



Research articles

Parallels between a system of coupled magnetic vortices and a ferromagnetic/nonmagnetic (FM/NM) multilayer system

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ABSTRACT

The coupling of interacting magnetic nanodisks with vortex configuration has interesting consequences, and has possible applications in the transport and processing of information. We show, through micromagnetic simulation and the use of Thiele's equation, that the coupling of magnetic vortices and the phenomenon of spin pumping in FM/NM multilayers, although arising from different physical mechanisms, show significant parallels. We study the variation of the effective damping (α_{eff}) of the system formed by a string of nanodisks with magnetic vortex structure, and Gilbert damping constant α_1 , interacting with another disk with Gilbert constant α_2 ($\alpha_2 \geq \alpha_1$). The result of varying the number of disks in the string is analogous to the observed variation of effective damping as a function of FM layer thickness, in FM/NM multilayers. The variation of the value of α_2 is analogous to the ability of the NM layer to act as a spin sink, for different NM materials. We also obtain that the effective damping goes through a maximum as a function of the value of damping α_2 ; an analogous effect is present in experiments with FM/NM systems, when the thickness of the NM layer is varied.

Introduction

The precession of the magnetization in a ferromagnetic layer (FM) produces an angular momentum current into an adjacent nonmagnetic (NM) layer, due to the interface coupling. This effect is known as spin pumping [1–5]. The presence of this phenomenon leads to an additional damping (enhanced damping) in the motion of the magnetic moments, that is quantified through the increased width of the FMR line, for example. Studies of the FM layer thickness effects on the enhancement of the damping reveal a complex phenomenology explained by a combination of spin-pumping, d-d hybridization and magnon scattering effect [6].

Magnetic nanostructure systems with exotic magnetic configurations, such as magnetic vortex nanodisks, have been proposed for many potential applications in Spintronics [7,8].

The vortex magnetic configuration is characterized by a curling magnetization in the plane of the disk, tangential to concentric circles. The circulation c defines the direction of curling: $c = +1$ for counter-clockwise (CCW) and $c = -1$ for clockwise (CW). The magnetization of the vortex core points perpendicularly to the plane of the disk: this characterizes the polarity p : $p = +1$ for magnetization pointing upwards, and $p = -1$, downwards [9,10].

The coupling between two nanodisks with magnetic vortex ground

states, essentially arising from magnetic dipolar interaction, has been studied both experimentally and through micromagnetic simulation (e.g., [11–14]). Interacting vortex disks show a loss-less transmission of energy [15,16], that makes this system a promising component of logic devices, e.g., [7,8].

Magnetic vortices also allow reproducing phenomena that are present in different areas of physics, e.g., Garcia et al. [17] and Sinnecker et al. [12] showed that it is possible to obtain magnetic vortex echoes using a matrix of nanodisks with magnetic vortex configuration, similar to spin echoes that are observed in nuclear magnetic resonance. Belanovsky et al. [18] showed that synchronization phenomenon can be modeled by a pair of coupled magnetic vortices.

The goal of this work is to show whether the dynamics of disks with magnetic vortex configuration can be viewed as an analogy to spin pumping in FM/NM interfaces. One string of disks with vortex structure, with intrinsic Gilbert damping parameter α_1 , interacts with another vortex disk of damping α_2 . We study the variation of the effective damping (α_{eff}) of the total system, as a function of the number of disks in the string of the FM identical disks, an analogy to the variation of the FM layer thickness in FM/NM multilayers, and its dependence with the variation of α_2 . The variation of α_2 models the differences in the NM layers, e.g., in composition, or thickness.

We used the open source software Mumax3 [19], with cell size

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$5 \times 5 \times L \text{ nm}^3$, where L is the thickness of the disks. The material used was Permalloy (NiFe) with parameters [20]: saturation magnetization $M_s = 8.6 \times 10^5 \text{ A/m}^2$, and exchange stiffness $A = 13 \text{ pJ/m}$. We employed in the simulations different values of the Gilbert damping constant α .

Results and discussion

Isolated disk

First, we have studied the effect of varying the Gilbert damping in the dynamics of one isolated disk. We have considered a disk with thickness $L = 20 \text{ nm}$ and diameter $D = 300 \text{ nm}$. In order to obtain the effective damping, we have initially applied an in-plane static magnetic field in the x -direction for a few nanoseconds, to displace the vortex core from the equilibrium position (center of disk). Then, the magnetic field was removed, allowing the vortex core to perform the gyrotropic motion [7,10]. This process is repeated for each value of Gilbert damping α (the parameter α was varied from $\alpha = 0.01$ to $\alpha = 0.1$ in 0.01 steps)¹. The time evolution of the x -component of the reduced magnetization m_x ($m_x = M_x/M_s$) during the gyrotropic motion is shown in the inset of Fig. 1 and can be fitted using an attenuated cosine function, as:

$$m_x = m_0 e^{-\beta t} \cos(2\pi f t). \quad (1)$$

The effective values of the damping (α_{eff}) are shown in Fig. 1. These were obtained using the equation:

$$\alpha_{\text{eff}} = \beta / (2\pi f), \quad (2)$$

where β and f parameters² were obtained from the fit using Eq. (1).

As expected, the effective damping increases with the increase of Gilbert damping α ; this increase indicates that the decay of the magnetization signal is faster as the value of Gilbert damping α increases. For our geometry, the effective damping obtained by micromagnetic simulation increases from approximately 0.017 (for $\alpha = 0.01$) to 0.165 (for $\alpha = 0.10$).

Coupled disks

In order to make an analogy with spin pumping systems, where ferromagnetic layers (FM) are in contact with nonmagnetic metals (NM), we have considered coupled identical disks separated by a fixed center to center distance $d = 310 \text{ nm}$, as shown in Fig. 2.

It is important to emphasize here that the coupling between magnetic vortices is different from the coupling between FM/NM layers. In oscillating closely situated coupled vortex systems, there appear higher order magnetic interactions, such as dipole-octupole, octupole-octupole, etc [22]. Whereas in FM/NM multilayer systems, the spin-orbit coupling, between the FM and NM layers, plays an important role, leading to complex mechanisms such as interface mixing conductance, interface spin flipping [5,2,4].

Although the physical origin of the magnetic interaction in these two systems is different, coupled magnetic vortex systems exhibit behaviors similar to those observed in FM/NM multilayers.

Effect of the variation of α_2 on the effective damping

We have considered five strings of disks, with number of disks varying from 2 (Fig. 2(a)) to 6 (Fig. 2(e)), as shown in Fig. 2. The disks left of the dotted vertical line, which are considered analogous to the ferromagnetic layers (FM), have a fixed Gilbert damping $\alpha_1 = 0.01$ and fixed polarity $p_1 = +1$. The disk right of the dotted line, which plays

¹ Large values of Gilbert damping in Permalloy can be obtained using a dopant, as already shown by Luo et al. [21].

² The value of the gyrotropic frequency is approximately 573 MHz.

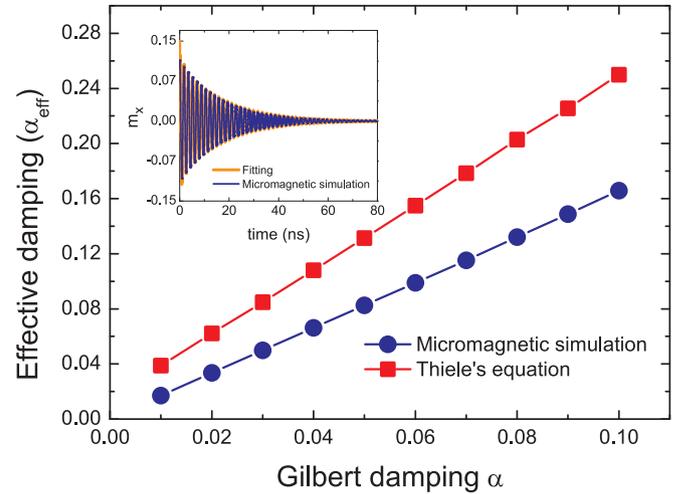


Fig. 1. Effective damping (α_{eff}) of an isolated Permalloy disk with $D = 300 \text{ nm}$, $L = 20 \text{ nm}$ vs. its Gilbert damping factor α . Blue circles represent data obtained by micromagnetic simulation and red squares data obtained from Thiele's equation (Eq. (4)). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

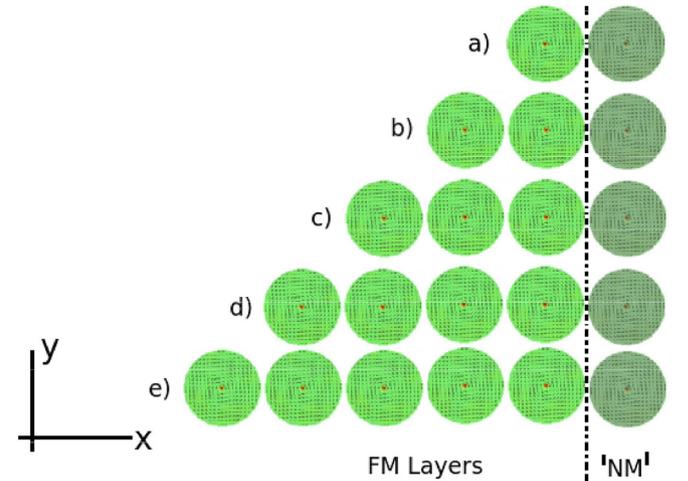


Fig. 2. Strings of disks representing the FM layer with magnetic vortex configuration (left of the dotted vertical line) interacting with the righthand side disk. The center to center separation of the disks is 310 nm. All lefthand side disks have the same Gilbert damping α_1 ; the righthand side disk that simulates the nonmagnetic layer, has Gilbert damping α_2 ($\alpha_2 \geq \alpha_1$).

the role of a nonmagnetic layer (NM)³, has Gilbert damping α_2 ; α_2 was varied from $\alpha_2 = 0.01$ to $\alpha_2 = 0.10$ in 0.01 steps, and from $\alpha_2 = 0.10$ to $\alpha_2 = 1$ in 0.10 steps, and polarity $p_2 = \pm 1$. All disks (FM and 'NM' layers) have fixed circulation⁴ $c = +1$.

In order to obtain the effective damping of the coupled systems (FM + 'NM'), we have followed the same procedure used in the case of an isolated disk. We have applied a DC in-plane magnetic field, in the x -direction, to all layers⁵ (FM and 'NM') to displace all the vortex cores

³ This is only an analogy. A permalloy disk is magnetic, but increasing its damping, we show here that it is possible to emulate the same role that the NM layer has in FM/NM systems, as explained in the text.

⁴ The circulation c only determines the phase relation between the coordinates of the vector positions of each disk, whereas the phase relations between the components of the magnetization of each disk are determined only by the polarity p [23]. For $p = +1$ and $c = \pm 1$ only a single mode can be excited with in-phase motion of all vortices. More details can be found in Ref. [24]

⁵ We have applied the magnetic field to all layers, since in FM/NM multilayer

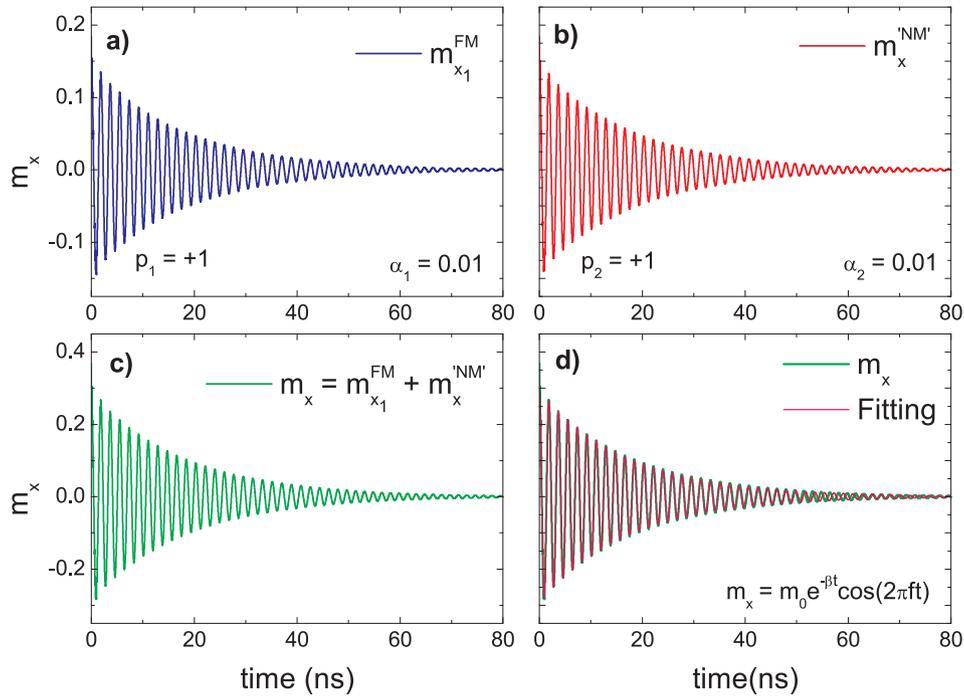


Fig. 3. Time evolution of the x-component of the magnetization (a) for the FM layer, (b) for the ‘NM’ layer, (c) x-component of the reduced magnetization m_x of the system, and (d) fitting using Eq. (1).

out of the equilibrium position, and immediately after, the magnetic field was turned off, allowing the vortex cores to perform the gyrotropic motion.

The values of the effective damping (α_{eff}) of the system (FM layer + ‘NM’ layer) were obtained in the following way: first, we have obtained the time evolution of the x-component of the magnetization m_x of the system (FM + ‘NM’) during the gyrotropic motion, defined by:

$$m_x = \sum_{i=1}^n m_{x_i}^{\text{FM}} + m_x^{\text{'NM'}} \quad (3)$$

where n = number of FM layers in the string.

Once obtained m_x of the system, we have used Eq. (1) and Eq. (2) to obtain the effective damping (α_{eff}).

For example, in Fig. 3(a) and Fig. 3(b), we show the time evolution of the x-component of the magnetization⁶ m_x in each disk, during the gyrotropic motion, for a string of two disks (one FM layer and one ‘NM’ layer), as shown in Fig. 2(a). In this case, $n = 1$. Thus, the time evolution of the x-component of the magnetization of the system is $m_x = m_{x_1}^{\text{FM}} + m_x^{\text{'NM'}}$ (Fig. 3(c)). In Fig. 3(d), m_x of the system is shown, as well as the fit using Eq. (1). The same procedure is repeated for the other strings.

The values of the effective damping (α_{eff}) for all strings of disks are shown in Fig. 4. The results show that the enhanced effective damping of the FM + ‘NM’ system varied as a function of α_2 , for $p = p_1, p_2 = \pm 1$. In every curve obtained in Fig. 4 there is a value of α_2 that leads to the maximum effective damping of the FM + ‘NM’ system. The curves show two regions: in the first region, the effective damping increases with the increase of α_2 , and in the second region the effective damping decreases with the increase of α_2 .

In the two regions the vortex cores perform the gyrotropic motion

(footnote continued)

spin pumping systems it is impossible to apply a magnetic field only to the FM layer. However, similar behaviors were obtained when the magnetic field was applied only to the FM layers (see item 3 in the Supplementary Material).

⁶The x-component of the magnetization of all disks are always in-phase between them for any value of p , as shown in Supplementary Material.

with their x-components of the magnetization in-phase (see Supplementary material), but with different amplitudes, due to the magnetic coupling.

The range of effective damping in the two regions varies, depending on the value of $p = p_1, p_2$ and the number of disks in the chain. For $p = p_1, p_2 = +1$, the first region ranges from $\alpha_2 = 0.01$ to $\alpha_2 = 0.08$, for a chain of two disks, from $\alpha_2 = 0.01$ to $\alpha_2 = 0.10$, and from $\alpha_2 = 0.01$ to $\alpha_2 = 0.10$ for the other cases.

For $p = p_1, p_2 = -1$, the first region ranges from $\alpha_2 = 0.01$ to $\alpha_2 = 0.20$ for a chain of two, from $\alpha_2 = 0.01$ to $\alpha_2 = 0.10$, for a chain of three and four disks, from $\alpha_2 = 0.01$ to $\alpha_2 = 0.09$, for a chain of five disks, and $\alpha_2 = 0.01$ to $\alpha_2 = 0.08$, for a chain of six disks. The second region ranges from the upper values of α_2 values previously mentioned.

The values of effective damping are higher for the case $p = p_1, p_2 = -1$ than in the case $p = p_1, p_2 = +1$. This is due to the stronger magnetic coupling in the case of antiparallel polarities⁷, allowing a decay of the magnetization signal faster than in the case of parallel polarities.

The dependence of the effective damping on α_2 is shown in Fig. 4. The existence of a maximum in the curve of effective damping versus α_2 has an analogy with the maximum found by Barati et al. [25] and Azzawi et al. [6] in the curve of calculated α_{eff} versus the thickness of the NM layer. These authors have found two regions in the graph of α_{eff} versus thickness of the NM layer; in the first region α_{eff} increases, in the second region it falls.

First region: we observe that the decay of the signal of the magnetization in each layer is faster with the increase of α_2 (see Supplementary material). That is, the vortex cores of both layers (FM and ‘NM’) move faster towards the equilibrium position (center of disk) as α_2 increases, causing a faster decay of the magnetization signal of the system, and increasing the values of its effective damping, as shown in Fig. 4. Thus, in our systems, the increase of α_2 in the ‘NM’ layer is

⁷In general, in a chain of vortices with different polarities, the effective damping of the system increases. This is due to the fact that the vortex cores perform the gyrotropic motion in different senses, allowing an interference between the different modes excited in each disk.

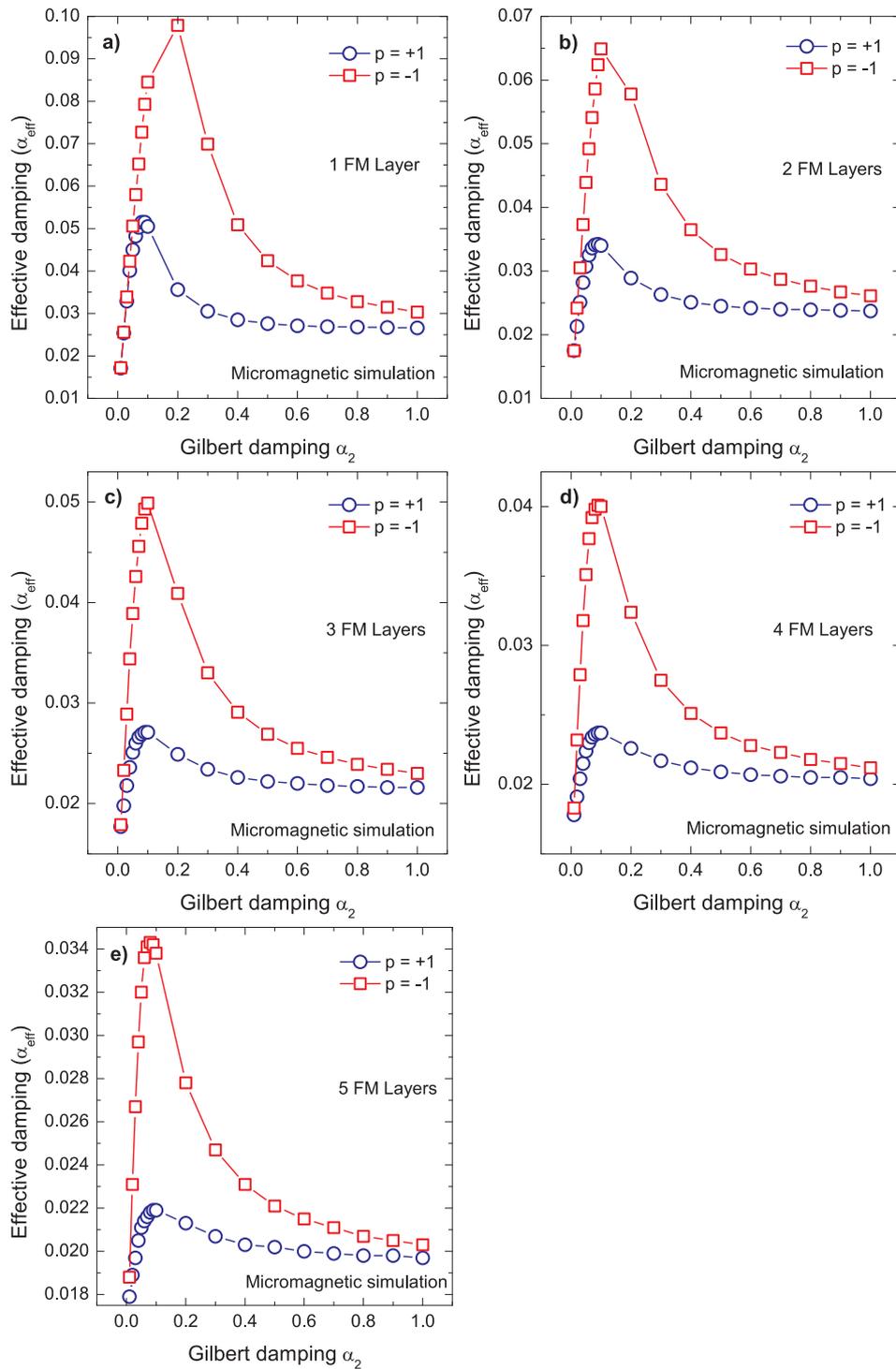


Fig. 4. Effective damping (α_{eff}) of the system of disk strings as a function of the damping of the ‘NM’ layer α_2 . (a) for one FM layer, (b) for two FM layers, (c) for three FM layers, (d) for four FM layers and, (e) for five FM layers.

responsible for the rapid decay of the magnetization of the FM layer, and also of the total magnetization (FM + ‘NM’).

We found an increase of the effective damping of approximately 200%, from $\alpha_{\text{eff}} = 0.017$ (for $\alpha_2 = 0.01$) to $\alpha_{\text{eff}} = 0.051$ (for $\alpha_2 = 0.08$), for the case of one FM layer and one ‘NM’ layer and $p = +1$. This increase is more drastic for $p = -1$, and is of approximately 470%, from $\alpha_{\text{eff}} = 0.017$ (for $\alpha_2 = 0.01$), to $\alpha_{\text{eff}} = 0.097$ (for $\alpha_2 = 0.20$).

It is important to note here that the role of the ‘NM’ layer in our system is analogous in its effects, although different in its mechanism, to that of the NM layer in FM/NM multilayer spin pumping systems. In

the multilayers, the FM layer loses magnetization to the NM layer, due to the transfer of angular momentum from the FM layer to the NM layer through interface coupling. This results in a faster decay of the magnetization signal in the FM layer, thus increasing the effective damping of the system. Additionally, the effect of increasing α_2 in our system is analogous to the effect of increasing the thickness of the NM layer in FM/NM multilayers, which leads to a change in the effective damping. In FM/NM multilayers, there is an optimum value of NM thickness that leads to a maximum in α_{eff} [25,6].

Second region: In this region, the system behaves in a manner

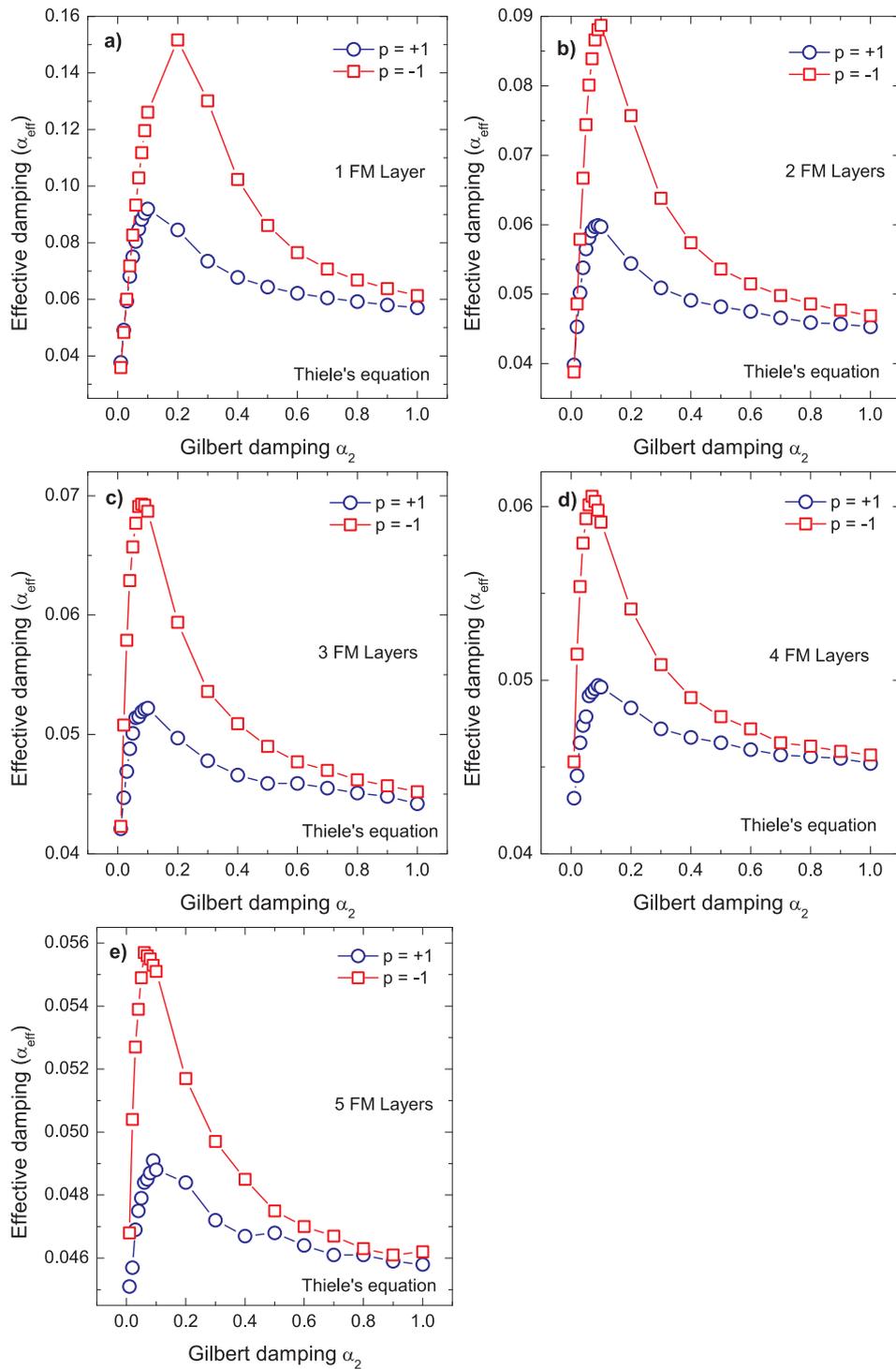


Fig. 5. Effective damping (α_{eff}) of the string of disks, obtained from Thiele's equation (Eq. (4)), as a function of the damping of the 'NM' layer α_2 . (a) for one FM layer, (b) for two FM layers, (c) for three FM layers, (d) for four FM layers, and (e) for five FM layers.

opposite to the behavior observed in the first region. With the increase of α_2 , the decay of the magnetization signal in the FM layers becomes slower, while the decay of the magnetization signal in the 'NM' layer becomes faster (see [Supplementary material](#)). This leads to a slower decay of the total magnetization signal in the system, and a drop in its effective damping, as shown in Fig. 4.

This behavior can be attributed to the following: a) the values of α_2 are so large that the vortex core of the 'NM' layer quickly goes to the equilibrium position, thus, its signal contribution to the effective damping is negligible, and b) the faster decay of the magnetization

signal reduces the time during which the FM and 'NM' layers interact. This behavior is also observed in FM/NM multilayers, as previously mentioned.

In the case when $\alpha_2 = 1$, the value of the effective damping is almost the same for $p = +1$ or $p = -1$ (see Fig. 4), since practically the only disks that interact magnetically with one another are the FM layers.

It is important to note that although higher values of α_2 reduce the contribution to the effective damping, the values of effective damping in the case when $\alpha_2 = 1$ (e.g., $\alpha_{\text{eff}} \approx 0.030$ for the case of two disks in the chain, one FM layer and one 'NM' layer) are not the same as in the

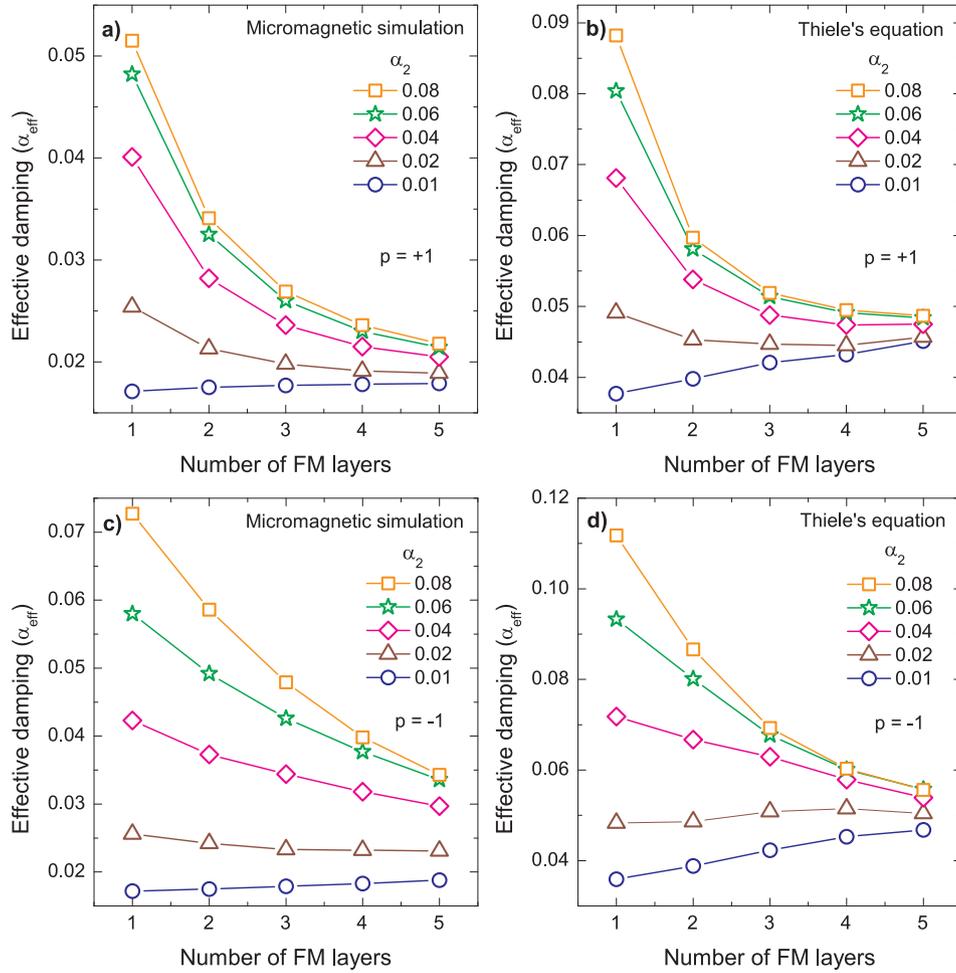


Fig. 6. Effective damping (α_{eff}) of the nanodisk systems (FM + ‘NM’ layers) (Fig. 2), obtained by micromagnetic simulation, as a function of the number of lefthand side disks (FM layers), for different values of the damping α_2 of the disk on the righthand side (‘NM’ layer). Fig. 2) and Fig. 6(c) refer to values of α_2 obtained by micromagnetic simulation, and Figs. 6(c) and 6(d) refer to those obtained by Thiele’s equation.

case of an isolated disk ($\alpha_{\text{eff}} \approx 0.022$). This can be attributed to the fact that there is still a weak magnetic interaction between the disks.

In the FM/NM multilayer systems, this behavior is attributed to the fact that, although the extrinsic contribution is negligible, intrinsic effects from the interface are the dominant contributions [25,6].

We have also considered the analytical approach based on Thiele’s equation [26]:

$$-\mathbf{G}_i \times \dot{\mathbf{X}}_i - D_i \dot{\mathbf{X}}_i + \frac{\partial W(\mathbf{X}_i)}{\partial \mathbf{X}_i} = 0, \quad (4)$$

where $\mathbf{X}_i = (x_i, y_i)$ is the vector position of the vortex core, $\mathbf{G}_i = -G_i \hat{z}$ is the gyrovector, and $G_i = 2\pi\mu_0 LM_s/\gamma$ is the gyrotropic constant, $D_i = -\alpha\pi\mu_0 LM_s \{2 + \log(R/R_c)\}/\gamma$ is the damping constant, γ is the gyromagnetic ratio, R_c is the vortex core radius and $W(\mathbf{X}_i)$ is the energy of each disk ($i = 1, 2, \dots, 6$).

The energy in the state of relaxation is given by the following expression:

$$W(\mathbf{X}_i) = \frac{1}{2} \kappa \mathbf{X}_i^2 + W_{\text{int}}, \quad (5)$$

where $\kappa_i = 0.81(40\pi M_s^2 L_i^2/9R_i)$ is the stiffness coefficient [27], and the second term is the magnetic interaction energy between neighbor disks, defined as follows [28]:

$$W_{\text{int}} = C_{j-1} C_j (\eta_x x_{j-1} x_j + \eta_y y_{j-1} y_j) + C_j C_{j+1} (\eta_x x_j x_{j+1} + \eta_y y_j y_{j+1}), \quad (6)$$

where $j = 2, 3, \dots, 5$. The quantities $\eta_{x,y}$ are the coupling integrals along x and y directions between neighbor disks. From their analytical

expressions [28], we have obtained $\eta_x = 1.44 \times 10^4 \text{J/m}^2$ and $\eta_y = -4.40 \times 10^4 \text{J/m}^2$.

The values of the effective damping obtained from Eq. (4) are shown in Fig. 1 for an isolated disk, and in Fig. 5 for coupled disks. These values were obtained using the y -component of the vortex core position, since $M_{xj} \propto y_j$ [24].

The results obtained from Thiele’s equation (Eq. (4)), are in qualitative agreement with those obtained from micromagnetic simulations, both for an isolated disk and coupled disks. They also show that there is a value of α_2 that induces the maximum effective damping of the FM + ‘NM’ system. However, there are quantitative discrepancies⁸.

Dependence of the effective damping on the number of FM layers

Finally, we show the dependence of the effective damping (α_{eff}) of the system on the number of FM layers, for different values of α_2 . We observe that the values of the effective damping decrease with the increase in the number of FM layers. This behavior is very similar to that obtained by Liu et al. [4] and Mizukami et al. [5] in multilayer spin

⁸ The analytical expressions for G and D work well when $L/R \ll 1$. For our geometry, we have $L/R = 0.13$; thus, quantitative differences are expected. Additionally, the expressions for the coupling integrals (Eq. (6)) are obtained in the rigid vortex model, which overestimates the frequency of the vortex motion [24]. Moreover, in the rigid vortex model, only surface charges are considered responsible for the magnetic interaction, but in the micromagnetic simulations the volumetric charges are also considered.

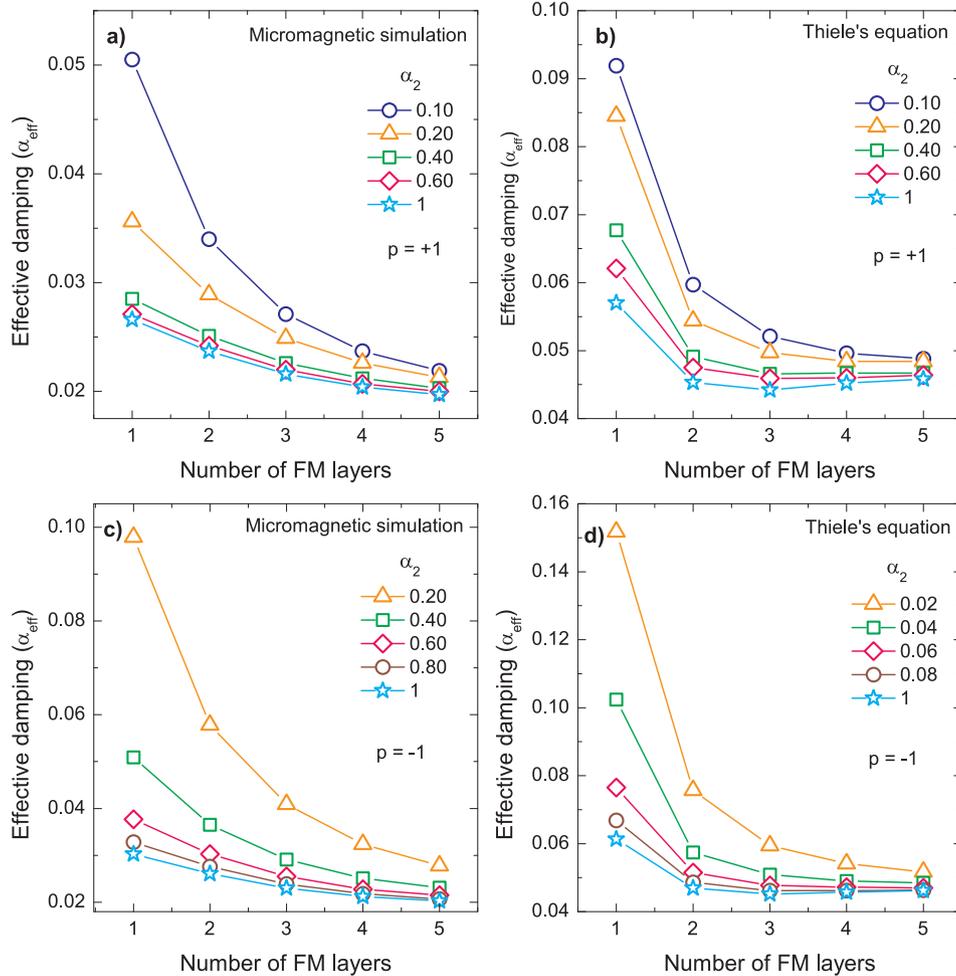


Fig. 7. Effective damping (α_{eff}) of the nanodisk systems (FM + ‘NM’ layers) (Fig. 2), obtained from Thiele’s equation, as a function of the number of lefthand side disks (FM layers), for different values of the damping α_2 of the disk on the righthand side (‘NM’ layer). Fig. 2), and Fig. 7(c) refer to values of α_2 obtained by micromagnetic simulation, and Fig. 7(c) and Fig. 7(d) refer to those obtained by Thiele’s equation.

pumping systems, when varying the thickness of the FM layer (Fig. 1 in Ref. [4] and Fig. 2 in Ref. [5]). These authors obtained that in α_{eff} there is a term inversely proportional to the thickness of the FM layer.

Fig. 6 corresponds to the first region, and Fig. 7 corresponds to the second region of the effective damping versus α_2 curves previously mentioned. We have in these figures exhibited the micromagnetic simulations in the left hand side, and the results from Thiele’s equation on the right.

We start by describing what happens in the first region and second region (Fig. 6). When FM layers and ‘NM’ layer have the same $\alpha_1 = \alpha_2 = 0.01$, the value of the effective damping remains almost constant with the increase in the number of FM layers.

We can compare this behavior with the one shown by Liu et al. [4] and Mizukami et al. [5], when the FM (Py) layer is coupled to NM (Cu) layer. In that case, the effective damping is small and almost constant, because there is no net angular momentum transfer from FM to NM.

In our case, this behavior is expected: for example, for a chain of two disks (one FM layer and one ‘NM’ layer), since both layers have the same value of Gilbert damping $\alpha_1 = \alpha_2 = 0.01$, the decay rate of the x-component of the magnetization signal m_x on every disk (FM and ‘NM’ layers) is practically the same. By increasing the number of FM layers, the amplitudes and frequencies of the x-component of the magnetization in each layer changes, but the decay rate of the total m_x is the same.

When α_2 increases, the effective damping of the system also increases. An analogous behavior is also shown by the previously mentioned authors, for NM = Ta, Pd, Pt.

In our system, the value of α_2 of the ‘NM’ layer has a role analogous to the parameter $G_{\text{eff}}^{\text{mix}}$ of the NM element in FM/NM multilayers. Lower values of $G_{\text{eff}}^{\text{mix}}$ do not lead to an enhanced effective damping, but higher values of $G_{\text{eff}}^{\text{mix}}$ have this effect [4].

For $\alpha_2 \neq \alpha_1$, as the number of FM layers increases, the values of the effective damping tend to decrease. For example, for $\alpha_2 = 0.08$ ($p = p_1, p_2 = +1$), the effective damping is approximately 0.052 for one FM layer (Fig. 6(a)). When the number of FM layers increases to two, the effective damping decreases to approximately 0.034, and keeps falling to a value of approximately 0.021 for 5 FM layers.

In our case, the decrease of the values of effective damping can be understood as follows: in the case of two disks in the chain (Fig. 2(a)), both vortex cores reach more rapidly the equilibrium position with the increase of α_2 , due to the magnetic coupling between the FM layer and the ‘NM’ layer. For the case of a string of three disks (two FM layers and one ‘NM’ layer), as shown in Fig. 2(b), it is also possible to observe that the decay of the magnetization signal is faster with the increase of α_2 . However, there is an important difference, now the first FM layer neighbor of the ‘NM’ layer interacts magnetically with another FM layer. This additional second FM layer also influences the gyrotropic motion of the vortex core of the first FM layer (and viceversa). The effect of this new magnetic interaction will be that the decay of the magnetization signal of the system will be slower compared to the case of a chain of two disks (1 FM layer and 1 ‘NM’ layer). This effect is more drastic when the number of FM layers in the chain increases, as shown in Fig. 6, because the disks that are further away from the ‘NM’ layer do

not feel its influence directly. This effect can be seen in the temporal evolution of the x-component of the magnetization in the [Supplementary material](#).

In the second region ([Fig. 7](#)), the FM layers feel less the effects of the ‘NM’ layer, as mentioned early, decreasing the rate of decay of their magnetization signals, and leading to a decrease in the effective damping with the increase of α_2 . This effect becomes more evident as the number of FM layer increases. This behavior is similar to that shown by Barati et al. [[25](#)] (see [Fig. 9](#) in this reference).

In [Fig. 6\(b\)](#), [Fig. 6\(d\)](#), [Fig. 7\(b\)](#), and [Fig. 7\(d\)](#), we show the variation of the values of effective damping obtained from Thiele’s equation (Eq. (4)) for different numbers of FM layers. In these figures, we can see that the behavior is similar to that shown in [Fig. 6\(a\)](#), [Fig. 6\(c\)](#), [Fig. 7\(a\)](#), and [Fig. 7\(c\)](#), showing the existence of two regions, the region where the effective damping increases with increasing α_2 , and the region where the effective damping decreases with the increase of α_2 .

Our results for the dependence of α_{eff} on the number of FM layers (or disks) with $\alpha_2 \leq 0.08$ ([Fig. 6](#)) and $\alpha_2 \leq 0.10$ ([Fig. 6\(c-d\)](#)) show larger values of α_{eff} for larger values of α_2 .

Conclusions

In this work, using micromagnetic simulation and Thiele’s equation, we have studied the influence of the increase of the Gilbert damping constant α in the effective damping of coupled disk systems with magnetic vortex configuration.

We have shown that increasing Gilbert damping constant α in one of the disks (‘NM’ layer) of the chain, leads to an enhanced effective damping of the system, and a similar dependence of effective damping with the number of layers, as found in FM/NM multilayer systems.

We have shown that further increasing α_2 in our ‘NM’ layer, leads to a decrease of the effective damping, similar to the effect of increasing the thickness of the NM layer in FM/NM multilayer systems.

Our results also showed that the dependence of the effective damping on the number of FM vortex layers parallels the dependence of α_{eff} on the thickness of the FM film in the multilayers.

It is also demonstrated that it is possible to control the decay of the magnetization signal in coupled nanodisk systems, tailoring α_2 in a single element that belongs to an array. This can be relevant for technological applications, where it is necessary to control the relaxation rates of the magnetization of systems with many elements.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.jmmm.2019.166009>.

References

- [1] L. Berger, Emission of spin waves by a magnetic multilayer traversed by a current, *Phys. Rev. B* 54 (1996) 9353–9358, <https://doi.org/10.1103/PhysRevB.54.9353>.
- [2] Y. Tserkovnyak, A. Brataas, G.E.W. Bauer, Enhanced Gilbert damping in thin ferromagnetic films, *Phys. Rev. Lett.* 88 (2002) 117601, <https://doi.org/10.1103/PhysRevLett.88.117601>.

- [3] A. Brataas, Y. Tserkovnyak, G. Bauer, P. Kelly, *Spin Pumping and Spin Transfer*, Oxford University Press, United Kingdom, 2012, pp. 87–135.
- [4] Y. Liu, Z. Yuan, R.J.H. Wesselink, A.A. Starikov, P.J. Kelly, Interface enhancement of Gilbert damping from first principles, *Phys. Rev. Lett.* 113 (2014) 207202, <https://doi.org/10.1103/PhysRevLett.113.207202>.
- [5] S. Mizukami, Y. Ando, T. Miyazaki, Ferromagnetic resonance linewidth for NM/80NiFe/NM films (NM=Cu, Ta, Pd and Pt), *J. Magn. Magn. Mater.* 226–230 (2001) 1640–1642, [https://doi.org/10.1016/S0304-8853\(00\)01097-0](https://doi.org/10.1016/S0304-8853(00)01097-0) proceedings of the International Conference on Magnetism (ICM 2000).
- [6] S. Azzawi, A. Ganguly, M. Toka, R.M. Rowan-Robinson, J. Sinha, A.T. Hindmarch, A. Barman, D. Atkinson, Evolution of damping in ferromagnetic/nonmagnetic thin film bilayers as a function of nonmagnetic layer thickness, *Phys. Rev. B* 93 (2016) 054402, <https://doi.org/10.1103/PhysRevB.93.054402>.
- [7] H. Jung, Y.-S. Choi, K.-S. Lee, D.-S. Han, Y.-S. Yu, M.-Y. Im, P. Fischer, S.-K. Kim, Logic operations based on magnetic-vortex-state networks, *ACS Nano* 6 (5) (2012) 3712–3717, <https://doi.org/10.1021/nn3000143>.
- [8] H. Vigo-Cotrina, A.P. Guimarães, Single array of magnetic vortex disks uses in-plane anisotropy to create different logic gates, *J. Magn. Magn. Mater.* 441 (2017) 14–20, <https://doi.org/10.1016/j.jmmm.2017.05.027>.
- [9] A.P. Guimarães, *Principles of Nanomagnetism*, second ed., Springer, Cham, 2017.
- [10] K.Y. Guslienko, Magnetic vortex state stability, reversal and dynamics in restricted geometries, *J. Nanosci. Nanotechnol.* 8 (2008) 2745–2760, <https://doi.org/10.1166/jnn.2008.003>.
- [11] J.-H. Kim, K.-S. Lee, H. Jung, D.-S. Han, S.-K. Kim, Information-signal-transfer rate and energy loss in coupled vortex-state networks, *Appl. Phys. Lett.* 101 (9) (2012) 092403, <https://doi.org/10.1063/1.4748885>.
- [12] J.P. Sinnecker, H. Vigo-Cotrina, F. Garcia, E.R.P. Novais, A.P. Guimarães, Interaction between magnetic vortex cores in a pair of nonidentical nanodisks, *J. Appl. Phys.* 115 (2014) 203902, <https://doi.org/10.1063/1.4878875>.
- [13] A. Vogel, A. Drews, T. Kamionka, M. Bolte, G. Meier, Influence of dipolar interaction on vortex dynamics in arrays of ferromagnetic disks, *Phys. Rev. Lett.* 105 (2010) 037201, <https://doi.org/10.1103/PhysRevLett.105.037201>.
- [14] S. Sugimoto, Y. Fukuma, S. Kasai, T. Kimura, A. Barman, Y. Otani, Dynamics of coupled vortices in a pair of ferromagnetic disks, *Phys. Rev. Lett.* 106 (2011) 197203, <https://doi.org/10.1103/PhysRevLett.106.197203>.
- [15] A. Barman, S. Barman, T. Kimura, Y. Fukuma, Y. Otani, Gyration mode splitting in magnetostatically coupled magnetic vortices in an array, *J. Phys. D: Appl. Phys.* 43 (42) (2010) 422001, <https://doi.org/10.1088/0022-3727/43/42/422001>.
- [16] H. Jung, K.-S. Lee, D.-E. Jeong, Y.-S. Choi, Y.-S. Yu, D.-S. Han, A. Vogel, L. Bocklage, G. Meier, M.-Y. Im, P. Fischer, S.-K. Kim, Tunable negligible-loss energy transfer between dipolar-coupled magnetic disks by stimulated vortex gyration, *Sci. Rep.* 1 (2011) 59–64, <https://doi.org/10.1038/srep00059>.
- [17] F. Garcia, J.P. Sinnecker, E.R.P. Novais, A.P. Guimarães, Magnetic vortex echoes, *J. Appl. Phys.* 112 (11) (2012) 113911, <https://doi.org/10.1063/1.4768446>.
- [18] A.D. Belanovsky, N. Locatelli, P.N. Skirdkov, F. Abreu Araujo, K.A. Zvezdin, J. Grollier, V. Cros, A.K. Zvezdin, Numerical and analytical investigation of the synchronization of dipolarly coupled vortex spin-torque nano-oscillators, *Appl. Phys. Lett.* 103 (12) (2013), <https://doi.org/10.1063/1.4821073>.
- [19] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, B. Van Waeyenberge, The design and verification of Mumax3, *AIP Adv.* 4 (2014) 107133, <https://doi.org/10.1063/1.4899186>.
- [20] K.Y. Guslienko, B.A. Ivanov, V. Novosad, Y. Otani, H. Shima, K. Fukamichi, Eigenfrequencies of vortex state excitations in magnetic submicron-size disks, *J. Appl. Phys.* 91 (2002) 8037–8039, <https://doi.org/10.1063/1.1450816>.
- [21] C. Luo, Z. Feng, Y. Fu, W. Zhang, P.K.J. Wong, Z.X. Kou, Y. Zhai, H.F. Ding, M. Farle, J. Du, H.R. Zhai, Enhancement of magnetization damping coefficient of permalloy thin films with dilute Nd dopants, *Phys. Rev. B* 89 (2014) 184412, <https://doi.org/10.1103/PhysRevB.89.184412>.
- [22] O.V. Sukhostavets, J. González, K.Y. Guslienko, Multipole magnetostatic interactions and collective vortex excitations in dot pairs, chains, and two-dimensional arrays, *Phys. Rev. B* 87 (2013) 094402, <https://doi.org/10.1103/PhysRevB.87.094402>.
- [23] M. Hanze, C.F. Adolff, M. Weigand, G. Meier, Tunable eigenmodes of coupled magnetic vortex oscillators, *Appl. Phys. Lett.* 104 (18) (2014) 182405, <https://doi.org/10.1063/1.4875618>.
- [24] O.V. Sukhostavets, J.M. Gonzalez, K.Y. Guslienko, Magnetic vortex excitation frequencies and eigenmodes in a pair of coupled circular dots, *Appl. Phys. Expre.* 4 (6) (2011) 065003, <https://doi.org/10.1143/apex.4.065003>.
- [25] E. Barati, M. Cinal, D.M. Edwards, A. Umerski, Gilbert damping in magnetic layered systems, *Phys. Rev. B* 90 (2014) 014420, <https://doi.org/10.1103/PhysRevB.90.014420>.
- [26] A.A. Thiele, Steady-state motion of magnetic domains, *Phys. Rev. Lett.* 30 (1973) 230–233, <https://doi.org/10.1103/PhysRevLett.30.230>.
- [27] M. Asmat-Uceda, X. Cheng, X. Wang, D.J. Clarke, O. Tchernyshyov, K.S. Buchanan, A comparison of numerical simulations and analytical theory of the dynamics of interacting magnetic vortices, *J. Appl. Phys.* 117 (12) (2015) 123916, <https://doi.org/10.1063/1.4916610>.
- [28] J. Shibata, K. Shigeto, Y. Otani, Dynamics of magnetostatically coupled vortices in magnetic nanodisks, *Phys. Rev. B* 67 (2003) 224404, <https://doi.org/10.1103/PhysRevB.67.224404>.