Modulation of whistler waves in nonthermal plasmas

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The modulation of whistler waves in nonthermal plasmas is investigated. The dynamics of the magnetized plasma is described by the fluid equations and the electron velocity distribution function is modeled via a nonthermal κ distribution. A multiscale perturbation analysis based on the Krylov–Bogoliubov–Mitropolsky method is carried out and the nonlinear Schrödinger equation governing the modulation of the high-frequency whistler is obtained. The effect of the superthermal electrons on the stability of the wave envelope and soliton formation is discussed and a comparison with previous results is presented. © *2011 American Institute of Physics*. [doi:10.1063/1.3556125]

I. INTRODUCTION

Since the discovery of whistler waves in the ionosphere over 100 years ago,[1] there has been a great interest in their complex properties, like strong dispersion and ducting. In space physics, the name "whistler" usually defines an electromagnetic (em) wave excited by lightning with all other excitation mechanisms leading to "whistler-mode waves." As in laboratory plasma physics, here we call whistler any wave propagating in the whistler-mode. Besides ongoing basic research on whistlers and whistler-related phenomena in space and laboratory plasmas, interesting applications of these waves have arisen in recent decades. Whistlers have been traditionally used for remote diagnostics of the Earth's magnetosphere and ionosphere,[2] but potential applications such as predictions of earthquakes and volcanic eruptions,[3] the current drive in toroidal fusion devices,[4] and a variety of industrial applications[5] have also been investigated. An interesting review about whistlers and related phenomena in both space and laboratory plasmas can be found in Ref. 6.

Whistlers are widely observed in the terrestrial magnetosphere and plasmasphere, where whistler-related modulated structures associated with density perturbations have been observed by recent satellite missions like Cluster,[7] Freja,[8] and Polar.[9] Ionospheric density irregularities related to whistlers have also been observed in natural events and in high-frequency heating experiments.[10] Whistler waves can be excited in the circumterrestrial plasma by wave-particle interactions, e.g., via the Landau resonance with propagating beams or via the cyclotron resonance in plasmas with electron temperature anisotropy.[11,12] On the other hand, wave-particle interactions can be responsible for the generation of high-energy tails in the velocity/energy distribution functions of the plasma particles.[13] Superthermal electrons and ions are a common feature in laboratory[14,15] and space plasmas.[16,17] However, which specific mechanism is behind the energization of the plasma particles is still an open question. It is believed that long-tail distributions are the result of wave turbulence[18] and that they represent stationary states far from thermal equilibrium.[19]

Plasmas containing particles with velocities exceeding the thermal velocity can present hard accelerated distributions, which can be conveniently modeled via a nonthermal distribution function. The family of κ distributions first discussed by Vasyliunas[20] has been proven to be appropriate for modeling non-Maxwellian plasmas. It has been employed to analyze and interpret data on different plasma environments, like the solar wind,[16,21] the Earth's magnetosphere[22] and ionosphere,[23] and the solar corona.[24] The reduced form of the standard κ distribution[25] is equivalent to the distribution function obtained from the maximization of the Tsallis entropy, the q distribution.[26] The parameters κ and q measure the deviation from the Maxwellian equilibrium ('nonthermality') and are related by the expression $-\kappa=1/(1-q)$. A distribution with a small value of κ describes a plasma with an excess of superthermal particles, while in the limit $\kappa\to\infty$ ($q\to 1$), the Maxwellian distribution is recovered. Tsallis distribution and statistics have also been employed to investigate many problems in plasma physics,[27–30] and some authors claim that they form the basis to understanding the observed nonthermal features in space plasmas.[25]

The dispersion relations predicted by the theory of waves in a κ plasma have been used to determine the index κ in space[31] and laboratory experiments.[14] A modified plasma dispersion function for isotropic κ distributions has been obtained[32] and successfully applied to the investigation of electrostatic and both parallel and perpendicularly propagating em waves.[33] Recently, the κ-Maxwellian distribution has been introduced as more suitable for modeling magnetized plasmas, where the existence of a magnetic field sets a preferred direction in space.[34] The related plasma dispersion function[34] has been used to study obliquely propagating waves in κ-Maxwellian plasmas,[33] and it is shown that the presence of energetic particles can significantly change the dispersion and damping of whistlers.[35]

In the present paper, we investigate the nonlinear modulation of a whistler wave propagating in a nonthermal plasma where electrons are modeled by a κ distribution. The modulation of whistlers due to the parametric coupling with low-frequency wave modes has been discussed in the past by many authors.[36–39] Here we analyze the modulation of the

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carrier wave due to the coupling with ion-acoustic perturbations, which are ponderomotively driven by the high-frequency em wave. The nonlinear coupling between the high-frequency whistler and the electrostatic perturbations produces an electric field envelope that, if unstable, undergoes a modulational instability. The presence of a high-energy tail in the distribution function should considerably change the rate at which particles and plasma waves exchange energy. Therefore, conditions for the onset of instabilities are also modified when compared to those in a Maxwellian plasma.

This paper is organized as follows: In Sec. II, we discuss the model and basic equations. The fluid model is used to describe the dynamics of the magnetized plasma, where electrons behave according to a nonthermal κ distribution and Maxwell's equations describe the self-consistent electromagnetic fields. The nonlinear Schrödinger (NLS) equation governing the modulation of the carrier wave is discussed in Sec. III, where we also review the derivation of the ponderomotive force acting on the plasma electrons. The NLS equation and the ponderomotive force are derived through a multiscale perturbation analysis based on the Krylov–Bogoliubov–Mitropolsky (KBM) method, with further details given in the Appendix. In Secs. IV and V, we discuss the role of electron nonthermality (via parameter κ) on the stability of the wave envelope and soliton formation, respectively. The influence of the superthermal particles on the growth rate of the modulational instability is investigated and a comparison with previous results is presented. Section VI is devoted to a summary of our results and some conclusions.

II. MODEL AND BASIC EQUATIONS

The family of isotropic (three-dimensional) κ distributions has the form

$$f_\kappa(v) = \frac{1}{(\pi\kappa\theta^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{1}$$

where v is the particle velocity, $\Gamma(x)$ is the gamma function, and $\theta=[(\kappa-3/2)/\kappa]^{1/2}v_T$ is a generalized thermal speed related to the usual thermal speed $v_T=\sqrt{2k_BT/m}$. Note that these distributions are properly defined only for $\kappa>3/2$. The correct form of the κ velocity distribution has been the subject of recent discussion.[40,41] The expansion of Eq. (1) in the limit $\kappa\to\infty$ reveals the similarity between the κ and Maxwellian distributions.[40]

We consider a whistler wave propagating in a nonthermal magnetized electron-ion plasma. Whistlers are right-hand circularly polarized electromagnetic electron-cyclotron waves propagating in magnetized plasmas at frequencies below the local electron-cyclotron frequency, $\omega<\omega_{ce}$, where $\omega_{ce}=eB_0/m_ec$ and B_0 is the strength of the local magnetic field. Assuming a field-aligned propagating wave, its dispersion relation is given by

$$\frac{c^2k^2}{\omega^2} = 1-\frac{\omega_{pe}^2/\omega^2}{(1-\omega_{ce}/\omega)}, \tag{2}$$

where $\omega_{pe}=(4\pi e^2N_0/m_e)^{1/2}$ and N_0 are the electron plasma frequency and equilibrium plasma density, respectively. As we can see, for whistlers, the phase velocity $v_\phi=\omega/k$ is always smaller than the speed of light c.

The em wave travels along the external magnetic field $\mathbf{B}=B_0\hat{\mathbf{z}}$ and as it propagates, low-frequency electron density fluctuations are generated due to the wave ponderomotive force. The electrons are then subject to a total potential $\varphi_e=\varphi_{sc}+\varphi_p$, where φ_{sc} and φ_p are the electrostatic and ponderomotive potentials, respectively. The ions follow the electron motion and are also subject to the space-charge potential, but their response to the high-frequency field can be neglected. Using the energy conservation relation, we can write the κ distribution function for the plasma electrons in the following form.

$$f_\kappa(v_e) = \frac{N_0}{(\pi\kappa_e\theta_e^2)^{3/2}}\frac{\Gamma(\kappa_e+1)}{\Gamma(\kappa_e-1/2)} \times\left(1+\frac{v_e^2-2e\varphi_e/m_e}{\kappa_e\theta_e^2}\right)^{-(\kappa_e+1)}. \tag{3}$$

Integrating the κ distribution [Eq. (3)] over velocity space, one obtains the electron number density,

$$N_e(\phi_e) = N_0\left[1-\frac{\phi_e}{(\kappa_e-3/2)}\right]^{-(\kappa_e-1/2)}, \tag{4}$$

where T_e is the electron temperature, $\phi_e=e\varphi_e/k_BT_e$, and k_B is the Boltzmann constant. The pressure is given by $P_e=m_e/3\int v_e^2f_\kappa(v_e)d^3v_e$,

$$P_e(\phi_e) = P_0\left[1-\frac{\phi_e}{(\kappa_e-3/2)}\right]^{-(\kappa_e-3/2)}, \tag{5}$$

where $P_0=N_0k_BT_e$ is the plasma pressure in the equilibrium state.

We now assume that the normalized potential ϕ_e is much smaller than one ($e\varphi_e\ll k_BT_e$). Therefore, we can expand Eqs. (4) and (5) around $\phi_e=0$ to obtain

$$N_e(\phi_e) = N_0(1+\alpha_0\phi_e+\alpha_1\phi_e^2+\alpha_2\phi_e^3\ldots), \tag{6}$$

$$P_e(\phi_e) = P_0(1+\phi_e+\beta_0\phi_e^2+\beta_1\phi_e^3\ldots), \tag{7}$$

where $\alpha_0,\alpha_1,\ldots,\beta_0,\beta_1,\ldots$ are constants depending on κ_e. Since we are considering the weak nonlinear regime, only the first nonlinear terms are kept, i.e.,

$$n_e = 1+\alpha_0\phi_e+\alpha_1\phi_e^2, \tag{8}$$

$$p_e = 1+\phi_e+\beta_0\phi_e^2, \tag{9}$$

where $n_e=N_e/N_0$ and $p_e=P_e/N_0k_BT_e$ are the normalized electron density and pressure, respectively, and

$$\alpha_0=(\kappa_e-1/2)/(\kappa_e-3/2),$$
$$\alpha_1=[\alpha_0(\kappa_e+1/2)]/[2(\kappa_e-3/2)], \tag{10}$$
$$\beta_0=\alpha_0/2.$$

Equations (8) and (9) together play the role of an 'equation of state' for the nonthermal electron plasma. As we will see, these equations are considered only when the longitudinal (slow) motion of the plasma is investigated. We would like to point out that Eqs. (8) and (9) are valid only when longitudinal waves with $v_\phi \ll v_{Te}$ are considered, since in the limit $\kappa_e \to \infty$, the Boltzmann distribution is recovered [Eq. (8)].

The wave equation governing the propagation of a transverse electron whistler in a magnetized plasma is

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}=\frac{4\pi e}{c}N_e\mathbf{v}_{e\perp}, \tag{11}$$

where $\mathbf{A}=(A_x,A_y,0)$ is the vector potential and the electromagnetic fields are given by $\mathbf{E}=-(1/c)\partial\mathbf{A}/\partial t$ and $\mathbf{B}=\nabla\times\mathbf{A}$. The electron quiver velocity $\mathbf{v}_{e\perp}=(v_{ex},v_{ey},0)$ obeys the transverse component of the electron equation of motion,

$$m_e\left(\frac{\partial}{\partial t}+\mathbf{v}_e\cdot\nabla\right)\mathbf{v}_e=-e\mathbf{E}-\frac{e}{c}[\mathbf{v}_e\times(\mathbf{B}_0+\mathbf{B})]-\frac{\nabla P_e}{N_e}, \tag{12}$$

with $\mathbf{v}_e=\mathbf{v}_{e\perp}+v_{ez}\hat{\mathbf{z}}$. The contribution of the generalized thermal pressure P_e has been taken into account since we plan to study the effect of the superthermal electrons on the propagation of the whistler wave. As mentioned earlier, the high-frequency wave travels along the z-direction. Here we consider the one-dimensional problem and assume that all quantities vary only with z, which turns Eq. (11) into

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}=\frac{4\pi e}{c}N_e\mathbf{v}_{e\perp}, \tag{13}$$

and the transverse component of Eq. (12) becomes

$$\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)\left(\mathbf{v}_{e\perp}-\frac{e\mathbf{A}}{m_ec}\right)=\omega_{ce}(\hat{\mathbf{z}}\times\mathbf{v}_{e\perp}). \tag{14}$$

As the em wave propagates, the electrons are also accelerated in the longitudinal direction due to the combined action of the ponderomotive and the restoring electrostatic forces. The longitudinal electron motion is described by the component z of Eq. (12),

$$\left(\frac{\partial}{\partial t}+v_{ez}\frac{\partial}{\partial z}\right)v_{ez}=-\frac{e}{m_e}E_z-\frac{e}{m_ec}\left(v_{ex}\frac{\partial A_x}{\partial z}+v_{ey}\frac{\partial A_y}{\partial z}\right)-\frac{1}{m_eN_e}\frac{\partial P_e}{\partial z}, \tag{15}$$

where E_z stands for the longitudinal electrostatic field. The quantities N_e and v_{ez} are related by the continuity equation,

$$\frac{\partial N_e}{\partial t}+\frac{\partial(N_ev_{ez})}{\partial z}=0. \tag{16}$$

The ions do not experience the ponderomotive force but also move in the longitudinal direction due to the electrostatic field,

$$\left(\frac{\partial}{\partial t}+v_{iz}\frac{\partial}{\partial z}\right)v_{iz}=\frac{e}{m_i}E_z, \tag{17}$$

and the ion continuity equation

$$\frac{\partial N_i}{\partial t}+\frac{\partial(N_iv_{iz})}{\partial z}=0, \tag{18}$$

relates the ion density to the ion velocity v_{iz}.

III. KBM METHOD AND NLS EQUATION

We now investigate the nonlinear modulation of the em wave by the low-frequency ion-acoustic perturbations. To analyze the long time behavior of the high-frequency field, we carry through a multiscale perturbation analysis based on the Krylov–Bogoliubov–Mitropolsky (KBM) method.[42] It is shown here that the KBM method is useful in obtaining the nonlinear Schrödinger (NLS) equation describing the amplitude modulation of the whistler wave and also the expression for the ponderomotive force acting on the plasma electrons. An advantage of this method is that it is conceptually natural: It consists of varying the amplitude of the wave so slowly that no secular terms can arise. First, all the transverse quantities are considered weakly nonlinear waves,

$$f=\varepsilon f_1(a,a^*,\psi)+\varepsilon^2f_2(a,a^*,\psi)+\varepsilon^3f_3(a,a^*,\psi)+\ \dots, \tag{19}$$

where f stands for any physical quantity. We can see that all the transverse quantities are functions of the complex amplitude a, its complex conjugate a^*, and the fast variable $\psi=kz-\omega t$. In the above expression, $\varepsilon\sim O(a)\ll 1$ and all the significant terms up to order ε^3 are kept. As we will see, this order of approximation is enough for our purpose in this paper. The complex amplitude of the em wave is assumed to be a slowly varying function of z and t through the relations

$$\begin{aligned}\partial_t a&=\varepsilon A_1(a,a^*)+\varepsilon^2A_2(a,a^*)+\varepsilon^3A_3(a,a^*)+\ \dots,\\ \partial_z a&=\varepsilon B_1(a,a^*)+\varepsilon^2B_2(a,a^*)+\varepsilon^3B_3(a,a^*)+\ \dots.\end{aligned} \tag{20}$$

Equivalent relations can be written for a^*. From Eq. (15), it is straightforward to see that the longitudinal quantities are connected to the transverse ones through the Lorentz force. Therefore, the lowest order term in the expansion of the longitudinal quantities originates from the product of two first order perturbations. The normalized electron density can then be written as

$$n_e=1+\varepsilon^2n_{e2}, \tag{21}$$

and all the longitudinal variables take the form $f=f_0+\varepsilon^2f_2$, where f_0 is the value of f for the equilibrium state. Here, the approximation up to order two is enough (also shown later). The low-frequency oscillations depend on the amplitude a

(and a^*) but not on the phase ψ, as will be seen when we analyze the longitudinal motion.

The perturbation analysis is first applied to the transverse equations. The procedure is very similar to the one presented in our previous work for the linearly polarized em waves[43] and the details are omitted here. Nonetheless, they can be found in the Appendix. After applying the perturbation method to the transverse equations, we get, to order ε^3,

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+\frac{1}{2}\frac{dv_g}{dk}\frac{\partial^2 a}{\partial z_1^2}-\frac{v_g\omega\omega_{pe}^2}{2kc^2(\omega-\omega_{ce})}\times\left[n_{e2}+\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}v_{ez2}\right]a=0, \tag{22}$$

where $t_2=\varepsilon^2 t=\varepsilon t_1$ and $z_2=\varepsilon^2 z=\varepsilon z_1$ (the amplitude a of the EM wave is normalized to $m_e c^2/e$). In the above equation, v_{ez2} is the second order longitudinal electron velocity ($v_{ez}=\varepsilon^2 v_{ez2}$), v_g is the group velocity of the whistler wave, $P=(1/2)dv_g/dk$ is the dispersion coefficient, and

$$\Delta=-\frac{v_g\omega\omega_{pe}^2}{2kc^2(\omega-\omega_{ce})}\left[n_{e2}+\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}v_{ez2}\right]. \tag{23}$$

The above nonlinear equation describes the modulation of the em wave envelope. It is very similar to those used in earlier works and which were derived by different perturbation methods[36–39] (in particular, this is the same equation used in Ref. 39). When compared to previous equations, the differences in Eq. (22) are seen to be caused by the absence of weak relativistic effects[38] and by our one-dimensional treatment.[36,37]

We now consider the equations describing the longitudinal motion. Applying the perturbation method to Eq. (15), we obtain $[O(\varepsilon^2)]$

$$m_e\frac{\partial v_{ez2}}{\partial t}=-eE_{z2}-\frac{1}{N_0}\frac{\partial P_{e2}}{\partial z}+f_{pz}, \tag{24}$$

where

$$\frac{P_{e2}}{P_0}=p_{e2}=\frac{(\kappa_e-3/2)}{(\kappa_e-1/2)}n_{e2}, \tag{25}$$

is derived from Eqs. (8) and (9) in their perturbed form and

$$f_{pz}=-m_e c\left(v_{ex1}\frac{\partial a_{x1}}{\partial z}+v_{ey1}\frac{\partial a_{y1}}{\partial z}\right), \tag{26}$$

is the ponderomotive force acting on the electrons. Here, $a_{x1}=ae^{i\psi}+c.c.$, $a_{y1}=iae^{i\psi}+c.c.$, v_{ex1}, and v_{ey1} are the first order components of the normalized vector potential and the electron quiver velocity, respectively (see Appendix), and $c.c.$ is the complex conjugate. If the definitions of a_{x1} and a_{y1} and the higher-order corrections to v_{ex1} and v_{ey1} are taken into account, Eq. (26) turns into

$$f_{pz}=-\frac{2m_e\omega c^2}{(\omega-\omega_{ce})}\left[\frac{\partial|a|^2}{\partial z}-\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}\frac{\partial|a|^2}{\partial t}\right]. \tag{27}$$

Further details about this calculation can be found in the Appendix. Equation (24) now becomes

$$\frac{\partial v_{ez2}}{\partial t}=-\frac{eE_{z2}}{m_e}-c_{se}^2\frac{\partial n_{e2}}{\partial z}-\frac{2\omega c^2}{(\omega-\omega_{ce})}\times\left[\frac{\partial|a|^2}{\partial z}-\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}\frac{\partial|a|^2}{\partial t}\right], \tag{28}$$

where E_{z2} is the second order electrostatic field and $c_{se}=(k_B T_e/m_e\alpha_0)^{1/2}$ is the generalized electron thermal speed.

From the above equation and the perturbed form of Eqs. (16)–(18), we obtain a wavelike equation for the plasma density fluctuations,

$$\left(\frac{\partial^2}{\partial t^2}-c_{si}^2\frac{\partial^2}{\partial z^2}\right)n_{e2}=\frac{2\mu_{ei}\omega c^2}{(\omega-\omega_{ce})}\left[\frac{\partial}{\partial z}-\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}\frac{\partial}{\partial t}\right]\frac{\partial|a|^2}{\partial z}, \tag{29}$$

where $\mu_{ei}=m_e/m_i$, $c_{si}=(\mu_{ei})^{1/2}c_{se}$, and the quasineutrality condition $n_{e2}\approx n_{i2}$ has been assumed. In deriving Eq. (29), the electron inertia in Eq. (28) has also been neglected. It is clear from the previous equation that the ion-acoustic oscillations are driven by the ponderomotive force.

Let us assume that the low-frequency perturbations are functions of the variable $\zeta=(z-v_g t)$. Then we can write $\partial/\partial t=-v_g\partial/\partial\zeta$, $\partial/\partial z=\partial/\partial\zeta$, and Eq. (29) becomes

$$(v_g^2-c_{si}^2)\frac{\partial^2 n_{e2}}{\partial\zeta^2}=\frac{2\mu_{ei}\omega c^2}{(\omega-\omega_{ce})}\left[1+\frac{k\omega_{ce}v_g}{\omega(\omega-\omega_{ce})}\right]\frac{\partial^2|a|^2}{\partial\zeta^2}. \tag{30}$$

Integrating the above equation and assuming a vanishing density perturbation at infinity, we get

$$n_{e2}=\frac{2\mu_{ei}c^2[\omega^2+\omega_{ce}(kv_g-\omega)](|a|^2-|a_\infty|^2)}{(\omega-\omega_{ce})^2(v_g^2-c_{si}^2)}. \tag{31}$$

Comparing the above equation with the expressions derived for n_{e2} in the case of a linearly polarized em wave [Eq. (56) in Ref. 43], it is easily seen that those are harmonics of the high-frequency field, different from the result obtained here for a circularly polarized wave.

Back to Eq. (22), we now consider waves whose phase velocity is small compared to the speed of light, $\omega/ck\ll 1$. In this limit, the dispersion relation [Eq. (2)] reduces to

$$\omega=\frac{\omega_{ce}c^2k^2}{(\omega_{pe}^2+c^2k^2)}, \tag{32}$$

a familiar expression for parallel propagating whistlers. This expression can be further reduced to $\omega\approx\omega_{ce}c^2k^2/\omega_{pe}^2$ if we consider the long-wavelength limit $k\ll\omega_{pe}/c$. The group velocity becomes $v_g=-2\omega(\omega-\omega_{ce})/\omega_{ce}k$, which leads us to write the coefficient Δ in the form

$$\Delta=-\frac{\omega_{pe}^2\omega^2}{\omega_{ce}k^2c^2}\left[n_{e2}-\frac{2v_{ez2}}{v_g}\right]. \tag{33}$$

If we assume that v_{ez2} is much smaller than the group velocity, Eq. (22) can be rewritten as

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+\frac{1}{2}\frac{dv_g}{dk}\frac{\partial^2 a}{\partial z_1^2}-\frac{2\mu_{ei}\omega_{pe}^2\omega^3(|a|^2-|a_\infty|^2)}{\omega_{ce}k^2(\omega-\omega_{ce})(v_g^2-c_{si}^2)}a=0, \quad (34)$$

where now

$$n_{e2}=-\frac{2\mu_{ei}\omega c^2(|a|^2-|a_\infty|^2)}{(\omega-\omega_{ce})(v_g^2-c_{si}^2)}. \quad (35)$$

Introducing the coordinate transformation $\xi=(1/\varepsilon)\times(z_2-v_g t_2)=\varepsilon\zeta$ and $\tau=t_2$, it is easily seen that Eq. (34) can be transformed into the nonlinear Schrödinger equation,

$$i\frac{\partial a}{\partial \tau}+P\frac{\partial^2 a}{\partial \xi^2}+Q(|a|^2-|a_0|^2)a=0, \quad (36)$$

where

$$P=\frac{v_g}{2k\omega_{ce}}(\omega_{ce}-4\omega), \quad (37)$$

and

$$Q=-\frac{2\mu_{ei}\omega_{pe}^2\omega^3}{\omega_{ce}k^2(\omega-\omega_{ce})(v_g^2-c_{si}^2)}, \quad (38)$$

is the nonlinear coefficient. A finite value for a as $\xi\to\pm\infty$ has been assumed in Eq. (36), $|a_\infty|\equiv|a_0|$. In going from Eq. (34) to Eq. (36), the term $-Q|a_\infty|^2a$ can be removed. This term is not so essential since it simply leads to a phase shift in a, and it can be eliminated by setting $a\to a\exp(-iQ|a_\infty|^2\tau)$. Equation (36) shows that, up to order ε^3, a wave packet traveling at the group speed has its amplitude a modified due to the linear dispersion and the action of nonlinear effects.

The diffusion of the wave packet due to the dispersion in a magnetized plasma can appear independently of nonlinear effects. The fact that in our analysis both terms appear in the same order can be understood if we analyze Eq. (22). The last term on the right-hand side (RHS) is clearly of order ε^3 since it results from the product of the amplitude a $[O(\varepsilon)]$ and quantities of order ε^2 (n_{e2} and v_{ez2}). The dispersion term becomes of order ε^3 due to our assumption that a varies slowly with space and time according to the expressions given in Eq. (20). This is equivalent to the slowly varying envelope approximation (SVEA), where one assumes that the envelope of a em wave pulse varies slowly in time and space compared to a period or wavelength, i.e.,

$$\left|\frac{\partial a}{\partial t}\right|\ll|\omega a|,\qquad\left|\frac{\partial a}{\partial z}\right|\ll|ka|. \quad (39)$$

For the space variable z, the previous approximation leads to $|a/L|\ll|ka|$, where L is the characteristic length of the wave packet. Thus, we can say that the implicit condition $1/kL\ll1$ is responsible for the appearance of the dispersion and nonlinear terms in the same order in ε. The first term on the RHS of Eq. (22) simply represents the undistorted transmission of the wave packet at the group speed v_g [an equivalent term appears when we consider the $O(\varepsilon^2)$ terms, see Appendix].

IV. MODULATIONAL INSTABILITY

A modulational instability arises when a whistler wave propagates in a nonlinear dispersive medium. As the wave travels through the plasma, it modifies those parameters which affect its dispersion and thereby modifies its propagation and amplitude.[6]

To investigate the effect of electron nonthermality on the stability of the modulation envelope described by the NLS equation, we follow the standard analysis of Hasegawa.[44] First, small perturbations on the phase and the amplitude of the wave envelope are considered. After that, a linear analysis shows that the mentioned perturbations cause a linear modulation with frequency Ω and wavenumber K obeying the dispersion relation

$$\Omega^2(K)=PK^2(PK^2-2Q|a_0|^2), \quad (40)$$

where $\Omega\ll\omega$ and $K\ll k$. From the above relation, we can see immediately that for QP (or Q/P) <0 the envelope is modulationally stable for any value of K. On the other hand, if $Q/P>0$, Ω^2 becomes negative for $K<K_{cr}=(2Q/P)^{1/2}|a_0|$. It means that, for a long-wavelength perturbation, the envelope becomes unstable and undergoes a modulational instability with a growth rate given by

$$\gamma=PK\left(2|a_0|^2\frac{Q}{P}-K^2\right)^{1/2}, \quad (41)$$

with the maximum growth attained for $K_{max}=(Q/P)^{1/2}|a_0|$. The nonlinear coefficient Q and the growth rate γ, as well as K_{cr} and K_{max}, depend on κ_e. Thus, some influence of the electron nonthermality on the behavior of the mentioned quantities is expected.

For the numerical examples in the sequel, we consider parameters valid for the Earth's magnetopause,[39,45,46] i.e., $N_0=10$ cm^{-3}, $k_BTe=8.5\times10^2$ eV, and $|a_0|=10^{-4}$. From Eqs. (37) and (38) in the previous section, we notice that to have $Q/P>0$, the condition

$$(\omega_{ce}-4\omega)(v_g^2-c_{si}^2)>0 \quad (42)$$

must be fulfilled. From the above condition, we detect two possible regions of instability: The first one, where $\omega<\omega_{ce}/4$ and $v_g>c_{si}$, and a second one, with $\omega>\omega_{ce}/4$ and $v_g<c_{si}$.

First, we analyze the frequency range $\omega<\omega_{ce}/4$. It implies $k<k_{cr}$, where $k_{cr}=\omega_{pe}/\sqrt{3}c\approx3.4\times10^{-4}$ m^{-1} for the given plasma density. In Fig. 1, we plot the normalized group velocity $\beta_g=v_g/c$ versus the wavenumber k of the em wave for $B_0=100$ nT (10^{-3} G). At the bottom of the figure are the curves representing c_{si}/c for extreme values of κ_e ($\kappa_e=1.6$ and 500). As we can see, v_g is larger than c_{si} in almost the whole range of interest ($\lambda=2\pi/k$ of the order of kilometers). However, $v_g\gg c_{si}$ for $k<k_{cr}$ and no effect of electron nonthermality is observable in this range. It is important to mention that this is the interval where the whistler waves become unstable due to the coupling with the ion-acoustic oscillations according to Ref. 39.

A second interval where the modulational instability can occur is given by $\omega_{ce}/4<\omega<\omega_{ce}$, which corresponds to $k>k_{cr}$. In Fig. 1 we observe that as k increases, the group

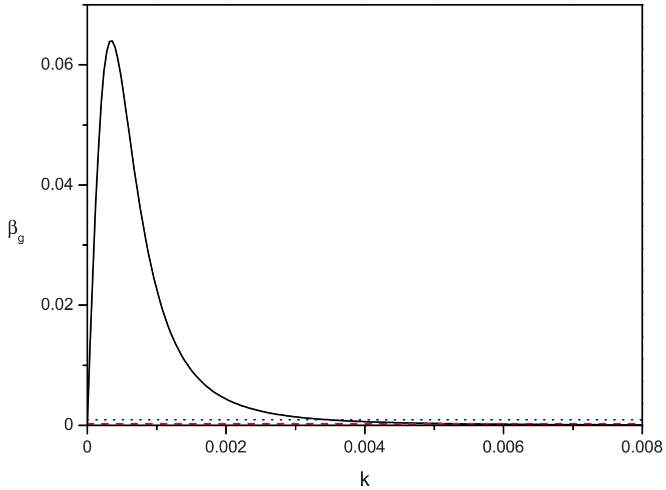

FIG. 1. (Color online) The normalized group velocity β_g for $B_0=100$ nT (solid line) compared to c_{si}/c evaluated for different values of κ_e.

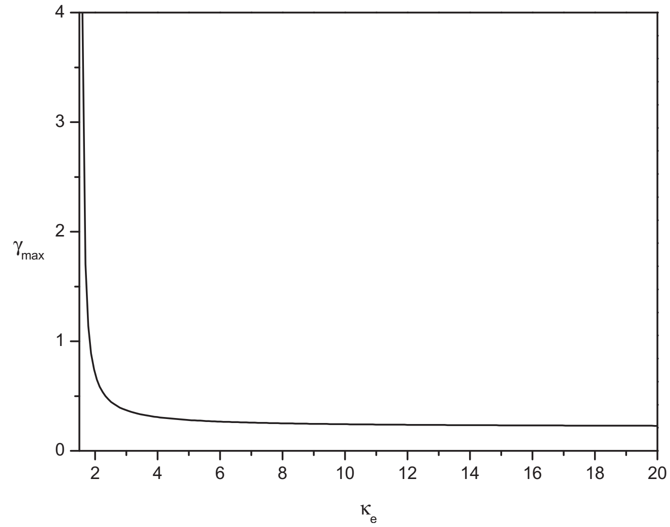

FIG. 3. γ_{max} (s^{-1}) vs κ_e for $k=6\times10^{-3}$ m^{-1} and $B_0=100$ nT.

velocity becomes comparable to the sound speed c_{si} and the influence of the energetic electrons starts to appear. In Fig. 2, we show how Q/P varies with k for three different values of κ_e (1.6, 2.8, and 5). We can see that the electron nonthermality affects the value of Q/P and also the wavenumber k_l where it changes its sign. The presence of the energetic particles seems to increase the value of Q/P and also the wavenumber (k_l) where the em envelope becomes unstable. Since Q/P is larger for plasmas containing superthermal electrons, smaller growth rates are expected for plasmas close to the thermal equilibrium ($\kappa_e\to\infty$). This can be observed in Figs. 3 and 4, where we plot the maximum growth rate and γ for $k=6\times10^{-3}$ m^{-1}, $B_0=100$ nT, and different values of κ_e. Hence, the effect of the superthermal electrons is to increase the growth rate and the instability window given by $0<K<K_{cr}$. We point out that our results are different from those obtained in Ref. 45, where the growth rate of the modulational instability of whistlers has been seen to decrease due to the influence of energetic particles. This kind of effect is observed when only the first term in the expression of the ponderomotive force [Eq. (27)] is considered; for example, what causes a change in the sign of Q/P.

V. BRIGHT SOLITONS

It is known that the NLS Eq. (36) admits localized solutions in the form of envelope solitons generated due to the combined effects of dispersion and nonlinearity.[44] In the case when the envelope is modulationally unstable ($Q/P>0$), wave collapse may lead to the formation of localized slowly varying structures called *bright* solitons. These localized solutions can be written in the form

$$a(\xi,\tau)=\sqrt{\rho(\xi,\tau)}e^{i\sigma(\xi,\tau)}, \tag{43}$$

where the real part is given by

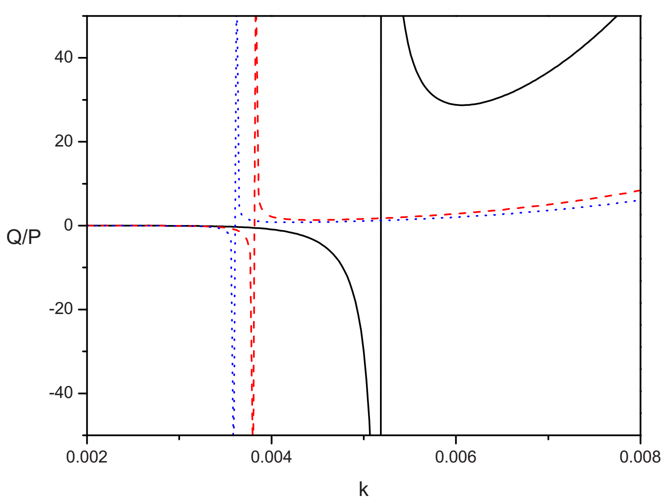

FIG. 2. (Color online) Q/P (m^{-2}) vs k (m^{-1}) for $B_0=100$ nT and $\kappa_e=1.6$ (solid line), $\kappa_e=2.8$ (dashed line), and $\kappa_e=5$ (dotted line).

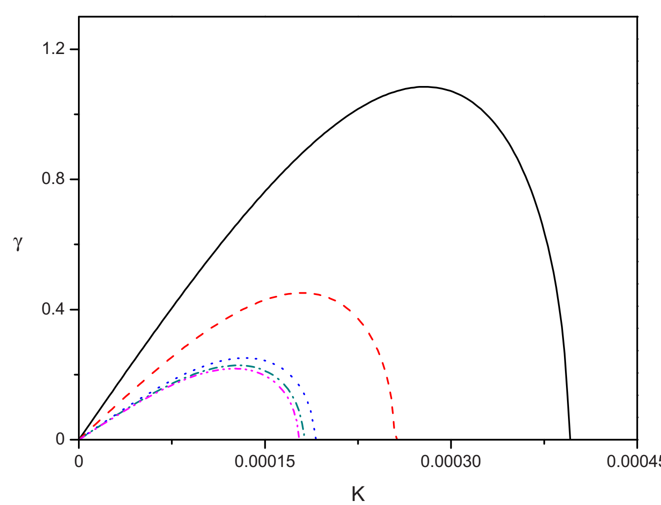

FIG. 4. (Color online) γ (s^{-1}) vs K (m^{-1}) for $k=6\times10^{-3}$ m^{-1}, $B_0=100$ nT and $\kappa_e=1.8$ (solid line), $\kappa_e=2.5$ (dashed line), $\kappa_e=8$ (dotted line), $\kappa_e=20$ (dot-dashed line), and $\kappa_e=100$ (dot-dot-dashed line, Maxwellian case).

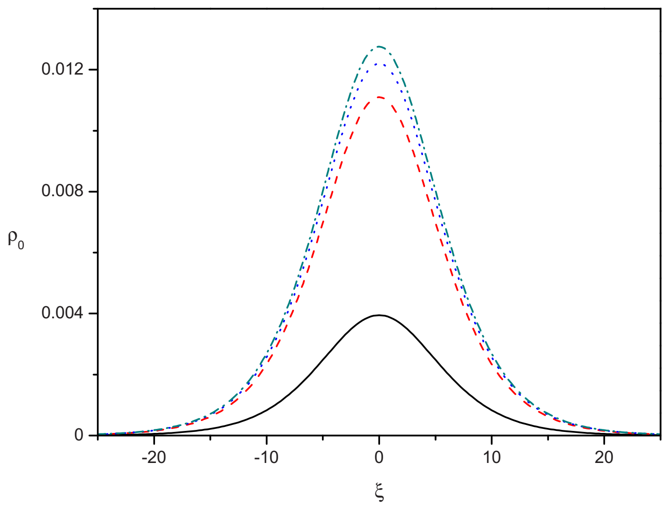

FIG. 5. (Color online) $\rho=|a|^2$ vs ξ for $k=6\times10^{-3}$ m^{-1}, $B_0=100$ nT and $\kappa_e=2$ (solid line), $\kappa_e=8$ (dashed line), $\kappa_e=20$ (dotted line), and $\kappa_e=100$ (dot-dashed line, Maxwellian case).

$$\rho=\rho_0\,\mathrm{sech}^2(\sqrt{\delta\rho_0}\xi). \tag{44}$$

In the above expression, ρ_0 is the soliton amplitude, $\delta=Q/P$, and $\Lambda=\rho_0\delta/2$. Like in the case of the growth rate γ, here, no effect of the superthermal particles is observed for $k<k_{cr}$. For the interval $k>k_{cr}$ (with $v_g<c_{si}$), the presence of the energetic electrons causes ρ_0 to decrease. This can be seen in Fig. 5, where the amplitude is displayed for $\Lambda=10^{-2}$, $k=6\times10^{-3}$ m^{-1}, $B_0=100$ nT, and four different values of κ_e.

VI. SUMMARY

In the present work, we investigate the modulation of whistler waves in nonthermal plasmas where electrons are modeled by a κ distribution. The nonlinear modulation of the high-frequency wave due to the coupling with low-frequency ion-acoustic perturbations is considered. The fluid model is used to describe the dynamics of the magnetized electron-ion plasma and a multiscale perturbation analysis based on the KBM method is carried out to derive the ponderomotive force and the NLS equation governing the modulation of the em wave. The influence of the superthermal electrons on the stability of the wave envelope is investigated and it is shown that the energetic particles significantly affect the wavenumber range where the em wave becomes unstable. Our results also show that the em envelope can become unstable for $\omega>\omega_{ce}/4$ and that for the considered conditions, the effect of electron nonthermality is to increase the growth rate of the modulational instability as well as the instability window. It is important to point out that the mentioned effects are noticeable only for large values of the wavenumber k (compared to $k_{cr}=\omega_{pe}/\sqrt{3}c$). The influence of the energetic particles on soliton formation is also discussed and we notice that the apparent effect of nonthermality is to decrease the amplitude of the bright solitons.

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APPENDIX: DERIVATION OF THE NLS EQUATION AND THE PONDEROMOTIVE FORCE

First, the transverse perturbations are written in the following form.

$$\begin{bmatrix} A_l \\ v_{el} \end{bmatrix} = \varepsilon \begin{bmatrix} A_{l1} \\ v_{el1} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} A_{l2} \\ v_{el2} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} A_{l3} \\ v_{el3} \end{bmatrix} + \dots, \tag{A1}$$

where $l=x$ or y. The perturbation technique is first applied to Eq. (13). Substituting Eqs. (21) and (A1)into Eq. (13) and separating powers of ε, we get, to the first order in ε,

$$\frac{\partial^2 a_{l1}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 a_{l1}}{\partial t^2} = \frac{\omega_{pe}^2}{c^3} v_{el1}, \tag{A2}$$

where $a_{l1}=eA_{l1}/m_e c^2$ $(l=x,y)$ are the components of the first order normalized vector potential $\mathbf{a}_1$ $(\mathbf{a}_1=a_{x1}\hat{\mathbf{x}}+a_{y1}\hat{\mathbf{y}})$. Following the same steps, we now apply the perturbation technique to Eq. (14) and obtain

$$\frac{\partial}{\partial t}(\beta_{ex1}-a_{x1}) = -\,\omega_{ce}\beta_{ey1}, \tag{A3}$$

and

$$\frac{\partial}{\partial t}(\beta_{ey1}-a_{y1}) = \omega_{ce}\beta_{ex1}, \tag{A4}$$

where $\beta_{el1}=v_{el1}/c$. As a starting solution to the first order normalized vector potential $\mathbf{a}_1$, we choose

$$\mathbf{a}_1 = a(\hat{\mathbf{x}}+i\hat{\mathbf{y}})e^{i\psi} + a^*(\hat{\mathbf{x}}-i\hat{\mathbf{y}})e^{-i\psi}. \tag{A5}$$

The electric field to the lowest order then becomes

$$\mathbf{E}_1 = \frac{im_e c\omega}{e}[a(\hat{\mathbf{x}}+i\hat{\mathbf{y}})e^{i\psi} - a^*(\hat{\mathbf{x}}-i\hat{\mathbf{y}})e^{-i\psi}]. \tag{A6}$$

From Eqs. (A2) and (A5), we obtain the first order electron quiver velocity,

$$\mathbf{v}_{e\perp1} = \frac{c(\omega^2-c^2k^2)}{\omega_{pe}^2}[a(\hat{\mathbf{x}}+i\hat{\mathbf{y}})e^{i\psi} + a^*(\hat{\mathbf{x}}-i\hat{\mathbf{y}})e^{-i\psi}]. \tag{A7}$$

For a solution different from the trivial one $(a_{x1}=a_{y1}=0)$, we obtain from Eqs. (A3) and (A4) the dispersion relation (2): $D(\omega,k)=\omega^2-c^2k^2-\omega\omega_{pe}^2/(\omega-\omega_{ce})=0$.

Back to the perturbed form of Eqs. (13) and (14), we now collect the terms of order ε^2 and get

$$-\left(\frac{\partial D}{\partial\omega}A_1 - \frac{\partial D}{\partial k}B_1\right)e^{i\psi} + \frac{\omega^2}{(\omega-\omega_{ce})}\frac{\partial^3 a_{x2}}{\partial\psi^3} + \omega\frac{\partial a_{x2}}{\partial\psi} - \frac{\omega\omega_{ce}}{(\omega-\omega_{ce})}\frac{\partial^2 a_{y2}}{\partial\psi^2} + c.c. = 0, \tag{A8}$$

$$-i\left(\frac{\partial D}{\partial \omega}A_1-\frac{\partial D}{\partial k}B_1\right)e^{i\psi}+\frac{\omega^2}{(\omega-\omega_{ce})}\frac{\partial^3 a_{y2}}{\partial \psi^3}+\omega\frac{\partial a_{y2}}{\partial \psi}+\frac{\omega\omega_{ce}}{(\omega-\omega_{ce})}\frac{\partial^2 a_{x2}}{\partial \psi^2}+c.c.=0, \tag{A9}$$

where $\partial D/\partial\omega=1-[2\omega(\omega-\omega_{ce})+\omega^2-c^2k^2]/\omega_{pe}^2$, $\partial D/\partial k=2kc^2(\omega-\omega_{ce})/\omega_{pe}^2$, and $c.c.$ is the complex conjugate to the first term in both previous equations. In finding Eqs. (A8) and (A9), we have used the operators

$$\partial_t=\varepsilon(A_1\partial_a+A_1^*\partial_{a^*})+\varepsilon^2(A_2\partial_a+A_2^*\partial_{a^*})-\omega\partial_\psi+O(\varepsilon^3),$$
$$\partial_z=\varepsilon(B_1\partial_a+B_1^*\partial_{a^*})+\varepsilon^2(B_2\partial_a+B_2^*\partial_{a^*})+k\partial_\psi+O(\varepsilon^3). \tag{A10}$$

In order to make the solutions a_{x2} and a_{y2} free from secular terms (ψ proportional terms), the first term in Eqs. (A8) and (A9) and its complex conjugate must be set equal to zero. It leads to the condition

$$A_1+v_gB_1=0, \tag{A11}$$

and its complex conjugate relation. Here,

$$v_g=-\frac{\partial D}{\partial k}\Big/\frac{\partial D}{\partial \omega}=\frac{d\omega}{dk}=\frac{2kc^2(\omega-\omega_{ce})}{[\omega^2-c^2k^2-\omega_{pe}^2+2\omega(\omega-\omega_{ce})]}, \tag{A12}$$

is the group velocity of the transverse whistler. From Eq. (20), it turns out that A_1 and B_1 can be written as $\partial a/\partial t_1$ and $\partial a/\partial z_1$ to the lowest order in ε, where $t_1=\varepsilon t$ and $z_1=\varepsilon z$. Thus, the above relation may be interpreted as $\partial a/\partial t_1+v_g\partial a/\partial z_1\cong 0$, which means that up to order ε^2, the amplitude a is constant in the rest frame of the wave packet. Solving now the coupled Eqs. (A8) and (A9) for a_{x2} and a_{y2} yields

$$\mathbf{a}_2=a_{x2}\hat{\mathbf{x}}+a_{y2}\hat{\mathbf{y}}=C_1(\hat{\mathbf{x}}+i\hat{\mathbf{y}})e^{i\psi}+C_1^*(\hat{\mathbf{x}}-i\hat{\mathbf{y}})e^{-i\psi}, \tag{A13}$$

where C_1 and C_1^* are arbitrary functions of a and a^*.

Proceeding further in the perturbation analysis, we get, to order ε^3,

$$-\left[\left(\frac{\partial D}{\partial \omega}A_2-\frac{\partial D}{\partial k}B_2\right)+\frac{i}{2}\left(v_g^2\frac{\partial^2 D}{\partial \omega^2}+2v_g\frac{\partial^2 D}{\partial \omega\,\partial k}+\frac{\partial^2 D}{\partial k^2}\right)\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)\right]e^{i\psi}+i\left[\omega n_{e2}+\frac{k\omega_{ce}}{(\omega-\omega_{ce})}v_{ez2}\right]ae^{i\psi}+\frac{\omega^2}{(\omega-\omega_{ce})}\frac{\partial^3 a_{x3}}{\partial \psi^3}+\omega\frac{\partial a_{x3}}{\partial \psi}-\frac{\omega\omega_{ce}}{(\omega-\omega_{ce})}\frac{\partial^2 a_{y3}}{\partial \psi^2}+c.c.=0, \tag{A14}$$

$$-i\left[\left(\frac{\partial D}{\partial \omega}A_2-\frac{\partial D}{\partial k}B_2\right)+\frac{i}{2}\left(v_g^2\frac{\partial^2 D}{\partial \omega^2}+2v_g\frac{\partial^2 D}{\partial \omega\,\partial k}+\frac{\partial^2 D}{\partial k^2}\right)\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)\right]e^{i\psi}-\left[\omega n_{e2}+\frac{k\omega_{ce}}{(\omega-\omega_{ce})}v_{ez2}\right]ae^{i\psi}+\frac{\omega^2}{(\omega-\omega_{ce})}\frac{\partial^3 a_{y3}}{\partial \psi^3}+\omega\frac{\partial a_{y3}}{\partial \psi}+\frac{\omega\omega_{ce}}{(\omega-\omega_{ce})}\frac{\partial^2 a_{x3}}{\partial \psi^2}+c.c.=0. \tag{A15}$$

In deriving Eqs. (A14) and (A15), we have used the operator

$$A_1\frac{\partial}{\partial a}+A_1^*\frac{\partial}{\partial a^*}=-v_g\left(B_1\frac{\partial}{\partial a}+B_1^*\frac{\partial}{\partial a^*}\right), \tag{A16}$$

and the relations $\partial^2D/\partial\omega^2=-2(3\omega-\omega_{ce})/\omega_{pe}^2$, $\partial^2D/\partial\omega\partial k=2kc^2/\omega_{pe}^2$, and $\partial^2D/\partial k^2=2c^2(\omega-\omega_{ce})/\omega_{pe}^2$, besides relation Eq. (A11). For the solutions a_{x3} and a_{y3} to be secular-free, it is necessary that

$$\left\{i\left(\frac{\partial D}{\partial \omega}A_2-\frac{\partial D}{\partial k}B_2\right)+\frac{1}{2}\frac{\partial D}{\partial \omega}\frac{dv_g}{dk}\left(B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}\right)+\left[\omega n_{e2}+\frac{k\omega_{ce}}{(\omega-\omega_{ce})}v_{ez2}\right]a\right\}e^{i\psi}=0, \tag{A17}$$

as well as its complex conjugate relation. In writing the above expression, we have introduced the definition

$$\frac{dv_g}{dk}=-\frac{(v_g^2\partial^2D/\partial\omega^2+2v_g\partial^2D/\partial k\,\partial\omega+\partial^2D/\partial k^2)}{\partial D/\partial\omega}. \tag{A18}$$

Dividing Eq. (A17) by $\partial D/\partial\omega$ and noting that

$$A_2=\frac{\partial a}{\partial t_2}-\frac{A_1}{\varepsilon},$$
$$B_2=\frac{\partial a}{\partial z_2}-\frac{B_1}{\varepsilon}, \tag{A19}$$
$$B_1\frac{\partial B_1}{\partial a}+B_1^*\frac{\partial B_1}{\partial a^*}=\frac{\partial^2 a}{\partial z_1^2},$$

where $t_2=\varepsilon t_1$ and $z_2=\varepsilon z_1$, one then obtains

$$i\left(\frac{\partial a}{\partial t_2}+v_g\frac{\partial a}{\partial z_2}\right)+\frac{1}{2}\frac{dv_g}{dk}\frac{\partial^2 a}{\partial z_1^2}-\frac{v_g\omega\omega_{pe}^2}{2kc^2(\omega-\omega_{ce})}\times\left[n_{e2}+\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}v_{ez2}\right]a=0. \quad \text{(A20)}$$

The above equation describes the modulation of the em wave envelope.

We now derive the expression for the ponderomotive force, Eq. (27). Back to Eq. (A7), the expressions for v_{ex1} and v_{ey1} were derived from Eqs. (A2) and (A5) keeping only the $O(\varepsilon)$ terms. Thus, higher-order corrections can be incorporated if we include the terms of order ε^2 and higher in the derivation of v_{ex1} and v_{ey1}. These corrections are important for the determination of the ponderomotive force acting on the plasma electrons, as will be shown soon. Using Eqs. (A2) and (A5) and keeping the higher-order terms, we get

$$v_{ex1}=\frac{c(\omega^2-c^2k^2)}{\omega_{pe}^2}ae^{i\psi}+\frac{2ic}{\omega_{pe}^2}\left(kc^2\frac{\partial a}{\partial z}+\omega\frac{\partial a}{\partial t}\right)e^{i\psi}+c.c., \quad \text{(A21)}$$

$$v_{ey1}=\frac{ic(\omega^2-c^2k^2)}{\omega_{pe}^2}ae^{i\psi}-\frac{2c}{\omega_{pe}^2}\left(kc^2\frac{\partial a}{\partial z}+\omega\frac{\partial a}{\partial t}\right)e^{i\psi}+c.c., \quad \text{(A22)}$$

where only the first order derivatives were maintained. Analyzing Eqs. (A21) and (A22), we observe that only the first term in each expression is a $O(\varepsilon)$ term; the other terms are of higher-order [see Eq. (20)]. If we now use the definition of the ponderomotive force, Eq. (26), the following result is obtained $[O(\varepsilon^3)]$.

$$f_{pz}=-\frac{2m_ec^2\varepsilon}{\omega_{pe}^2}[(\omega^2-k^2c^2)(a^*B_1+aB_1^*)+2k^2c^2(a^*B_1+aB_1^*)+2\omega k(a^*A_1+aA_1^*)], \quad \text{(A23)}$$

where the higher-order terms in the derivatives of a_{x1} and a_{y1} have been kept. It is easy to check that if only the terms of order ε in v_{ex1}, v_{ey1}, $\partial_z a_{x1}$, and $\partial_z a_{y1}$ are considered, the ponderomotive force becomes null. The second term in the RHS of Eq. (A23) can be rewritten using Eqs. (20), (A11), and (A12) and becomes $-(2k^2c^2/v_g)(a^*A_1+aA_1^*)$. With the help of Eq. (2) and writing $A_1^{(*)}=\partial a^{(*)}/\partial t_1$ and $B_1^{(*)}=\partial a^{(*)}/\partial z_1$, we finally obtain the well-known expression for the ponderomotive force,

$$f_{pz}=-\frac{2m_e\omega c^2}{(\omega-\omega_{ce})}\left[\frac{\partial|a|^2}{\partial z}-\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}\frac{\partial|a|^2}{\partial t}\right]. \quad \text{(A24)}$$

The above equation can also be expressed in terms of the electric field,

$$f_{pz}=-\frac{\omega_{pe}^2}{16\pi N_0\omega(\omega-\omega_{ce})}\left[\frac{\partial|\mathbf{E}|^2}{\partial z}-\frac{k\omega_{ce}}{\omega(\omega-\omega_{ce})}\frac{\partial|\mathbf{E}|^2}{\partial t}\right], \quad \text{(A25)}$$

where $\mathbf{E}_1=1/2\mathbf{E}e^{i\psi}+c.c.$ and $\mathbf{E}=(2im_ec\omega/e)[a(\hat{\mathbf{x}}+i\hat{\mathbf{y}})]$. This is the same expression derived by Karpman and Washimi in Ref. 37 for a right-hand circularly polarized em electron-cyclotron wave propagating along the magnetic field in the z-direction.

[1] W. H. Preece, Nature (London) **49**, 554 (1894).
[2] S. S. Sazhin and M. Hayakawa, Planet. Space Sci. **40**, 681 (1992); S. S. Sazhin, M. Hayakawa, and K. Bullough, Ann. Geophys. **10**, 293 (1992).
[3] O. A. Molchanov, O. A. Mazhaeva, A. N. Golyavin, and M. Hayakawa, Ann. Geophys. **11**, 431 (1993); F. Nemec, O. Santolik, M. Parrot, and J. J. Berthelier, Geophys. Res. Lett. **35**, L05109, doi:10.1029/2007GL032517 (2008).
[4] M. K. Paul and D. Bora, J. Appl. Phys. **105**, 013305 (2009).
[5] F. F. Chen, Phys. Plasmas **3**, 1783 (1996); M. I. Panevsky and R. Bengtson, *ibid.* **11**, 4196 (2004); D. F. Berisford, R. D. Bengtson, and L. L. Raja, *ibid.* **17**, 033503 (2010) and references therein.
[6] R. L. Stenzel, J. Geophys. Res. **104**, 14379, doi:10.1029/1998JA900120 (1999).
[7] O. Moullard, A. Masson, H. Laakso, M. Parrot, P. Décréau, O. Santolik, and M. André, Geophys. Res. Lett. **29**, 1975, doi:10.1029/2002GL015101 (2002).
[8] G.-L. Huang, D.-Y. Wang, and Q.-W. Song, J. Geophys. Res. **109**, A02307, doi:10.1029/2003JA010137 (2004).
[9] R. L. Arnoldy, M. J. Engebretson, R. E. Denton, J. L. Posch, M. R. Lessard, N. C. Maynard, D. M. Ober, C. J. Farrugia, C. T. Russell, J. D. Scudder, R. B. Torbert, S. -H. Chen, and T. E. Moore, J. Geophys. Res. **110**, A07229, doi:10.1029/2005JA011041 (2005).
[10] S. P. Kuo and E. Koretzky, Phys. Plasmas **8**, 277 (2001).
[11] T. Stix, *Waves in Plasmas* (AIP, New York, 1992).
[12] W. Baumjohann and R. A. Treumann, *Basic Plasma Space Physics* (Imperial College Press, London, 1996).
[13] G. Manfredi, Phys. Rev. Lett. **79**, 2815 (1997).
[14] M. A. Hellberg, R. L. Mace, R. J. Armstrong, and G. Karlstad, J. Plasma Phys. **64**, 433 (2000).
[15] J. M. Urrutia, R. L. Stenzel, and K. D. Strohmaier, Phys. Plasmas **15**, 062109 (2008).
[16] V. Pierrard, M. Maksimovic, and J. Lemaire, J. Geophys. Res. **104**, 17021, doi:10.1029/1999JA900169 (1999); M. Maksimovic, S. P. Gary, and R. M. Skoug, *ibid.* **105**, 18337, doi:10.1029/2000JA900039 (2000) and references therein.
[17] J. T. Steinberg, J. T. Gosling, R. M. Skoug, and R. C. Wiens, J. Geophys. Res. **110**, A06103, doi:10.1029/2005JA011027 (2005).
[18] C. Ma and D. Summers, Geophys. Res. Lett. **25**, 4099, doi:10.1029/1998GL900108 (1998); **26**, 181, doi:10.1029/1998GL900310 (1999); **26**, 1121 (1999) and references therein.
[19] R. A. Treumann, Astrophys. Space Sci. **277**, 81 (2001); R. A. Treumann, C. H. Jaroschek, and M. Scholer, Phys. Plasmas **11**, 1317 (2004).
[20] V. M. Vasyliunas, J. Geophys. Res. **73**, 2839, doi:10.1029/JA073i009p02839 (1968).
[21] M. R. Collier, D. C. Hamilton, G. Gloeckler, P. Bochsler, and R. B. Sheldon, Geophys. Res. Lett. **23**, 1191, doi:10.1029/96GL00621 (1996).
[22] S. P. Christon, D. G. Mitchell, D. J. Williams, L. A. Frank, C. Y. Huang, and T. E. Eastman, J. Geophys. Res. **93**, 2562, doi:10.1029/JA093iA04p02562 (1988); S. P. Christon, D. J. Williams, D. G. Mitchell, C. Y. Huang and L. A. Frank, *ibid.* **96**, 1 (1991) and references therein.
[23] S. Saito, F. R. E. Forme, S. C. Buchert, S. Nozawa, and R. Fujii, Ann. Geophys. **18**, 1216 (2000); H. Bahcivan, R. B. Cosgrove, and R. T. Tsunoda, J. Geophys. Res. **111**, A07306, doi:10.1029/2005JA011595 (2006).
[24] J. D. Scudder, Astrophys. J. **427**, 446 (1994); M. Maksimovic, V. Pierrard, and J. F. Lemaire, Astron. Astrophys. **324**, 725 (1997); C. Vocks, G. Mann, and G. Rausche, *ibid.* **480**, 527 (2008).
[25] M. P. Leubner, Astrophys. Space Sci. **282**, 573 (2002).
[26] C. Tsallis, J. Stat. Phys. **52**, 479 (1988); C. Tsallis, Physica A **221**, 277 (1995).
[27] A. Esquivel and A. Lazarian, Astrophys. J. **710**, 125 (2010).
[28] L. F. Burlaga and A. F. Viñas, Geophys. Res. Lett. **31**, L16807, doi:10.1029/2004GL020715 (2004); L. F. Burlaga, A. F. Viñas, and C. Wang, J. Geophys. Res. **112**, A07206, doi:10.1029/2006JA012213 (2007).
[29] G. Balasis, I. A. Daglis, C. Papadimitriou, M. Kalimeri, A. Anastasiadis, and K. Eftaxias, J. Geophys. Res. **114**, A00D06, doi:10.1029/2008JA014035 (2009).
[30] B. Liu and J. Goree, Phys. Rev. Lett. **100**, 055003 (2008).
[31] A. F. Viñas, R. L. Mace, and R. F. Benson, J. Geophys. Res. **110**, A06202, doi:10.1029/2004JA010967 (2005).

[32]R. L. Mace and M. A. Hellberg, Phys. Plasmas **2**, 2098 (1995).
[33]M. Hellberg, R. Mace, and T. Cattaert, Space Sci. Rev. **121**, 127 (2005).
[34]M. A. Hellberg and R. L. Mace, Phys. Plasmas **9**, 1495 (2002).
[35]T. Cattaert, M. A. Hellberg, and R. L. Mace, Phys. Plasmas **14**, 082111 (2007).
[36]A. Hasegawa, Phys. Rev. A **1**, 1746 (1970); Phys. Fluids **15**, 870 (1972).
[37]V. I. Karpman and H. Washimi, Zh. Eksp. Teor. Fiz. **71**, 1010 (1976); J. Plasma Phys. **18**, 173 (1977).
[38]P. K. Shukla and L. Stenflo, Phys. Rev. A **30**, 2110 (1984).
[39]I. Kourakis and P. K. Shukla, Phys. Plasmas **12**, 012902 (2005).
[40]L.-N. Hau and W.-Z. Fu, Phys. Plasmas **14**, 110702 (2007).
[41]M. A. Hellberg, R. L. Mace, T. K. Baluku, I. Kourakis, and N. S. Saini, Phys. Plasmas **16**, 094701 (2009); L.-N. Hau, W.-Z. Fu, and S.-H. Chuang, *ibid.* **16**, 094702 (2009).
[42]T. Kakutani and N. Sugimoto, Phys. Fluids **17**, 1617 (1974).
[43]L. A. Rios and R. M. O. Galvão, Phys. Plasmas **17**, 042116 (2010); Phys. Plasmas **17**, 119901 (2010) and references therein.
[44]A. Hasegawa, *Plasma Instabilities and Nonlinear Effects* (Springer-Verlag, Berlin, 1975).
[45]H. Abbasi and H. Hakimi Pajouh, Phys. Plasmas **15**, 092902 (2008).
[46]C. A. Cattell, J. Crumley, J. Dombeck, J. Wygant, and F. S. Mozer, Geophys. Res. Lett. **29**, 1065, doi:10.1029/2001GL014046 (2002).