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Modeling default probabilities: The case of Brazil

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ABSTRACT

Using disaggregated data from the Brazilian stock market, we calculate default probabilities for 30 different economic sectors. Empirical results suggest that domestic macroeconomic factors can explain these default probabilities. In addition, we construct the Minimum Spanning Tree (MST) and the ultrametric hierarchical tree with the MST based on default probabilities to disclose common trends, which reveals that some sectors form clusters. The results of this paper imply that macroeconomic variables have distinct effects on default probabilities, which is important to take into account in credit risk modeling and the generation of stress test scenarios.

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1. Introduction

One of the primary goals of financial regulation is to maintain economic stability, e.g., to avoid crises and sudden adverse changes in the financial system. Historically, banking crises have proven to generate significant costs to the real economy (Hoggarth et al., 2002), which are usually less severe in countries with regulated banking systems (Angkinand, 2009).

We have recently seen that the 2008 subprime crisis brought us the need of a robust risk management culture (Ackermann, 2008). For investors and financial institutions, detecting risk trends and making comparisons across countries are important to minimize risks. Financial instability not only

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diminishes the welfare of the economy with losses in the GDP, but affects consumption and aggravates uncertainty (Barrell et al., 2006). Thus, considering the inherent risk of the economic agents, as well as the possibility of contagion effects, policy actions from authorities are necessary to accomplish stability and avoid damages. Therefore, predicting crises and assessing the degree of risk of the institution/country concerned provides important information to regulators.

Probabilities of default are valuable pieces of information for supervisors when assessing the health of the financial system. They are usually calculated with stock market data and used to identify and predict upcoming crises as early as possible, as an attempt to minimize its negative effects. Furthermore, as a rule, authorities and regulators must primarily remain watchful not to the actual value of the probability, but to movements in the probabilities of failure, as to detect upward trends and avoid failure (Clare, 1995).

In this context, one important branch of the recent financial literature has focused on the determinants of the probabilities of default. For instance, probabilities of default were found to be influenced by domestic macroeconomic factors (such as inflation, production and interest) as well as a market portfolio and the international risk (Clare and Priestley, 1998), by financial deregulation (Clare and Priestley, 2002), by the size of the company (Dietsch and Petey, 2004), by a variety of capital markets factors (such as interest and exchange rates and credit spreads) (Berardi et al., 2004) and by the institutional environment of the country (such as the quality of governance, the degree of law and order) (Byström, 2004). Furthermore, Ammer and Packer (2000), in their turn, point out that although the risk is usually assigned in accordance with the issuer (sovereigns, municipal governments, industrial firms, and financial institutions located in many countries), the default determinants can differ also across industrial sector and geographical localization, so that maintaining the consistency across sectors usually is not easy.

Another important branch of the financial literature has tried to detect the existence of contagion effects among countries. We know that the Russian crisis (1998) increased the probability of Brazilian domestic bank failure, whereas the Argentinean crisis (2001) did not, providing evidence that contagion has decreased since 1998, due to the introduction of a floating exchange rate regime and the inflation-targeting framework (Tabak and Staub, 2007). In the United States, while geographic distance of the solvent banks' head offices from the head offices of the failed banks and capital adequacy are found to be negatively correlated to the magnitude of the contagion effect, size is positively related, supporting the existence of information-based contagion (Aharony and Swary, 1996). Byström et al. (2005) analyze Thai firms and banks and observe a significant increase in market based default probabilities around the Asian crisis (1997–1998), but with a slow return to pre-crisis levels.

Parallel to to this literature that is devoted to understand the determinants of probabilities of default and the contagion among countries, a literature based on complex networks analysis has been developed as an intersection of several fields from graph theory to statistical physics to provide a unified view of dynamic systems that may be described by complex web-like structures and non-parametric statistics (Albert and Barabasi, 2002; Boccaletti et al., 2006; Costa et al., 2007). The modeling of financial networks using tools provided by the theory of complex networks can provide important insights on the understanding of financial links between banks and for the development of better financial regulation (Boss et al., 2004; Iori et al., 2006; Nier et al., 2007; Cajueiro and Tabak, 2008; Cajueiro et al., 2009). In this paper, we are particularly interested in identifying the hierarchy present in the network formed with correlations of the probabilities of default.

We construct the Minimum Spanning Tree (MST) and the ultrametric hierarchical tree associated with the MST for this purpose (Mantegna, 1999). The networks property of hierarchy is useful because it allow us to observe that the networks often have structure in which vertices cluster together into groups that then join to form groups of groups, from the lowest levels of organization up to the level of the entire network. Furthermore, the use of MST analysis is adequate for extracting relevant information when a large number of markets are being studied as it provides a parsimonious representation of the network of all possible interconnectedness and can greatly reduce complexity by showing only the most important non-redundant connections in a graphical manner (Coelho et al., 2007). One may note that that there is a large body of literature that have studied the emergence of complex patterns in networks formed by correlations of stocks (Onnela et al., 2002; Coelho et al., 2006), interest rates (Matteo et al., 2004, 2005; Tabak et al., 2009b) and interbank activities (Tabak et al., 2009a).

Our paper derives implied default probabilities for different economic sectors following the work of (Byström, 2004) and contribute to the discussion about the determinants of the probabilities of default in two different ways. First, we show that the implied default probabilities for different sectors present common trends using recent econometric and the above-mentioned clustering methods. To the best of our knowledge this is the first paper that studies the network formed by correlations of default probabilities, which may prove useful for credit risk management. Second, we present important evidence linking default probabilities to macroeconomic variables, which may be used to stress risk within these sectors. Therefore, we provide new evidence suggesting that macro-financial variables (Clare and Priestley, 1998; Berardi et al., 2004) may be used as determinants of probabilities of default, using disaggregated data. One also should note that we focus our analysis on one of the most important markets in Latin America, Brazil. Brazil ranks as one of the most important stock markets in Latin America both by size of the market and liquidity. We focus on economic sectors that comprise the Brazilian domestic traded firms within the Brazilian stock market. Many of these shares are also traded in the New York Stock Exchange as American Depositary Receipts (ADRs) and may be seen as an important source for international diversification. Despite the economic significance of the Brazilian stock market the literature on this particular market is scant.

The paper is divided as follows. The next section presents the methodology used to estimate the probabilities of default and to study the topology of the networks of correlations of probabilities of default. Section 3 presents the data and the main empirical results of this paper. Finally, Section 4 concludes the paper summarizing the main findings of this paper.

2. Methodology

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2.1. Estimation of default probabilities

In our approach, we estimate probabilities of default (PD's) based on a conditional version of the Capital Asset Pricing Model (CAPM), following the work of Byström (2004).

The share price of a firm is given by:

$$S_{it} = \frac{\sum_{l=1}^{N} P_{lt} X_{lt}}{N},$$
(1)

where *N* is the number of issued ordinary shares, P_l is the price of asset/liability *l* and X_l represents asset/liability *l*.

The excess return on stock *i* is given by:

$$R_{it} = \beta_t E(R_{mt}) + \varepsilon_{it}, \tag{2}$$

where ε_{it} is assumed to be a white noise error term.

The conditional form of the CAPM shows that the return of a stock *i* depends on the time-varying market price of the risk λ_t , scaled by the time-varying conditional covariance between the excess return on stock *i* and the stock return on the market portfolio:

$$R_{it} = \lambda_t E(u_{mt}, \varepsilon_{it}) + \varepsilon_{it}.$$
(3)

The conditional variance (that is, the variability in the market value of the bank's capital around it's expected value) of firm capital at time t as measured at time t - 1 is:

$$E_{t-1}(S_t N - E_{t-1}(S_t N))^2 = (S_{t-1} N)\sigma_{\mathcal{E}_{it}}^2,$$
(4)

where $\sigma_{\varepsilon_{it}}^2$ is the variance of ε_i . This expression can be interpreted as the difference between the actual and expected value of a firm's capital at time *t*, where ε_{it} is the rational expectation forecast error.

We can thus develop a measure of the probability of default as the number of standard deviations the value of capital represents at time t - 1 which is given by the following expression:

$$\frac{S_{it-1}N}{(S_{it-1}N)\sigma_{\varepsilon_{it}}} = \frac{1}{\sigma_{\varepsilon_{it}}}.$$
(5)

Assuming normality on the error term, we use the normal distribution to construct the default probability.

In order to estimate Eqs. (2) and (3) we first calculate the conditional variance σ_{ε_t} using a bivariate EGARCH, as described below:

$$E(\sigma_{\hat{\varepsilon}_{t}}^{2}) = \omega_{1}^{2} + \beta_{1}^{2}\sigma_{t-1}^{2} + \alpha_{1}^{2}\varepsilon_{t-1}^{2} + \gamma_{1}I\varepsilon \\ E(\sigma_{v_{t}}^{2}) = \omega_{3}^{2} + \omega_{2}^{2} + \beta_{2}^{2}\sigma_{t-1}^{2} + \alpha_{2}^{2}\upsilon_{t-1} + \gamma_{2}I\upsilon \\ E(\sigma_{\varepsilon_{t},v_{t}}) = \omega_{1}\omega_{2} + \beta_{2}\beta_{1}E(\sigma_{\varepsilon_{t-1},v_{t-1}}) + \alpha_{2}\alpha_{1}\varepsilon_{t-1}\upsilon_{t-1},$$
(6)

where $E(\sigma_{\varepsilon_t}^2)$ and $E(\sigma_{\upsilon_t}^2)$ are the conditional variances of ε_t and υ_t , $E(\sigma_{\varepsilon_t,\upsilon_t})$ is the covariance between ε_t and υ_t , $I\varepsilon$ ($I\upsilon$) are dummy variables that are equal to 1 when $\varepsilon_{t-1} < 0$ ($\upsilon_{t-1} < 0$) and 0 otherwise.

Our choosing of a bivariate EGARCH is based on an asymmetric conditional volatility, i.e., falling prices will lead to a higher increase in the volatility of a stock's rate of return.

We add up the calculated daily $\sigma_{\varepsilon_{it}}^2$ estimates on each month and create a monthly default measure $1/\sqrt{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \cdots + \sigma_{\varepsilon_{21}}^2}$. Afterwards, we use an annualized measure defined as $1/\sqrt{12\sigma_{\varepsilon_1}^2}$.

2.2. Construction of Minimum Spanning Tree (MST) and hierarchical tree from default probabilities

From the probabilities of default we build a Minimum Spanning Tree (MST) to study the topology of the network. However, the MST requires the use of a variable that can be interpreted as distance, satisfying the three axioms of Euclidian distance. Therefore, we transform this matrix in order to build a distance matrix. To build the probability of default network we employ the metric distance $d_{i,j} = \sqrt{2(1 - \rho_{i,j})}$ proposed by Mantegna and Stanley (1999), where $\rho_{i,j}$ is the correlation between changes in default probabilities *i* and *j*.¹

The MST is a graph that connects all the *n* nodes of the graph with n - 1 edges, such that the sum of all edge weights $\sum_{i,j\in D} d_{i,j}$ is a minimum, where *D* is the distance matrix. The MST extracts significant information from the distance matrix and it reduces the information space from $n \times (n-1)/2$ correlations to n - 1 tree edges. It is the spanning tree of the shortest length using the Kruskal algorithm of the $d_{i,j}$ and is a graph without cycles connecting all nodes with links.²

Define the maximal distance $d_{i,j}^*$ between two successive commodities when moving from PD_i to PD_j over the shortest path of the MST connecting these two commodities.³ The distance $d_{i,j}^*$ satisfies the above axioms of Euclidian distance and also the following ultrametric inequality:

$$d_{i,j} \le \max[d_{i,k}, d_{k,j}]. \tag{7}$$

Networks have many properties that help researchers to understand the interactions between agents in a complex system. They have the property of hierarchy which is useful to observe that the networks often have structure in which vertices cluster together into groups that then join to form groups of groups, from the lowest levels of organization up to the level of the entire network.

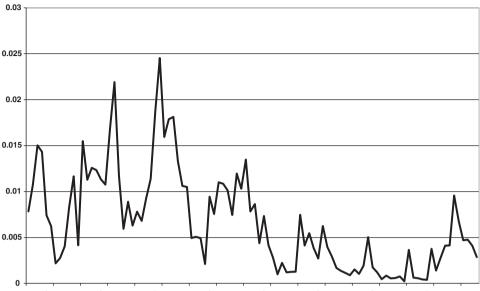
The ultrametric hierarchical tree uses the single-linkage clustering method, which builds up clusters by starting with distinct objects and linking them based on similarity. The major issue with this method is that while it is robust for strongly clustered networks, it has a tendency to link poorly clustered groups into chains by successively joining them to their nearest neighbors. This ultrametric hierarchical tree provides useful information to investigate the number and nature of the common factors that affect the different sectors.

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¹ This metric satisfies the three axioms of Euclidian distance: (i) $d_{i,j} = 0$ if and only if i = j, (ii) $d_{i,j} = d_{j,i}$, and (iii) $d_{i,j} \leq d_{i,k} + d_{k,j}$.

² The Kruskal algorithm has the following steps: (1) choose a pair of commodities with the nearest distance and connect with a line proportional to this distance, (2) connect a pair with second nearest distance, (3) connect the nearest pair that is not connected by the same tree, and (4) repeat step three until all commodities are connected in one tree.

³ This distance is called subdominant ultrametric distance and a space connected by these distances provides a topological space that has associated a unique indexed hierarchy.



Mar-00 Sep-00 Mar-01 Sep-01 Mar-02 Sep-02 Mar-03 Sep-03 Mar-04 Sep-04 Mar-05 Sep-05 Mar-06 Sep-06 Mar-07 Sep-07 Mar-08

Fig. 1. Average of the probabilities of default.

3. Empirical results and data

3.1. Individual regressions

We estimate default probabilities for 30 markets over the period from March 1, 2000 to June 30, 2008. In this period, default probabilities remained very low in most market sectors. "Media" and "Broadcast and Entertainment" have had the highest average default probability (around 6%). Table 1 presents the descriptive statistics of the estimated probabilities of default, Fig. 1 shows the average of probabilities of default over time and Figs. 2–5 present the evolution of the probabilities of default in different market sectors over time.

The first step in our modeling procedure was to test whether default probabilities for the different sectors had unit roots. We find evidence suggesting that this is indeed the case and therefore the dependent variables were changes in default probabilities. Second, we also test whether macrofinancial variables contained unit roots and employ the changes of these variables as well. An additional step was to estimate whether the macro-financial variables were statistically correlated and to employ auxiliary regressions to build a set of orthogonal macro-financial variables. In order to do so we employ the residuals of the regressions relating two or more of these macro-financial variables as a proxy for the original variable.

We model these default probabilities by regressing them for each sector on a number of macro-financial variables. We included 8 variables to explain the probabilities of default: inflation, interest rate, oil price (Brent), exchange rate (Real/Dollar), interest rates spread, stock market index (Ibovespa–Sao Paulo Stock Exchange Index) as well as the Brazilian industrial production and the consumer confidence index.

Results presented in Tables 2 and 3 indicate that some of these probabilities can be explained in terms of domestic macroeconomic factors, although a significant part remains unknown as the mean adjusted *R*-squared is low (around 0.126). An interesting feature is that different sectors have different sensitivities to these macro-financial variables, which is an important finding in order to build coherent credit risk models.

Table 1	
Descriptive statistic	s.

Market	Mean	Maximum	Minimum	Std. dev.	Skewness	Kurtosis	Jarque-Bera
Media	6.4121%	0.2825	9.84E-04	0.05909	1.3166	4.70	40.87578 [*]
Broadcast and entertainment	6.3935%	0.2823	9.86E-04	0.05895	1.3230	4.73	41.63556*
Retail	1.3392%	0.0972	4.90E-08	0.02163	2.0179	6.54	119.945*
Broadline retail	0.6059%	0.0478	2.20E-05	0.00954	1.9689	6.80	124.6459*
Water	0.5987%	0.0255	1.15E-03	0.00418	1.9420	7.75	156.9765*
Gas/water/multiutilities	0.5702%	0.0343	4.15E-04	0.00541	2.4144	10.91	358.0002*
Tobacco	0.4962%	0.0339	1.97E-04	0.00573	2.3957	9.86	291.9157*
Chemicals	0.4899%	0.0959	3.42E-09	0.01477	4.8665	27.70	2936.999*
Alternative electricity	0.3970%	0.0262	6.99E-07	0.00493	1.9285	7.15	133.746*
Industrial good and services	0.3604%	0.1011	1.27E-09	0.01331	5.7956	38.85	5913.484 [*]
Iron and steel	0.3388%	0.0899	7.67E-07	0.00982	7.1571	61.99	15351.15*
International oil and gas	0.3276%	0.0375	3.50E-05	0.00601	3.4984	16.40	951.9748 [*]
Industrial metal and mines	0.3100%	0.0926	1.09E-07	0.01012	7.2482	62.88	15813.79 [*]
Oil and gas production	0.3075%	0.0348	2.98E-05	0.00579	3.4647	15.76	878.8194*
Electricity	0.1755%	0.0194	9.19E-09	0.00292	3.2169	16.35	915.2544 [*]
Brewers	0.1522%	0.0315	9.37E-09	0.00444	4.4613	25.43	2428.358^{*}
Beverages	0.1477%	0.0310	8.01E-09	0.00440	4.4610	25.19	2382.577^{*}
Utilities	0.1376%	0.0158	3.48E-09	0.00237	3.3208	17.14	1017.049^{*}
Specification chemicals	0.1206%	0.0176	1.29E-07	0.00284	3.9555	19.35	1375.074^{*}
Consumer electricity	0.0841%	0.0196	2.26E-06	0.00247	5.6604	39.00	5933.256 [*]
Basic resource	0.0673%	0.0103	8.79E-09	0.00180	3.6761	16.60	995.8931 [*]
Paper	0.0545%	0.0168	9.08E-09	0.00230	5.8910	37.79	5620.706 [*]
Financials	0.0377%	0.0126	2.09E-08	0.00163	6.4280	44.82	7977.1*
Personal and household goods	0.0321%	0.0033	2.06E-06	0.00051	2.9255	14.63	706.5487*
Food and drug retail	0.0277%	0.0072	2.74E-13	0.00102	4.9371	28.84	3188.671*
Telecom	0.0206%	0.0031	2.80E-08	0.00056	4.0631	19.34	1388.173 [*]
Fixed line telecommunications	0.0192%	0.0026	1.22E-07	0.00047	3.8555	18.22	1212.877*
Banks	0.0129%	0.0062	2.12E-08	0.00064	8.6251	80.54	26288.89*
Speciality financials	0.0085%	0.0008	1.07E-07	0.00017	2.7221	9.90	322.0301*
Forestry and paper	0.0075%	0.0016	1.82E-09	0.00024	4.6063	24.81	2336.458*

Probabilities of default, ranked in decreasing order.

* Statistical significance at the 1% level.

The regressions of default probabilities in each sector are shown in Tables 2 and 3. The coefficients for the variable which measures inflation, IPCA, vary between -2.40 and 2.55, where 17 of them are significant. The results for the interest rate are limited between 0.82 and 15.53, 13 of those are significant and all of them have positive signal. Regarding the variable for oil price we can find coefficients varying from -8.87 to 3.35, where only one is significant and negative. Other 15 negative coefficients can be found within the list. Therefore, this is the variable with the most heterogenous impact. There are 17 significant exchange rate coefficients, all of them positive. In total, 26 of the 30 coefficients for this variable have positive signal, which implies that exchange rate shocks are an important source of systemic risk within the Brazilian economy.

The variable with the most homogenous impact is the interest rates spread, which is significant for 26 sectors. Also, all of these 26 coefficients are positive and the range of impact varies from 1.32 to 20.27. Again, it seems that different sector have very different sensitivities to these macro-financial variables. The Ibovespa index coefficients are in the interval between -16.98 and 2.21, 10 of those are significant, all of them part of the 25 with positive signal from the 30. The index for the Brazilian Industrial production varies from -8.55 to 34.41, 21 of them are positive, but only two significant. Finally, the monthly change in the consumer confidence index presents coefficients in the range from -9.00 to 0.39. Seven of them are significant and negative. Other 21 sectors also presents the same signal direction. The adjusted degree of explanation of the regressions ranges between 0.05 and 0.33.

3.2. Panel data regressions

We test for the significance of the macro-financial variables within a panel data framework. We estimate three different models to check for consistency and robustness of results. We run a random

Table 2

Regressions of default probabilities in different market sectors.

Market	Statistics	С	IPCA	$\Delta(R_t)$	$\Delta(\text{Oil}_t)$	$\Delta(\text{ER}_t)$	$\Delta(\text{Spread}_t)$	$\Delta(\text{IBOV}_t)$	Δ (Production)	Δ (Confidence)	\overline{R}^2
Industrial metal and mines	Coef.	-7.562828*	-1.304888	14.71741**	-2.719689	-8.254438	8.567546**	-7.014548	19.67821	-8.070765***	0.203601
	p-Value	0.0000	0.1783	0.0391	0.3966	0.3648	0.0390	0.1651	0.1856	0.0897	
Iron and steel	Coef. p-Value	-7.370158 0.0000	-0.578399 0.4490	13.90868 0.0123	-2.501015 0.3544	1.768415 0.8268	9.236813 [*] 0.0069	-5.466020 0.2336	16.96743 0.1747	-6.675053 0.1021	0.220039
Industrial good and services	Coef.	-10.88001^{*}	0.661191	10.50364	-8.868970^{**}	33.31009	20.27092*	-2.398362	13.78652	0.044178	0.192211
	p-Value	0.0000	0.4296	0.1447	0.0358	0.1060	0.0027	0.7692	0.5456	0.9948	
Beverages	Coef. p-Value	-11.15409 [*] 0.0000	1.508119 [*] 0.0038	11.71608 ^{**} 0.0274	-0.331912 0.9346	21.56031 0.1307	12.19394 ^{**} 0.0125	-13.61594 ^{***} 0.0652	34.25593 0.1193	-9.002506 0.1835	0.176335
Brewers	Coef. p-Value	-11.10309^{*} 0.0000	1.580907 [*] 0.0017	11.22537** 0.0311	-0.304574 0.9390	20.92299 0.1388	11.84304 ^{**} 0.0141	-13.57833 [*] 0.0634	34.41303 0.1187	-9.009758 0.1784	0.172613
Oil and gas production	Coef.	-7.351073*	0.895007*	6.150756*	-0.521231	11.75572**	5.220727*	-4.651838**	6.122886	-1.381934	0.233445
	p-Value	0.0000	0.0001	0.0020	0.7082	0.0272	0.0029	0.0345	0.4200	0.5651	
Personal and household goods	Coef.	-9.158406 [*]	-0.146870	5.877077**	3.005556	5.759820	4.342761**	1.029457	3.102345	-2.241658	0.064985
-	p-Value	0.0000	0.6892	0.0222	0.0855	0.3003	0.0438	0.7568	0.7826	0.5294	
Tobacco	Coef. p-Value	-5.834734^{*} 0.0000	-0.027463 0.9086	3.406483 ^{***} 0.0944	0.574752 0.5958	8.362448 ^{**} 0.0363	3.962020 [*] 0.0018	-0.569164 0.7828	1.344648 0.8399	-1.864926 0.4019	0.122386
Retail	Coef. p-Value	-7.944058 [*] 0.0000	2.388779 [*] 0.0046	6.372544 0.2165	-0.996323 0.8052	27.16017 ^{**} 0.0169	14.20563 [*] 0.0064	-7.582603 0.1799	-2.466235 0.8581	-1.518712 0.7132	0.224313
Food and drug retail	Coef.	-15.84976*	1.298060	2.653975	-1.176713	30.12627	16.90471**	-16.97882	-0.342065	-9.781423	0.045478
	p-Value	0.0000	0.3638	0.8390	0.8494	0.2209	0.0718	0.2084	0.9910	0.2873	
Broadline retail	Coef. p-Value	-7.118793 [*] 0.0000	0.632811 0.2213	4.947176 0.1826	-0.764007 0.7437	15.13527 0.1988	12.70140 [*] 0.0011	-4.192759 0.2963	-0.612040 0.9522	0.392901 0.9052	0.263384
Media	Coef. p-Value	-3.916665 [*] 0.0000	1.146558 [*] 0.0000	3.516254 ^{***} 0.0751	0.257559 0.8298	12.56217 [*] 0.0007	4.283809 [*] 0.0161	0.212118 0.9346	4.791558 0.4554	-3.136591 ^{***} 0.0775	0.263298

Table 2 (Continued)

Market	Statistics	С	IPCA	$\Delta(R_t)$	$\Delta(\text{Oil}_t)$	$\Delta(\text{ER}_t)$	$\Delta(\text{Spread}_t)$	$\Delta(\text{IBOV}_t)$	Δ (Production)	Δ (Confidence)	\overline{R}^2
International oil and gas	Coef.	-7.301793 [*]	0.945812*	5.955206*	-0.565238	12.01459**	5.461347*	-4.95113**	6.385547	-1.541000	0.256625
0	p-Value	0.0000	0.0000	0.0028	0.6731	0.0234	0.0017	0.0195	0.3908	0.5236	
Broadcast and entertain- ment	Coef.	-3.921331*	1.150493*	3.515430***	0.237541	12.53776 [*]	4.293837**	0.223423	4.696265	-3.119995***	0.264847
	p-Value	0.0000	0.0000	0.0746	0.8426	0.0007	0.0156	0.9310	0.4623	0.0783	
Telecom	Coef. p-Value	-11.13277 [*] 0.0000	-0.138400 0.8102	7.036593 0.1140	-0.473647 0.8735	24.04516 ^{**} 0.0308	11.82580 ^{***} 0.0046	-6.954333 0.2110	1.831297 0.8913	-8.400662*** 0.0653	0.210666
Fixed line telecommu- nications	Coef.	-10.60529*	-0.123934	4.861569	-0.252541	21.40344**	9.240069*	-4.915916	-0.674042	-8.085648**	0.200829
	p-Value	0.0000	0.8022	0.1769	0.9149	0.0183	0.0059	0.3122	0.9554	0.0379	
Utilities	Coef. p-Value	-9.356747 [*] 0.0000	1.612017 [*] 0.0020	5.059278 0.3110	0.887485 0.7683	19.42822 ^{**} 0.0279	11.04391** 0.0169	-6.596252 0.2138	-4.297158 0.7643	-3.591418 0.3423	0.141034
Electricity	Coef. p-Value	-8.925697^{*} 0.0000	1.512000^{*} 0.0026	5.164055 0.2837	1.081171 0.6944	17.45961 ^{**} 0.0331	10.55152** 0.0161	-5.332927 0.2936	-4.769552 0.7220	-3.233894 0.3692	0.135576

Standard errors are corrected for heteroscedasticity and autocorrelation (Newey and West, 1987). *T*-statistics are provided in parentheses. We use IPCA as a measure for inflation, $\Delta(R_t)$ as the monthly change of the 12-months interest rate, $\Delta(Oil_t)$ as the monthly change of the price of crude oil (Brent), $\Delta(ER_t)$ as the monthly change of the exchange rate, $\Delta(Spread_t)$ as monthly change of the spread, $\Delta(IBOV_t)$ the monthly change of the index for Sao Paulo Stock Market, $\Delta(Production_t)$ as the monthly change for the Brazilian industrial production, $\Delta(Confidence_t)$ as the monthly change of the consumer confidence index and \overline{R}^2 is the adjusted degree of explanation. The exchange rate, interest rates spreads and the Bovespa index are correlated. Therefore, before we use intermediary regressions to derive orthogonal explanatory variables for the regression to explain default probabilities. The $\Delta(IBOV_t)$, $\Delta(ER_t)$ and $\Delta(Spread_t)$ are the orthogonal residuals of intermediary regressions, avoiding multicolinearity.

* Statistical significance at 1% level.

** Statistical significance at 5% level.

*** Statistical significance at 10% level.

Market	Statistics	С	IPCA	$\Delta(R_t)$	$\Delta(\text{Oil}_t)$	$\Delta(\text{ER}_t)$	$\Delta(\text{Spread}_t)$	$\Delta(\text{IBOV}_t)$	Δ (Production)	Δ (Confidence)	\overline{R}^2
Consumer electricity	Coef.	-9.162521*	0.789416*	0.817395	0.571575	-0.155139	1.319393	1.884568	-6.976217	-0.215461	-0.027502
electricity	p-Value	0.0000	0.0078	0.8189	0.7367	0.9831	0.6673	0.6439	0.5501	0.9344	
Alternative electricity	Coef.	-7.502533^{*}	1.383370*	3.022822	0.943870	13.99614**	7.256406**	-4.738684	-3.011976	-3.599439	0.141389
5	p-Value	0.0000	0.0005	0.4289	0.6500	0.0323	0.0258	0.2196	0.7784	0.1735	
Gas/water/ multiutilities	Coef.	-5.797405^{*}	0.484764*	1.814557	-0.605172	10.96514*	2.470702**	-4.637747**	3.979922	-2.962761**	0.292547
	p-Value	0.0000	0.0009	0.2134	0.6093	0.0000	0.0254	0.0102	0.5279	0.0270	
Water	Coef. <i>p</i> -Value	-5.468333^{*} 0.0000	0.285445 [*] 0.0064	1.498355 0.1229	-0.259834 0.7603	8.207801 [*] 0.0000	1.188850 ^{***} 0.0863	-3.390596 [*] 0.0099	3.833202 0.4190	-2.226941 ^{**} 0.0339	0.282767
Financials	Coef. p-Value	-12.08784^{*} 0.0000	1.170075 ^{***} 0.0624	2.969787 0.5392	1.838052 0.5104	9.288916 0.4991	2.945843 0.5405	-4.010657 0.5223	16.8571 0.4187	-8.648136 0.1327	-0.011525
Banks	Coef. <i>p</i> -Value	-12.24178^{*} 0.0000	0.519403 0.3580	8.951088 [*] 0.0090	3.355064 0.1097	6.375743 0.5694	7.219296 ^{***} 0.0998	-2.698823 0.6200	13.48979 0.3055	-5.989863 0.1970	0.066957
Speciality financials	Coef.	-10.86604^{*}	-0.864553	6.663401	2.214343	-8.028583	4.773156	2.21446	19.91171	-0.935939	0.048837
	p-Value	0.0000	0.1373	0.2868	0.3400	0.3246	0.1461	0.6744	0.1322	0.7803	
Chemicals	Coef. p-Value	-9.783136 [*] 0.0000	2.548403 [*] 0.0008	1.342104 0.8280	2.102501 0.5031	29.71079 ^{**} 0.0156	5.572278 0.2242	-13.28910 ^{**} 0.0244	3.04594 0.8549	-7.188978 0.1225	0.148086
Specification chemicals	Coef.	-10.39271^{*}	1.839875*	6.688077	1.904138	38.54339*	14.35309*	-5.620960	25.28971***	-2.782904	0.335084
	p-Value	0.0000	0.0002	0.0815	0.5711	0.0022	0.0018	0.2592	0.0645	0.5298	

Table 3Regressions of default probabilities in different market sectors.

Table 3	(Continued)
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Market	Statistics	С	IPCA	$\Delta(R_t)$	$\Delta(\text{Oil}_t)$	$\Delta(\text{ER}_t)$	$\Delta(\text{Spread}_t)$	$\Delta(\text{IBOV}_t)$	Δ (Production)	Δ (Confidence)	\overline{R}^2
Basic resource	Coef. p-Value	-9.301607 [*] 0.0000	-2.100143 ^{**} 0.0236	15.53166 ^{**} 0.0278	-0.297732 0.9271	-17.93778 0.1108	7.472978 ^{***} 0.0872	-9.938328 [*] 0.0806	17.54630 0.3046	-6.233412 0.2343	0.204365
Forestry and paper	Coef.	-12.67129^{*}	0.049076	8.999254*	-0.349742	28.85039**	18.03276*	-11.6049^{*}	5.763899**	-2.948842	0.327440
paper	p-Value	0.0000	0.9102	0.0089	0.8969	0.0134	0.0000	0.0286	0.6770	0.5956	
Paper	Coef. p-Value	-11.35351 [*] 0.0000	0.620603 0.2024	6.975848 ^{***} 0.0877	-1.683909 0.6127	25.32640 ^{**} 0.0333	13.42433 [*] 0.0008	-14.78334 [*] 0.0091	-8.547073 0.5794	-7.200355 0.2167	0.221552

Standard errors are corrected for heteroscedasticity and autocorrelation (Newey and West, 1987). *T*-statistics are presented in parentheses. We use IPCA as a measure for inflation, $\Delta(R_t)$ as the monthly change of the 12-months interest rate, $\Delta(Oil_t)$ as the monthly change of the price of crude oil (Brent), $\Delta(ER_t)$ as the monthly change of the exchange rate, $\Delta(Spread_t)$ as monthly change of the spread, $\Delta(IBOV_t)$ the monthly change of the index for Sao Paulo Stock Market, $\Delta(Production_t)$ as the monthly change for the Brazilian industrial production,

 Δ (Confidence_t) as the monthly change of the consumer confidence index and \overline{R}^2 is the adjusted degree of explanation. The exchange rate, interest rates spreads and the Bovespa index are correlated. Therefore, before we use intermediary regressions to derive orthogonal explanatory variables for the regression to explain default probabilities. The Δ (IBOV_t), Δ (ER_t) and Δ (Spread_t) are the orthogonal residuals of intermediary regressions, avoiding multicolinearity.

* Statistical significance at 1% level.

* Statistical significance at 5% level.

*** Statistical significance at 10% level.

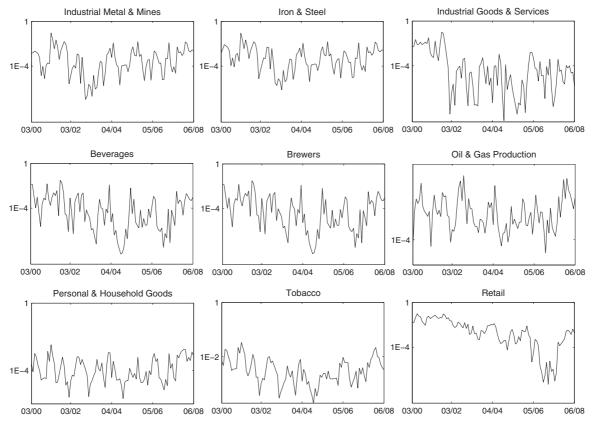


Fig. 2. Probabilities of default in different market sectors, shown in logarithmic scale.

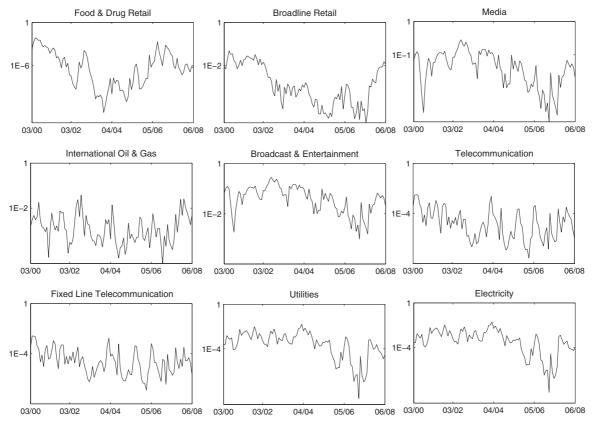


Fig. 3. Probabilities of default in different market sectors, shown in logarithmic scale.

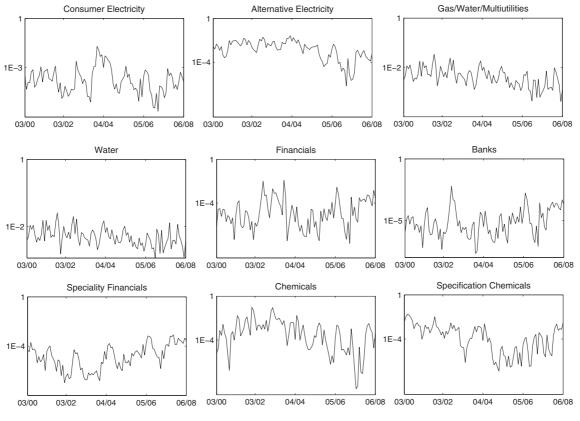
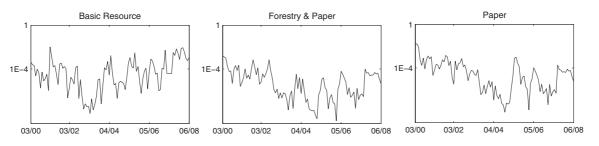


Fig. 4. Probabilities of default in different market sectors, shown in logarithmic scale.



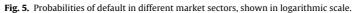


Table	e 4
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	Panel	regressions	for	default	probabilities.
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Variables	(1) PD	(2) PD	(3) PD
PD_{t-1}	0.643*	0.701*	0.918*
	(0.0142)	(0.0132)	(0.00751
IPCA	0.369*	0.213*	-0.0063
	(0.0868)	(0.0750)	(0.0545)
$\Delta(R_t)$	5.156 [*]	5.066*	3.185*
	(0.609)	(0.568)	(0.428)
$\Delta(\text{Oil}_t)$	-0.693	-0.707***	-0.679^{**}
	(0.434)	(0.420)	(0.321)
$\Delta(\text{ER}_t)$	10.09*	9.013 [*]	5.634*
	(1.222)	(1.131)	(0.847)
$\Delta(\text{IBOV}_t)$	-5.100^{*}	-4.986^{*}	-3.398*
	(0.759)	(0.728)	(0.555)
$\Delta(\text{Spread}_t)$	0.724	0.420	-1.292^{*}
	(0.518)	(0.455)	(0.319)
$\Delta(\text{Production}_t)$	11.22 [*]	10.26 [*]	6.432 [*]
	(2.280)	(2.305)	(1.810)
$\Delta(\text{Confidence}_t)$	-1.567**	-1.512**	-0.491
	(0.655)	(0.648)	(0.502)
С	-3.315*	-2.743*	-0.589^{*}
	(0.173)	(0.133)	(0.0704)
Observations	2940	2940	2940
\overline{R}^2		0.561	
Number of sectors	30	30	30
Wald test			16002*
Modified wald test			6478*
Hausman specification test		97.33*	

This table presents the results regressing macro variables on default probabilities using the (1) random effects model with AR(1) disturbance, (2) fixed effects model, (3) FGLS method, allowing for serial correlation and heteroscedasticity. The values in parenthesis are the standard errors of each variable. We use IPCA as a measure for inflation, $\Delta(R_t)$ as the monthly change of the 12-months interest rate, $\Delta(\text{Oil}_t)$ as the monthly change of the price of crude oil (Brent), $\Delta(\text{ER}_t)$ as the monthly change of the exchange rate, $\Delta(\text{Spread}_t)$ as monthly change of the spread, $\Delta(\text{IBOV}_t)$ the monthly change of the index for Sao Paulo Stock Market, $\Delta(\text{Production}_t)$ as the monthly change for the Brazilian industrial production, $\Delta(\text{Confidence}_t)$ as the monthly change of the consumer confidence index and \overline{R}^2 is the adjusted degree of explanation.

* Statistical significance at 1% level.

** Statistical significance at 5% level.

*** Statistical significance at 10% level.

effects model with a first-order autoregressive (AR(1)) disturbance, a fixed effects model and also a Feasible Generalized Least Squares (FGLS) method, allowing for serial correlation and heteroscedasticity in the residuals (White, 1980; Arellano, 2003). We have 100 time periods and only 30 sectors and therefore we do not employ the usual Arellano-Bond method that corrects for the bias in dynamic panel data models (Arellano and Bond, 1991).

Table 4 presents the results of the panel regressions applied to the 30 sectors providing us the sensitivities of default probabilities to the macro-financial variables. The coefficient of the lag default probability ranges between 0.643 and 0.918 according to the method, indicating historically strong persistence of these probabilities. We expect that increases in interest rates spread should imply in greater default probabilities and the first two methods give us positive coefficients although none of those being significant at a 10% level.

Moreover, we expect that rises in inflation (measured by IPCA index) and in the consumer confidence should rise and diminish, respectively, the default probabilities. All coefficients in the three models show this desired relation, varying, for the first, between -0.00630, which is not significant to 0.37 and, for the second, between -1.57 and -0.49. Also, the rise in price of crude oil should diminish the default probabilities in Brazilian economic sectors, as the country has achieved self-sufficiency at this commodity. All three coefficients are negative, although, in the first model one is almost significant at the 10% level, varying between -0.68 and -0.71. Another hypothesis is that the interest

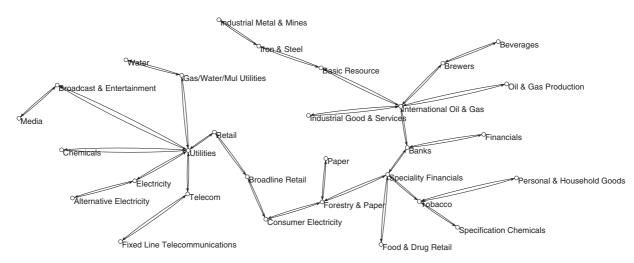


Fig. 6. Plot of the MST of a network connecting the full sample of probabilities of default for the period from January 3, 2000 to June 30, 2008.

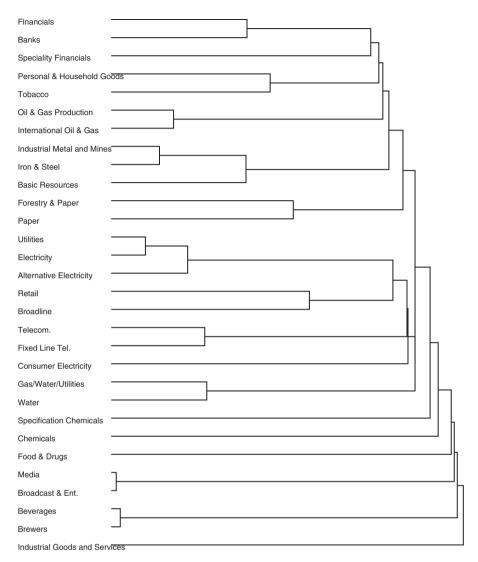


Fig. 7. Plot of the Taxonomy Hierarchical Tree of the subdominant ultrametric associated to the MST of the full sample of probabilities of default.

rate should have positive impact in default probabilities. The three methods used provide significant coefficients with a positive relationship, in accordance to theoretical predictions. Also the results show that for the monthly change in the production index we obtain the coefficients 11.22, 10.26 and 6.432, respectively, from the first to the third method.

As Brazilian firms are net exporters, our expectation is that a rise of the exchange rate should imply in greater default probability, this effect is significantly captured in all coefficients of this variable in the models, varying from 5.634 to 10.09. Furthermore, the stock market index is a leading indicator for economic activity, therefore if the stock market index increases we should expect positive growth of the economy and lower default probabilities. Our results show that in the three methods applied the signal of the coefficient is the one desired, negative and the values are -5.100, -4.986 and -3.398, for the first to the third model, respectively.

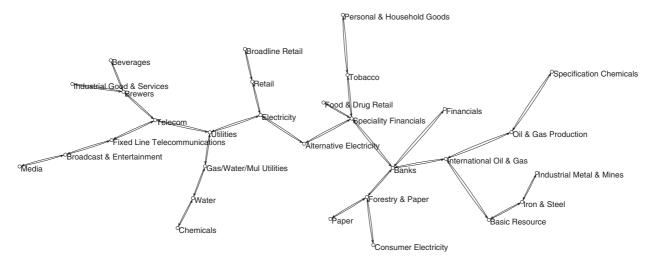


Fig. 8. Plot of the MST of a network connecting the full sample of probabilities of default for the period from 2000 to 2004.

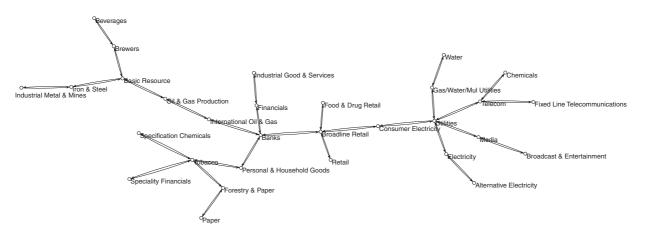


Fig. 9. Plot of the MST of a network connecting the full sample of probabilities of default for the period from 2004 to 2008.

3.3. Minimum Spanning Tree

We use the probabilities of default to construct the Minimum Spanning Tree (MST) and the ultrametric hierarchical tree, to identify clusters and connections between market sectors. As well as it can be seen with large complex financial institutions, including stock and equity markets, and interest rates (Hawkesby et al., 2007; Coelho et al., 2007; Huang et al., 2009; Tabak et al., 2009b), shocks in the economy that affect a specific sector tend to affect the entire cluster spreading to near neighbors.

Our search of these topological arrangements, which are present between the stocks of a given portfolio, is intended to provide empirical evidence about the existence of economic factors which drive the time evolution of stock prices (Mantegna, 1999). This graphical tool based in the matrix of correlation between probabilities of default can be used to minimize risks for a given portfolio return by optimizing the asset weights. Stocks of the minimum risk portfolio are found on the outskirts of a graph, thus it is expected that larger graphs lead to a greater diversification potential, as the scope of the stock market tends to eliminate specific risks (Onnela et al., 2003; Jung et al., 2006).

A careful look at the MST and of the Taxonomy Hierarchical Tree show mainly two groups of stocks, centralized by International Oil & Gas and Utilities. Both of these sectors, as we can observe in the individual regressions, suffer great impact due to variations of the exchange rate and of the industrial production index which are highly significant variables of all the three panel regressions of Table 4. The connection between those two groups is made by the Banks and the Speciality Financials sectors playing its role as financial intermediaries.

Also, we can observe in Figs. 6 and 7 that there is a small cluster centralized by Speciality Financials connecting Forestry & Paper and the Tobacco sectors. Moreover, many intuitive direct connections are identifiable such as that of Brewer and Beverages sectors, Broadcast & Entertainment and Media, Forestry & Paper and Paper or Iron & Steel and Industrial Metal & Mines sectors. All of the latter when compared tend to have similar individual regressions indicating great coherence in the graph that is able to illustrate such relation. This is expected as in some cases, specific sectors may possess similar stocks and our procedure is able to identify these sectors as they have a large correlation between themselves.

The observed groups are homogeneous with respect to industry (although with a few exceptions) and can be divided into subgroups. It is argued that stocks belonging to the same clusters carry detectable economic information in the sense that their prices respond, in a statistical point of view, to similar economic factors. In order to gain some benefit from diversification one should build portfolios using stocks that are dissimilar and are not fully connected with other sectors. Furthermore, these results suggests which sectors may be used to hedge against adverse price movements in specific sectors.

From a credit risk point of view diversification is crucial. Therefore, lending to sectors that belong to specific clusters may imply higher credit risk, which can be avoided by focusing on distant sectors belonging to different clusters (Bauerle, 2002).

In Figs. 8 and 9, we also present the MST for two distinct periods—2000–2004 and 2004–2008. Our results suggests that the links between clusters may weaken in specific cases and in others they may reinforce. Therefore, these results suggest that the method has to be updated within a certain frequency in order to gain the full benefits of it. It is a visual method that allows to establish which sectors should be targeted in order to mitigate risks or enhance investment performance.

4. Conclusions

In this paper we have estimated default probabilities for 30 market sectors using an approach based on stock market behavior. The measure is based on a conditional version of the CAPM and provides failure probabilities for each of the market sectors over the last 8 years. From 2000 to 2008, we observe a declining trend for the average of the probabilities. After, we try to improve our understanding on the sources of systematic risk in Brazil. Domestic macroeconomic factors such as the exchange rate and spread were found to be the most significant variables to explain these probabilities.

We have also estimated panel regressions for the default probabilities using three different methods. We could observe great significance in variables such as the exchange and the interest rate, the national stock market index and also in the industrial production index. Furthermore, the fixed effects model is able to explain 56% of the change in the default probabilities. The MST and the Taxonomy Hierarchical Tree analysis designed with these probabilities also tell us that the Brazilian sectors cluster together in groups centralized by the International Oil & Gas and the Utilities sectors.

We believe that the results obtained in this paper are of vital importance to risk management. With the measure of default probabilities it is possible to assess the risk associated to the economic sectors that were analyzed. Moreover, with the design of the network involving each sector, the researcher can have a broader view of the system, being able to observe how sectors are interconnected and whether they form clusters. This analysis opens possibilities of minimizing risks associated to loans and investments through diversification.

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