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PLASMA INSTABILITY

Generation of Zonal Flows by Kinetic Alfvén Waves

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Abstract—The generation of zonal flows by kinetic Alfvén waves is analyzed. It is noted that the basic approach underlying the existing theory of this phenomenon is too simplified because it attributes the generation of zonal flows to instabilities of an individual monochromatic wave packet of kinetic Alfvén waves. It is shown that, when a monochromatic wave packet is stable, it is necessary to analyze a more complicated situation with a double-peak packet (or, in the simplest case, with two pump waves). It is found that, for a double-peak packet of kinetic Alfvén waves, there is a new class of instabilities of zonal flows and that these instabilities are analogous to two-stream instabilities in linear theory. The main types of such instabilities are investigated.

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1. INTRODUCTION AND GENERAL REVIEW OF THE PROBLEM

Among the plasma physics problems actively discussed over the past decade is the problem of the secondary instabilities [1] that give rise to zonal flows [2] (in what follows, we will use the term "instability of zonal flows"). Interest in these instabilities stems primarily from the fact that zonal flows reduce anomalous transport in a magnetized plasma [2].

There are two main lines of research in the theory of the generation of zonal flows. The first is a detailed investigation of the flow instabilities whose existence has already been proved and is now generally accepted. The second is to reveal new types of instability of zonal flows. The best example of fairly well studied instabilities is the instability of zonal flows generated by electrostatic drift waves (for more detail, see [2] and the literature cited therein and also recent papers [3, 4]). The present work is devoted to the second line of research. We analyze whether zonal flows can be generated by the so-called kinetic Alfvén waves (KAWs).

KAWs are one of the main wave types in a magnetized plasma. They are described by the dispersion relation

$$\omega^2 = k_z^2 v_A^2 (1 + \hat{\rho}^2 k_\perp^2). \qquad (1.1)$$

Here, $\hat{\rho}^2 = \rho_s^2 + (3/4)\rho_i^2$, ω and $\mathbf{k}$ are the frequency and wave vector of the primary modes, k_z and $\mathbf{k}_\perp$ are the longitudinal and transverse components of the wave vector $\mathbf{k}$ with respect to the equilibrium magnetic field, v_A is the Alfvén speed, ρ_i is the ion Larmor radius, and ρ_s is the so-called sonic ion Larmor radius (i.e., the radius calculated in terms of the electron temperature). The parameter β (the ratio of plasma to magnetic field pressure) lies within the range $M_e/M_i < \beta < 1$, where M_e and M_i are the masses of an electron and of an ion, respectively. The term "kinetic Alfvén waves" was introduced by Hasegawa and Chen [5], but it should be noted that, within the framework of the original papers on drift-Alfvén instabilities (see [6, 7]), dispersion relation (1.1) can be derived by ignoring drift effects and without expanding the Bessel functions in powers of $k_\perp^2 \rho_i^2$.

The question then naturally arises of whether "pure" Alfvén modes (i.e., those described by dispersion relation (1.1) in which the dispersion term with $k_\perp^2$ is discarded) can generate zonal flows. This question was discussed in [8] (see also [9]), where it was noted, in particular, that, in the case of pure Alfvén modes, the Reynolds stress, which gives rise to zonal flows in the case of electrostatic drift modes, is completely counterbalanced by the Maxwell stress, so the resulting driving force for a zonal flow is zero. Hence, the dispersion term in dispersion relation (1.1) plays the role of an unbalancing factor and as such describes a nonzero driving force.

This stage of research faces the question of whether the driving force is stabilizing or destabilizing. Previous investigations gave contradictory answers to this question. The essence of the contradiction can be explained as follows.

Note, first of all, that the bulk of papers on KAWs deal with a particular case of cold ions, i.e., use not dispersion relation (1.1) but the dispersion relation

$$\omega^2 = k_z^2 v_A^2 (1 + k_\perp^2 \rho_s^2). \tag{1.2}$$

There are a number of papers aimed at studying the generation of zonal flows by Alfvén modes coupled to electron drift modes [10–12]. If, in those papers, the drift effects were ignored, the primary modes would be described by dispersion relation (1.2). It then follows from [12] that KAWs are stable against the generation of zonal flows. As for papers [10, 11], they give quite an uncertain answer to this question (see below for details). On the other hand, in a more recent paper by Shukla [13], it was asserted that KAWs can be unstable against the generation of zonal flows. The cited papers refer to the general and fusion plasma theories. However, there also is a work by Pokhotelov et al. [14], carried out within astrophysical research. This work, which is a continuation of the original paper by Sagdeev [15], was devoted to considering the generation of zonal flows by a particular type of KAWs described by dispersion relation (1.2). It was shown there that such KAWs are stable against the generation of zonal flows. Hence, from [10–12], as well as from [14], it follows that the primary modes described by dispersion relation (1.2) cannot generate zonal flows; in contrast, according to [13], such a generation should occur. This is the essence of the contradiction at hand.

Unlike in papers dealing with a particular type of KAWs described by dispersion relation (1.2), Onishchenko et al. [16] considered the generation of KAWs by the primary modes described by dispersion relation (1.1), i.e., they accounted for the ion dispersion in KAWs. The result they obtained was rather intriguing: zonal flows can be generated if the ion dispersion predominates over the electron dispersion, $(3/4)T_i > T_e$. In the context of what was said above, it seems, however, important to reexamine this results of [16].

We also make some comments on the investigation of finite ion temperature effects that was carried out by Guzdar et al. [17]. They considered the primary modes to be drift-Alfvén modes, so, in the limiting case in which drift effects are ignored, their problem reduces to that of the generation of zonal flows by KAWs. However, just like in the above-cited papers [10, 11], passing over to this limiting case yields a fairly contradictory answer to the question of whether zonal flows can be generated by KAWs. This is why it seems expedient to check the validity of the results obtained in [10, 11], as well as in [17].

We are primarily interested in KAWs with $k_x \gg k_y$, where k_x and k_y are the radial and poloidal components of the wave vector. Our interest stems from the fact that such KAWs are most important for magnetic confinement systems, whereas KAWs with a very small ratio of the radial to the transverse wave vector components, $k_x/k_\perp$, cannot occur there because of the magnetic shear effects (see [7] for details). For the KAWs of interest to us, dispersion relation (1.1) reads

$$D(\omega, \mathbf{k}) \equiv \omega^2 - k_z^2 v_A^2 (1 + \hat{\rho}^2 k_x^2) = 0. \tag{1.3}$$

Analogously, the version of dispersion relation (1.2) that is most important for fusion applications has the form

$$\omega^2 = k_z^2 v_A^2 (1 + \rho_s^2 k_x^2). \tag{1.4}$$

At the same time, the results of [14, 16, 17] were obtained for $k_y \gg k_x$. This is why we will also consider KAWs with an arbitrary ratio k_y/k_x.

Note that the aforementioned result of [12], which dealt with dispersion relation (1.4), is expressed in terms of the radial derivative of the group velocity of the primary modes, $\partial V_g(\mathbf{k})/\partial k_x$ (where $V_g = \partial\omega/\partial k_x$), i.e., through $\partial^2\omega/\partial k_x^2$. According to [12], the criterion for stability of KAWs against the generation of zonal flows becomes

$$\partial^2\omega/\partial k_x^2 > 0. \tag{1.5}$$

On the other hand, from [18] we can see that the second derivative $\partial^2\omega/\partial k_x^2$ enters into the Lighthill stability criterion, which characterizes the self-modulation of waves and has the form

$$\alpha\partial^2\omega/\partial k_x^2 > 0, \tag{1.6}$$

where α is a factor dependent on the type of primary modes. We then can assume that, for the KAWs described by dispersion relation (1.4), this factor is positive, $\alpha > 0$.

At this point, it is helpful to take a brief look at the problem about the generation of zonal flows by weakly dispersive electrostatic drift waves [2, 19] from the standpoint of the Lighthill criterion. In the case of cold ions and KAWs with $k_x \gg k_y$, we are dealing not with dispersion relation (1.2) but with the dispersion relation

$$\omega = k_y V_{*e} (1 - k_x^2 \rho_s^2), \tag{1.7}$$

where V_{*e} is the electron diamagnetic drift velocity in terms of the density gradient and it is assumed that $k_x^2 \rho_s^2 \ll 1$. In contrast to criterion (1.5), we find from dispersion relation (1.7) that, for $\omega > 0$, the following criterion should be satisfied:

$$\partial^2\omega/\partial k_x^2 < 0. \tag{1.8}$$

It is well known that drift waves can generate zonal flows. Consequently, we can suppose that the possibility for one or another type of primary modes to generate zonal flows is directly related to the sign of the second derivative $\partial^2\omega/\partial k_x^2$ calculated for the corresponding modes.

In contrast to dispersion relation (1.1), weakly dispersive Alfvén waves at $\beta < M_e/M_i$ are described by the dispersion relation (see, e.g., [7], Eq. (3.14), and [20], Eq. (14.87) with no drift terms taken into account)

$$\omega^2 = k_z^2 v_A^2 (1 - c^2 k_\perp^2/\omega_{pe}^2), \tag{1.9}$$

where ω_{pe} is the electron plasma frequency and c is the speed of light. For such waves, the derivative $\partial^2\omega/\partial k_x^2$ satisfies criterion (1.8) (for simplicity, we set $k_x \gg k_y$). On the other hand, in [21, 22], it was shown that these modes, which were there called inertial Alfvén waves, are unstable against the generation of zonal flows (convective cells). This example also demonstrates the aforementioned direct relationship between the sign of the quantity $\partial^2\omega/\partial k_x^2$ and the possibility for primary modes to generate zonal flows.

The objective of the present paper is to investigate the generation of zonal flows by the KAWs described by dispersion relation (1.1). In the analysis to follow, we resolve the above contradiction, which concerns dispersion relation (1.2), and confirm the conclusion of [16] about the role of ion dispersion. We also discuss some results of [10, 11, 17].

All the aforementioned results of the cited papers were obtained for monochromatic packets of primary modes. This raises the question of how completely these results describe the problem of the generation of zonal flows by wave packets of arbitrary shapes.

According to [23], the dispersion relation for zonal flows generated by weakly dispersive nonmonochromatic drift wave packets can be represented as

$$1 + \int \frac{F(\mathbf{k})d\mathbf{k}}{[\Omega - q_x V_g(\mathbf{k})]^2} = 0. \tag{1.10}$$

Here, Ω and q_x are the oscillation frequency and radial wave vector of a zonal flow, $F(\mathbf{k})$ is a positive definite function, and $V_g(\mathbf{k})$ is the radial group velocity of the primary modes (see above). For a monochromatic wave packet, we have $F(\mathbf{k}) \sim \delta(\mathbf{k} - \mathbf{k}_0)$, where $\mathbf{k}_0$ is the wave vector of the packet. Dispersion relation (1.10) then becomes

$$1 + \frac{\Omega_0^2}{[\Omega - q_x V_g(\mathbf{k}_0)]^2} = 0, \tag{1.11}$$

where Ω_0^2 is a positive constant. This relation has a root with $\mathrm{Im}\,\Omega > 0$, which corresponds to an unstable zonal flow. On the other hand, according to Section 4.1 of [24] (see also [25]), which is devoted to studying the negative-mass instability of a slightly relativistic plasma with a δ-shaped velocity distribution, the instability of a zonal flow is described by the dispersion relation

$$1 + \frac{\omega_0^2}{(\omega - \omega_B)^2} = 0, \tag{1.12}$$

where $\omega_0^2 > 0$ is the effective plasma frequency squared and ω_B is the electron gyrofrequency. A comparison between dispersion relations (1.11) and (1.12) shows that the instability of a zonal flow that is driven by a weakly dispersive monochromatic drift wave packet is analogous to the negative-mass instability. On the other hand, this instability of a zonal flow is consistent with criterion (1.8). On the whole, we can conclude that the results obtained in the monochromatic wave packet approximation are sufficiently representative for primary modes satisfying criterion (1.8).

Our analysis implies that, for KAWs described by dispersion relation (1.3), we are dealing not with dispersion relation (1.10) but with the relation

$$1 - \int \frac{F(\mathbf{k})d\mathbf{k}}{[\Omega - q_x V_g(\mathbf{k})]^2} = 0. \tag{1.13}$$

Here, as in dispersion relation (1.10), $F(\mathbf{k})$ is a positive definite function, but unlike in dispersion relation (1.10), the integral has a plus, rather than minus, sign. For a monochromatic wave packet, we then have not dispersion relation (1.11) but the relation

$$1 - \frac{\Omega_0^2}{[\Omega - q_x V_g(\mathbf{k}_0)]^2} = 0. \tag{1.14}$$

Let us consider dispersion relations (1.13) and (1.14) in the context of the theory of two-stream instabilities. According to Sections 1–3 of [26], dispersion relation (1.14) is an analogue of the dispersion relation for an individual cold beam and dispersion relation (1.13) formally coincides with that in the case of an arbitrary particle velocity distribution. We thus can conclude that KAWs having a double-peak spectrum can generate a zonal flow by an instability mechanism analogous to that for a two-stream instability. In particular, in the simplest case of two monochromatic packets of KAWs, dispersion relation (1.13) is reduced to

$$1 - \frac{\Omega_1^2}{[\Omega - q_x V_{g1}]^2} - \frac{\Omega_2^2}{[\Omega - q_x V_{g2}]^2} = 0. \tag{1.15}$$

Formally, this is the dispersion relation in the case of two cold beams. It is well known [26] that one of the roots of this dispersion relation is such that $\mathrm{Im}\,\Omega > 0$, which corresponds to the simplest case of a two-stream instability.

Hence, the situation described by criterion (1.5) is analogous to that with the negative-mass instability, while the situation described by criterion (1.8) should be analyzed in analogy with the theory of two-stream instabilities. It is thus clear that the results obtained in the monochromatic wave packet approximation are unrepresentative for dispersion relation (1.13) (see also criterion (1.8)). In order for criterion (1.8) to provide an adequate description of the generation of zonal flows, it is necessary to consider primary modes having a double-peak spectrum, or two pump waves in the simplest formulation of the problem. From a recent paper by Erokhin et al. [27], which predicted the decay of an individual KAW into two KAWs (see also recent papers [28–30] and the literature cited therein), we can infer that KAWs with a double-peak spectrum are of great practical interest.

It follows from the aforesaid that, in order to give an adequate description of the generation of zonal flows by KAWs, a theory of this phenomenon should be developed for KAWs having an arbitrary spectrum. Most previous investigations of different types of primary modes were carried out based on the wave kinetic equation—an approach that was originally developed by Vedenov et al. [31] (see, e.g., [2] and the references cited therein). In an earlier paper [32], devoted to the generation of zonal flows by small-scale drift-Alfvén modes, it was shown that an approach based on the notion of convective cells [15, 19] (which usually deals with monochromatic wave packets) can be fairly simply generalized to wave packets having arbitrary spectra (see also [22]). The generalization is done by summing (or integrating) the contributions of each primary mode in the wave spectrum to the evolutionary equation for zonal flows. We perform this generalization and obtain a dispersion relation for zonal flows generated by KAWs with an arbitrary spectrum that are described by dispersion relation (1.1).

Our paper is organized as follows. In Section 2, we present the basic plasmodynamic equations, namely, the nonlinear equations that were derived in [33–36] in the theory of Alfvén vortices. By analogy with [32], in Section 2, we also carry out preliminary transformations of these plasmodynamic equations; introduce the functions characterizing zonal flows, as well as primary modes and their satellites; and write out the basic evolutionary equations for the flows and the satellites. In Section 3, we describe the procedure for deriving the dispersion relation for zonal flows. The procedure mostly consists in calculating the satellites. The details of the calculations are given in the Appendix. In Section 4, we analyze the dispersion relation relationship for zonal flows. In the conclusion, we discuss the results obtained.

2. BASIC EQUATIONS AND THEIR PRELIMINARY TRANSFORMATIONS

2.1. Basic Plasmodynamic Equations

From [33–36], we have the equations

$$\left(\frac{\partial}{\partial t}+\frac{c}{B_0}(\nabla\phi\times\nabla)_z\right)\left(\nabla_\perp^2\phi+\frac{3}{4}\rho_i^2\nabla_\perp^4\phi\right)$$
$$+\frac{v_A^2}{c}\left(\frac{\partial}{\partial z}-\frac{1}{B_0}(\nabla A\times\nabla)_z\right)\nabla_\perp^2 A = 0, \quad (2.1)$$

$$\frac{\partial A}{\partial t}+c\left(\frac{\partial}{\partial z}-\frac{1}{B_0}(\nabla A\times\nabla)_z\right)\left(\phi-\frac{T_e}{en_0}n\right) = 0, \quad (2.2)$$

$$\left(\frac{\partial}{\partial t}+\frac{c}{B_0}(\nabla\phi\times\nabla)_z\right)n$$
$$+\frac{c}{4\pi e}\left(\frac{\partial}{\partial z}-\frac{1}{B_0}(\nabla A\times\nabla)_z\right)\nabla_\perp^2 A = 0. \quad (2.3)$$

Equation (2.1) is the vorticity equation, Eq. (2.2) is the longitudinal Ohm's law (the equation of longitudinal electron motion), and Eq. (2.3) is the electron continuity equation. The function ϕ is the electrostatic potential, defined by the equality $\mathbf{E}_\perp = -\nabla_\perp\phi$, where $\nabla_\perp$ is the transverse gradient operator, $\mathbf{E}_\perp$ is the transverse electric field, $\mathbf{B}_0$ is the equilibrium magnetic field, and $\mathbf{e}_z$ is a unit vector directed along $\mathbf{B}_0$. The rest of the notation is as follows: the function A is the transverse vector potential, which is related to the perturbed magnetic field $\mathbf{B}_\perp$ by the formula

$$\mathbf{B}_\perp = \nabla A\times\nabla e_z; \quad (2.4)$$

n is the perturbed plasma density; n_0 is the equilibrium density; and e is the charge of an ion.

2.2. Transformation of the Basic Plasmodynamic Equations

2.2.1. Separation of variables. By analogy with [32], we represent each of the perturbed quantities $X = (\phi, A, n)$ as

$$X = \tilde{X}+\hat{X}+\bar{X}, \quad (2.5)$$

where $\tilde{X}$, $\hat{X}$, and $\bar{X}$ describe the primary modes, the secondary small-scale modes, and the zonal flows, respectively. The functions $\bar{X}$ are chosen to have the form

$$\bar{X} = \bar{X}_0\exp(-i\Omega t+iq_x x)+\text{c.c.}, \quad (2.6)$$

where $\bar{X}_0 \equiv (\bar{\phi}_0, \bar{A}_0, \bar{n}_0)$ are the electrostatic and vector potentials of the zonal flow and the plasma density corresponding to the flow, respectively, and the symbol c.c. stands for the complex conjugate.

We represent the functions $\tilde{X} = (\tilde{\phi}, \tilde{A}, \tilde{n})$, which characterize the primary modes, as

$$\tilde{X} = \sum_{\mathbf{k}} [\tilde{X}_+(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega_{\mathbf{k}}t) + \tilde{X}_-(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega_{\mathbf{k}}t)], \tag{2.7}$$

assuming that $\tilde{X}_-(\mathbf{k}) = \tilde{X}_+^*(\mathbf{k})$, where the asterisk denotes the complex conjugate. The sum in representation (2.7) is taken over all the primary modes.

The functions $\hat{X} \equiv (\hat{\phi}, \hat{A}, \hat{n})$ are represented as

$$\hat{X} = \sum_{\mathbf{k}} [\hat{X}_+(\mathbf{k})\exp(i\mathbf{k}_+\cdot\mathbf{r} - i\omega_+t) + \hat{X}_-(\mathbf{k})\exp(i\mathbf{k}_-\cdot\mathbf{r} - i\omega_-t) + \text{c.c.}], \tag{2.8}$$

where $\omega_\pm = \Omega \pm \omega_{\mathbf{k}}$, $\mathbf{k}_\pm = (q_x \pm k_x, \pm k_y, \pm k_z)$, and $\hat{X}_\pm(\mathbf{k})$ are the amplitudes of the satellites.

We restrict our analysis to very large-scale zonal flows assuming that $q_x \ll k_x$, which is analogous to the corresponding assumption in the standard theory of the generation of zonal flows by electrostatic drift waves (see, e.g., [2, 32]).

2.2.2. Evolutionary equations for zonal flows. Under the above assumptions, we see from Eqs. (2.1)–(2.3) that the functions $\bar{\phi}_0$, $\bar{A}_0$, and $\bar{n}_0$ satisfy the evolutionary equations

$$-i\Omega\bar{\phi}_0 = R_\perp, \tag{2.9}$$

$$-i\Omega\bar{A}_0 = R_\parallel, \tag{2.10}$$

$$-i\Omega\bar{n}_0 = R_n, \tag{2.11}$$

where

$$R_\perp = \frac{c}{B_0}\frac{1}{q_x^2}\left(\left\langle(\nabla\phi\times\nabla\nabla_\perp^2\phi)_z - \frac{v_A^2}{c^2}(\nabla A\times\nabla\nabla_\perp^2 A)_z + \frac{3}{4}\rho_i^2(\nabla\phi\times\nabla\nabla_\perp^4\phi)_z\right\rangle\right), \tag{2.12}$$

$$R_\parallel = \frac{c}{B_0}\left\langle\left[\nabla A\times\nabla\left(\phi - \frac{T_e}{en_0}n\right)\right]_z\right\rangle, \tag{2.13}$$

$$R_n = -\frac{c}{B_0}\left\langle(\nabla\phi\times\nabla n)_z - \frac{1}{4\pi e}(\nabla A\times\nabla\nabla_\perp^2 A)_z\right\rangle, \tag{2.14}$$

and the angle brackets $\langle\ldots\rangle$ stand for averaging over small-scale oscillations.

We convert expressions (2.12)–(2.14) into the form

$$R_\perp = \frac{c}{B_0}\left\langle\frac{\partial\phi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{v_A^2}{c^2}\frac{\partial A}{\partial y}\frac{\partial A}{\partial x} - \frac{3}{2}\rho_i^2\nabla_\perp^2\phi\frac{\partial^2\phi}{\partial x\partial y}\right\rangle, \tag{2.15}$$

$$R_\parallel = \frac{icq_x}{B_0}\left\langle A\frac{\partial}{\partial y}\left(\phi - \frac{T_e}{en_0}n\right)\right\rangle, \tag{2.16}$$

$$R_n = \frac{icq_x}{B_0}\left\langle n\frac{\partial\phi}{\partial y}\right\rangle. \tag{2.17}$$

In expressions (2.15) and (2.17), we have omitted the terms with q_x^2, which are unimportant for our calculations because they are small quantities on the order of q_x^2/k_x^2.

2.2.3. Primary modes. From Eqs. (2.2) and (2.3), we see that the amplitudes $\tilde{A}_\pm$ and $\tilde{n}_\pm$ are expressed in terms of $\tilde{\phi}_\pm$ through the relationships

$$\tilde{A}_\pm = \frac{ck_z}{\omega}\tilde{\phi}_\pm(1+\delta_e), \tag{2.18}$$

$$\tilde{n}_\pm = \frac{c^2k_z^2k_\perp^2}{4\pi e\omega^2}\tilde{\phi}_\pm(1+\delta_e), \tag{2.19}$$

where the quantity δ_e, which characterizes the electron dispersion in the primary modes, is defined by

$$\delta_e = k_\perp^2\rho_s^2. \tag{2.20}$$

Using expression (2.18), we obtain from Eq. (2.1) dispersion relation (1.1) for the zonal flows. For simplicity, in expressions (2.18) and (2.19), as well as in the subsequent formulas, we use ω in place of $\omega_{\mathbf{k}}$.

2.2.4. Driving forces in terms of $\tilde{X}_\pm$ and $\hat{X}_\pm$. In terms of the quantities $\tilde{X}_\pm$ and $\hat{X}_\pm$, expression (2.15) reads

$$R_\perp = \sum_{\mathbf{k}}\frac{ck_y}{B_0}\{\Lambda_0 + \Lambda_i\}, \tag{2.21}$$

where

$$\Lambda_0 = 2k_x\left[\hat{\phi}_+\tilde{\phi}_- + \hat{\phi}_-\tilde{\phi}_+ - \frac{v_A^2}{c^2}(\hat{A}_+\tilde{A}_- + \hat{A}_-\tilde{A}_+)\right] + q_x\left[\hat{\phi}_+\tilde{\phi}_- - \hat{\phi}_-\tilde{\phi}_+ - \frac{v_A^2}{c^2}(\hat{A}_+\tilde{A}_- - \hat{A}_-\tilde{A}_+)\right], \tag{2.22}$$

$$\Lambda_i = -2\delta_i[2k_x(\hat{\phi}_+\tilde{\phi}_- + \hat{\phi}_-\tilde{\phi}_+) + (1 + 2k_x^2/k_\perp^2)q_x(\hat{\phi}_+\tilde{\phi}_- - \hat{\phi}_-\tilde{\phi}_+)]. \tag{2.23}$$

The parameter δ_i accounts for ion dispersion and is defined as

$$\delta_i = (3/4)\rho_i^2 k_\perp^2. \quad (2.24)$$

Analogously, expressions (2.16) and (2.17) become

$$R_\parallel = \sum_{\mathbf{k}} \frac{ck_y q_x}{B_0}\left[\hat{A}_+\left(\tilde{\phi}_- - \frac{T_e}{en_0}\tilde{n}_-\right) - \hat{A}_-\left(\tilde{\phi}_+ - \frac{T_e}{en_0}\tilde{n}_+\right)\right.$$
$$\left. + \tilde{A}_+\left(\hat{\phi}_- - \frac{T_e}{en_0}\hat{n}_-\right) - \tilde{A}_-\left(\hat{\phi}_+ - \frac{T_e}{en_0}\hat{n}_+\right)\right], \quad (2.25)$$

$$R_n = \sum_{\mathbf{k}} \frac{ck_y q_x}{B_0}(\hat{n}_+\tilde{\phi}_- - \hat{n}_-\tilde{\phi}_+ + \hat{\phi}_-\tilde{n}_+ - \hat{\phi}_+\tilde{n}_-). \quad (2.26)$$

In expression (2.26), as in expression (2.17), the terms with q_x^2 are ignored.

2.2.5. Equations for the amplitudes of the satellites. From Eqs. (2.1)–(2.3) we have

$$\omega_\pm\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\hat{\phi}_\pm \mp \frac{k_z v_A^2}{c}\hat{A}_\pm = Y_\pm^\phi, \quad (2.27)$$

$$\omega_\pm \hat{A}_\pm \mp ck_z\left(\hat{\phi}_\pm - \frac{T_e}{en_0}\hat{n}_\pm\right) = Y_\pm^A, \quad (2.28)$$

$$\omega_\pm \hat{n}_\pm \pm \frac{ck_z}{4\pi e}k_{\perp\pm}^2\hat{A}_\pm = Y_\pm^n, \quad (2.29)$$

where

$$Y_\pm^\phi = \pm\frac{ic}{B_0}q_x k_y \frac{k_\perp^2}{k_{\perp\pm}^2}\left[\tilde{\phi}_\pm\bar{\phi}_0(1-\delta_i) - \frac{v_A^2}{c^2}\tilde{A}_\pm\bar{A}_0\right], \quad (2.30)$$

$$Y_\pm^A = \mp i q_x k_y \frac{c}{B_0}\left[\bar{A}_0\left(\tilde{\phi}_\pm - \frac{T_e}{en_0}\tilde{n}_\pm\right)\right.$$
$$\left. - \tilde{A}_\pm\left(\bar{\phi}_0 - \frac{T_e}{en_0}\bar{n}_0\right)\right], \quad (2.31)$$

$$Y_\pm^n = \pm\frac{ic}{B_0}q_x k_y\left(\bar{\phi}_0\tilde{n}_\pm - \bar{n}_0\tilde{\phi}_\pm + \frac{k_\perp^2}{4\pi e}\bar{A}_0\tilde{A}_\pm\right). \quad (2.32)$$

The amplitudes $\tilde{A}_\pm$ and $\tilde{n}_\pm$ are expressed in terms of $\tilde{\phi}_\pm$ through relationships (2.18) and (2.19).

3. DERIVATION OF THE DISPERSION RELATION FOR ZONAL FLOWS

3.1. General Expressions for the Amplitudes of the Satellites

From Eqs. (2.27)–(2.29) we find

$$\hat{\phi}_\pm = \frac{1}{\omega_\pm D_\pm}\left[(\omega_\pm^2 - k_z^2 k_{\perp\pm}^2 v_A^2 \rho_s^2)Y_\pm^\phi\right.$$
$$\left. \pm \omega_\pm \frac{k_z v_A^2}{c}Y_\pm^A - \frac{T_e}{en_0}k_z^2 v_A^2 Y_\pm^n\right], \quad (3.1)$$

$$\hat{A}_\pm = \frac{1}{D_\pm}\left[\pm ck_z Y_\pm^\phi\right.$$
$$\left. + \left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\left(\omega_\pm Y_\pm^A \mp \frac{ck_z T_e}{en_0}Y_\pm^n\right)\right], \quad (3.2)$$

$$\hat{n}_\pm = \frac{1}{\omega_\pm D_\pm}\left\{-\frac{c^2 k_z^2 k_{\perp\pm}^2}{4\pi e}Y_\pm^\phi\right.$$
$$\mp \omega_\pm\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\frac{ck_z k_{\perp\pm}^2}{4\pi e}Y_\pm^A \quad (3.3)$$
$$\left. + \left[\omega_\pm^2\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right) - k_z^2 v_A^2\right]Y_\pm^n\right\},$$

where

$$D_\pm = \left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\omega_\pm^2 - k_z^2 v_A^2(1 + \rho_s^2 k_{\perp\pm}^2). \quad (3.4)$$

Substituting expressions (2.30)–(2.32) into expressions (3.1)–(3.3) yields

$$\hat{\phi}_\pm = \pm\frac{ic}{B_0}\frac{q_x k_y}{\omega_\pm D_\pm}(Z_\pm^{\phi\phi}\bar{\phi}_0 + Z_\pm^{\phi A}\bar{A}_0 + Z_\pm^{\phi n}\bar{n}_0), \quad (3.5)$$

$$\hat{A}_\pm = \frac{ic}{B_0}\frac{q_x k_y}{D_\pm}(Z_\pm^{A\phi}\bar{\phi}_0 + Z_\pm^{AA}\bar{A}_0 + Z_\pm^{An}\bar{n}_0), \quad (3.6)$$

$$\hat{n}_\pm = \pm\frac{ic}{B_0}\frac{q_x k_y}{\omega_\pm D_\pm}(Z_\pm^{n\phi}\bar{\phi}_0 + Z_\pm^{nA}\bar{A}_0 + Z_\pm^{nn}\bar{n}_0). \quad (3.7)$$

The expressions for $Z_\pm^{ik}$ $(i, k = \phi, A, n)$ are presented in the Appendix.

3.2. Expansions of the Amplitudes of the Satellites in Power Series in Ω and q_x

Using relationship (3.4), we obtain the following expansion for $D_\pm$:

$$D_\pm = \pm 2\omega D^{(0)} + D^{(1)}, \quad (3.8)$$

where

$$D^{(0)} = \Omega(1-\delta_i) - q_x k_x \omega\hat{\rho}^2, \quad (3.9)$$

$$D^{(1)} = \Omega^2(1-\delta_i) - 3\rho_i^2\omega\Omega k_x q_x - q_x^2 k_z^2 v_A^2\hat{\rho}^2. \quad (3.10)$$

Taking into account expansions (3.8), (3.9), and (A.13)–(A.15), we find that the leading-order amplitudes of the satellites in expressions (3.5)–(3.7) have the form

$$\hat{\phi}_{\pm}^{(0)} = \pm\frac{icq_xk_y\tilde{\phi}_{\pm}}{B_0D^{(0)}}\frac{k_z^2v_A^2(1+\delta_e)}{\omega^2}\times\left(\bar{\phi}_0-\frac{\omega}{ck_z}\bar{A}_0-\frac{T_e}{2en_0}\delta_e\bar{n}_0\right), \tag{3.11}$$

$$\hat{A}_{\pm}^{(0)} = \pm\frac{ic^2k_zq_xk_y\tilde{\phi}_{\pm}}{B_0D^{(0)}\omega}(1+\delta_e-\delta_i)\times\left(\bar{\phi}_0-\frac{\omega}{ck_z}\bar{A}_0-\frac{T_e\delta_e}{2en_0}\bar{n}_0\right), \tag{3.12}$$

$$\hat{n}_{\pm}^{(0)} = \mp\frac{ic^3k_z^2k_\perp^2q_xk_y\tilde{\phi}_{\pm}}{B_0D^{(0)}\omega^2}(1+\delta_e-\delta_i)\left(\bar{\phi}_0-\frac{\omega}{ck_z}\bar{A}_0\right). \tag{3.13}$$

3.3. Elimination of the Zonal Flow Variables $\bar{A}_0$ and $\bar{n}_0$

The driving forces $R_\parallel$ and R_n, defined by the expressions (2.25) and (2.26), can be calculated to zero order in the amplitudes of the satellites, $\hat{\phi}_{\pm}$, $\hat{A}_{\pm}$, and $\hat{n}_{\pm}$, given by expressions (3.11)–(3.13). Discarding the terms that are as small as q_x^2/k_x^2, we obtain

$$R_\parallel = 0, \tag{3.14}$$

$$R_n = 0. \tag{3.15}$$

As a result, Eqs. (2.10) and (2.11) give

$$\bar{A}_0 = 0, \tag{3.16}$$

$$\bar{n}_0 = 0. \tag{3.17}$$

We can see that, in the approximation at hand, the zonal components of the vector potential and plasma density are not produced, so expressions (3.5) and (3.6) for $\hat{\phi}_{\pm}$ and $\hat{A}_{\pm}$ become

$$\hat{\phi}_{\pm} = \pm\frac{ic}{B_0}\frac{q_xk_y\bar{\phi}_0}{D_{\pm}}\tilde{\phi}_{\pm}(1-\delta_i)\left(\frac{k_\perp^2}{k_{\perp\pm}^2}\omega_{\pm}\pm\omega\right), \tag{3.18}$$

$$\hat{A}_{\pm} = \pm\frac{ic}{B_0}\frac{q_xk_y\bar{\phi}_0}{D_{\pm}}\tilde{\phi}_{\pm}\frac{ck_z}{\omega}\times\left\{\omega_{\pm}\left(1+\delta_e-\frac{3}{4}\rho_i^2k_{\perp\pm}^2\right)\pm\omega\left[\frac{k_\perp^2}{k_{\perp\pm}^2}(1-\delta_i)+\delta_e\right]\right\}. \tag{3.19}$$

3.4. Calculation of the Force $R_\perp$ and Derivation of the Dispersion Relation for Zonal Flows

In order to calculate the force $R_\perp$, it is necessary to incorporate first-order corrections to the amplitudes of the satellites $\hat{\phi}_{\pm}$ and $\hat{A}_{\pm}$, i.e., to set

$$\hat{\phi}_{\pm} = \hat{\phi}_{\pm}^{(0)}+\hat{\phi}_{\pm}^{(1)}, \tag{3.20}$$

$$\hat{A}_{\pm} = \hat{A}_{\pm}^{(0)}+\hat{A}_{\pm}^{(1)}. \tag{3.21}$$

Expressions (2.22) and (2.23) then are reduced to

$$\Lambda_0 = 2k_x\left[\hat{\phi}_+^{(1)}\tilde{\phi}_-+\hat{\phi}_-^{(1)}\tilde{\phi}_+-\frac{v_A^2}{c^2}(\hat{A}_+^{(1)}\tilde{A}_-+\hat{A}_-^{(1)}\tilde{A}_+)\right] + q_x\left[\hat{\phi}_+^{(0)}\tilde{\phi}_--\hat{\phi}_-^{(0)}\tilde{\phi}_+-\frac{v_A^2}{c^2}(\hat{A}_+^{(0)}\tilde{A}_--\hat{A}_-^{(0)}\tilde{A}_+)\right], \tag{3.22}$$

$$\Lambda_i = -2\delta_i[2k_x(\hat{\phi}_x^{(1)}\tilde{\phi}_-+\hat{\phi}_-^{(1)}\tilde{\phi}_+) + (1+2k_x^2/k_\perp^2)q_x(\hat{\phi}_+^{(0)}\tilde{\phi}_--\hat{\phi}_-^{(0)}\tilde{\phi}_+)], \tag{3.23}$$

and expressions (3.18) and (3.19) yield

$$\hat{\phi}_{\pm}^{(1)} = -\frac{ic}{B_0}\frac{q_xk_y\bar{\phi}_0\tilde{\phi}_{\pm}}{2D^{(0)^2}}(1-\delta_i)\times\left(2\Omega\frac{q_x}{k_x}(1-3\delta_i)+\frac{\Omega}{\omega^2}q_xk_xk_z^2v_A^2\hat{\rho}^2-\frac{3}{\omega}q_x^2k_z^2v_A^2\hat{\rho}^2\right), \tag{3.24}$$

$$\hat{A}_{\pm}^{(1)} = -\frac{ic^2}{B_0}\frac{q_xk_yk_z\bar{\phi}_0\tilde{\phi}_{\pm}}{2\omega D^{(0)^2}}(1-\delta_i)\times\left(2\Omega\frac{q_x}{k_x}(1-2\delta_i)+\Omega q_xk_x\hat{\rho}^2-3\omega q_x^2\hat{\rho}^2\right). \tag{3.25}$$

Inserting expressions (3.11), (3.12), (3.24), and (3.25) into expressions (3.22) and (3.23), we obtain

$$\Lambda_0 = -\frac{ick_yq_x^2\bar{\phi}_0I_{\mathbf{k}}\Omega}{B_0D^{(0)^2}}(\delta_e-\delta_i), \tag{3.26}$$

$$\Lambda_i = -2\delta_i\frac{ick_yq_x^2\bar{\phi}_0I_{\mathbf{k}}\Omega}{B_0D^{(0)^2}}\left(\frac{2k_x^2}{k_\perp^2}-1\right), \tag{3.27}$$

where

$$I_{\mathbf{k}} = 2\tilde{\phi}_+\tilde{\phi}_-. \tag{3.28}$$

With relationships (3.26) and (3.27), expression (2.21) simplifies to

$$R_\perp = -i\Omega\bar{\phi}_0\int\frac{F(\mathbf{k})}{(\Omega-q_xV_g)^2}d\mathbf{k}, \tag{3.29}$$

where

$$F(\mathbf{k}) = \frac{c^2 k_y^2 q_x^2 I_\mathbf{k}}{B_0}\left[\rho_s^2 + \frac{3}{4}\rho_i^2\left(4\frac{k_x^2}{k_\perp^2} - 3\right)\right]. \tag{3.30}$$

For $k_x \gg k_y$, relationship (3.30) becomes

$$F(\mathbf{k}) = c^2 k_y^2 q_x^2 I_\mathbf{k} k_x^2 \hat{\rho}^2 / B_0^2. \tag{3.31}$$

Substituting expressions (3.29) and (3.31) into Eq. (2.9), we arrive at dispersion relation (1.13) for zonal flows.

From expression (3.30) we see that, for finite values of $k_y^2/k_\perp^2$, the rigid relationship between the type of linear dispersion and the sign of the function $F(\mathbf{k})$ fails when

$$\frac{k_x^2}{k_y^2} \le \frac{(9/4)T_i - T_e}{(3/4)T_i + T_e}. \tag{3.32}$$

In accordance with what was said in the Introduction, from dispersion relations (1.10) and (1.13) we find that, under inequality (3.32), we are dealing with negative-mass instabilities rather than with two-stream instabilities of a zonal flow.

For $k_x \longrightarrow 0$, inequality (3.32) is reduced to the condition

$$T_i > (4/9)T_e, \tag{3.33}$$

which is a refined version of the condition $T_i > (4/3)T_e$, obtained in [17]. On the whole, the onset of the instabilities that are analogous to negative-mass instabilities and occur under condition (3.32) can be called the Onishchenko–Pokhotelov–Sagdeev (OPS) effect.

4. ANALYSIS OF THE DISPERSION RELATION FOR ZONAL FLOWS

4.1. The Case of a Single-Peak Wave Packet

4.1.1. Monochromatic wave packet. For a single-peak (monochromatic) packet of KAWs with $k_x \gg k_y$, we are dealing with dispersion relation (1.14) for a zonal flow. A remarkable property of this dispersion relation is that it describes zonal oscillating branches

$$\Omega = q_x V_g(k_0) \pm \Omega_0. \tag{4.1}$$

By analogy with the linear oscillating branches of plasma waves [26], the interaction between these branches with other zonal branches can give rise to two-stream instabilities.

4.1.2. Nonmonochromaticity effects. In analogy with [32], we consider an individual nonmonochromatic packet of KAWs with a Gaussian intensity spectrum $I_\mathbf{k}$,

$$I_\mathbf{k} = \frac{1}{\pi^{1/2}\Delta k_x}\exp\left[-\frac{(k_x - k_{x0})^2}{(\Delta k_x)^2}\right] I_{\mathbf{k}_0}, \tag{4.2}$$

where k_{x0} is the centering wave vector component k_x of the wave packet and Δk_x is its characteristic width. In this case, the radial group velocity V_g can be approximated by the expression

$$V_g = V_{g0} + V'_{g0}(k_x - k_{x0}), \tag{4.3}$$

where the prime denotes the derivative with respect to k_x and the subscript 0 stands for the values calculated at $k_x = k_{x0}$. It follows from dispersion relation (1.13) that, for small values of the ratio $q_x V'_{g0}\Delta k_x/\hat{\Omega}$ (where $\hat{\Omega} \equiv \Omega - q_x V_{g0}$), zonal flows are described not by dispersion relation (1.14) but by the relation

$$1 = \frac{\Omega_0^2}{\hat{\Omega}^2}\left[1 + \frac{3}{2}\frac{q_x^2 V_{g0}'^2}{\hat{\Omega}^2}(\Delta k_x)^2\right], \tag{4.4}$$

from which we find

$$\hat{\Omega}^2 = \hat{\Omega}_0^2\left[1 + \frac{3}{2}\frac{q_x^2 V_{g0}'^2}{\hat{\Omega}_0^2}(\Delta k_x)^2\right]. \tag{4.5}$$

We thus can conclude that sufficiently weak nonmonochromaticity effects do not suppress the oscillating branches defined by expression (4.1).

4.1.3. General dispersion relation for zonal flows in the case of a Gaussian wave packet. In analogy with [32], in the case of an arbitrarily nonmonochromatic wave packet of KAWs, dispersion relation (4.4) becomes

$$1 = -\Omega_0^2\frac{\partial}{\partial\hat{\Omega}}\left[\frac{1}{\hat{\Omega}}Z\left(\frac{\hat{\Omega}}{|q_x V'_{g0}|\Delta k_x}\right)\right], \tag{4.6}$$

where $Z(x)$ is the plasma dispersion function given by the expression [37]

$$Z(x) = 2xe^{-x^2}\int_0^x e^{t^2}dt - i\pi^{1/2}xe^{-x^2}. \tag{4.7}$$

4.1.4. Landau damping of zonal flows generated by an individual nonmonochromatic wave packet. Setting $\hat{\Omega}^2 \gg (q_x V'_{g0}\Delta k_x)^2$ and taking into account the exponentially small term in expression (4.7), we reduce dispersion relation (4.4) to

$$1 = \frac{\Omega_0^2}{\hat{\Omega}^2}\left(1 + \frac{3}{2x^2} - 2ixe^{-x^2}\right), \tag{4.8}$$

where $x \equiv \hat{\Omega}/(|q_x V'_{g0}|\Delta k_x)$. We can see that, because of the nonmonochromatic nature of the wave packet, zonal modes described by expression (4.5) are subject to a sort of Landau damping at the rate

$$\mathrm{Im}\,\Omega \simeq -\frac{\Omega_0^4}{|q_x V'_{g0}\Delta k_x|^3}\exp\left[-\frac{\Omega_0^2}{(q_x V'_{g0}\Delta k_x)^2}\right]. \quad (4.9)$$

4.2. The Case of Two Monochromatic Wave Packets

This case is described by dispersion relation (1.15). It is convenient to consider this dispersion relation in the following two limits: $\Omega_2^2 = \Omega_1^2$, by analogy with a system of two beams of the same density, and $\Omega_2^2 = \alpha_b\Omega_1^2$ (where $\alpha_b \ll 1$), by analogy with the interaction of a low-density beam with a high-density plasma (the parameter α_b is an equivalent of the beam-to-plasma density ratio).

4.2.1. Two wave packets of equal intensity. For $\Omega_2^2 = \Omega_1^2$, dispersion relation (1.15) becomes

$$1 - \frac{\Omega_1^2}{(\Omega - q_x V_{g1})^2} - \frac{\Omega_1^2}{(\Omega - q_x V_{g2})^2} = 0. \quad (4.10)$$

According to Section 1.5.1 of [26], one of the roots of dispersion relation (4.10), namely, the root determined by the expression

$$q_x 6 \le q_{x\max} = \frac{2^{1/2}\Omega}{|V_{g2} - V_{g1}|} \quad (4.11)$$

corresponds to instability, $\mathrm{Im}\,\Omega > 0$. The instability growth rate is maximum at

$$q_x = (3/8)^{1/2} q_{x\max}, \quad (4.12)$$

which is equivalent to

$$(\mathrm{Im}\,\Omega)_{\max} = \Omega_1/2. \quad (4.13)$$

4.2.2. A system of two wave packets of high and low intensity. For $\Omega_2^2 = \alpha_b\Omega_1^2$ with $\alpha_b \ll 1$, dispersion relation (1.15) is reduced to

$$1 - \frac{\Omega_1^2}{\hat{\Omega}_1^2} - \frac{\alpha_b\Omega_1^2}{(\hat{\Omega}_1 - q_x\Delta V_g)^2} = 0, \quad (4.14)$$

where $\hat{\Omega}_1 = \Omega - q_x V_{g1}$ and $\Delta V_g = V_{g2} - V_{g1}$. An analysis of dispersion relation (4.14) in accordance with Section 1.5.2 of [26] shows that it describes a sort of two-stream instability. The maximum instability growth rate is achieved at

$$q_x = \Omega_1/\Delta V_g \quad (4.15)$$

and is equal to

$$(\mathrm{Im}\,\Omega)_{\max} = (3^{1/2}/2^{4/3})\alpha_b^{1/3}\Omega_1. \quad (4.16)$$

4.3. High-Intensity Monochromatic Packet and Low-Intensity Fuzzy Packet

Using dispersion relation (4.6), we can show that, for a low-intensity wave packet having a large width, dispersion relation (4.14) should be replaced by the relation

$$1 - \frac{\Omega_1^2}{\hat{\Omega}_1^2} + \frac{\alpha_b\Omega_1^2}{(q_x V'_{g2}\Delta k_x)^2}\left[1 + \frac{i\pi^{1/2}(\hat{\Omega}_1 - q_x\Delta V_g)}{|q_x V'_{g2}|\Delta k_x}\right] = 0. \quad (4.17)$$

From Section 3.2 of [26], we see that, for the q_x value determined by expression (4.15), there is an instability with the growth rate

$$\mathrm{Im}\,\Omega = \frac{\alpha_b\Omega_1}{2}\left(\frac{\Delta V_g}{V'_{g2}\Delta k_x}\right)^2. \quad (4.18)$$

This expression for the growth rate is valid for low-intensity wave packets whose width is not too small,

$$V'_{g2}\Delta k_x > \alpha_b^{1/3}\Delta V_g. \quad (4.19)$$

For packets with narrower spectra, we must use expression (4.16) rather than expression (4.18).

4.4. Low-Intensity Monochromatic Packet and High-Intensity Packet with a Large Width

In this case, in place of dispersion relation (4.17), we have

$$1 - \frac{\alpha_b\Omega_1^2}{(\Omega - q_x V_{g2})^2} + \frac{2\Omega_1^2}{(q_x V'_{g1}\Delta k_x)^2}\left(1 + \frac{i\pi^{1/2}\hat{\Omega}_1}{|q_x V'_{g1}|\Delta k_x}\right) = 0. \quad (4.20)$$

An analysis of this dispersion relation in accordance with Section 3.3 of [26] shows that it formally coincides with dispersion relation (3.19) of that paper. As a result, we can conclude that dispersion relation (4.20) describes an instability whose maximum growth rate is achieved at

$$\Delta V_g \simeq \alpha_b^{1/2}|V'_{g1}|\Delta k_x \quad (4.21)$$

and is equal to

$$(\mathrm{Im}\,\Omega)_{\max} \simeq \alpha_b^{1/2}\frac{\Delta V_g}{|V'_{g1}\Delta k_x|}\Omega_1. \quad (4.22)$$

4.5. Effect of Nonmonochromaticity on Instabilities of Zonal Flows Like the Negative-Mass Instability

4.5.1. Decrease in the growth rate of a hydrodynamic instability for relatively weak nonmonochromaticity. For an instability like the negative-mass one, we obtain from dispersion relation (1.10) not expression (4.5) but the expression

$$\hat{\Omega}^2 = -\Omega_0^2\left[1 - \frac{3}{2}\frac{q_x^2 V_{g0}'^2}{\Omega_0^2}(\Delta k_x)^2\right]. \qquad (4.23)$$

In analogy with [32], we can see that nonmonochromaticity substantially reduces the growth rate when

$$\Delta k_x \gtrsim \Omega_0/(q_x V_{g0}'). \qquad (4.24)$$

4.5.2. Kinetic instability and its suppression by increasing nonmonochromaticity. Under condition (4.24), expression (4.23) should be replaced by the expression

$$\hat{\Omega} = \frac{i}{\pi^{1/2}}|q_x V_{g0}'|\Delta k_x\left[1 - \frac{1}{2}\frac{(q_x V_{g0}'\Delta k_x)^2}{\Omega_0^2}\right], \qquad (4.25)$$

which describes the kinetic instability of zonal flows. This instability is suppressed when

$$\Delta k_x > 2^{1/2}\Omega_0/(q_x V_{g0}'). \qquad (4.26)$$

5. DISCUSSION OF THE RESULTS

Our analysis shows that the basic approach underlying the existing theory of the generation of zonal flows by KAWs is too simplified because the results obtained for a monochromatic packet of KAWs are considered as covering the entire pattern of the generation of the flows. The main question discussed in the existing theory was that of whether such a wave packet can generate zonal flows. In the context of our analysis, this question is certainly important. Nevertheless, when a monochromatic wave packet is stable, it is necessary to examine a more complicated situation with a double-peak packet, the simplest case of which is a system of two pump waves. Hence, investigation can be carried out in two directions. If a monochromatic wave packet is found to be unstable, then the investigation should be terminated (or continued with examining the role of the nonmonochromaticity, see below for details). If a monochromatic wave packet is found to be stable, then the investigation should be continued with examining double-peak wave packets.

The investigation performed in [16] and our analysis both demonstrate that the possibility for a monochromatic packet of KAWs to generate zonal flows depends on the ion-to-electron temperature ratio T_i/T_e. We have shown that, for small T_i/T_e values, a monochromatic packet of KAWs is stable for arbitrary k_y/k_x values and that, for small k_y/k_x values, such a packet is stable for arbitrary T_i/T_e values. Nevertheless, if the ratio k_y/k_x exceeds a critical value determined by inequality (3.32), then the wave packet at hand is unstable at finite T_i/T_e values satisfying condition (3.33). We have called the onset of the corresponding instabilities the OPS effect.

Let us compare the results of our work with the results obtained in earlier papers on the subject. The results obtained in the present paper can be seen to agree with the results of [12], which were obtained for $T_i = 0$ and for small k_y/k_x values, and with the results of [14], obtained for $T_i = 0$ and $k_y \gg k_x$. Our results also agree qualitatively with [17], in which the ratio T_i/T_e was arbitrary and it was assumed that $k_y \gg k_x$. At the same time, our results differ from those of [10, 11, 13, 17]. Let us discuss the reasons for this disagreement.

In [10, 11, 17], consideration was given to the case $k_x = 0$. It follows from Section 3 that, in this case, as well as for an arbitrary ratio between k_x and k_y, the zonal component of the vector potential, $\bar{A}_0$, is not generated, provided that the small terms on the order of $q_x^2/k_\perp^2$ are ignored (see Eq. (3.16)). For the case $k_y \gg k_x$, the same result was obtained in [14, 16] (see [14], Eqs. (25) and (26), and [16], Eqs. (27) and (28)). The point of disagreement with [10, 11, 17] in that some equations in those papers contradict Eq. (3.16). At the same time, if, in [10, 11], the effects associated with $\bar{A}_0$ (the term with M_B in those papers) were ignored, then monochromatic KAWs would be seen to be stable in accordance with the conclusion that was reached in our study and in [14].

In [13], an analysis was carried out for $T_i = 0$ and for an arbitrary ratio between k_x and k_y. Taking the limit $\rho_i^2 \longrightarrow 0$ in Eq. (2.1), we can see that the basic equations of [13] coincide with Eqs. (2.1)–(2.3) in the particular case $T_i = 0$. The expression for the amplitudes of the satellites $\hat{\phi}_\pm$ (see [13], expression (13)) coincides with expression (3.18) with $T_i = 0$. Paper [13] contains neither expression for the amplitudes of the satellites of the vector potential, $\hat{A}_\pm$, nor expression analogous to (3.19), but at the same time, makes use of an approximate relationship between the total vector and electrostatic potentials, A and ϕ (see [13], relationship (10)). We can see that the expression for $\hat{A}_\pm$ that follows from relationships (10) and (13) of [13] differs from expression (3.19) with $T_i = 0$. This is why we think that our disagreement with [13] stems from the fact that the expression for A used in that paper (see [13], expression (10)) was too simplified.

In considering KAWs with a double-peak spectrum, we have revealed a new class of instabilities of zonal flows that are analogous to linear two-stream instability (see Section 4). These instabilities can be called two-stream instabilities of zonal flows. They can also be revealed in problems concerning other types of primary modes, such as highly dispersive electrostatic waves and Rossby waves.

Let us now consider the problem of how to choose the basic equations with which to describe nonlinear KAWs. The most nontrivial aspect of this problem is how to describe ion dispersion, both linear and nonlinear (see the term with ρ_i^2 in Eq. (2.1)). In [34], it was shown that this effect can be described correctly by the Grad hydrodynamics [38], whereas the Braginskii hydrodynamics [39] yields the numerical factor 1 in place of the factor 3/4 in Eq. (2.1) (see also [40]). It should be noted that the replacement $3/4 \longrightarrow 1$, which corresponds to the Braginskii hydrodynamics, transforms the basic vorticity equation (2) from [17] to Eq. (2.1). This is why dispersion relation (16) for primary modes from [17] differs from the standard relation, i.e., dispersion relation (1.1).

There are at least two approaches that provide a correct description of ion dispersion in the vorticity equation. One of them is the so-called dispersive ion-drift hydrodynamics, which was developed in [36]. The other is the heuristic approach of [41, 42], which was developed based on the idea of expressing the perturbed ion density n through the electrostatic potential ϕ by means of the Boltzmann kinetic equation in the linear approximation and substituting the result into the ion continuity equation. This heuristic approach was used in [16]. It was shown in [34] that, for $k_\perp^2\rho_i^2 \ll 1$, this approach yields the same vorticity equation as in the Grad hydrodynamics, namely, Eq. (2.1).

In accordance with what was said in the Introduction, the above approaches to describing nonlinear KAWs at a finite ion temperature were originally formulated mainly in connection with the problem of Alfvén vortices (the approach used in [41, 42] was developed in connection with the anomalous heat transport problem). At the same time, the aforementioned problem of the decay of an individual KAW into two KAWs at $T_i \neq 0$ should be solved with allowance for ion dispersion, both linear and nonlinear. In [28], one more approach to deriving nonlinear equations for KAWs was developed in order to study the same problem, specifically, the approach based on direct solution of the nonlinear Boltzmann kinetic equation. With this approach, it is a fairly complicated matter to obtain the term having the numerical factor 3/4 in Eq. (2.1). On the other hand, it may well be that, with Eqs. (2.1)–(2.3), the analysis by Voitenko [28] of the aforementioned decay of an individual KAW into two KAWs at $T_i \neq 0$ would be greatly simplified. The construction of a simplified scheme for analyzing this decay process may be the subject of ongoing investigations.

The idea of [40, 41] to use the linear relationship between the perturbed ion density n and the electrostatic potential ϕ was exploited in [14] for the case $T_i = 0$. This is why, in [14, 16], there are only two basic equations relating the functions ϕ and A, rather than three equations (2.1)–(2.3). Our analysis shows that this idea is legitimate.

Our expressions (3.18) and (3.19) for the amplitudes of the satellites $\hat{\phi}_\pm$ and $\hat{A}_\pm$ differ from the corresponding expressions in [14, 16] (see [14], expressions (14)–(17), and [16], expressions (16)–(19)). Note also that the expressions for $\hat{\phi}_\pm$ in [14, 16] differ from the corresponding expressions in [13] (see, e.g., the aforementioned expression (14) from that paper). We think that the results of [14, 16] are not quite correct and, accordingly, the OPS effect in [16] was calculated inexactly (see Section 3 for details).

We have shown that zonal flows with $k_x \gg k_y$ are described by dispersion relation (1.13) with the function $F(\mathbf{k})$ given by expression (3.31) and that zonal flows with an arbitrary ratio $k_x^2/k_\perp^2$ are described by the same dispersion relation but with the function given by more general expression (3.30). The function $F(\mathbf{k})$ given by expression (3.31) is positive definite for an arbitrary ratio T_i/T_e, but the general expression (3.30) for this function is negative for the values of k_y^2/k_x^2 satisfying inequality (3.32). This inequality can hold only for finite T_i/T_e values determined by condition (3.33). Physically, condition (3.32) describes the onset of instabilities of zonal flows that are analogous to the negative-mass instability rather than to the two-stream instabilities, which occur at $k_y \longrightarrow 0$. In Section 3, this phenomenon has been called the OPS effect.

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APPENDIX

Auxiliary Relationships for Calculating the Amplitudes of the Satellites

The quantities $Z_\pm^{ik}$ $(i, k = \phi, A, n)$ in expressions (3.5)–(3.7) are given by the relationships

$$Z_\pm^{\phi\phi} = k_\perp^2\left[\frac{\omega_\pm^2}{k_{\perp\pm}^2}(1-\delta_i) - v_A^2 k_z^2 \rho_s^2\right]\tilde{\phi}_\pm \pm \omega_\pm \frac{k_z v_A^2}{c}\tilde{A}_\pm - k_z^2 v_A^2 \frac{T_e}{en_0}\tilde{n}_\pm, \tag{A.1}$$

$$Z_\pm^{\phi A} = -\omega_\pm \frac{v_A^2}{c^2}\left[\frac{k_\perp^2}{k_{\perp\pm}^2}\omega_\pm \tilde{A}_\pm \pm ck_z\left(\tilde{\phi}_\pm - \frac{T_e}{en_0}\tilde{n}_\pm\right)\right], \tag{A.2}$$

$$Z_\pm^{\phi n} = \frac{k_z^2 v_A^2 T_e}{en_0}\left(\tilde{\phi}_\pm \mp \frac{\omega_\pm}{ck_z}\tilde{A}_\pm\right), \tag{A.3}$$

$$Z_\pm^{A\phi} = ck_z \frac{k_\perp^2}{k_{\perp\pm}^2}(1-\delta_i)\tilde{\phi}_\pm \pm \omega_\pm\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\tilde{A}_\pm - \frac{ck_z T_e}{en_0}\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\tilde{n}_\pm, \tag{A.4}$$

$$Z_\pm^{AA} = -\left\{\pm\omega_\pm\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\left(\tilde{\phi}_\pm - \frac{T_e}{en_0}\tilde{n}_\pm\right) + \frac{v_A^2}{c}k_z k_\perp^2\left[\frac{1}{k_{\perp\pm}^2} + \rho_s^2\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\right]\tilde{A}_\pm\right\}, \tag{A.5}$$

$$Z_\pm^{An} = \frac{T_e}{en_0}\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)(\mp\omega_\pm \tilde{A}_\pm + ck_z\tilde{\phi}_\pm), \tag{A.6}$$

$$Z_\pm^{n\phi} = -\frac{c^2 k_z^2 k_\perp^2}{4\pi e}(1-\delta_i)\tilde{\phi}_\pm \mp \omega_\pm \frac{ck_z k_{\perp\pm}^2}{4\pi e}\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\tilde{A}_\pm + \left[\omega_\pm^2\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right) - k_z^2 v_A^2\right]\tilde{n}_\pm, \tag{A.7}$$

$$Z_\pm^{nA} = \frac{\omega_\pm}{4\pi e}\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right) \times \left[\omega_\pm k_\perp^2 \tilde{A}_\pm \pm ck_z k_{\perp\pm}^2\left(\tilde{\phi}_\pm - \frac{T_e}{en_0}\tilde{n}_\pm\right)\right], \tag{A.8}$$

$$Z_\pm^{nn} = \pm\omega_\pm \frac{ck_z k_{\perp\pm}^2 T_e}{4\pi e^2 n_0}\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right)\tilde{A}_\pm - \left[\omega_\pm^2\left(1 - \frac{3}{4}\rho_i^2 k_{\perp\pm}^2\right) - k_z^2 v_A^2\right]\tilde{\phi}_\pm. \tag{A.9}$$

The general expressions for the leading-order amplitudes of the satellites (3.5)–(3.7) have the form

$$\hat{\phi}_\pm^{(0)} = \pm\frac{icq_x k_y}{2B_0\omega^2 D^{(0)}}(Z_\pm^{\phi\phi(0)}\bar{\phi}_0 + Z_\pm^{\phi A(0)}\bar{A}_0 + Z_\pm^{\phi n(0)}\bar{n}_0), \tag{A.10}$$

$$\hat{A}_\pm^{(0)} = \pm\frac{icq_x k_y}{2B_0\omega D^{(0)}}(Z_\pm^{A\phi(0)}\bar{\phi}_0 + Z_\pm^{AA(0)}\bar{A}_0 + Z_\pm^{An(0)}\bar{n}_0), \tag{A.11}$$

$$\hat{n}_\pm^{(0)} = \pm\frac{icq_x k_y}{2B_0\omega^2 D^{(0)}}(Z_\pm^{n\phi(0)}\bar{\phi}_0 + Z_\pm^{nA(0)}\bar{A}_0 + Z_\pm^{nn(0)}\bar{n}_0). \tag{A.12}$$

Relationships (A.1)–(A.9) yield

$$(Z_\pm^{\phi\phi(0)}, Z_\pm^{\phi A(0)}, Z_\pm^{\phi n(0)}) = 2k_z^2 v_A^2 \tilde{\phi}_\pm(1+\delta_e)\left(1, -\frac{\omega}{ck_z}, -\frac{1}{2}\frac{T_e}{en_0}\delta_e\right), \tag{A.13}$$

$$(Z_\pm^{A\phi(0)}, Z_\pm^{AA(0)}, Z_\pm^{An(0)}) = 2ck_z\tilde{\phi}_\pm(1+\delta_e-\delta_i)\left(1, -\frac{\omega}{ck_z}, -\frac{1}{2}\frac{T_e}{en_0}\delta_e\right), \tag{A.14}$$

$$(Z_\pm^{n\phi(0)}, Z_\pm^{nA(0)}, Z_\pm^{nn(0)}) = -\frac{c^2 k_z^2 k_\perp^2 \tilde{\phi}_\pm}{2\pi e}(1+\delta_e-\delta_i)\left(1, -\frac{\omega}{ck_z}, 0\right). \tag{A.15}$$

Here, we have ignored the small quantity $Z_\pm^{nn(0)}$, which is unimportant for our calculations.

Substituting relationships (A.13)–(A.15) into relationships (A.10)–(A.12), we arrive at expressions (3.11)–(3.13).

REFERENCES

1. V. N. Oraevskii, in *Basic Plasma Physics*, Ed. by A. A. Galeev and R. N. Sudan (North-Holland, Amsterdam, 1984; Énergoatomizdat, Moscow, 1983, 1984), Vol. 2.
2. P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Controlled Fusion **47**, R35 (2005).
3. A. B. Mikhailovskii, A. I. Smolyakov, E. A. Kovalishen, et al., Phys. Plasmas **13**, 032502 (2006).

4. A. B. Mikhailovskii, A. I. Smolyakov, E. A. Kovalishen, et al., Phys. Plasmas **13**, 052 516 (2006).
5. A. Hasegawa and L. Chen, Phys. Rev. Lett. **35**, 370 (1975).
6. A. B. Mikhailovskii and L. I. Rudakov, Zh. Éksp. Teor. Fiz. **44**, 912 (1963) [Sov. Phys. JETP **17**, 621 (1963)].
7. A. B. Mikhaĭlovskiĭ, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1965), Vol. 3.
8. A. I. Smolyakov, P. H. Diamond, and Y. Kishimoto, Phys. Plasmas **9**, 3826 (2002).
9. G. G. Craddock and P. H. Diamond, Phys. Rev. Lett. **67**, 1535 (1991).
10. P. N. Guzdar, R. G. Kleva, and P. K. Kaw, Phys. Rev. Lett. **87**, 015 001 (2001).
11. P. N. Guzdar, R. G. Kleva, A. Das, and P. K. Kaw, Phys. Plasmas **8**, 3907 (2001).
12. V. P. Lakhin, Fiz. Plazmy **29**, 157 (2003) [Plasma Phys. Rep. **29**, 137 (2003)].
13. P. K. Shukla, Phys. Plasmas **12**, 012310 (2005).
14. O. P. Pokhotelov, O. G. Onishchenko, R. Z. Sagdeev, et al., J. Geophys. Res. **109**, A03305 (2004).
15. R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Zh. Éksp. Teor. Fiz. **27**, 361 (1978) [JETP Lett. **27**, 340 (1978)].
16. O. G. Onishchenko, O. A. Pokhotelov, R. Z. Sagdeev, et al., J. Geophys. Res. **109**, A03 306 (2004).
17. P. H. Guzdar, R. G. Kleva, and N. Chakrabarti, Phys. Plasmas **11**, 3324 (2004).
18. A. A. Galeev and R. Z. Sagdeev, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Atomizdat, Moscow, 1973; Consultants Bureau, New York, 1979), Vol. 7.
19. R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Fiz. Plazmy **4**, 551 (1978) [Sov. J. Plasma Phys. **4**, 306 (1978)].
20. A. B. Mikhailovskii, *Electromagnetic Instabilities in an Inhomogeneous Plasma* (Énergoatomizdat, Moscow, 1991; Institute of Physics, Bristol, 1992).
21. O. P. Pokhotelov, O. G. Onishchenko, R. Z. Sagdeev, and R. A. Treumann, J. Geophys. Res. **108**, A1291 (2003).
22. O. G. Onishchenko, O. A. Pokhotelov, M. A. Balikin, et al., Phys. Plasmas **11**, 4954 (2004).
23. A. I. Smolyakov, P. H. Diamond, and V. I. Shevchenko, Phys. Plasmas **7**, 1349 (2000).
24. A. B. Mikhailovskii, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Atomizdat, Moscow, 1972; Consultants Bureau, New York, 1975), Vol. 6.
25. N. S. Erokhin and S. S. Moiseev, Private communication (1968).
26. A. B. Mikhailovskii, *Theory of Plasma Instabilities* (Atomizdat, Moscow, 1971; Consultants Bureau, New York, 1974), Vol. 1.
27. N. S. Erokhin, S. S. Moiseev, and V. V. Mukhin, Fiz. Plazmy **4**, 1172 (1978) [Sov. J. Plasma Phys. **4**, 656 (1978)].
28. Y. M. Voitenko, J. Plasma Phys. **60**, 497 (1998).
29. Y. M. Voitenko and M. Goossens, Astron. Astrophys. **357**, 1073 (2000).
30. Y. M. Voitenko and M. Goossens, J. Geophys. Res. **110**, A01S01 (2005).
31. A. A. Vedenov, A. V. Gordeev, and L. I. Rudakov, Plasma Phys. **9**, 119 (1967).
32. A. B. Mikhailovskii, A. I. Smolyakov, E. A. Kovalishen, et al., Phys. Plasmas, **13**, 042507 (2006).
33. V. P. Lakhin, A. B. Mikhailovskii, and O. G. Onishchenko, Fiz. Plazmy **13**, 188 (1987) [Sov. J. Plasma Phys. **13**, 104 (1987)].
34. V. P. Lakhin, A. B. Mikhailovskii, and A. I. Smolyakov, Zh. Éksp. Teor. Fiz. **92**, 1601 (1987) [Sov. Phys. JETP **65**, 898 (1987)].
35. A. B. Mikhailovskii, V. P. Lakhin, G. D. Aburdzhaniya, et al., Plasma Phys. Controlled Fusion **29**, 1 (1987).
36. V. P. Lakhin, S. V. Makurin, A. B. Mikhailovskii, and O. G. Onishchenko, J. Plasma Phys. **38**, 387 (1987).
37. V. D. Shafranov, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1965), Vol. 3.
38. H. Grad, Commun. Pure Appl. Math. **2**, 311 (1949).
39. S. I. Braginskii, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1965), Vol. 1.
40. V. P. Lakhin, Plasma Phys. Controlled Fusion **46**, 877 (2004).
41. B. B. Kadomtsev and O. P. Pogutse, Pis'ma Zh. Éksp. Teor. Fiz. **39**, 225 (1984) [JETP Lett. **39**, 269 (1984)].
42. B. B. Kadomtsev and O. P. Pogutse, in *Nonlinear and Turbulent Processes in Physics*, Ed. by R. Z. Sagdeev (Harwood, New York, 1984), Vol. 3, p. 257.

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