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Creativity: The moment between fascination and knowledge

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Abstract

John Keats wrote in 1819 "Beauty is truth, truth beauty – that is all Ye know on earth, and all Ye need to know". Pope Benedict XVI said in 2008 "La verita ci rende buoni, e la bonta e` vera." These wise words point to the basic triangle of creativity, Truth-Beauty-Goodness. Truth is the Utopia of Science: "What is the meaning of what I see?"; Beauty is the Utopia of Art: "What is the meaning of what I feel?"; and Goodness is the Utopia of Ethics: "What is the meaning of what I do?" The way I see them, altogether, indissolubly entangled, constitute the frame within which emerges and evolves human creativity. We may distinguish three essential steps in any act of discovery or invention. The first step consists in the fascination, the wonder caused by the sudden perception of something unexpected, inside or outside our minds, involving some sort of beauty, of elegance, of basic truth. The second step is when creativity might come in, through the analysis of what we perceived from a novel, uncompromised perspective. It is the moment when we spontaneously look for consistency between the unexpected, presumptive reality, and our mind, our interior, our psyche. The third step is when knowledge enters in scene: either objective knowledge, in which case it offers itself as a scientific new aspect of truth, or subjective knowledge, in which case it contributes to the universal feelings, the texture, the plectics of human culture. These steps, essential to the human condition, naturally incline us towards sharing this new (real or imaginary) "toy" with the others – our friends, our colleagues, our family, teachers and disciples. This is the primary source of generosity, of friendship, of goodness. In the way I perceive this complex and happy process, it is the dynamical emphasis of its various angles, its various facets, that constitutes the pedagogy for stimulating creative education in schools (Σχολέ, schole, meaning spare time, leisure) and academic institutions, where the future original, talented, rigorous and innovative scientists might naturally grow. The impact on the evolution of individuals and of human societies of such attitudes and initiatives can hardly be overestimated.

1. Introduction

Along the history of humanity, very many efforts and words have been dedicated to what appears to me as being an essential triangle, namely that formed by the concepts of Truth-Beauty-Goodness (see Figure 1). Good part of what humans have constructed and transmited to their children is based on this triangle. And sadly enough (although not surprisingly) good part of their failures and inglorious acts comes from the negation of one or more of its elements.

Since Plato, and most probably even before, the deep relationship between truth and beauty has struck humanity. John Keats wrote in 1819, "Beauty is truth, truth is beauty\ that is all Ye know on earth, and all Ye need to know."

A few years later, lonely Emily Dickinson (a reader of John Keats) shared

I died for beauty, but was scarce
Adjusted in the tomb,
In an adjoining room.
He questioned softly why I failed?
"For beauty," I replied.
"And I for truth, - the two are one;
We brethren are", he said.
And so, as kinsmen met at night,
We talked between the rooms,
Until the moss had reached our lips,
And covered up our names.

Jules Henri Poincaré wrote "*Le savant n'etudie pas la nature parce que cela est utile; il l'etudie parce qu'il y prend plaisir et il y prend plaisir parce qu'elle est belle. Si la nature n'etait pas belle, elle ne vaudrait pas la peine d'etre connue, la vie ne vaudrait pas la peine d'etre vecue*"[1]. Scientific truth tends to be closer to objective knowledge, whereas beauty has a grand component of subjectivism. If truth and beauty are almost synonyms, almost two faces of the same coin, so ought to be objective and subjective knowledges, two categories that have emerged, in virtually all languages, to characterize opposites. Strange? At first sight, surely yes! But this first impression does not really resist a deeper, more fundamental analysis. Indeed, what we call an objective fact is never totally free from a subjective background, constructed on human conventions (about space, time, structures, what is to be considered as contradictory[2], and so on). Even the most elementary notion in physics or mathematics is never totally free from some primitive, undefined concepts or ideas (e.g., the point is, since Euclid, taken to be a primitive notion in axiomatic geometry), or from some conscious or unconscious conventions. These primitive notions or these shared conventions are usually reasonable. But sometimes they can even be shocking, be it at the intellectual level or when contrasted to our daily intuition and perceptions - far outside the scales below some microns or above a few thousands of kilometers, the scales below a few milliseconds or above a few millenia that we definitively know to exist! *Επίστήμε*, the objective knowledge, never totally escapes some degree of interpretation, never scapes *Δόχα*, the subjective knowledge.

Episteme and doxa undoubtedly (or at least "beyond any reasonable doubt" as some jurists like to say) are intimately inter-twined. This is not only inevitable, it might even be seen as a marvelous, magnificent convergence. A sea is more, indescribably more, than the fortuitous confluence into condensed matter of an astronomic number of elementary particles. A sea

1 The scientist does not study nature because it is useful to do so. He studies it because he takes pleasure in it, and he takes pleasure in it because it is beautiful. If nature were not beautiful it would not be worth knowing, and life would not be worth living.

2 Wave and particle were considered excluding concepts during centuries and centuries, by Isaac Newton and James Clerk Maxwell in particular. Nowadays, because of the impressive success of quantum mechanics, university professors everywhere teach to their students that they must consider those two concepts as "two faces of the same reality". They explain to them that, in the Young experiment, a particle - an electron - simultaneously passes through two separate holes thanks to its wave nature!

is also the 'Argonauts' and the 'Golden Fleece'. It is also the western-most point of Europe, Cabo da Roca, where fly on stone the words of Luis Vaz de Camoes "*Aqui... onde a terra se acaba e o mar começa...*"[3]. This is where science meets art, where truth meets beauty. Have you ever noticed that the books carried by professors of humanities have in big letters the name of the author, and in small letters the title? And that the books about the so called hard and natural sciences are the other way around, with big letters for the title and small letters for the author? This is so because episteme reigns in hard and natural sciences, whereas it is *doxa* that reigns in humanities. But, have you ever seen a good book without having in the cover or the first page both the author and the title?

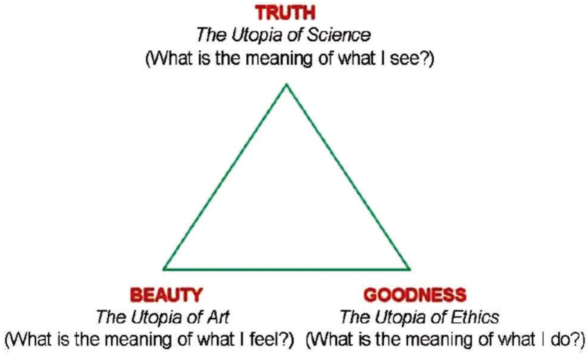

Figure 1: Truth, Beauty and Goodness: the three basic concepts involved in fascination, creativity and knowledge.

Moreover, the interpretation of *doxa* as "opinion" comes in fact from ancient Greek. In modern Greek, the meaning of doxa has evolved: it is now better translated as "glory". Your glory comes from your opinion! From your interpretation of the world, from the singular manner you integrate, for you and for the others, things, thoughts, feelings and acts into an unified and unique philosophical conception of reality! What makes the eternal glory of Socrates is not the fact that he refused to try to escape and freely accepted to drink the cicuta. These are the proofs of his courage. His highest glory, however, does not come from these circumstances. It comes from the fact that he did so because of his opinion, his intimate belief, that laws ought to be respected. That was his ultimate, his supreme lesson for his disciples, and for us.

We have up to now explored the connection between truth and beauty. What about their connection with goodness? This point is in fact addressed in the talk that Pope Benedict XVI could not present at his lecture scheduled at the University of Rome La Sapienza for the 17th January, 2008. He intended to say: "*La verità ci rende buoni, e la bontà è vera.*"[4]

The discovery (or re-discovery) of truth in nature, the contemplation of beauty, spontaneously generates the desire of sharing: some unique kind of noblesse, of generosity of the spirit, of the heart. The scientist impatiently wants to share with his close colleagues, friends and family, the new, almost unbelievable, view of reality that he has attained. The feeling of a

3 Here... where the earth comes to its end and starts the sea...
4 Truth makes us good, and goodness is true.

discovery, either sudden or gradual, is accompanied by some sort of perplexity, like some gift coming from who knows where, like some form of unmerited luck. The whole process gives the sensation that nature, or reality, or something undefined, is being very generous with you. Unavoidably, you tend to walk along the same path that has emerged in front of you, you naturally tend to being generous with others. One may try to qualify and summarize the whole process by saying that it ultimately is a contact with some form of goodness.

If we admit, as argued above, the links between Truth and Beauty, and between Beauty and Goodness, then transitivity guarantees that Truth and Goodness are one. This closes the triangle Truth-Beauty-Goodness, as indicated in figure 1. As one more pragmatical argument, let me add that, in many languages (e.g., Portuguese, Greek), several words or expressions exist which are indistinctively used to indicate that a person is intelligent, beautiful or of good character.

Still in figure 1, the word utopia (in Greek, nowhere, no-place) is repeatedly used. This demands some clarification. Utopia has at least two meanings. The first one, directly from its Greek etymology, refers to something which does not exist, something which in some sense is "out of reality" (even if, pholosophically speaking, the concept of reality itself is subject to controversy - see for instance [1]). The second meaning (an evolution from the primitive original Greek word) refers to utopia like something that we can approach more and more, something which guides our human, finite steps, but which always remains inaccessible, unattainable. Something like the sum of a series with infinite terms (the final result of which could be finite, or infinite, or oscillating, or even more complex). It is with this second meaning, of a scope never fully attained but which nevertheless guides our (inexorably finite) mouvements, that the word utopia has been introduced in the figure. In the words of Lucius Annaeus Seneca (ca. 4 BC – AD 65): "*Ignoranti quem portum petat nullus suus ventus est.*"[5]

Or in the words of the Uruguayan writer Eduardo Hughes Galeano:

> *La utopia està en el horizonte. Me acerco dos pasos, ella se aleja dos pasos. Camino diez pasos y el horizonte se desplaza diez pasos mas alla`. Por mucho que camine, nunca la alcanzarè. Para que` sirve la utopia? Para eso: sirve para caminar.*[6]

Notice, by the way, that there is some relevant difference between Seneca's and Galeano's words. In the first case, the port will one day be touched. In the second case, it will never be touched. It is not this difference that we want to emphasize in the present occasion. It is the fact that in both ways of thinking, utopia does guide you! In fact, this role of utopia in science and in the acts of scientists is quite intriguing. It frequently occurs like if the final result was achieved before the gradual steps leading to it. As if "knowing" the result was previous to "proving" the result. In the words of Alexandre Koyrè: "*La bonne physique*

5 The wind is never favorable to those who do not know where are they going to.
6 Utopia is in the horizon. I approach it two steps, it receeds two steps. I walk ten steps and the horizon moves ten steps further on. No matter how long I walk, I will never reach it. Utopia, what is it good for? Precisely for that: to walk.

se fait a priori.". In a more pragmatical sense, Galileo himself was convinced that the fact of knowing with certainty some conclusion is by no means neglectable when one wants to discover its proof.

2. Fingerprints of good science and good education

"Imagination is more important than knowledge", said Albert Einstein. In what sense would that be true? Well, it belongs to the people's wisdom that it is far better to teach how to fish than to provide with some fishes. With knowledge one can solve and handle some class of problems. With imagination and creativity we might attack, and possibly solve, several classes of problems, quite frequently including the one that we just focused on. A central question becomes, therefore, how to stimulate imagination? The royal path goes through metaphors. At this point, it is a must to quote Aristoteles. In his *Ars Poetica*, he wrote: "By far the greatest thing is to be a master of metaphor. It is the one thing that cannot be learned from others. It is a sign of genious, for a good metaphor implies an intuitive perception of similarity among dissimilars." Aristotle must have used permanently metaphors in his teaching at his School (see figure 2). Researchers and educators should never loose the oportunities of making good methaphors, either for themselves or for others!

Figure 2 Top: The entrance of the School of Aristoteles, near Thessaloniki-Greece.
Center:Aristoteles believed that the study of nature (Φΰσις) ought to be done preferentially in contact with it.
Bottom: Aristoteles and his disciples were probably sitting here to talk and rest.

Not always necessarily, but quite often we must go step by step, as if we were climbing a mountain. "*Io stimo più il trovar un vero benchè di cosa leggiera che l'disputar lungamente delle massime questioni senza conseguir verità nissuna*"[7], writes Galileo Galilei. All important things started one day as tiny little things, which did not seem particularly valuable:

Of my base Metal may be filed a Key,
That shall unlock the Door he howls without

writes Omar Khayyam (1048-1122) in 'The Rubaiyat'.

7 I esteem more to find a truth even in a light thing than to argue lengthly on the maximal questions without reaching any truth at all.

Some degree of freedom and of poetry are also fundamental - practically sine qua non ingredients - for doing creative science or technology. The words of Aristoteles are relevant at this point: "Poetry is more elevated and more philosophical than history; for poetry expresses the universal, and history only the particular. History tells us the events as they happened, whereas poetry tells them as they could or should have happened." Or those of Michel Eyquem de Montaigne (1533-1592): "*Si l'action n'a quelque splendeur de liberte, elle n'a point de grace ni d'honneur*"[8], in his Essais.

Or those of the founding father of statistical mechanics, the magnificent Austrian physicist Ludwig Eduard Boltzmann (1844-1906): "*Die Phantasie ist die Wiege der Theorie, der beobachtende Verstand ihr Erzieher.*", and of the french physicist Philippe Nozieres: "*...j'ai appris la curiosite et l'enthousiasme, une certaine forme de reve et de fantaisie aussi, sans lesquels il n'est pas de vraie recherche.*" [9]

Courage and determination, some form of self-confidence (not the arrogant one, but the audacious one), must be cultivated as well, by educators, students, researchers - or should I say everybody? Galileo writes, in his *Dialogo dei massimi sistemi*:

> *Simplicio: Che dunque voi non n'avette fatte cento, non che una prova, e l'affermate così francamente per sicura?*
>
> *Salviati: Io senza esperienza son sicuro che l'effetto seguirà come vi dico perché Così è necessario che segua.*[10]

In the words of the Brazilian poet Carlos Drummond de Andrade (1902-1987): "*Os senhores me desculpem, mas devido ao adiantado das horas, eu me sinto anterior as fronteiras.*" or in those of the Brazilian politician Ruy Barbosa (1849-1923): "*Creio que o nosso dever è cortar, quanto ser possivel alias possa, os favores jà outorgados que empenharem o credito da naçao, e nunca aumenta-los.*"[11] Or still, as expressed by Galeano: "*Somos lo que hacemos, pero sobre todo somos lo que hacemos para cambiar lo que somos*"[12].

Concomitantly with all the above, one must be prepared to see the emergence of controversy, of all types of attacks – the high-level, and the low-level ones as well. The German philosopher Arthur Schopenauer (1788-1860) said that "All truths pass through three stages: first, they are considered ridiculous, second, they are violently adversed, third, they are accepted and considered self-evident."

8 If action has not some splendor of freedom, it has no grace nor honor.
9 ... I learnt the curiosity and the enthusiasm, some form of dream and phantasy also, without which there is no true research.
10 Simplicio: So you have not done one hundred, not even one proof, and you state it so frankly as sure? Salviati: Me, without experience I am sure that the effect will follow as I tell you, because it is necessary that it so follows.
11 I believe that our duty is to cut, as much as possible, more precisely as much as I can, the already given favors that damage the credit of the nation, and never to increase them.
12 We are what we do, but over all we are what we do to change what we are.

We can read in Il Principe (Chapter VI) the peculiar thoughts of Niccolo Machiavelli (1469-1527):

> *E debbasi considerare come non è cosa più difficile a trattare, nè più dubia a riuscire, nè pi pericolosa a maneggiare, che farsi capo ad introdurre nuovi ordini. Perché lo introduttore ha per nimici tutti quelli che delli ordini vecchi fanno bene, et ha tepidi defensori tutti quelli che delli ordini nuovi farebbono bene. La quale tepidezza nasce, parte per paura delli avversarii, che hanno le leggi dal canto loro, parte dalla incredulita` delli uomini; li quali non credano in verita` le cose nuove, se non ne veggono nata una ferma esperienza.*[13]

As brilliantly described by the American intellectual Thomas Samuel Kuhn (1922-1996), new scientific paradigms require the reformulation of previous hypothesis and the re-evaluation of previous facts. This is an uneasy and time consuming task, and it almost unavoidably becomes the target of strong resistance by the established community. In his 'The Structure of Scientific Revolutions', Kuhn writes "*The road to a firm research consensus is extraordinarily hard". But we should also keep in mind that he also writes that "a scientist's world is qualitatively transformed [and] quantitatively enriched by fundamental novelties of either fact or theory.*"

Indeed, Antoine-Laurent Lavoisier (1743-1794), the founding father of modern chemistry, wrote in his '*Reflexions sur le Phlogistique*': "I do not expect my ideas to be adopted all at once. [...] It is the passage of time, therefore, which must confirm or destroy the opinions I have presented. Meanwhile, I observe with great satisfaction that the young people are beginning to study the science without prejudice..."

Serenity and good humor comes sometimes from what is so deliciously expressed by the French novelist Marcel Pagnol (1895-1974): "*Tout le monde savait que c'etait impossible. Il est venu un imbecile qui ne le savait pas... et qui l'a fait!*"[14] ; Or, in the version attributed to the French writer Jean Cocteau (1889-1963), "*Il ne savait pas que c'etait impossible et il l'a fait*"[15]

3. From my own experience

Let me focus in this section on my present line of research. Statistical mechanics is one of the monuments of contemporary physics. It was founded by Boltzmann, together with the Scottish physicist James Clerk Maxwell (1831-1879) and the American mathematician and physicist Josiah Willard Gibbs (1839-1903). This branch of physics focuses on the connection between the natural laws at different scales. More precisely between the

13 We must consider that nothing is harder to implement, of more uncertain success, nor more dangerous to deal with, than to initiate a new order of things. Because the one who introduces the novelties finds enemies in all those who profit from the old order and tepid defenders in all those who would profit from the new order. This tepidity comes in part from their fear of their adversaries, who have the laws on their side, and in part from the incredulity of people, who do not really believe in new things until they have solid experience of them. (Translation by C. Tsallis and M. Gell-Mann).

14 Everybody knew that it was impossible. A stupid arrived who did not know... and he did it!.

15 He did not know that it was impossible and he did it.

microcosmos (atoms and molecules, for instance) and the macrocosmos (materials, a piece of iron for instance). It emerged through deep controversies at the end of the XIX century, the historically most important papers being those of Boltzmann during the period 1872-1877 [2,3]. A crucial point at the heart of the controversies was whether atoms exist or not, since, according to Boltzmann and followers, it would be them which would be the microscopic agents of matter.

A few years earlier, in 1865, the German physicist Rudolf Julius Emanuel Clausius (1822-1888) had introduced the concept of entropy, noted S, which, together with that of energy, constitutes the two building blocks of thermodynamics, the science of the macroscopic world. What primarily Boltzmann and Gibbs did was to identify the connection between this macroscopic entropy and the W configurations of the microscopic constituents of the system. In modern language, this connection can be written as follows:

$$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i \,, \tag{1}$$

where BG stands for Boltzmann-Gibbs, and $0<p_i<1$ is the probability of the i-th configuration to occur. These probabilities naturally satisfy

$$\sum_{i=1}^{W} p_i = 1 \,. \tag{2}$$

The constant k is usually taken to be the Boltzmann constant, one of the universal constants of contemporary physics (the others being the velocity of light c, the Newton gravitational constant G, and the Planck constant h). All units that exist in science and technology can be expressed in terms of these four constants.

The form S_{BG} has an important mathematical property, namely additivity. An entropy S is said additive [4] if, for two probabilistically independent systems A and B (such that $p_i(A+B)=p_i(A)p_j(B)$, for all (i,j)),

$$S(A+B) = S(A) + S(B) \,. \tag{3}$$

We straightforwardly verify that S_{BG} is additive. In particular, if we have a system composed by N independent elements, we trivially verify that

$$S_{BG}(N) = N S_{BG}(1) \propto N \,. \tag{4}$$

The BG entropy is not the only additive entropy, however. Renyi entropy S_q^R, defined as follows

$$S_q^R = k \frac{\ln \sum_{i=1}^{W} p_i^q}{1-q} \quad (q \in \mathcal{R};\, S_1^R = S_{BG}) \,, \tag{5}$$

is also additive, for all q. The entropy S_q^R with $q>1$ is, however, inadequate for thermodynamical purposes since it violates concavity.

In 1985 I was participating in a Brazilian-French-Mexican workshop in Mexico City. During a coffee-break it came to my mind that, using p_1^q, it would be possible to define an entropic form which would generalize S_{BG} in such a way that the BG statistical mechanics itself, based on S_{BG}, could be generalized as well. The corresponding paper was published in 1988 [5]. This entropy is defined as follows:

$$S_q = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad (q \in \mathcal{R};\ S_1 = S_{BG}) \,, \tag{6}$$

and it can be shown that, for two independent systems A and B, it satisfies

$$\frac{+B)}{\;} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} \,. \tag{7}$$

Therefore, for q ^{1} 1, this entropy is nonadditive. S_q is related with S_q^R as follows

$$S_q^R = k\frac{\ln[1+(1-q)S_q/k]}{1-q} \,, \tag{8}$$

but is concave, for all $q > 0$ (and convex, for all $q < 0$). The question arises therefore whether S_q could be used as the basis for generalizing the successful BG theory. It turns out that it can, and this more general theory is referred to in the literature as "nonextensive statistical mechanics" [6-11].

The story of this theory is plenty of points that are centrally relevant to the present short essay. Its almost instantaneous conception, based just on the beauty of having probabilities raised to a power, q, that would emphasize the rare events, or the frequent events (notice that p_i^q is larger, smaller or equal to p_i, according to q being smaller, larger, or equal to unity respectively), its amazing development along the last two decades[16], the controversies it has raised among some members of the community (going from interesting and fairly posed scientific questions and objections, down to personal or collective offenses), all these features pedagogically illustrate how progress in science and technology proceeds. It is out of the scope of the present brief account to describe and analyze the whole process. I will therefore concentrate in a couple of points that are, in some sense, paradigmatic.

Let us start with the meaning of the words additive and extensive with regard to entropy. To understand this important point, let us refer to an interesting story of Ancient Egipt. At the time of the great Pharaoh Thutmosis III (three and a half millennia ago), the North was named 'along the stream', referring of course to the stream of the Nile, the only river known by them at the time, and the South was 'against the stream'. Then the Pharaoh conquered the regions where the Euphrates flows - basically along the direction opposite to that of the Nile (see figure 3). This fact strongly intrigued the astronomers of the time. When the Pharaoh came back to Egypt, an obelisk was erected in his honor.

It was there witten "That strange river that when you go along the stream, you go against

16 At the time at which these lines are being written, the number of publications significantly related to it overcomes 2600 papers by more than 2000 scientists from sixty-three countries; the number of citations overcomes 9000, of which close to 2000 refer to the 1988 paper.

it"! Of course, two completely different concepts, namely the sense of flows of the rivers on Earth and the relative motion of the stars, were being confused. This is were we come to the words 'additive' and 'extensive' in order to qualify entropy. Clausius entropy S is a macroscopic concept which, for mathematical consistency of standard thermodynamics, ought to be extensive for normal systems (e.g., a gas, a piece of metal, some water), i.e., such that *S(N) ~N* for large *N*. This desirable macroscopic extensivity is a concept a priori totally independent from the mathematical form which might connect, through the probabilities of the microscopic configurations (or complexions, as Boltzmann used to call them), S with the microscopic world.

However, this important independence has not been perceived, or has been very weakly perceived, during 130 years, from Boltzmann's first articles in the subject (1872-1877) until recent years. Indeed, the Boltzmann connection, as provided by Eq. 8, which satisfies additivity, expressed in Eq. 5, provided the ground for the confusing identification of two different properties, namely additivity and extensivity. The situation is indicated in table 1. The satisfaction or violation of additivity depends only on the mathematical form of S in terms of probabilities, whereas the satisfaction or violation of extensivity depends on that, but also on the system (more precisely on the type of space and/or time correlations present in the system). Therefore, for normal systems (those for which the BG statistical mechanics is legitimately applicable), the additivity of S_{BG} guarantees its macroscopic extensivity. But, for anomalous systems, the additivity of S_{BG} precisely precludes its extensivity! It is for those anomalous systems that the nonadditive entropy S_q (for a special value of q differing from unity) can be extensive, as required in classical thermodynamics!

TABLE 1

SYSTEM	ENTROPY S_{BG} (additive)	ENTROPY S_q $(q < 1)$ (nonadditive)
Short-range interactions, weakly entangled blocks, etc	**EXTENSIVE**	NONEXTENSIVE
Long-range interactions (QSS)*, strongly entangled blocks, etc	NONEXTENSIVE	**EXTENSIVE**

**QSS stands for quasi-stationary state ([13, 14]and references therein).*

Two different words, two different concepts: many years have been necessary to really appreciate the important distinction between them. As Wolfgang von Goethe suggested, when humans do not understand something, a word quickly emerges and everybody remains satisfied. Even if they still do not understand! Mephistopheles says to Faust: "*Denn eben wo Begriffe fehlen, Da stellt ein Wort zur rechten Zeit sich ein*"[17]

17 "Just there where terms are missing, just then a word appears" (Translation by S. Thurner) or "When the thought is vague and fleeting, Comes the word to give it shape." (Non literal translation)., in Faust I, Vers 1995, Schuelerszene, 1808.

Figure 3 Thutmosis III, the Nile, the Euphrates, and the obelisk in his memory

If somebody would ask me "What are you doing?! Are you violating our familiar and well established thermodynamical property that a double of some substance has the double of entropy? In other words, are you violating the extensivity of the entropy?", I would answer "By no means, I am violating the additivity of the entropy in order not to violate its extensivity, precisely!" A nice analytical illustration of this fact has been recently presented for a strongly quantum-entangled subsystem [12,13, 14]. For a d=1 first-neighbor-interacting quantum ferromagnet (belonging to the universality class associated with a central charge, c) at criticality (as a function of a transverse magnetic field) at zero temperature, we have that

$$S_{BG}(L) \propto \ln L \neq L \;\; (L \to \infty, \, \forall c \geq 0)\,, \tag{9}$$

whereas

$$S_q(L) \propto L \;\; (L \to \infty)\,, \tag{10}$$

with

$$q = \frac{\sqrt{c^2+9}-3}{c} \in [0,1] \quad (\forall c \geq 0)\,, \tag{11}$$

L being the linear size of a subsystem of an infinitely large system. In other words, since L is proportional to the total number of particles N of the one-dimensional fermionic-like (subsystem) under consideration, we have that the Boltzmann-Gibbs-von Neumann additive entropy is nonextensive (indeed, $S_{BG}(N) \sim ln(N)$), whereas the nonadditive entropy S_q is extensive for that special value of q (indeed, $S_q(N) \sim N$). This remarkable result, as well as other numerical and analytical evidences, have suggested the following conjecture for d-dimensional anomalous systems or subsystems [15]:

$$S_{BG}(L) \propto \frac{L^{d-1}-1}{d-1} \;\; (d \geq 1;\, L \to \infty)\,, \tag{12}$$

whereas a special value of q (depending on d, the fermionic/bosonic nature of the particles or quasi-particles, and other details of the system) might exist such that

$$S_q(L) \propto L^d \ \ (d \geq 1;\ L \to \infty)\,, \tag{13}$$

where L is the linear size of the system. Since $N \sim L^d$, we obtain $S_{BG}(N) \sim ln(L) \sim ln(N)\ ^{1} N$, for $d = 1$, and the so called 'area law' $S_{BG}(N) \sim L^{(d-1)}$ hence $S_{BG}(N) \sim N^{(d-1)/d}\ ^{1}N$ *for* $d > 1$. In all these circumstances we obtain, however, $S_q(N) \sim N$, where $q=1$ for the normal systems, and q^1 *1* for the anomalous ones.

The longstanding intriguing feature that black holes have an entropy which violates thermodynamics [16] is reformulated as follows: The BG entropy of a black hole is "strange" since it is proportional to its area instead of being proportional to its volume; but the (nonadditive) entropy of a black hole might be perfectly consistent with classical thermodynamics, since it is expected to be proportional to the volume. Detailed calculations addressing this interesting issue would be very welcome. If such q exists, what is its value? Is it for example q=1/2, as intriguingly emerging in [17]?

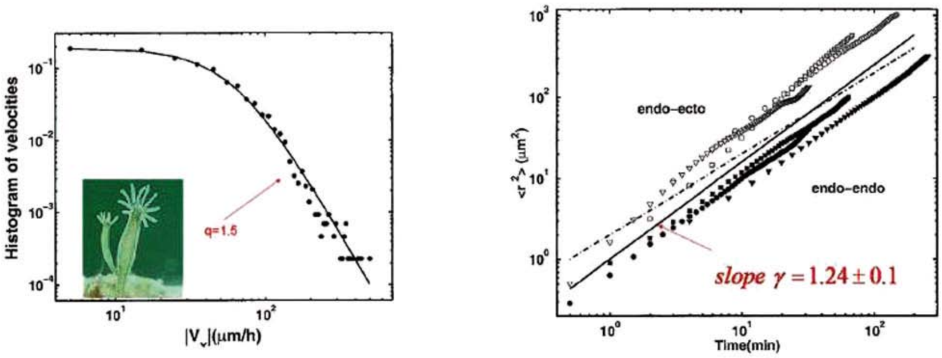

Figure 4 Hydra viridissima: Distribution of velocities, fitted with a q-Gaussian with q=1.5 (left), and anomalous diffusion, characterized with a slope 1.24±0.1 (right). From [18]

Let us describe now another typical situation which requires, in order to be satisfactorily addressed, a reformulation of the pre-established ideas - a sort of Gestalt image-background re-arrangement of reality, a sort of discovery of Bersanelli's strawberries [18] -, a typical illustration of creativity in science. This story happened during 1998 in the Physics Department of Notre-Dame University, USA, and the protagonists were Arpita Upadhyaya, at the time a young PhD student of James Glazier, and myself.

I was visiting the Department for a few days, by invitation of James Glazier. Arpita was showing to me her interesting measurements of the velocities of a one-millimeter-long organism named Hydra viridissima. With the help of an appropriate camera, she was filming the motion of these organisms, and constructing the histograms of those velocities. This distribution of velocities was clearly non-Maxwellian. What was it then? Arpita showed to me, on the computer screen, her experimental results as well as her tentative fittings. She was using stretched-exponentials (i.e., $p(|v|) \sim exp\{-\beta|v|\alpha\}$, with $\beta > 0$ and $0< \alpha <2$) to fit. The reason was, as far as I can remember, that she had read some theoretical work leading to those distributions. She showed to me the first decade of velocities of figure 4. And the fitting was reasonably good. I asked her whether she had experimental points at larger velocities, say one more decade. She said that she had, but added that "the points were not so good". I asked why. The answer was very revealing: "*They cannot be fitted by a stretched-exponential*"! I insisted, and she also showed the rest of her measurements (basically what is seen in figure 4, left). I then recognized the typical (and familiar to me) shape of a q-Gaussian (*p(|v|) µ 1/[1+(q-1) β |v|2]{1/(q-1)}*). I then asked her to fit her data with this form, which I wrote for her on a paper.

The result was what you can appreciate in figure 4 (left). With astonishment, she crossed the corridor and called her supervisor to see the "surprise"! I would guess that what happened in her mind was a reconstruction of the type than can be seen in a Gestalt image. The "stretched-exponential theory" was replaced by the "q-exponential theory", the experimental evidence having re-acquired the primacy it should have never lost! Two years later, I was doing a visit at MIT-USA by invitation of Seth Lloyd, and I met once again Arpita, by then already a PhD. We analyzed together (at the top-level Cafeteria at one of the ends of the infinite corridor) her data on Hydra viridissima, this time having also at hand her results for anomalous diffusion. We found that she had a slope $\gamma \sim 1.24 \pm 0.1$, which, together with q=1.5 (her fitting of the data for the velocities) is perfectly compatible with $\gamma = 2/(3-q)$, a specific scaling predicted within q-statistics [19]. The new paradigm was, in some sense, entering into her mind: I guess she started to consider it "admissible". Her paper was published one year later [20] This is how emerged the paper which constitutes the first experimental evidence of the just mentioned scaling prediction, verified by now in many other complex systems. It is unavoidable to agree with many of the statements made by Thomas Kuhn [21] and by Bruno Latour [1] about the paths of the evolution of sciences: the flavor of their thoughts is in there!

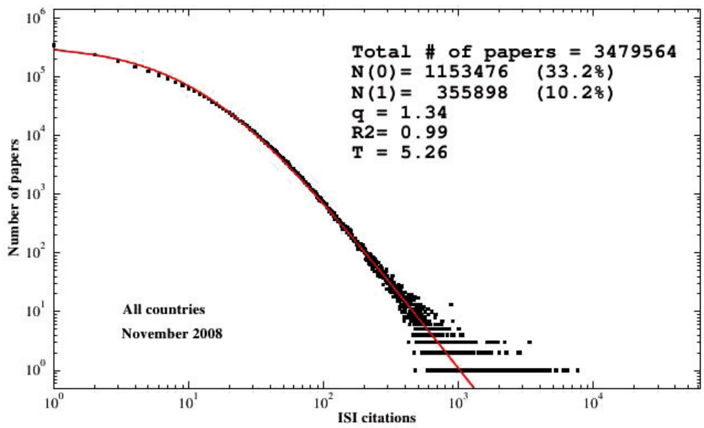

Figure 5 Distribution of ISI citations of the scientific production of 13 countries, since the end of the Second World War. The continuous line represents the number of papers N(c) that have received c citations, q is the entropic index, and T is the effective "temperature". The q index of these 13 countries separately is virtually the same, i.e., q ~ 4/3. From [22].

A central question in the theories that we are discussing here is when should we apply BG statistical mechanics, and when nonextensive statistical mechanics? The full answer to this important question still eludes us. Nevertheless, part of it is today known. For example, for classical systems (either conservative or dissipative), the basic criterium consists in checking whether the maximal Lyapunov exponent is positive or zero - if it is negative, there is no place for statistical mechanical methods, we must just use the methods of mechanics. If it is positive, strong chaos is present.

Therefore, for Hamiltonian systems, there is mixing and ergodicity (i.e., ensemble and time averages coincide). This is the realm of BG concepts. We must therefore use q=1. If the maximal Lyapunov exponent is instead zero, then the q-concepts are in order. We are not saying that q-statistics becomes mandatory, but for sure it constitutes a very strong "candidate". A typical dissipative system intensively studied nowadays is any unimodal map at its edge of chaos, e.g., the logistic map. As said to me by Ricardo Ferreira, we may say (at least as a first, very rough approximation) that the BG theory primarily is the statistical mechanics of inanimate matter, whereas the nonextensive theory primarily is the statistical mechanics of living matter, or living "systems" [22]. Some years later, related arguments were advanced to me by the Brazilian physicist Paulo Murilo Castro de Oliveira. Of course, by "living systems" we mean a variety of natural, artificial and social complex systems which share relevant properties with biological systems.

Let us end by showing a recent and typical illustration [23] of the emergence of q-functions in complex systems. In figure 5 we see the distribution of ISI citations (Web of Science), during the period 1945-2008, for all sciences, of thirteen countries of Latin America (Brazil, Argentina, Mexico and Chile), Africa (South Africa) and Europe (Italy, Spain, Switzerland, Austria, Hungary, Greece, Portugal and Romania) for which large data bases are available. In this histogram, all countries are represented together. It is clear that this empirical result constitutes a first approach to the problem. A next desirable step would be to construct a model in order to improve the understanding of the phenomenon. It is nevertheless suggestive the fact that q=4/3 (i.e., log-log slope is -3) precisely corresponds to the most common degree distribution of (asymptotically) scale-free networks.

Conclusion

The stimulation of creativity in science, technology, and overall in education is a most delicate and powerful task. Its ingredients are multifaceted. They have to do with paradigms, poetry, intellectual rigor and pleasure, courage, learning of several languages, freedom, determination of character, celebration of good ideas (even if modest), to mention some of them. In one way or another they turn around the concepts of Truth, Beauty and Goodness. In the present short essay, we have tried to illustrate these various aspects through quotes of great thinkers, as well as through the analysis of personal experiences. Although part of the illustrations concern statistical mechanics, we believe that their basic content is universal, in the sense that it emerges similarly in all times, places, and cultures.

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