Viscous relaxation of drift-Alfvén waves in tokamaks and its application for triggering improved confinement regimes

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Viscous damping of the drift-Alfvén modes in tokamaks and its application for triggering improved confinement regimes are considered. It is shown that these waves are effectively damped by the neoclassical viscosity with the upper limit for their frequency ω equal to the collision frequency of slow ions ν_i/ϵ, where ϵ is the inverse aspect ratio. Quasistationary plasma poloidal and toroidal velocities and current drive produced by damping of these waves are estimated. Evaluations of the viscosity effects on the Alfvén waves which were induced by the dynamic ergodic divertor in the Tokamak Experiment for Technology Oriented Research tokamak [K. H. Finken *et al.*, Phys. Rev. Lett. **94**, 015003 (2005)] are made. © *2007 American Institute of Physics*.
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Triggering a transition to improved confinement regimes in tokamaks (see Ref. 1, and references therein) by external sources of rf waves[2] is attractive because of the flexibility of this method in allowing the waves to be launched on arbitrary magnetic surfaces, control over the profiles of launched and absorbed power, and the possibility of choosing the required wave power to trigger the transition. This is to be contrasted with triggering by zonal flows,[3] which have to rely on turbulence or indirect coupling to be excited. The transition can be realized via induced poloidal, $U_{i\theta}$, and/or toroidal, $U_{i\zeta}$, plasma (ion) flows to create sheared cross-field velocity, $V_E=cE_r/B_0$ (E_r is the radial electric field, B_0 is equilibrium magnetic field, and c is the speed of light), through the relation

$$E_r \approx \frac{B_0}{c}\left(-U_{i\theta}+\frac{\epsilon}{q}U_{i\zeta}+U_{pi}\right). \tag{1}$$

Here $\epsilon=r/R$ is the inverse aspect ratio, R is the tokamak major radius, q is the safety factor, $U_{pi}=(c/e_in_0B_0)\partial p_i/\partial r$ is the ion drift velocity related to the radial ion pressure gradient, $\partial p_i/\partial r$, and n_0 is the equilibrium plasma number density.

Low-frequency (LF) magnetohydrodynamic modes, with the characteristic frequency ω of the order of $\omega \lesssim v_{Ti}/qR$ ($v_{Ti}=\sqrt{2T_i/M_i}$ is the ion thermal velocity, T_i and M_i are the ion temperature and mass, respectively), e.g., drift-Alfvén modes,[4] are of special interest[5] as they can be absorbed in the tokamak plasma due to the neoclassical viscosity in different collisionality regimes.[6–10] In Ref. 5 it was demonstrated that low-frequency modes induced by the dynamic ergodic divertor (DED) can be efficient to induce toroidal plasma rotation in tokamaks. However, an absorption mechanism of these waves related to the neoclassical viscosity was not discussed in this reference. The perpendicular viscosity effect on plasma rotation was discussed in Ref. 11 in relation to analogous experiments, but this effect was weak.

Initially, we obtain the dispersion relation of the drift-Alfvén waves taking into account viscous damping and find the intervals of plasma parameters for which this mechanism is effective. In the sequel we estimate the radial localization of the absorption region of these modes for the neoclassical viscosity. The next problem considered is to calculate the time- and magnetic-surface-averaged LF forces that can induce poloidal and toroidal plasma flows sufficient for triggering transition to improved confinement regimes.

Here we follow the one-fluid approach in all collisionality regimes (see, e.g., Ref. 12). Thus our basic equations are the current continuity equation[13]

$$\nabla \cdot \mathbf{j}=0, \tag{2}$$

the ion continuity equation

$$\frac{\partial n}{\partial t}+\nabla \cdot (n\mathbf{V}_{i\perp})=0, \tag{3}$$

and the heat balance equation

$$\frac{3}{2}\frac{\partial p_i}{\partial t}+\frac{5}{2}p_i \nabla \cdot \mathbf{V}_{i\perp}+\nabla \cdot \mathbf{q}_i^{\wedge}=0. \tag{4}$$

The perpendicular component of the current density is given by

$$\mathbf{j}_\perp=\frac{c}{B}\left[\mathbf{h}\times\left\{M_in\frac{d\mathbf{V}_{i\perp}}{dt}+\nabla p_\perp+(\nabla\cdot\boldsymbol{\pi})_\perp\right\}\right], \tag{5}$$

where

$$p_\perp=p-\tfrac{1}{2}\pi_\parallel, \tag{6}$$

is the perpendicular pressure,

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$$(\nabla \cdot \boldsymbol{\pi})_{\perp} = -\mathbf{F}^{\pi} + \nabla \cdot \boldsymbol{\pi}^{\wedge}, \tag{7}$$

$p=p_i+p_e$, and $\pi_\parallel$ is the parallel viscosity. The perpendicular ion velocity is used in the form[13]

$$\mathbf{V}_{i\perp} = \frac{c}{B}[\mathbf{E} \times \mathbf{h}] + \frac{c}{e_i n B}[\mathbf{h} \times \nabla p_i] \tag{8}$$

and

$$\mathbf{q}_i^{\wedge} = \frac{5}{2}\frac{cnT_i}{e_i B}[\mathbf{h} \times \nabla T_i]. \tag{9}$$

For axially symmetric magnetic confinement configuration, the viscous force related to the parallel viscosity, $\pi_\parallel$, is

$$\mathbf{F}^{\pi} = \tfrac{3}{2}\pi_\parallel \nabla \ln B \tag{10}$$

with the magnetic surface averaged poloidal

$$F_\theta^\pi = -\gamma_\theta M_i n (U_{i\theta} - \kappa U_{Ti}) \tag{11}$$

and parallel

$$F_\parallel^\pi = \frac{\epsilon}{q} F_\theta^\pi \tag{12}$$

components; $U_{Ti}=U_{pi}\kappa_T/\kappa_p$, where $\kappa_p=\partial \ln p_i/\partial r$ and $\kappa_T=\partial \ln T_i/\partial r$ and $\pi^\wedge$ is the gyroviscosity.[13–15]

In Eqs. (11) and (12) the values γ_θ and κ are equal to[16,17]

$$\gamma_\theta = \frac{3}{4}0.96\frac{v_{Ti}^2}{\nu_i R^2}; \quad \kappa = -1.83 \quad \text{for} \quad \nu_i > \{v_{Ti}/qR, \omega\} \tag{13}$$

in the Pfirsch-Schlüter regime[18,19] and

$$\gamma_\theta = \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\frac{qv_{Ti}}{R}; \quad \kappa = -0.5 \quad \text{for}$$
$$\epsilon^{3/2}v_{Ti}/qR < \{\nu_i, \omega\} < v_{Ti}/qR \tag{14}$$

and

$$\gamma_\theta = 2.10\frac{\nu_i q^2}{\epsilon^{3/2}}; \quad \kappa = 1.17 \quad \text{for} \quad \omega < \frac{\nu_i}{\epsilon} < \epsilon^{1/2}v_{Ti}/qR \tag{15}$$

in the plateau and banana regimes, respectively.[20] Here ν_i is the ion-ion collision frequency.[13]

From Eqs. (2)–(11) renormalization of plasma inertia, $\omega \to \omega + i\gamma_\theta$, following Refs. 6 and 7, leads to the dispersion relation for the drift-Alfvén waves and their effective damping

$$\omega(\omega - \omega_{pi}^*) + i\gamma_\theta(\omega - \omega_{pi}^* + \kappa\omega_{Ti}^*) - k_\parallel^2 c_A^2 = 0, \tag{16}$$

where $\omega_{pi}^*=mU_{pi}/r$ and $\omega_{Ti}^*=mU_{Ti}/r$ are the drift frequencies related to the ion pressure and ion temperature gradients, respectively, and m is the poloidal mode number in the phase factor of the perturbed quantity, $\sim \exp[-i\omega t + i(m\theta - s\zeta)]$. Here we omitted the plasma parallel untwisting effects,[8–10] which introduce terms of the order of $(1+i\epsilon^2\gamma_\theta/\omega)^{-1}$ in Eq. (16). We should also note that one more banana regime is possible,

$$\frac{\nu_i}{\epsilon} < \omega < \epsilon^{1/2}v_{Ti}/qR, \tag{17}$$

but in this regime $\gamma_\theta \sim -i\omega/\sqrt{\epsilon}$ (see Ref. 7) and there is no damping in Eq. (16). Thus, the effective damping of the drift-Alfvén waves by the neoclassical viscosity is only possible for drift-Alfvén wave frequencies $\omega < \nu_i/\epsilon$.

Next we estimate the spatial region of drift-Alfvén wave damping. For this, we first calculate components of the permittivity tensor. From Eqs. (3) and (4) we find for perturbed values

$$\frac{n'}{n_0} = \frac{\kappa_n}{\kappa_p}\frac{p_i'}{p_{i0}} = \frac{\kappa_n}{\kappa_p}\frac{p'}{p_0} = \frac{\kappa_n}{\kappa_T}\frac{T_i'}{T_{i0}} = -\frac{\kappa_n}{\omega}\frac{icE_\theta'}{B_0}. \tag{18}$$

In addition, here we assume $p_i'=p_e'$ and $T_{i0}=T_{e0}$. Substituting these relations into Eq. (5), we arrive at the permittivity tensor components

$$\varepsilon_{rr} = 1 + \frac{c^2}{c_A^2}\left[1 - \frac{\omega_{pi}^*}{\omega} + \frac{i\gamma_\theta}{\omega}\left(1 - \frac{\omega_{pi}^*}{\omega} + \kappa\frac{\omega_{Ti}^*}{\omega}\right)\right], \tag{19}$$

$$\varepsilon_{\theta\theta} = 1 + \frac{c^2}{c_A^2}\left(1 - \frac{\omega_{pi}^*}{\omega}\right), \quad \varepsilon_{r\theta} = -\varepsilon_{\theta r} = -i\frac{\omega_p^*\omega_{ci}}{\omega^2}\frac{c^2}{c_A^2}, \tag{20}$$

where $\omega_p^*=mc/(re_i n_0 B_0)\partial p_0/\partial r$ and $p_0=p_{0i}+p_{0e}$.

Assuming that the wave frequency is real and $k_r \gg m/r$, we find an equation for the spatial absorption of the drift-Alfvén waves[21]

$$k_r^2 = \varepsilon_\parallel\left(\frac{\omega^2}{c^2} - \frac{k_\parallel^2}{\varepsilon_\perp}\right), \tag{21}$$

where

$$\varepsilon_\parallel = 1 - \frac{\omega_{pe}^2}{(\omega + i\nu_e)\omega}\left(1 - \frac{\omega_{pe}^*}{\omega} - 0.71\frac{\omega_{Te}^*}{\omega}\right), \tag{22}$$

$\omega_{pe}^*=mc/(re_e n_0 B_0)\partial p_{0e}/\partial r$ and $\omega_{Te}^*=mc/(re_e B_0)\partial T_{0e}/\partial r$ are the electron drift frequencies, $\omega_{pe}^2=4\pi n_0^2/M_e$ is the plasma frequency, and ν_e is the electron-ion collision frequency. The last term in the brackets of Eq. (22), related to the thermal force, follows from the parallel component of the electron momentum equation.[13] From Eqs. (19)–(22) it follows that these waves will be effectively absorbed due to the effect of the neoclassical viscosity only in the vicinity of the rational magnetic surface where $k_\parallel \sim \omega/c_A \sim \omega^*/c_A$. They implies that, if a magnetic island is present on this magnetic surface, there is a possibility to affect it by the drift-Alfvén waves damped on this surface.

Let us apply the obtained equations to explain the experimental results of Ref. 5. The low-frequency waves were absorbed in the vicinity of the rational magnetic surface $q=3$ in that work. In this region the plasma was collisional satisfying the inequality $\omega > \gamma_\theta$. Then we can assume that these waves transformed into the short-wave mode near the rational magnetic surface

$$\omega(\omega - \omega_{pi}^*) - k_\parallel^2 c_A^2 = 0 \tag{23}$$

and then were absorbed due to the neoclassical viscosity with the damping coefficient γ_θ according to Eq. (13).

Substituting Eq. (19) in this equation with $\varepsilon_\perp = \varepsilon_{rr}$ and taking into account that, as it follows from Eq. (13), $\gamma_\theta < \omega$, we find the characteristic region for absorption of these waves from the relation

$$\int_{r_A}^{r_A+\Delta r_A} k_r dr \approx 1. \tag{24}$$

Here r_A and Δr_A are the wave transformation point and the characteristic length of the wave absorption, correspondingly. As a result, we have from Eqs. (21) and (24)

$$\Delta r_A \lesssim \sqrt[3]{\frac{r_A}{2}\frac{c_A^2}{\omega^2}\left|\frac{\varepsilon_{rr}}{\varepsilon_\parallel}\right|}. \tag{25}$$

Using parameters of the Tokamak Experiment for Technology Oriented Research (TEXTOR) (Ref. 5) and the Tokamak Chauffage Alfvén Brazil (TCABR) (Ref. 25), we find from Eq. (25) $\Delta r_A \approx 1$ cm and $\Delta r_A \approx 0.5$ cm, correspondingly.

Let us now estimate the possible effect of the drift-Alfvén waves on poloidal and toroidal plasma velocities and, consequently, on the radial electric field. Magnetic-surface-averaged equations for the poloidal and toroidal velocities evolution in tokamaks are well-known[16,17]

$$(1+2q^2)M_i n_0 \frac{\partial U_{i\theta}}{\partial t} = F_\theta^\pi + F_\theta^h \tag{26}$$

and

$$M_i n_0 \frac{\partial U_{i\zeta}}{\partial t} = 4\epsilon q M_i n_0 \frac{\partial U_{i\theta}}{\partial t} + F_\zeta^\pi + F_\zeta^h. \tag{27}$$

In a stationary state we have

$$F_\theta^\pi + F_\theta^h = 0 \tag{28}$$

and

$$F_\zeta^\pi + F_\zeta^h = 0. \tag{29}$$

The viscous force F_θ^π has been defined by Eq. (11) and for F_ζ^π we can use the expression[22]

$$F_\zeta^\pi = \frac{\partial}{\partial r}\left\{\mu_{i\perp}\left(\frac{\partial U_{i\zeta}}{\partial r} - 0.107\frac{q^3}{\epsilon}\frac{\partial \ln T_{0i}}{\partial r}U_{i\theta}\right)\right\}, \tag{30}$$

where $\mu_{i\perp} = 6p_{0i}\nu_i/(5\omega_{ci}^2)$.

Thus time- and magnetic-surface-averaged rf forces are needed to be calculated as far as these forces can induce the abovementioned flows. These forces are also important for conventional rf current drive.[23] For this calculation, for weakly collisional regimes in tokamaks (banana and plateau regimes), a hydrodynamic approach can be utilized.[24,25] From Ref. 24 we have

$$\bar{F}_{e\parallel}^h \approx \frac{i}{4\omega}\left[\frac{1}{r}\frac{\partial}{\partial r}(j_e^{*r}E_\parallel^{ef}) + \frac{1}{qR}E_m^{ef}\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta}\right)j_e^{*m} - \text{c.c.}\right], \tag{31}$$

where

$$E_k^{ef} = E_k' + \frac{4\pi i}{\omega}\frac{\omega^2}{\omega_{pe}^2}j_{ek}'.$$

The rf force, Eq. (31), enters the magnetic surface-averaged Ohm's law[26,27]

$$\langle J_\parallel B_0\rangle = \sigma_\parallel\left(\langle E_{0\parallel}B_0\rangle - \frac{1}{e_e N_e}\langle \mathbf{B}_0\cdot(\boldsymbol{\nabla}\cdot\hat{\boldsymbol{\pi}}_{\parallel e}\rangle + \frac{1}{e_e N_e}\langle \mathbf{F}_e^h\cdot\mathbf{B}_0\rangle\right), \tag{32}$$

where

$$\bar{F}_{\alpha\parallel} = \langle F_{\alpha\parallel}B_0\rangle/B_s \tag{33}$$

and

$$\langle\cdots\rangle = \oint(\cdots)\frac{dl}{B_0}\Big/\oint\frac{dl}{B_0}. \tag{34}$$

The rf forces F_θ^h and F_ζ^h entering Eqs. (28) and (29) can be found from Ref. 25 in the simplified form

$$F_\theta^h = \frac{i}{4\omega}\sum_\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_{\alpha\theta}^{ef}j_\alpha^{*r}) - \frac{j_\alpha^{*k}}{r}\frac{\partial}{\partial\theta}E_{\alpha k}^{ef} - \text{c.c.}\right] \tag{35}$$

and

$$F_\zeta^h = \frac{i}{4\omega}\sum_\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_{\alpha\zeta}^{ef}j_\alpha^{*r}) - \frac{j_\alpha^{*k}}{R}\frac{\partial}{\partial\zeta}E_{\alpha k}^{ef} - \text{c.c.}\right], \tag{36}$$

where

$$E_{\alpha k}^{ef} = E_k' + \frac{4\pi i\omega}{\omega_{p\alpha}^2}j_{\alpha k}', \quad \alpha = i,e \tag{37}$$

and $\omega_{p\alpha}^2 = 4\pi n e_\alpha^2/M_\alpha$. These expressions can be used for calculating the quasistationary plasma poloidal and toroidal velocities, radial electric field, and current drive produced by these waves damping. We use these equations to compare with results obtained in recent experiments.[5]

In this relation, we estimate the toroidal velocity induced by the drift-Alfvén modes damping. As far as these waves are absorbed by ions, for estimations we can rewrite Eq. (36) in the simplified form

$$F_\zeta^h = \frac{s}{R\omega}P_w, \tag{38}$$

where the absorbed power

$$P_w = \tfrac{1}{4}(\mathbf{j}_i^*\mathbf{E}' + \text{c.c.}). \tag{39}$$

The viscous force, Eq. (30), we use in the form

$$F_\zeta^\pi \approx \mu_{i\perp}\frac{U_{i\zeta}}{(\Delta r_A)^2}, \tag{40}$$

where Δr_A is the characteristic size of the drift-Alfvén modes absorption region Eq. (25). Hence we have for the induced toroidal velocity

$$U_{i\zeta} \approx \frac{(\Delta r_A)^2}{\mu_{i\perp}}\frac{s}{R\omega}P_w. \tag{41}$$

Exploring the relation

$$P_w = \frac{\omega}{8\pi}\,\text{Im}\,\varepsilon_{rr}|E_r'|^2, \tag{42}$$

where

$$\mathrm{Im}\,\varepsilon_{rr} = \frac{c^2}{c_A^2}\frac{\gamma_\theta}{\omega}\left(1 - \frac{\omega_{pi}^*}{\omega} + \kappa\frac{\omega_{Ti}^*}{\omega}\right), \tag{43}$$

and assuming $\omega \approx \omega_{pi}^*$, it is useful to represent Eq. (28) in the form

$$U_{i\theta} \approx \kappa U_{Ti}\left(1 + \frac{c^2}{2U_{pi}^2 B_0^2}|E_r'|^2\right) = \kappa U_{Ti}\left(1 + \frac{|v_\theta'|^2}{2U_{pi}^2}\right). \tag{44}$$

Here

$$v_\theta' = \frac{c}{B_0}E_r'. \tag{45}$$

Estimating

$$E_r' \simeq \frac{c_A}{c}B_\theta', \tag{46}$$

we find for TEXTOR,[5] with $B_\theta' \approx 20$ G,

$$v_\theta' = 10^6 \text{ cm/s} \gtrsim U_{pi}. \tag{47}$$

We note that, from Eq. (44) it follows that the viscous coefficient γ_θ disappears from the expression for the poloidal velocity.

From Eqs. (41) and (42) we see that the toroidal velocity induced by the drift-Alfvén waves in tokamaks depends on the ratio of the poloidal and toroidal viscous coefficients

$$U_{i\zeta} \approx \kappa\frac{\gamma_\theta(\Delta r)^2 M_i n}{2\mu_{i\perp}}\frac{\kappa_T}{\kappa_p}\frac{s}{R\omega}\frac{c^2}{B_0^2}|E_r'|^2. \tag{48}$$

The last equation can be used only for qualitative evaluations.

In conclusion, viscous damping of the drift-Alfvén modes in tokamaks and its application for triggering improved confinement regimes are considered. It is shown that these waves are effectively damped by the neoclassical viscosity with the upper limit for their frequency ω equal to the collision frequency of slow ions ν_i/ϵ, where ϵ is the inverse aspect ratio. Quasistationary plasma poloidal and toroidal velocities and current drive produced by these waves damping are estimated. Estimations of the viscosity effects on the Alfvén waves which were induced by the dynamic ergodic divertor in TEXTOR tokamak are made.

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