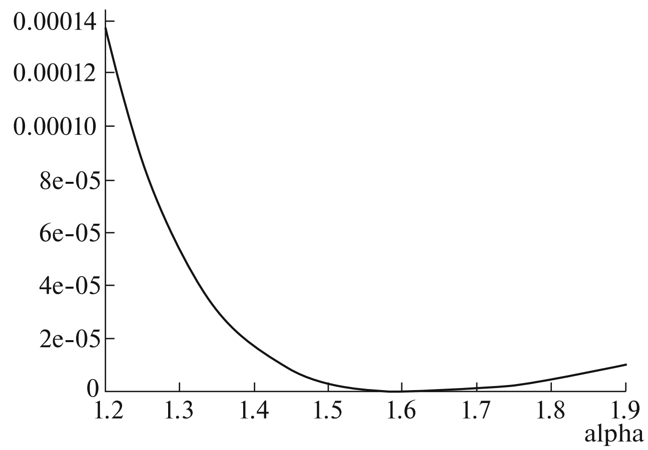

Fig. 12. The minimum of the effective potential for $\sigma = 0.9$ has shifted to $\alpha = 1.6104$. The trajectory with the initial conditions $\alpha(0) = 1.4$, $\phi(0) = 0$ "has found a place" to close.

$$+\left(\frac{\partial S(\phi,\alpha)}{\partial \phi}\right)^2 + Q(\phi,\alpha) = 0, \tag{39}$$

from which, using the guidance equations, we obtain

$$0 = \dot{\alpha}^2 - \dot{\phi}^2 - Q\,e^{-6\alpha}, \tag{40}$$

or equivalently, using $\alpha \equiv \log a$

$$0 = \left(\frac{\dot{a}}{a}\right)^2 - \dot{\phi}^2 - \frac{Q}{a^6}. \tag{41}$$

Equation (40), which is nothing else than the quantum version of the Friedmann equation, can be interpreted as representing a system with the total energy (given by the l.h.s., which is constant and in this case is zero due to the assumed flat geometry) equal to the sum of the "kinetic energy," presented by the first term in the r.h.s., plus the "effective potential energy" given by the other terms of the r.h.s.. This resembles a particle moving under the action of a classical potential, although not exactly equivalent, since the "effective potential energy" ($= -\dot{\phi}^2 - Qe^{-6\alpha}$) includes the quantum potential, which is strongly nonlocal and nonlinear.

We know that the singular points, i.e., centers and nodes appear along the axis $\phi = 0$. Then, in order to have a qualitative idea of what is happening, we can consider $\phi = \mathrm{const}$ (a small but nonzero value because otherwise the equation reduces to $0 = 0$), so we have a system depending on only one coordinate, namely, α.

Then the local minimums of $V_q \equiv -Q\,e^{-6\alpha}$ will determine, as in a dynamical system, the centers, and the poles will determine unreachable regions, that

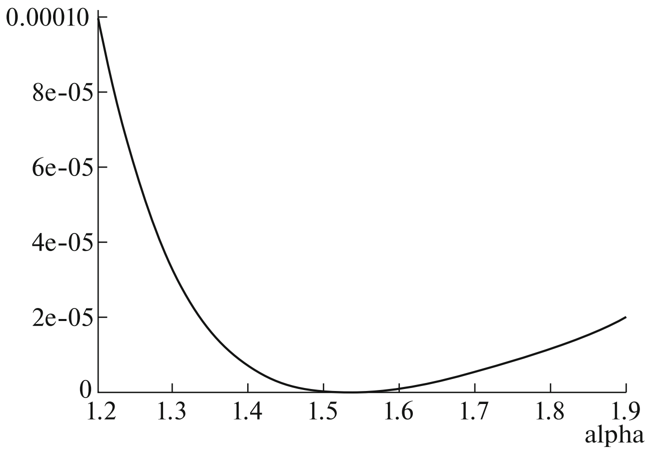

Fig. 13. The minimum of the effective potential for $\sigma = 1$ has shifted to $\alpha = 1.522$, and another trajectory becomes closed (the one with the initial conditions $\alpha(0) = 1.3$, $\phi(0) = 0$.

is, the nodes. In Figs. 10–13 present a cut of the effective potential as a function of α.

We see that as the parameter σ increases, the position of the minimum (which, as we said, determines the centers) moves away from the node (which, as we know, remains fixed), so that there are open trajectories that "acquire space" to close (the node prevented it). This could be explained as in a dynamic system through the energy balance, but now we have a quantum potential, which is strongly nonlinear in the considered region and makes it difficult to obtain an analytical result.

5. CONCLUSION

We have considered the procedure of discarding negative-frequency solutions, usual in quantum cosmology, and which is made invoking a type of "superselection". Discarding negative-frequency solutions in QFT brings about the absence of antiparticles, which, after all, means violation of 4-inversion symmetry $(x \to -x, t \to -t)$ which is an (improper) Lorentz transformation. As a heuristic discussion, suppose you have a theory of quantum gravity which lacks negative-frequency solutions. Taking some limit in this theory in order to obtain a weak (or null) gravitational regime, the result is a theory that does not respect Lorentz symmetry and has no place for antiparticles. That is, a relativistic QFT is not obtained, as it should be.

In the case of a model of quantum cosmology, we have shown that if we ignore negative-frequency solutions, the rich processes of creation/annihilation of universes at the Planck scale are lost. In fact, we were able to obtain Bohmian trajectories described by solutions of the Wheeler-DeWitt equation for a simple model. We considered initially a positive-frequency solution and studied numerically the behavior of trajectories while including negative frequencies. We have shown that when negative frequencies are considered on equal footing with positive frequencies, as is required in the general solution to any Klein-Gordon type equation, there emerge new processes, previously absent: cyclic universes of Planckian size, which can be interpreted as processes of creation-annihilation of universes that exist for a very short duration. This is a natural feature of any quantum relativistic field theory. In this way our results have led us to believe that this superselection rule can not exist.

We have verified numerically that, for a given trajectory, there is a threshold of negative-frequency solutions, above which cyclic universes are obtained, i.e., processes of creation-annihilation. We see that this strongly depends on the initial conditions, i.e., it is different for each of this type of trajectory. Moreover, it is clear that not every trajectory becomes that of a cyclic universe by adding negative frequencies. However it could be possible to determine the set for which this is possible. This could be a subject of future research. Another important point for a new investigation would be to analyze the more general case of a potential $V(\alpha, \phi)$, in which null coordinates in WDW equation are not completely separated.

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