Zonal flows generated by small-scale drift-Alfvén modes

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(Received 18 January 2006; accepted 6 March 2006; published online 18 April 2006)

The generation of zonal flows by small-scale drift-Alfvén (SSDA) modes is investigated. It is shown that these zonal flows can be generated by a monochromatic wave packet of SSDA modes propagating in the ion diamagnetic drift direction. The corresponding zonal-flow instability resembles a hydrodynamic one. Its growth rate depends on the spectrum purity of the wave packet; it decreases for relatively weak spectrum broadening and the instability turns into a resonant one, and eventually is suppressed, as the broadening increases. A general conclusion of this work is that the SSDA modes are less effective for driving zonal flows than standard drift modes.
 [DOI: 10.1063/1.2192755]

I. INTRODUCTION

Zonal flows driven by drift-type turbulence have been intensively investigated in recent years because of their efficacy on reducing the anomalous transport generated by such turbulence in magnetic confinement systems.[1–4] Most of the previous theoretical investigations are focused on the regime of large scale turbulence, i.e., turbulence generated by drift-type waves for which the characteristic perpendicular scale length is larger than the ion Larmor radius. However, in inhomogeneous plasmas with $\beta > M_e/M_i$, small-scale drift Alfvén (SSDA) modes, with characteristic perpendicular scale length smaller than the ion Larmor radius, also play an important role in driving the ubiquitous turbulence. Therefore, their effectiveness on generating zonal flows has to be thoroughly evaluated. The parameter β is the ratio between the kinetic and magnetic field pressures and M_j is the mass of species j ($j=e,i$ for electrons and ions, respectively). In a plasma slab model, with the magnetic field directed along the z axis and the density gradient along the x axis, the local dispersion relation for the SSDA modes is given by[5]

$$D(\omega,\mathbf{k}) \equiv (\omega-\omega_{*i})(\omega-\omega_{*e}) - (1+T_e/T_i)k_z^2k_x^2\rho_i^2v_A^2 = 0, \tag{1}$$

where ω is the mode frequency, ω_{*j} and T_j are the diamagnetic drift frequency and temperature for species j, v_A is the Alfvén speed, and k_z and k_x are the parallel and "radial" components of the wave vector, respectively. The above dispersion relation is valid within the approximation of localized short wavelength modes, i.e., $k_x\rho_i \gg 1$, where ρ_i is the ion Larmor radius. For simplicity, we have also assumed $k_x^2 \gg k_y^2$ where k_y is the "poloidal" wave number in the drift direction y. This approximation can be trivially accounted for by the substitution $k_x^2 \rightarrow k_\perp^2 = k_x^2 + k_y^2$ in Eq. (1).

The nonlinear properties of SSDA modes attracted a great deal of interest in the literature, viz., the occurrence of dipole vortices associated to them,[6–9] their Kolmogorov spectra,[10–12] and the realization that these modes can evolve as small-scale magnetic islands,[13] lately referred to as microislands.[14–16] In particular, the relevance of these islands effect on the anomalous electron heat conductivity in tokamaks was investigated by Kadomtsev.[14]

An important contribution to the nonlinear theory of SSDA modes was the investigation of zonal flows generated by them, carried out originally by Smolyakov *et al.*[17] and later continued by Lakhin.[18] The theory presented in Ref. 17 is particularly relevant because it develops the basic mathematical framework to describe the generation of zonal flows in two distinct approaches. The first is a direct perturbative calculation of the growth rate of zonal flows within the weak turbulence approximation. The essence of the second is the derivation of a generalized wave action which is invariant for SSDA turbulence modulated by zonal flows; their growth rate is then obtained by using this action in a wave kinetic equation (WKE).[19]

Recent theoretical work on zonal flow generation has shown that, in addition to the two approaches formulated in Ref. 17, it is also possible to employ the formulation of parametric instabilities developed much earlier for studying weak plasma turbulence.[20–22] Many different investigations have been carried out on the basis of this formulation, viz., zonal flow generation by kinetic Alfvén waves,[23] zonal flow generation by drift waves in the presence of a scalar nonlinearity,[24] and the effect of curvature on zonal flow generation by drift-Alfvén modes.[4] The generation of zonal flows by the ion-temperature-gradient (ITG) and related modes, in the presence of neoclassical viscosity, was investigated by Mikhailovskii *et al.* employing an approach similar to the parametric one.[3]

The goal of the present paper is to further develop the study of zonal flow generation by SSDA modes initiated in of Ref. 17. The basic mathematical formulation of the problem is introduced and the growth rates of zonal flow instabilities and the conditions for driving them are determined. The parametric formulation is employed to investigate the instabilities of zonal flows driven by a monochromatic wave packet of primary modes. The determination of their growth rates is useful as benchmark for the more general theory of zonal flow generation including nonmonochromatic wave packets of primary modes. Actually, we show that the parametric formulation can be trivially generalized to the case of wave packets with arbitrary spectrum broadening. The procedure, which is described in detail in Secs. II and III, consists basically in introducing driving forces for the electrostatic and vector potentials of the zonal flows in terms of summation over contributions of the primary modes of the wave packet.

One recent publication on zonal flows, which is somewhat related to this paper, is the work of Shukla on the generation of a mean magnetic field by small scale turbulence.[25] However, he addresses the turbulence described within the model of electron magnetohydrodynamics, whereas we consider both the effects of electrons and ions.

We present the basic equations in Sec. II and make some preliminary transformations on them before introducing the variables characterizing the zonal flows, the primary modes generating the flows, and the side-band amplitudes, which depend on both the primary modes and zonal flows. Then, we present the expressions for the Maxwell stress tensor, mean electrostatic potential and electromotive force, and prepare the basic equations to calculate the side-band amplitudes.

In Sec. III we calculate the driving forces of zonal flows, which are the mean Maxwell stress and electromotive force. The result of the calculations is a system of coupled linear equations for the mean electrostatic and vector potentials from which the zonal flow dispersion relation is obtained.

We consider the zonal-flow instability of a monochromatic wave packet in Sec. IV and the effect of a spectrum broadened wave packet in Sec. V. Our study of monochromatic wave packets succeeds former work by different authors,[2,3,23,24,26] and many others using the parametric formulation, and the study on the effect of spectrum broadening is preceded by the work of Smolyakov and collaborators.[27,28] We use the technique of the plasma dispersion function[29] to describe these effects.

We discuss the main results of the paper in Sec. VI and, for the sake of better knowing the general properties of the SSDA modes, we show in the Appendix that their nonlinear interaction does not change their total energy, within the approximations made in this work.

II. STARTING EQUATIONS AND THEIR PRELIMINARY TRANSFORMATIONS

A. Starting plasmadynamical equations

We consider a simple collisionless plasma model described by the fluid equations. The electron inertia is neglected and the ions are assumed to respond adiabatically to the field perturbations. Then the electron continuity equation (the vorticity equation) can be written as

$$\partial n/\partial t + \mathbf{V}_E \cdot \nabla n_0 - \nabla_\parallel J/c = 0, \tag{2}$$

and, from the momentum conservation equation, we obtain

$$0 = - e n_0 E_\parallel - T_e \nabla_\parallel (n + n_0), \tag{3}$$

where n is the perturbed plasma number density defined by the ion Boltzmannian,

$$n = - e \phi n_0 / T_i. \tag{4}$$

In these equations ϕ is the electrostatic potential, $\mathbf{V}_E$ is the cross-field drift velocity given by $\mathbf{V}_E = c[\mathbf{e}_z \times \nabla \phi]/B_0$, n_0 and B_0 are the equilibrium number density and magnetic field, respectively, $\nabla_\parallel$ is the nonlinear parallel gradient defined by

$$\nabla_\parallel = \partial/\partial z + \mathbf{B}_\perp \cdot \nabla / B_0, \tag{5}$$

$\mathbf{B}_\perp$ is the perturbed perpendicular magnetic field expressed in terms of the parallel vector potential A by $\mathbf{B}_\perp = [\nabla A \times \mathbf{e}_z]$, $E_\parallel$ is the parallel electric field related to ϕ and A by

$$E_{\parallel} = -\nabla_{\parallel}\phi - \frac{1}{c}\frac{\partial A}{\partial t}, \tag{6}$$

T_i and T_e are the equilibrium ion and electron temperatures, respectively, assumed to be constant, e is the ion charge, and c is the light velocity. The function J is the parallel electric current expressed in terms of A by

$$J = -c\nabla_{\perp}^2 A/4\pi, \tag{7}$$

where $\nabla_{\perp}$ is the perpendicular gradient.

Using Eqs. (2)–(7) reduce to

$$\left(\frac{\partial}{\partial t} + V_{*i}\frac{\partial}{\partial y}\right)\phi - \frac{cT_i}{4\pi e^2 n_0}\left(\frac{\partial}{\partial z} - \frac{1}{B_0}[\nabla A \times \nabla]_z\right)\nabla_{\perp}^2 A = 0, \tag{8}$$

$$\left(\frac{\partial}{\partial t} + V_{*e}\frac{\partial}{\partial y}\right)A + c\left(1 + \frac{1}{\tau}\right)\left(\frac{\partial}{\partial z} - \frac{1}{B_0}[\nabla A \times \nabla]_z\right)\phi = 0. \tag{9}$$

Here $\tau = T_i/T_e$, $V_{*j} = T_j c\kappa_n/(e_j B_0)$ is the diamagnetic drift velocities for species j and $\kappa_n = \partial \ln n_0/\partial x$ is the density inverse scale length.

Equations (8) and (9) have been initially formulated in Ref. 6, where one can find their generalization for the cases of finite β and finite perpendicular electric current.

B. Equations for the coupled modes

The nonlinear term in Eqs. (8) and (9), coming from the parallel gradient, introduces coupling between different modes. We consider a standard three-wave coupling scenario, in which the coupling between the pump SSDA and side-band modes drives low-frequency large-scale modes without poloidal variation, i.e., the zonal flows. Accordingly, the perturbed quantities $X = (\phi, A)$ in Eqs. (8) and (9) are split in three components:

$$X = \tilde{X} + \hat{X} + \bar{X}, \tag{10}$$

where

$$\tilde{X} = \sum_{\mathbf{k}} [\tilde{X}_+(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega_{\mathbf{k}}t) + \tilde{X}_-(\mathbf{k}) \times\exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega_{\mathbf{k}}t)] \tag{11}$$

describes the spectrum of pump SSDA modes,

$$\hat{X} = \sum_{\mathbf{k}} [\hat{X}_+(\mathbf{k})\exp(i\mathbf{k}_+\cdot\mathbf{r} - i\omega_+ t) + \hat{X}_-(\mathbf{k}) \times\exp(i\mathbf{k}_-\cdot\mathbf{r} - i\omega_- t) + \text{c.c.}] \tag{12}$$

describes the spectrum of side-band modes, and

$$\bar{X} = \bar{X}_0 \exp(-i\Omega t + iq_x x) + \text{c.c.} \tag{13}$$

describes the zonal flow modes. Energy and momentum conservation is imposed by requiring that $\omega_{\pm} = \Omega \pm \omega_{\mathbf{k}}$ and $\mathbf{k}_{\pm} = q_x\mathbf{e}_x \pm \mathbf{k}$, the pairs $(\omega_{\mathbf{k}}, \mathbf{k})$ and $(\Omega, q_x\mathbf{e}_x)$ represent the frequency and wave vector of the SSDA pump and zonal flow modes, respectively. The amplitude $\bar{X}_0 \equiv (\bar{\phi}_0, \bar{A}_0)$ of the zonal flow mode is assumed constant, within the local approximation.

Following the standard quasilinear procedure, we substitute Eq. (11)–(13) into Eqs. (8) and (9) and neglect the contribution of the small nonlinear term in the relations for the high frequency, but not for the low frequency zonal flow modes. Then, equations for the SSDA modes become

$$(\omega_{\mathbf{k}} - \omega_{*i})\tilde{\phi}_{\pm}(\mathbf{k}) - \frac{cT_i k_z k_x^2}{4\pi e^2 n_0}\tilde{A}_{\pm}(\mathbf{k}) = 0 \tag{14}$$

and

$$\tilde{A}_{\pm}(\mathbf{k}) = \frac{c(1+1/\tau)k_z}{\omega_{\mathbf{k}} - \omega_{*e}}\tilde{\phi}_{\pm}(\mathbf{k}), \tag{15}$$

where $\omega_{*j} = k_y V_{*j}$. Solving this homogeneous system one arrives at the dispersion relation for the SSDA modes given in Eq. (1).

The relations for the amplitude of the zonal flow modes are obtained by substituting Eqs. (10)–(13) into Eqs. (8) and (9) and averaging out over the fast small-scale fluctuations; then we obtain

$$-i\Omega\bar{\phi}_0 = R_{\perp} \tag{16}$$

and

$$-i\Omega\bar{A}_0 = R_{\parallel}, \tag{17}$$

where $R_{\perp}$ and $R_{\parallel}$ are the mean Maxwell stress and mean electromotive force, respectively, defined by

$$R_{\perp} = -\frac{cT_i q_x^2}{4\pi e^2 n_0 B_0}\left\langle \frac{\partial\tilde{A}}{\partial y}\frac{\partial\hat{A}}{\partial x} + \frac{\partial\tilde{A}}{\partial x}\frac{\partial\hat{A}}{\partial y}\right\rangle \tag{18}$$

and

$$R_{\parallel} = \frac{icq_x(1+1/\tau)}{B_0}\left\langle \tilde{A}\frac{\partial\hat{\phi}}{\partial y} + \hat{A}\frac{\partial\tilde{\phi}}{\partial y}\right\rangle, \tag{19}$$

where $\langle\ldots\rangle$ represents the average over fast oscillations. Using the Fourier decompositions given by Eqs. (11) and (12), these quantities can be written as

$$R_{\perp} = -\frac{cT_i q_x^2}{4\pi e^2 n_0 B_0}\sum_{\mathbf{k}} k_y r_{\perp}(\mathbf{k}) \tag{20}$$

and

$$R_{\parallel} = \frac{cq_x(1+1/\tau)}{B_0}\sum_{\mathbf{k}} k_y r_{\parallel}(\mathbf{k}), \tag{21}$$

where

$$r_{\perp}(\mathbf{k}) = q_x(\tilde{A}_-\hat{A}_+ - \tilde{A}_+\hat{A}_-) + 2k_x(\tilde{A}_-\hat{A}_+ + \tilde{A}_+\hat{A}_-), \tag{22}$$

$$r_{\parallel}(\mathbf{k}) = \tilde{\phi}_-\hat{\lambda}_+ - \tilde{\phi}_+\hat{\lambda}_-, \tag{23}$$

and $\hat{\lambda}_{\pm}$ are the auxiliary side-band amplitudes determined by

$$\hat{\lambda}_{\pm} = \hat{A}_{\pm} - \frac{ck_z(1+1/\tau)}{\omega_{\mathbf{k}} - \omega_{*e}}\hat{\phi}_{\pm}. \tag{24}$$

In order to calculate the functions $r_\perp$ and $r_\parallel$, we should find the side-band amplitudes $\hat{A}_\pm$, $\hat{\phi}_\pm$. Turning to Eqs. (8) and (9), these amplitudes satisfy the relations

$$(\omega_\pm \mp \omega_{*i})\hat{\phi}_\pm \mp \frac{cT_i k_z k_{x\pm}^2}{4\pi e^2 n_0}\hat{A}_\pm = \mp \alpha_\pm \frac{cT_i k_z (k_x^2 - q_x^2)}{4\pi e^2 n_0(\omega - \omega_{*e})}, \tag{25}$$

$$\mp ck_z(1+1/\tau)\hat{\phi}_\pm + (\omega_\pm \mp \omega_{*e})\hat{A}_\pm = \mp \alpha_\pm\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right). \tag{26}$$

Here

$$\alpha_\pm = \frac{ic}{B_0}k_y q_x(1+1/\tau)\bar{A}_0\tilde{\phi}_\pm. \tag{27}$$

Thus, our following program is finding $\hat{\phi}_\pm$ and $\hat{A}_\pm$ and calculating $R_\perp$ and $R_\parallel$.

III. CALCULATION OF DRIVING FORCES OF EQUATIONS FOR ZONAL FLOWS

A. Solution of equations for side-band amplitudes

We obtain from Eqs. (25) and (26):

$$\hat{\phi}_\pm = \mp \frac{\alpha_\pm}{D_\pm}\frac{cT_i k_z}{4\pi e^2 n_0}\left[\frac{\omega_\pm \mp \omega_{*e}}{\omega - \omega_{*e}}(k_x^2 - q_x^2) \pm k_{x\pm}^2 \times\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right)\right] \tag{28}$$

and

$$\hat{A}_\pm = \mp \frac{\alpha_\pm}{D_\pm}\left[(\omega_\pm \mp \omega_{*i}) \times\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right) \pm \frac{c^2 T_i (k_x^2 - q_x^2)k_z^2(1+1/\tau)}{4\pi e^2 n_0(\omega - \omega_{*e})}\right], \tag{29}$$

where

$$D_\pm = D(\Omega \pm \omega, \mathbf{q} \pm \mathbf{k}). \tag{30}$$

Using Eqs. (28) and (29), the expression for the auxiliary side-band amplitudes, Eq. (24), takes the form

$$\hat{\lambda}_\pm = \mp \frac{\alpha_\pm}{D_\pm}\left\{\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right) \times\left[\Omega - \frac{(\omega - \omega_{*i})q_x}{k_x}\left(2 \pm \frac{q_x}{k_x}\right)\right] - \Omega\frac{\omega - \omega_{*i}}{\omega - \omega_{*e}}\right\}. \tag{31}$$

In this expression we have neglected terms of order $\Omega(q_x/k_x)^2$ because their contribution is not important for the problem.

Considering Ω and q_x to be small parameters, Eq. (30) for $D_\pm$ can be expressed in a perturbative expansion, i.e.,

$$D_\pm = \pm D^{(0)} + D^{(1)}, \tag{32}$$

where

$$D^{(0)} = \Omega(2\omega - \omega_{*i} - \omega_{*e}) - 2\frac{q_x}{k_x}(\omega - \omega_{*e})(\omega - \omega_{*i}) \tag{33}$$

and

$$D^{(1)} = \Omega^2 - \frac{q_x^2}{k_x^2}(\omega - \omega_{*e})(\omega - \omega_{*i}). \tag{34}$$

Similarly, one has

$$\hat{A}_\pm = \hat{A}_\pm^{(0)} + \hat{A}_\pm^{(1)}, \tag{35}$$

where

$$\hat{A}_\pm^{(0)} = \mp \frac{\alpha_\pm(\omega - \omega_{*i})}{D^{(0)}}\left(2 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right) \tag{36}$$

and

$$\hat{A}_\pm^{(1)} = \mp \frac{D^{(1)}}{D^{(0)}}\hat{A}_\pm^{(0)} - \frac{\alpha_\pm \Omega}{D^{(0)}}\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right). \tag{37}$$

Turning to Eq. (31), we find

$$\hat{\lambda}_\pm^{(0)} = 0. \tag{38}$$

This equation describes the remarkable fact that the main contributions of the "magnetic" and "electrostatic" side-band amplitudes to the evolution equation of the mean magnetic field are mutually cancelled [see Eqs. (17), (19), (21), and (23)]. Therefore, one has the expansion

$$\hat{\lambda}_\pm = \hat{\lambda}_\pm^{(1)} + \hat{\lambda}_\pm^{(2)}. \tag{39}$$

The terms $\hat{\lambda}_\pm^{(1)}$ do not contribute to Eq. (23) for $r_\parallel$ and we can omit the expression for them. The terms $\hat{\lambda}_\pm^{(2)}$ are given by

$$\hat{\lambda}_\pm^{(2)} = \pm \frac{\alpha_\pm \Omega}{D^{(0)2}}\left\{\left(1 - \frac{\bar{\phi}_0}{\bar{A}_0}\frac{c(1+1/\tau)k_z}{\omega - \omega_{*e}}\right) \times\left[D^{(1)} + (\omega - \omega_{*i})\frac{q_x}{k_x}\left(\frac{q_x}{k_x}(2\omega - \omega_{*i} - \omega_{*e}) - 2\Omega\right)\right] - \frac{\omega - \omega_{*i}}{\omega - \omega_{*e}}D^{(1)}\right\}. \tag{40}$$

In terms of $\hat{\lambda}_\pm^{(2)}$, Eq. (23) takes the form

$$r_\parallel = \tilde{\phi}_- \hat{\lambda}_+^{(2)} - \tilde{\phi}_+ \hat{\lambda}_-^{(2)}. \tag{41}$$

B. Derivation of zonal-flow dispersion relation

Using Eqs. (32)–(41) and (27) we transform Eqs. (22) and (23) to

$$r_\perp = \frac{ic^2(1+1/\tau)^2 q_x k_z k_y k_x \Omega}{D^{(0)2} B_0(\omega-\omega_{*e})} I_{\mathbf{k}} \times \left(f_\perp^A \bar{A}_0 + \frac{c(1+1/\tau)k_z}{\omega-\omega_{*e}} f_\perp^\phi \bar{\phi}_0 \right) \tag{42}$$

and

$$r_\parallel = \frac{ick_y q_x \Omega(1+1/\tau)}{D^{(0)2} B_0} I_{\mathbf{k}} \left(f_\parallel^A \bar{A}_0 - \frac{c(1+1/\tau)k_z}{\omega-\omega_{*e}} f_\parallel^\phi \bar{\phi}_0 \right), \tag{43}$$

where

$$f_\perp^A = 2(\omega_{*e}-\omega_{*i})\left(\Omega - \frac{q_x}{k_x}(\omega-\omega_{*i}) \right), \tag{44}$$

$$f_\perp^\phi = 2\Omega(\omega-\omega_{*e}) - \frac{q_x}{k_x}(\omega-\omega_{*i})(2\omega+\omega_{*i}-3\omega_{*e}), \tag{45}$$

$$f_\parallel^A = \Omega^2 \frac{\omega_{*i}-\omega_{*e}}{\omega-\omega_{*e}} - 2\Omega\frac{q_x}{k_x}(\omega-\omega_{*i}) + 2\frac{q_x^2}{k_x^2}(\omega-\omega_{*i})^2, \tag{46}$$

$$f_\parallel^\phi = \left(\Omega - \frac{q_x}{k_x}(\omega-\omega_{*i}) \right)^2, \tag{47}$$

and

$$I_{\mathbf{k}} = 2\tilde{\phi}_+ \tilde{\phi}_-. \tag{48}$$

Using Eqs. (42), (43), (20), and (21), Eqs. (16) and (17) reduce to

$$\bar{\phi}_0 = I_\perp^\phi \bar{\phi}_0 + I_\perp^A \bar{A}_0, \tag{49}$$

$$\bar{A}_0 = I_\parallel^\phi \bar{\phi}_0 + I_\parallel^A \bar{A}_0. \tag{50}$$

Here

$$(I_\perp^\phi, I_\perp^A, I_\parallel^\phi, I_\parallel^A) = \sum_{\mathbf{k}} \frac{(a_\perp^\phi, a_\perp^A, a_\parallel^\phi, a_\parallel^A)}{(\Omega - q_x V_g)^2}. \tag{51}$$

These values can be called the transport coefficients. The function V_g is the zonal-flow group velocity given by

$$V_g = \frac{2}{k_x}\frac{(\omega-\omega_{*e})(\omega-\omega_{*i})}{2\omega-\omega_{*i}-\omega_{*e}}. \tag{52}$$

The functions $(a_\perp^\phi, a_\perp^A, a_\parallel^\phi, a_\parallel^A)$ mean

$$a_\perp^\phi = \left(1+\frac{1}{\tau}\right)^2 \frac{q_x}{k_x}\frac{(\omega-\omega_{*i})\Gamma_0^2}{(\omega-\omega_{*e})(2\omega-\omega_{*i}-\omega_{*e})^2} f_\perp^\phi, \tag{53}$$

$$a_\perp^A = \frac{1}{c}\left(1+\frac{1}{\tau}\right)\frac{q_x}{k_x k_z}\frac{(\omega-\omega_{*i})\Gamma_0^2}{(2\omega-\omega_{*i}-\omega_{*e})^2} f_\perp^A, \tag{54}$$

$$a_\parallel^\phi = c\left(1+\frac{1}{\tau}\right)^3 \frac{k_z\Gamma_0^2}{(\omega-\omega_{*e})(2\omega-\omega_{*i}-\omega_{*e})^2} f_\parallel^\phi, \tag{55}$$

$$a_\parallel^A = -\left(1+\frac{1}{\tau}\right)^2 \frac{\Gamma_0^2}{(2\omega-\omega_{*i}-\omega_{*e})^2} f_\parallel^A, \tag{56}$$

where

$$\Gamma_0^2 = c^2 q_x^2 k_y^2 I_{\mathbf{k}}/B_0^2. \tag{57}$$

By means of Eqs. (48) and (49), we arrive at the zonal-flow dispersion relation

$$1 - (I_\perp^\phi + I_\parallel^A) + I_\perp^\phi I_\parallel^A - I_\parallel^\phi I_\perp^A = 0. \tag{58}$$

Thus, in general, we deal with a biquadratic zonal flow dispersion relation with respect to $\Omega - q_x V_g$. However, as will be shown in the sequel, Eq. (58) reduces to a quadratic one, in the most interesting case.

IV. ZONAL-FLOW INSTABILITIES OF MONOCHROMATIC WAVE PACKET

In this section we analyze the zonal-flow dispersion relation, Eq. (58), in the case of monochromatic wave packet of the primary modes. In other words, we consider a single wave vector on the right-hand sides of Eqs. (51). In addition, we allow for the value Γ_0^2 being a small parameter. In this case the right-hand sides of these equations are relevant only if the value $\Omega - q_x V_g$ is also a small parameter. Then the functions $f_\perp^A, f_\perp^\phi, f_\parallel^A$, and $f_\parallel^\phi$ defined by Eqs. (44)–(47), can be calculated for $\Omega \approx q_x V_g$.

As a result, we find

$$f_\perp^\phi = \frac{q_x}{k_x}\frac{(\omega-\omega_{*i})(\omega_{*i}-\omega_{*e})^2}{2\omega-\omega_{*i}-\omega_{*e}}, \tag{59}$$

$$f_\perp^A = -2 f_\perp^\phi, \tag{60}$$

$$f_\parallel^\phi = \frac{q_x^2}{k_x^2}\frac{(\omega-\omega_{*i})^2(\omega_{*i}-\omega_{*e})^2}{(2\omega-\omega_{*i}-\omega_{*e})^2}, \tag{61}$$

$$f_\parallel^A = 2 f_\parallel^\phi. \tag{62}$$

Using Eqs. (51)–(56) and (59)–(62), we arrive at

$$I_\perp^\phi + I_\parallel^A = -\left(1+\frac{1}{\tau}\right)^2 \frac{(\omega-\omega_{*i})^2(\omega_{*i}-\omega_{*e})^3\Gamma_0^2}{D^{(0)2}(2\omega-\omega_{*i}-\omega_{*e})^2(\omega-\omega_{*e})}\frac{q_x^2}{k_x^2}, \tag{63}$$

$$I_\perp^\phi I_\parallel^A - I_\parallel^\phi I_\perp^A = 0. \tag{64}$$

Then Eq. (58) reduces to

$$1 - (I_\perp^\phi + I_\parallel^A) = 0. \tag{65}$$

It hence follows that

$$(\Omega - q_x V_g)^2 = -\Gamma^2, \tag{66}$$

where Γ^2 means the squared zonal-flow growth rate defined by

$$\Gamma^2 = \left(1+\frac{1}{\tau}\right)^2 \frac{(\omega-\omega_{*i})^2(\omega_{*i}-\omega_{*e})^3\Gamma_0^2}{(\omega-\omega_{*e})(2\omega-\omega_{*i}-\omega_{*e})^4}\frac{q_x^2}{k_x^2}. \tag{67}$$

According to Eq. (66), the zonal-flow instability condition is as follows:

$$\frac{\omega_{*i}}{\omega-\omega_{*e}} > 0. \tag{68}$$

This condition is satisfied for the primary modes propagating in the ion diamagnetic drift direction.

Using Eq. (65), it follows from Eq. (50) that

$$\bar{A}_0/\bar{\phi}_0 = I_{\parallel}^{\phi}/I_{\perp}^{\phi}. \tag{69}$$

Substituting here Eqs. (51) and allowing for Eqs. (53), (55), (59), and (61), we arrive at

$$\frac{\bar{A}_0}{\bar{\phi}_0} = \frac{c(1+1/\tau)k_z}{2\omega-\omega_{*i}-\omega_{*e}}. \tag{70}$$

Thus, in the considered driven zonal-flow instability, both mean electric and magnetic fields are generated.

V. EFFECTS OF NONMONOCHROMATICITY OF WAVE PACKETS ON GENERATION OF ZONAL FLOWS

A. Problem statement and starting equations

Let us take the function $I_{\mathbf{k}}$ in the Gaussian form (cf. Ref. 28)

$$I_{\mathbf{k}} = \frac{1}{\pi^{1/2}\Delta k_x}\exp\left(-\frac{(k_x-k_{x0})^2}{(\Delta k_x)^2}\right)I_{\mathbf{k}_0}. \tag{71}$$

Here k_{x0} is the centered radial wave vector of the wave packet, and $\Delta k_x > 0$ is the radial characteristic wave-packet width. The poloidal and parallel projections of the wave vector k_y and k_z are assumed to be the same for all modes of the wave packet, $k_y = k_{y0}$, $k_z = k_{z0}$. The sums over $\mathbf{k}$ on the right-hand sides of Eqs. (51) are now understood as the integrals over k_x. Then we allow for that the primary mode frequency $\omega = \omega_{\mathbf{k}}$ and the zonal-flow radial group velocity V_g to be functions of k_x, $\omega = \omega(k_x)$, $V_g = V_g(k_x)$. The value Δk_x is assumed to be small compared with k_{x0}, $\Delta k_x/k_{x0} \ll 1$.

We suggest that the most important effect of the nonmonochromaticity of wave packets is a modification of the "resonant denominator" $(\Omega - q_x V_g)^{-2}$ in Eqs. (51). The fact is that, for finite $\Delta k_x/k_{x0}$, in contrast to Sec. IV, these values are functions of the variable k_x. Then the zonal-flow dispersion relation given by Eq. (66) proves to be invalid. Our goal is to generalize this dispersion relation for the case of finite $\Delta k_x/k_{x0}$. Thus, instead of Eqs. (51), we now use

$$(I_{\perp}^{\phi}, I_{\perp}^{A}, I_{\parallel}^{\phi}, I_{\parallel}^{A}) = (a_{\perp 0}^{\phi}, a_{\perp 0}^{A}, a_{\parallel 0}^{\phi}, a_{\parallel 0}^{A})\left\langle \frac{1}{(\Omega - q_x V_g)^2}\right\rangle_{k_x}. \tag{72}$$

Here $(\ldots)_0 \equiv (\ldots)_{k_x = k_{x0}}$, while

$$\langle(\ldots)\rangle_{k_x} = \frac{1}{\pi^{1/2}\Delta k_x}\int(\ldots)\exp\left(-\frac{(k_x-k_{x0})^2}{(\Delta k_x)^2}\right)dk_x. \tag{73}$$

Expressing $\omega_{\mathbf{k}}$ in terms of k_x, Eq. (52) reduces to

$$V_g = V_g(k_x) = \pm\frac{k_x}{(1+a_0)^{1/2}}\frac{k_z^2 v_A^2 \rho_s^2}{|\omega_{*e}|}, \tag{74}$$

where

$$a_0 = \frac{4k_z^2 v_A^2 k_x^2 \rho_s^2}{(1+\tau)\omega_{*e}^2}. \tag{75}$$

We expand V_g in series in Δk_x obtaining

$$V_g = V_{g0} + V'_{g0}\Delta k_x. \tag{76}$$

Here the subscript "0" denotes that the corresponding function is taken for $k_x = k_{x0}$ and the prime is the derivative with respect to k_x.

B. Wave-packet spectrum broadening effect on hydrodynamic zonal-flow instability

Assuming the broadening of the wave packet to be sufficiently small and carrying out integration over k_x, we find

$$\left\langle\frac{1}{(\Omega - k_x V_g)^2}\right\rangle_{k_x} = \frac{1}{\hat{\Omega}^2}\left(1 + \frac{3}{2}\frac{q_x^2 {V'_{g0}}^2}{\hat{\Omega}^2}(\Delta k_x)^2\right), \tag{77}$$

where

$$\hat{\Omega} \equiv \Omega - q_x V_{g0}. \tag{78}$$

Then, instead of Eq. (66), we arrive at the zonal-flow dispersion relation

$$\hat{\Omega}^2 = -\Gamma^2\left(1 + \frac{3}{2}\frac{q_x^2 {V'_{g0}}^2}{\hat{\Omega}^2}(\Delta k_x)^2\right). \tag{79}$$

It hence follows that the broadening can be neglected only if, in order of magnitude,

$$\frac{\Delta k_x}{k_{x0}} < \frac{\Gamma}{q_x V_{g0}} \simeq \left|\frac{q_x}{k_{x0}}\right|\left|\frac{\tilde{V}_{Ey}}{V_*}\right|. \tag{80}$$

Here $\tilde{V}_{Ey}$ is the y component of the cross-field drift velocity of the primary modes, $V_* \simeq (V_{*e}, V_{*i})$. Treating the second term in the large parentheses of Eq. (79) as a small correction, one can see that this correction leads to decreasing the growth rate of hydrodynamic instability.

C. Resonant zonal-flow instabilities and their suppression for strong broadening

For arbitrary Δk_x one has, instead of Eq. (77),

$$\left\langle\frac{1}{(\Omega - q_x V_g)^2}\right\rangle_{k_x} = -\frac{\partial}{\partial\hat{\Omega}}\left[\frac{1}{\hat{\Omega}}Z\left(\frac{\hat{\Omega}}{|q_x V'_{g0}|\Delta k_x}\right)\right], \tag{81}$$

where[29]

$$Z(x) = 2xe^{-x^2}\int_0^x e^{t^2}dt - i\pi^{1/2}xe^{-x^2}. \tag{82}$$

Then, for $\hat{\Omega} \ll |q_x V'_{g0}|\Delta k_x$,

$$\left\langle\frac{1}{(\Omega - q_x V_g)^2}\right\rangle_{k_x} = -\frac{2}{(q_x V'_{g0})^2(\Delta k_x)^2}\left(1 + \frac{i\pi^{1/2}\hat{\Omega}}{|q_x V'_{g0}|\Delta k_x}\right). \tag{83}$$

As a result, we arrive at the zonal-flow dispersion relation

$$1=\frac{2\Gamma^2}{(q_xV'_{g0})^2(\Delta k_x)^2}\left(1+\frac{i\pi^{1/2}\hat{\Omega}}{|q_xV'_{g0}|\Delta k_x}\right). \quad (84)$$

It hence follows that

$$\hat{\Omega}=\frac{i}{\pi^{1/2}}|q_xV'_{g0}|\Delta k_x\left(1-\frac{1}{2}\frac{(q_xV'_{g0}\Delta k_x)^2}{\Gamma^2}\right). \quad (85)$$

Then we find instability condition

$$\Gamma^2>\frac{1}{2}(q_xV'_{g0}\Delta k_x)^2. \quad (86)$$

Qualitatively, this condition means the same as Eq. (80).

VI. DISCUSSIONS

Dealing with SSDA modes described by Eqs. (8) and (9), we have modified the parametric approach[20] for the problem of zonal-flow generation assuming the spectrum of primary modes to be arbitrary [see Eq. (11)]. Then, instead of the side-band amplitude for a single wave vector **k**, we have dealt with a spectrum of such amplitudes [see Eq. (12)]. In this approach, the driving forces of zonal flows are represented as summation (or integration) over the spectrum of the primary modes [see Eqs. (20) and (21)]. Thereby, we have suggested a rather simple mathematical apparatus, which is an alternative of the standard weak-turbulence approach used in Ref. 17. One can think that our approach can be effectively used for studying generation of zonal flows by different types of primary modes.

One more our methodical achievement, which can be used in the problems of different primary modes, is the analysis of the case of zonal-flow generation by the Gaussian wave packets [see Eq. (71)]. According to our analysis, it seems reasonable to distinguish the limiting cases of a sufficiently small spectrum broadening, describing it in terms of an additive to the monochromatic resonant denominator [see Eq. (77)] and a strong broadening, when the spectrum spread is larger than this denominator [see Eq. (83)]. Such an approach to studying the broadening effects is an alternative to the so-called "box approximation" considered in Ref. 27. Note also that our understanding of the Gaussian wave packet differs from that of Ref. 28, where, in contrast to Eq. (71), a "two-hamped" Gaussian packet has been analyzed [see a non-numbered formula of Ref. 28 before its Eq. (15)].

Our analysis has shown several remarkable properties of the problem considered. Thus, both driving forces, $R_\perp$ and $R_\parallel$, prove to be proportional to the zonal-flow mode frequency Ω [see Eqs. (42) and (43)]. This frequency is cancelled with that entering the left-hand side of the evolution equations, Eqs. (16) and (17). As a result, these equations transit to simpler Eqs. (49) and (50), in which the above factor disappears. One more remarkable property of our problem is revealed in the approximation of small resonant denominator, $\Omega\approx q_xV_g$. Then, the transport coefficients entering Eqs. (49) and (50) prove to be interrelated by Eq. (64). As a result, our rather complicated biquadratic zonal-flow dispersion relation, Eq. (58), reduce to an essentially simpler quadratic dispersion relation given by Eq. (65).

Note also that, in the above approximation, all the transport coefficients, $I_\perp^\phi$, $I_\perp^A$, $I_\parallel^\phi$, and $I_\parallel^A$, prove to be proportional to the squared difference between the ion and electron diamagnetic drift frequencies [see Eqs. (59)–(62)], while the squared zonal-flow growth rate, Γ^2, is proportional to the third degree of this difference [see Eq. (67)]. Thereby, according to our analysis, zonal-flow generation by the SSDA modes is possible only in allowing for drift effects, while starting nonlinear equations for these modes, Eqs. (8) and (9), are valid also in neglecting these effects.

One of our main physical results is the fact that zonal flows can be generated only by the branch of the SSDA modes propagating in the ion diamagnetic drift direction [see Eq. (68)]. The maximum growth rate of such a generation is reached for the case of monochromatic wave packet. Then one has an instability of hydrodynamic type similar to that studied in Ref. 26 for the case of drift monochromatic wave packet. In contrast to the hydrodynamic instability of that reference, the growth rate in our problem proves to be proportional to the small parameter q_x/k_x, what is a consequence of that the driving forces containing the small parameter $(q_x/k_x)^2$. The broadening of the wave packet can be neglected for the condition given by Eq. (80). Then, as in the limit of monochromatic wave packet, the instability looks as hydrodynamic but its growth rate decreases [see Eq. (79)]. With increasing the broadening the instability transits to the resonant one described by Eq. (84). It is possible only if the broadening is not too strong [see Eq. (86)]. Otherwise, the instability is suppressed by this effect.

In accordance with that said in Sec. I, the first step in studying generation of the zonal flows by the SSDA modes has been made in Ref. 17. Meanwhile, this reference has used a series of simplifying approximations. Our analysis allows one to check whether these approximations are adequate.

One of the most important approximation of Ref. 17 was the assumption that the mean magnetic field of the zonal flow vanishes, i.e., in our definitions, $\bar{A}_0=0$. Our analysis shows that this approximation is invalid, see, e.g., Eq. (70). One more simplifying approximation of this reference was neglecting of the mean electromotive force given by Eq. (19), $R_\parallel=0$. Turning to Eq. (17), one can see that, if one takes $R_\parallel=0$, one arrives at $\bar{A}_0=0$, i.e., the second assumption of Ref. 17 corroborates the first one. Meanwhile, according to our analysis, the second assumption of Ref. 17 is also invalid. This fact can be shown, in particular, from Eq. (50).

Allowing for the mean magnetic field of zonal flow, $\bar{A}_0\neq 0$, is important, in particular, in calculating the side-band amplitudes. This fact can be seen from Eq. (25), where the parameters $\alpha_\pm$ are proportional to $\bar{A}_0$ [see Eq. (27)]. Therefore, Ref. 17 has dealt with an incorrect equation for the side-band amplitudes, which differs from Eq. (25) by the omitted term with $\alpha_\pm$. Meanwhile, the side-band amplitudes govern the mean Maxwell stress, see Eqs. (20) and (21). Thus, the expression for the mean Maxwell stress found in Ref. 17 is inadequate. As a result, instead of correct two equations for the mean electrostatic and vector potentials, $\bar{\phi}_0$ and $\bar{A}_0$, Eqs. (49) and (50), Ref. 17 has dealt with a single equation for the mean electrostatic potential similar to Eq. (49) with the omitted term proportional to $\bar{A}_0$ and an incorrect expression for $I_{\perp}^{\phi}$.

Meanwhile, if one deals with incorrect expressions for the side-band amplitudes, one can not describe our problem even on a qualitative level. This is demonstrated by the fact that only for correct side-band amplitudes $\hat{A}_{\pm}$ and $\hat{\phi}_{\pm}$, given by Eqs. (28) and (29), one can reveal the remarkable property of the auxiliary side-band amplitudes $\hat{\lambda}_{\pm}$ to be vanishing in the zero order of expansion in the series in q_x and Ω [see Eq. (38)]. Such a vanishing is a result of mutual cancellation of the main contributions of $\hat{A}_{\pm}$ and $\hat{\phi}_{\pm}$ into $\hat{\lambda}_{\pm}$.

At the same time, Ref. 17 contains valuable results of a heuristic character. Thus, our analysis has confirmed the fact revealed in Ref. 17 that the mean Maxwell stress is proportional to the small parameter $(q_x/k_x)^2$ [see, e.g., Eq. (37) of Ref. 17]. The discussion of Ref. 17 on the electron polarization current seems to be valuable.

As a whole, it is reasonable to conclude that the SSDA modes have less practical importance for the problem of zonal-flow generation than the standard drift waves since even weak spectrum broadening suppresses such a generation. This situation can be changed if one modifies Eqs. (8) and (9) by allowing for additional physical effects. One such effect was discussed in Ref. 17: it is the above-mentioned Reynolds stress related to the electron polarization current significant for $k_x^2c^2/\omega_{pe}^2 \simeq 1$, where ω_{pe} is the electron plasma frequency. An analysis of the role of additional physical effects can be the topic of following studies; see also Ref. 18.

ACKNOWLEDGMENTS

This work was supported in part by the Russian Foundation for Basic Research, Grant No. 06–02–16767, the Russian Federal Program on Support of Leading Scientific School Researches, Grant No. 2024.2003.2, by the Department of Atomic Science and Technology of the Russian Agency of Atomic Industry, by the US Civilian Research and Development Foundation for the Independent States of the Former Soviet Union, Grant No. BRHE REC-011, by NSERC of Canada and a NATO Science Program collaborative linkage grant, by the National Council of Scientific and Technological Development (CNPq), and by the State of São Paulo Research Foundation (FAPESP), Brazil.

APPENDIX: ENERGY INTEGRAL FOR SMALL-SCALE DRIFT-ALFVÉN MODES

Let us multiply Eqs. (8) and (9) by $[4\pi e^2 n_0(1+1/\tau)T_i]\phi$ and $(-\nabla_{\perp}^2 A)$, respectively, sum both the results, and integrate the obtained relation over $\mathbf{r}$. Then we arrive at

$$\frac{\partial}{\partial t}\int w d\mathbf{r} = 0, \tag{A1}$$

where

$$w = \frac{4\pi e^2 n_0}{T_i}\left(1+\frac{1}{\tau}\right)\phi^2 + (\nabla_{\perp} A)^2. \tag{A2}$$

Now we show that Eq. (A1) can be treated as the energy integral of the SSDA modes. Then we turn to Eq. (15.13) of Ref. 29 for the energy of monochromatic wave packet of arbitrary type, W,

$$W = \frac{\partial(\omega\varepsilon'_{\alpha\beta})}{\partial\omega}\frac{E_\alpha^* E_\beta}{16\pi} + \frac{\mathbf{B}^2}{16\pi}. \tag{A3}$$

Here $\varepsilon'_{\alpha\beta}$ is the Hermitian part of the dielectric permittivity tensor, $\mathbf{E}$ and $\mathbf{B}$ are the perturbed electric and magnetic fields, $(\alpha,\beta)=(x,y,z)$. In our case $(\alpha,\beta)=(x,y)$ and

$$\varepsilon'_{\alpha\beta} = \frac{4\pi e^2 n_0}{k_{\perp}^2 T_i}\left(1+\frac{1}{\tau}\right)\delta_{\alpha\beta}. \tag{A4}$$

In addition, we have

$$E_\alpha^* E_\beta \to 2\mathbf{k}_{\perp}^2 \phi\phi^*, \tag{A5}$$

$$\mathbf{B}^2 = 2\mathbf{k}_{\perp}^2 AA^*.$$

Therefore,

$$W = w/(16\pi), \tag{A6}$$

where w is given by Eq. (A2). Thus, Eq. (A1) is the energy conservation law for the SSDA modes. It is a remarkable fact that nonlinear interaction of these modes described by Eqs. (8) and (9) does not result in a change of their total energy.

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