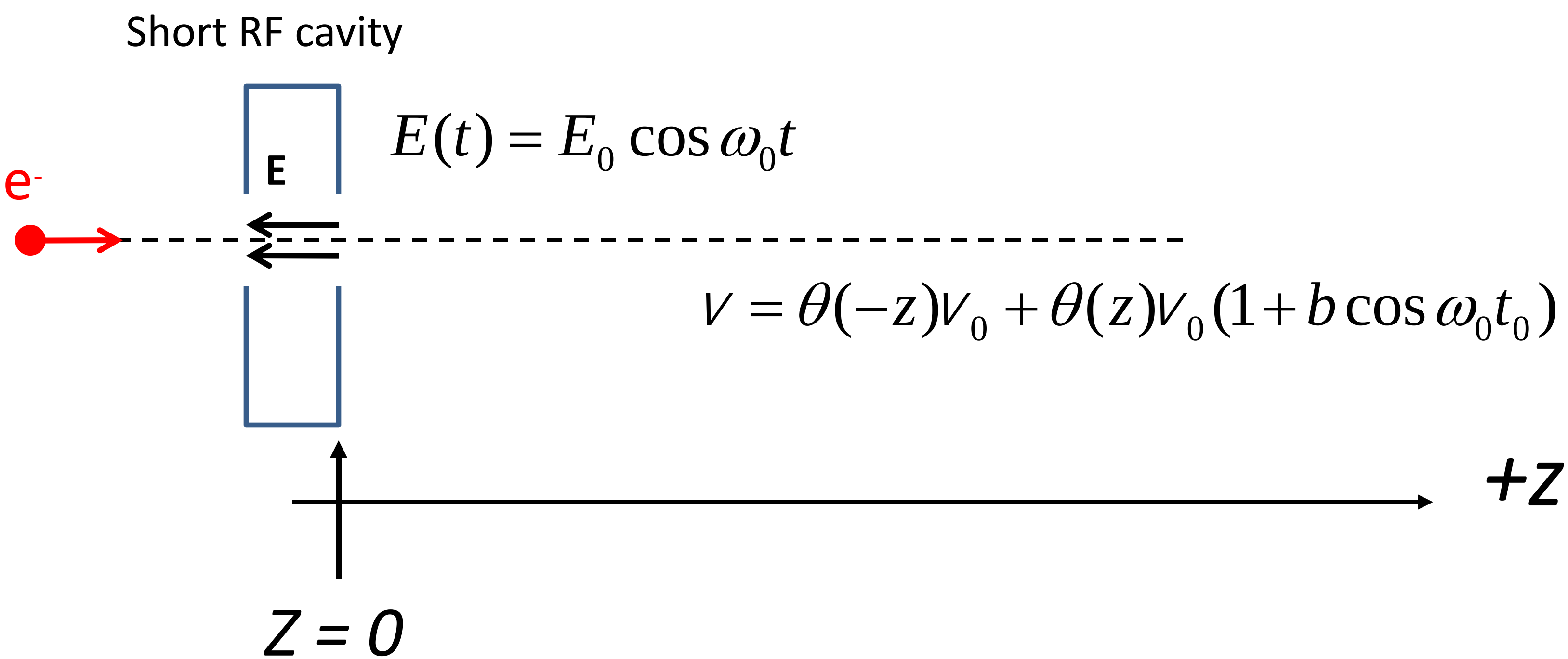
Velocity modulation and bunching of an electron beam

Electron velocity modulator



$$E(t) = E_0 \cos \omega_0 t$$

$$v = \theta(-z) v_0 + \theta(z) v_0 (1 + b \cos \omega_0 t_0)$$

t_0 = the time at which the electron exits the short cavity at z =0

b = modulating parameter ω_0 = angular frequency

Time for electron to move from z = 0 to z = z

$$t(z,t_0) = t_0 + \frac{1}{V_0}\int_0^z \frac{dz''}{(1+b\cos\omega_0 t_0)} = t_0 + \frac{z}{V_0(1+b\cos\omega_0 t_0)}$$

For $b \ll 1$

$$t(z,t_0) \cong t_0 + \frac{z}{V_0}(1-b\cos\omega_0 t_0)$$

Beam current

$$I(z,t,t_0) = \sum_{n=-\infty}^{\infty} I_n(z,t_0) e^{-in\omega_0 t} dt$$

$$I_n(z,t_0) = \frac{\omega_0}{2\pi} \int_{-T_0/2}^{T_0/2} I(z,t,t_0) e^{in\omega_0 t} dt$$

Beam current is periodic in time with period $T_0 = 2\pi/\omega_0$ and can be represented by a Fourier series

Construction of the beam current generated by a single electron

$$J_z = \boldsymbol{V}(t)\rho(z,t) = |e|\boldsymbol{V}(t)\delta(x)\delta(y)\delta(z - z_0(t))$$

$$I(z,t,t_0) = \int dxdy|e|\boldsymbol{V}(t)\delta(x)\delta(y)\delta(z - z_0(t))$$

$$I(z,t,t_0) = |e|\boldsymbol{V}(t)\delta(z - z_0(t))$$

$$\delta(z - z_0(t)) = \frac{\delta(t - t(z_0, t_0)}{v_0(1 + b\cos\omega_0 t_0)}$$

Fourier components of the electron current

$$I_n(z,t_0)=\frac{\omega_0}{2\pi}\int_{-T_0/2}^{T_0/2}|e|\delta(t-t(z,t_0))e^{in\omega_0 t}dt$$

$$I_n(z,t_0)=\frac{\omega_0|e|}{2\pi}e^{in\omega_0 t(z,t_0)}$$

$$I_n(z,t_0)=\frac{\omega_0|e|}{2\pi}e^{in\omega_0\left[t_0+\frac{z}{V_0}-\frac{zb}{V_0}\cos\omega_0 t_0\right]}$$

$$I_n(z,t_0)=\frac{\omega_0|e|}{2\pi}e^{in\frac{z\omega_0}{V_0}}e^{in\left[\theta-\frac{zb\omega_0}{V_0}\cos\theta\right]}$$

Bessel Function

Using

$$e^{im(\theta-\alpha\cos\theta)} = \sum_{s=-\infty}^{\infty} J_s(s\alpha)(-1)^s e^{i(m-s)\theta} \qquad \alpha = \frac{zb\omega_0}{V_0}$$

$$I_n(z,t_0) = \frac{\omega_0 |e|}{2\pi} e^{in\frac{z\omega_0}{V_0}} \sum_{-\infty}^{\infty} J_s(s\alpha)(-1)^s e^{i(n-s)\theta}$$

Time dependence of average current at position z

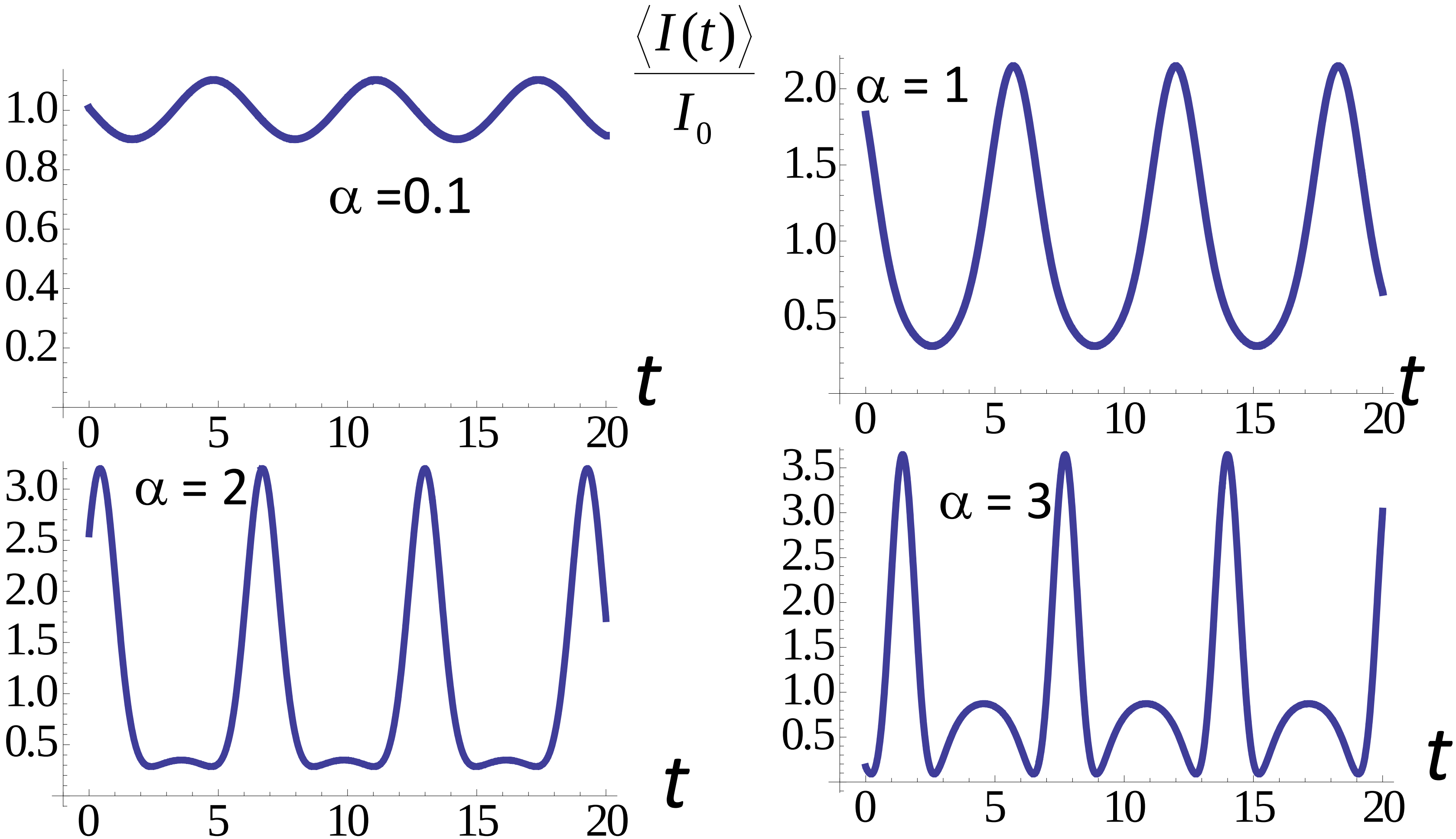
Average current over one RF period of a uniformly distributed electron beam

$$\frac{1}{2\pi}\int_{-2\pi/\omega_0}^{2\pi/\omega_0} e^{i(n-s)\theta}\,d\theta = \delta_{n,s}$$

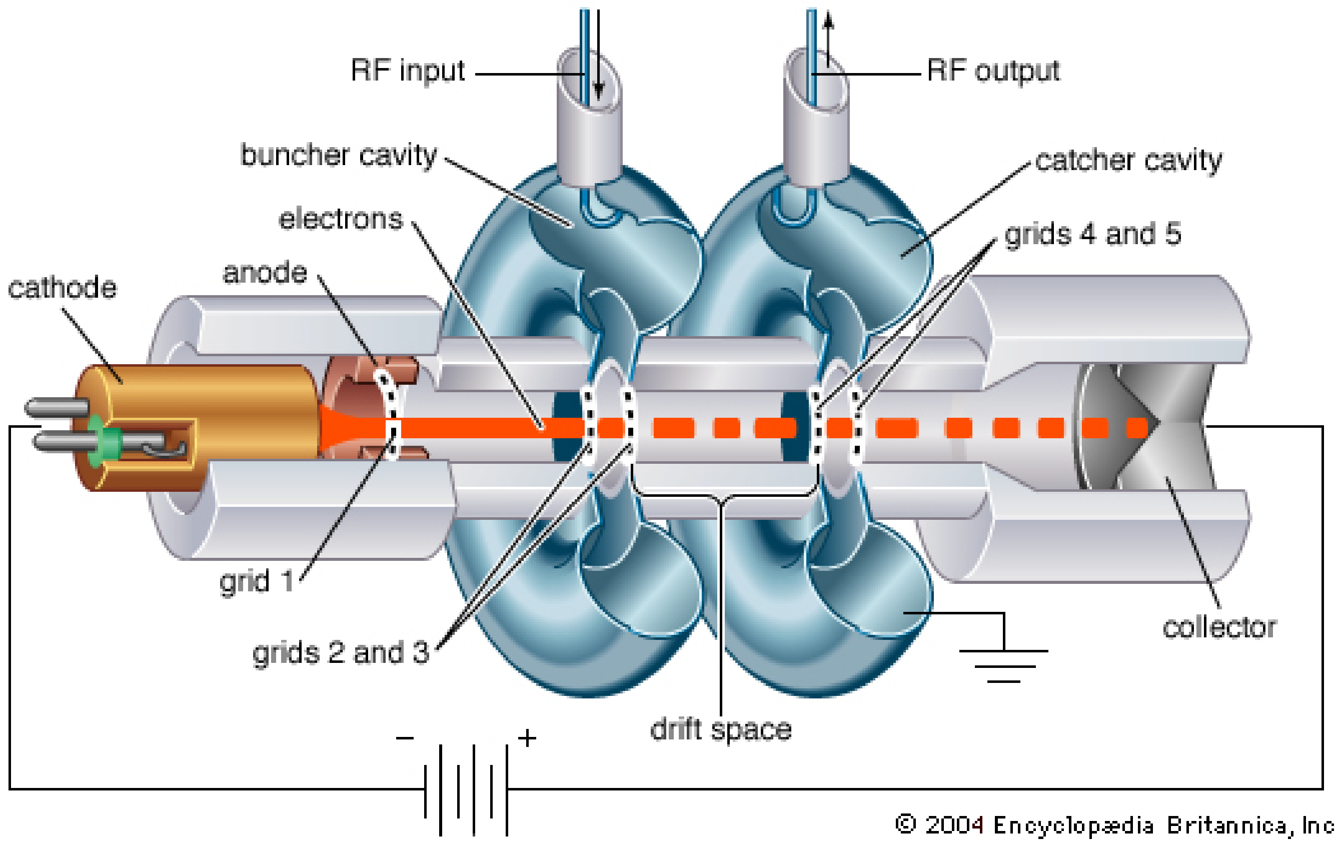
$$\left\langle I_n(z,t_0)\right\rangle_{t_0} = \frac{\omega_0|e|}{2\pi} e^{in\frac{z\omega_0}{V_0}} \sum_{-\infty}^{\infty} J_s(s\alpha)(-1)^s \qquad \alpha = \frac{zb\omega_0}{V_0}$$

$$\left\langle I(t,z)\right\rangle = I_0\left[1 + 2\sum_{s=1}^{\infty} J_s(s\alpha)\cos\left\{s(\omega_0 t - \alpha + \frac{\pi}{2})\right\}\right]$$

Average current as a function of time



Two-cavity klystron amplifier



Coherent Radiation

With a bunched beam undulator spontaneous radiation energy and directionality can increase significantly while reducing spectral homogeneity over spontaneous radiation.