Comment on "Debye shielding in a nonextensive plasma" [Phys. Plasmas 18, 062102 (2011)]

L. A. Rios,[1,a)] R. M. O. Galvão,[1,2] and L. Cirto[1]
[1]*Centro Brasileiro de Pesquisas Físicas and Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[2]*Instituto de Física, Universidade de São Paulo, 05508-900 São Paulo, Brazil*

(Received 19 January 2012; accepted 9 March 2012; published online 30 March 2012)

Gougam and Tribeche [Phys. Plasmas **18**, 062102 (2011)] revisit the phenomenon of Debye shielding within the theoretical framework of Tsallis statistical mechanics. The effect of electron nonextensivity on the Bohm sheath criterion is also investigated, and a modified critical Mach number is derived. However, their results are questionable, since they are based on a somewhat unjustified assumption about the parameter that specifies the width of the distribution function.
 [http://dx.doi.org/10.1063/1.3697818]

In a recent paper,[1] Gougam and Tribeche considered the phenomenon of Debye shielding within the theoretical framework of Tsallis statistical mechanics. A modified Bohm sheath criterion is also derived, and due to electron nonextensivity the critical Mach number may become less than unity (for $q > 1$), allowing ions with speed smaller than the ion-acoustic speed to enter the sheath. However, the results presented are based on the assumption that β, the parameter that specifies the width of the distribution function, is equal to $1/k_BT$, which is not always true in the Tsallis formalism.

The one-dimensional Tsallis distribution function for a variable x has the following form[2]

$$f(x) \propto \left[1-\beta(1-q^*)x^2\right]^{1/(1-q^*)}. \qquad (1)$$

Tsallis distribution and statistics have been employed to investigate a variety of nonequilibrium systems, with long-range interactions and memory effects. They have been successfully applied to analyze different problems in plasma physics, from anomalous diffusion in dusty plasmas to turbulence in the interstellar medium.[3]

If we consider a two-component plasma with nonextensive ions and electrons, the velocity distribution function for both species can be written as[4]

$$f_j(v_j) = A_{q_j}\left\{1-\beta_j(q_j-1)\left[\frac{m_jv_j^2}{2}+Q_j\phi\right]\right\}^{1/(q_j-1)}, \qquad (2)$$

where electrons and ions are subject to a potential ϕ and

$$A_{q_j} = \begin{cases} n_0\sqrt{\frac{m_j\beta_j(1-q_j)}{2\pi}}\dfrac{\Gamma\left[\dfrac{1}{(1-q_j)}\right]}{\Gamma\left[\dfrac{1+q_j}{2(1-q_j)}\right]}, & -1<q_j<1, \\ n_0\sqrt{\frac{m_j\beta_j(q_j-1)}{2\pi}}\frac{(1+q_j)}{2}\dfrac{\Gamma\left[\dfrac{1+q_j}{2(q_j-1)}\right]}{\Gamma\left[\dfrac{1}{(q_j-1)}\right]}, & q_j>1. \end{cases}$$

In the above equation, n_0 is the equilibrium number density, $j=e(i)$ for the electrons(ions), Q_j is the electric charge, and we have defined $q^* = 2-q$. It is easy to see that Eq. (3) in Ref. 1 is obtained from Eq. (2) above, if we consider $\beta_e = 1/k_BT_e$. However, as mentioned before, this last assumption—in principle, a natural one—is not always valid in the framework of nonextensive statistical mechanics. In fact, the connection of β with the equilibrium temperature T of the system is still unknown. As in traditional thermodynamics, β is related to the dispersion of the distribution function and its value has been derived for different plasma systems analyzed in the context of Tsallis statistics.[3,5] In some particular cases, analytic expressions for β can be derived, with some of them showing an explicit dependence on the "entropic index" q.[4]

We now exemplify how an incorrect assumption about the value of β can lead to spurious results. For simplicity, we assume that the ions just form a neutralizing background. Thus, for the interval $-1<q<1$, the velocity distribution function for the electrons can be written as

$$f_\kappa(v_e) = \frac{n_0}{v_{th}}\sqrt{\frac{\Theta}{\pi\kappa}}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\beta}{\kappa}\left(\frac{m_ev_e^2}{2}-e\phi\right)\right]^{-\kappa}, \qquad (3)$$

for $\kappa > 1/2$. In the above expression, $\Theta = k_BT_e\beta$, $v_{th} = \sqrt{2k_BT_e/m_e}$ is the electron thermal speed and $\kappa = 1/(1-q)$. For the interval $-1<q<1$, the Tsallis distribution is a long-tail distribution function, whose behavior is quite different from the one observed for $q>1$. When $q\to1$, the Maxwellian distribution is recovered. With Eq. (3) in mind, we can follow the steps of Ref. 1 and derive the electron density,

$$n_e(\phi) = n_0\left(1-\frac{\beta}{\kappa}e\phi\right)^{-(\kappa-1/2)}, \qquad (4)$$

and the Debye length,

$$\lambda_{\kappa e} = \lambda_{De}\sqrt{\frac{\kappa}{(\kappa-1/2)\Theta}}, \qquad (5)$$

[a)]Electronic mail: rios@cbpf.br.

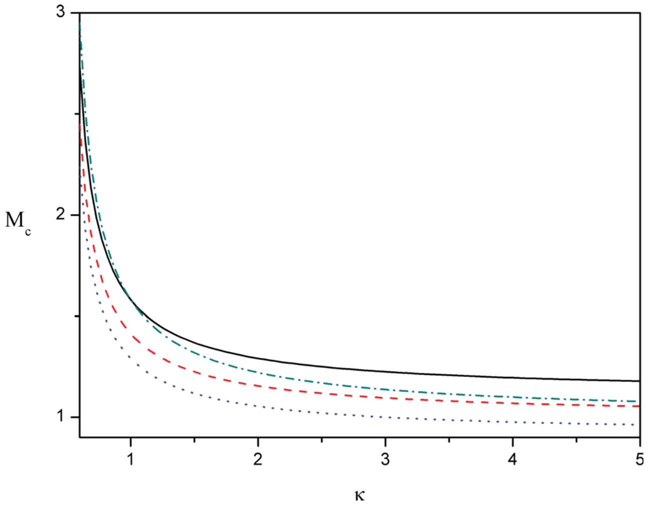

FIG. 1. $M_c(\Lambda_e)$ vs κ for $\Theta = 0.8$ (solid line), $\Theta = 1$ ($\beta = 1/k_BT_e$, dashed line), $\Theta = 1.2$ (dotted line) and $\Theta = A^{1/\kappa}$ ($A = 0.8$, dotted-dashed line).

where λ_{De} is the classical Debye length. For the cold ions to enter the sheath region from the background plasma we must have

$$v_i \geq c_s\sqrt{\frac{\kappa}{(\kappa - 1/2)\Theta}}, \tag{6}$$

where $c_s = \sqrt{k_BT_e/m_i}$ is the ion-acoustic speed. The modified Bohm criterion thus establishes that the Mach number $M = v_i/c_s$ has to be greater or equal to $M_c = \sqrt{\kappa/[(\kappa - 1/2)\Theta]}$. For $\beta = 1/k_BT_e$ ($\Theta = 1$), this is the same result obtained in Ref. 1. We notice that the normalized Debye length $\Lambda_e = \lambda_{\kappa e}/\lambda_{De}$ and the critical Mach number M_c have the same behavior, which is shown in Fig. 1 for different values of Θ. As in Figs. 4 and 8 of Ref. 1, Λ_e and M_c decrease as κ grows ($q \to 1, -1 < q < 1$). However, it is clear that a wrong choice for β can lead to the underestimation/overestimation of the mentioned quantities. The dotted-dashed line in Fig. 1 illustrates the behavior of Λ_e and M_c for $\beta = \gamma A^{1/\kappa}$, as given in Ref. 5 ($\gamma = 1/k_BT_e$ and $A = 0.8$). A similar analysis can be carried out for $q > 1$.

Based on Eq. (3), it is straightforward to derive the three-dimensional velocity distribution function, which also leads to results (4)-(6). The 3D version of Eq. (3) with $\beta = 1/k_BT_e$ denotes a reduced form of the so called κ-distribution, which has been used for many decades in astrophysics and can be written as[6]

$$f_\kappa(v_e) = A_\kappa\left[1 + \frac{1}{\kappa}\left(\frac{v_e^2}{v_{th}^2} - \frac{e\phi}{k_BT_e}\right)\right]^{-(\kappa+1)}, \tag{7}$$

for electrons in the presence of an electric potential, with $A_\kappa = \frac{n_0}{(v_{th}\sqrt{\pi\kappa})^3}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}$. Equation (7) also leads to expression (4) for the electron density, with $\beta = 1/k_BT_e$. It means that $\Lambda_e \equiv M_c = \sqrt{\kappa/(\kappa - 1/2)}$ in this case. Here, we take the opportunity to call the attention to some contradictory results related to the κ-distributions. The differences between expression (7) and the more conventional form of the κ-distribution, i.e.,

$$f_\kappa(v_e) = A_\kappa\left\{1 + \frac{1}{(\kappa - 3/2)}\left[\frac{v_e^2}{v_{th}^2} - \frac{e\phi}{k_BT_e}\right]\right\}^{-(\kappa+1)}, \tag{8}$$

with $A_\kappa = \frac{n_0}{(\pi\kappa\theta^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}$ and $\theta = [(\kappa - 3/2)/\kappa]^{1/2}v_{th}$, have already been discussed[7] and we are not going into details here. However, we would like to point out some discrepancies in the results obtained from the two different forms presented above. Considering Eq. (8), we get for Λ_e and M_c the following expression

$$M_c \equiv \Lambda_e = \sqrt{\frac{(\kappa - 3/2)}{(\kappa - 1/2)}}, \tag{9}$$

for $\kappa > 3/2$. It is clear that far from thermal equilibrium (κ) the expressions obtained for $M_c(\Lambda_e)$ for the two different distribution functions above have opposite behaviors. Notice that M_c can become less than unity for the distribution given in Eq. (8). Since the physical mechanism responsible for the origin of long-tail distributions in plasmas is a subject under investigation, a good explanation for the discrepancies presented above is still not available.

To conclude, we emphasize that in the framework of Tsallis statistical mechanics the connection of β with the equilibrium temperature of the system is still not known. It should be kept in mind that a wrong assumption about β can lead to the incorrect estimation of important plasma parameters, like the Debye length and the critical Mach number. Such parameters are related to the plasma velocity distribution function, and the explanation for the discrepancies discussed above relies on the physics behind the origin of long-tail distributions in nonthermal plasmas.

We are grateful to Dr. Constantino Tsallis (at CBPF) for valuable discussions and remarks. This work has been supported by the Brazilian Agencies CAPES, FAPERJ, and CNPq (INCT-SC).

[1] L. A. Gougam and M. Tribeche, Phys. Plasmas **18**, 062102 (2011).
[2] C. Tsallis, *Introduction to Nonextensive Statistical Mechanics* (Springer Science, New York, 2009).
[3] B. Liu and J. Goree, Phys. Rev. Lett. **100**, 055003 (2008); A. Esquivel and A. Lazarian, Astrophys. J. **710**, 125 (2010).
[4] R. Silva, Jr., A. R. Plastino, and J. A. S. Lima, Phys. Lett. A **249**, 401 (1998).
[5] L. F. Burlaga, N. F. Ness, and M. H. Acuña, Astrophys. J. **691**, L82 (2009).
[6] M. P. Leubner, Astrophys. Spac. Sci. **282**, 573 (2002).
[7] L.-N. Hau and W.-Z. Fu, Phys. Plasmas **14**, 110702 (2007); M. A. Hellberg, R. L. Mace, T. K. Baluku, I. Kourakis, and N. S. Saini, *ibid*. **16**, 094701 (2009); L.-N. Hau, W.-Z. Fu, and S.-H. Chuang, *ibid*. **16**, 094702 (2009).