Nonlinear stationary structures in nonthermal plasmas

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Abstract. In plasmas with two distinct populations of hot and cold electrons waves with wave frequency larger than the ion plasma frequency can be generated. In the nonlinear regime, the existence of rarefaction waves and shocks in two-electron temperature isothermal plasmas has been analyzed in the past. In the present work we investigate the conditions for the existence of rarefaction waves and shocks in nonthermal two-electron plasmas. Here the cold and hot electrons are modeled by the Maxwellian and κ distributions, respectively. Some preliminary results are presented, and the influence of electron nonthermality is discussed.

1. Introduction

It is well known that laboratory and space plasmas can contain distinct populations of hot and cold electrons [1]. In two-electron plasmas, electron-acoustic waves (EAWs) with wave frequency larger than the ion plasma frequency can be generated [2]. In their classical paper of 1978, Bezzerides, Forslund and Lindman [3] investigate the nonlinear regime and analyze the existence of rarefaction waves and shocks in a two-electron temperature isothermal plasma. The study of rarefaction waves (and shocks) is important for a variety of problems in plasma physics, including the so-called current-free double layers [4].

A double layer (DL) consists of a positive/negative Debye sheath, connecting two quasineutral regions of a plasma. These nonlinear structures can be found in a variety of plasmas, from discharge tubes to space plasmas. A DL may be regarded as a BGK equilibrium in some cases, for which certain conditions must be fulfilled. The best known of these structures is the strong Langmuir DL, which is characterized by two counterstreaming plasmas, carrying a large electric current across the DL. The current-free double layer (CFDL) constitutes a different group, for which there is no trapped ion population. Contrary to the Langmuir DL, the CFDL is weak, with $\varphi < k_B T_h/e$ (φ is the potential drop across the layer and T_h is the temperature of the hot electron population). It is worth to mention that in general the plasma distributions near a DL are strongly non-Maxwellian [5].

As a preparatory step for a deeper investigation of CFDLs, we follow the steps of reference [3] and analyze the conditions for the existence of rarefaction waves and shocks in nonthermal two-electron plasmas. The dynamics of the plasma is described by the fluid equations, with the cold and hot electrons modeled by the Maxwellian and κ distributions, respectively. The family of κ distributions has been employed to analyze and interpret data on different plasma

environments, like the solar wind [6] and the Earth's ionosphere [7]. The reduced form of the standard κ distribution is equivalent to the distribution function obtained from the maximization of the Tsallis entropy, the q distribution [8], with the parameter κ measuring the deviation from the Maxwellian equilibrium (in the limit $\kappa \to \infty$ the Maxwellian distribution is recovered). Here some preliminary results of our work are presented, and the influence of the superthermal electrons present in the long tails of the κ distribution is discussed.

2. Model equations

First we consider the general rarefaction wave problem. The ions are assumed to obey the cold hydrodynamic equations, and the ion and electron densities are related through Poisson's equation

$$\frac{\partial^2 \varphi}{\partial z^2} = -4\pi e \left[N_i - N_e\left(\varphi\right) \right]. \tag{1}$$

The electron number density can be written as

$$N_e\left(\varphi\right) = N_c\left(\varphi\right) + N_h\left(\varphi\right), \tag{2}$$

where

$$N_c\left(\varphi\right) = N_{c0} e^{e\varphi/k_B T_c}, \tag{3}$$

$$N_h\left(\varphi\right) = N_{h0} \left[1 - \frac{e\varphi}{\left(\kappa - 3/2\right) k_B T_h} \right]^{-\left(\kappa - 1/2\right)}, \tag{4}$$

with $\kappa > 3/2$. In the above equations T_c is the temperature of the cold electrons and $N_{c0} + N_{h0} = N_0$. In the limit $\kappa \to \infty$ we obtain the Boltzmann distribution (3) also for the hot electrons .

Introducing the similarity parameter $\xi = (z/t)/c_h$, where $c_h = (k_B T_h/m_i)^{1/2}$, and following reference [3] we obtain the equation that governs the electrostatic potential ϕ, i.e.

$$\frac{d\phi}{d\xi} \left(1 + \frac{1}{2} \frac{dc_s^2}{d\phi} \right) + c_s = 0, \tag{5}$$

where $\phi = e\varphi/k_B T_h$ and

$$c_s^2 = \frac{dP_e}{dn_e} \equiv \frac{n_e\left(\phi\right)}{\left\{ n_c\left(\phi\right)\tau + n_h\left(\phi\right) \frac{\left(\kappa-1/2\right)}{\left(\kappa-3/2\right)} \left[1 - \frac{\phi}{\left(\kappa-3/2\right)} \right]^{-1} \right\}}, \tag{6}$$

with $n_e(\varphi) = N_e(\varphi)/N_0$ and $n_i = N_i/N_0$. The above expression is the square of the normalized speed of sound, where $\tau = T_h/T_c$ and $P_e \equiv P_e(\phi)$ is the normalized pressure, which obeys the relation $dP_e(\phi)/d\phi = n_e(\phi)$. A complete solution for (5) exists provided ϕ is a single valued function of ξ. However, multiple valued solutions occur when

$$\frac{dc_s^2}{d\phi} + 2 \leq 0, \tag{7}$$

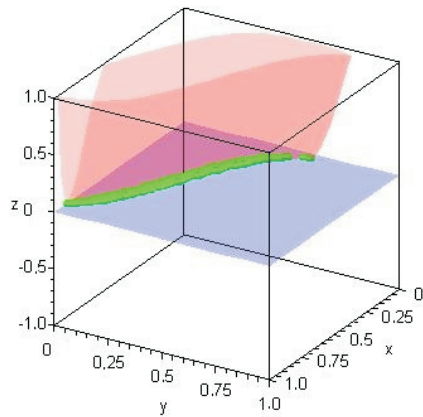with the equality defining the onset of the singularity in the rarefaction wave.
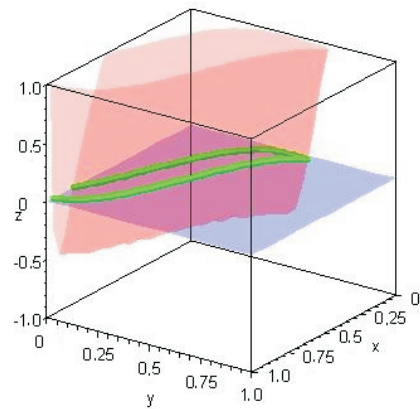
Figure 1. $\kappa = 500$ and $\tau = 10$.

Figure 2. $\kappa = 500$ and $\tau = 12$.

3. Rarefaction waves and shocks

As discussed by Bezzerides, Forslund and Lindman, the threshold for rarefaction shocks and the onset of the singularity in the rarefaction wave are expressed by the same condition, $dc_s^2/d\phi + 2 = 0$. When both electron populations are modeled via Maxwellian distributions, condition (7) reduces to $\tau \geq 5 + \sqrt{24} \approx 9.9$ [3]. This can be seen in figure 1, where we plot $z = 2 + dc_s^2/d\phi$ as a function of $x = \alpha = N_{h0}/N_0$ and $y = -\phi$ for $\kappa = 500$ (Maxwellian limit). For $\tau = 10$ we observe that $z = 0$ (onset of the singularity) for a broad range of α's. For $\tau = 12$ (figure 2) we notice that $z \leq 0$ also for a broad range of α's, with the formation of the shock between the two extremes of ϕ. However, as κ decreases (figure 3, $\kappa = 5$), we notice the singularity starts to appear (for a short range of α's) only for $\tau \approx 11$. As τ increases (figure 4), we have the formation of the shock for all the values of α. For longer tails (figure 5), the singularity
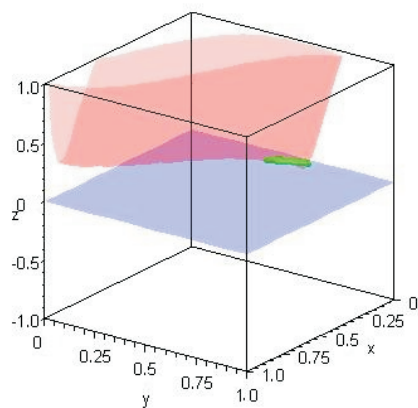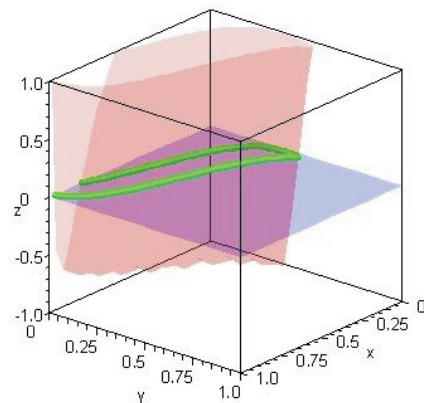

Figure 3. $\kappa = 5$ and $\tau = 11$.

Figure 4. $\kappa = 5$ and $\tau = 16$.

appears only for a larger value of τ (≈ 13). For the same value of τ, we observe that electron nonthermality seems to "disturb" the formation of the shock: as κ decreases, the shock does not
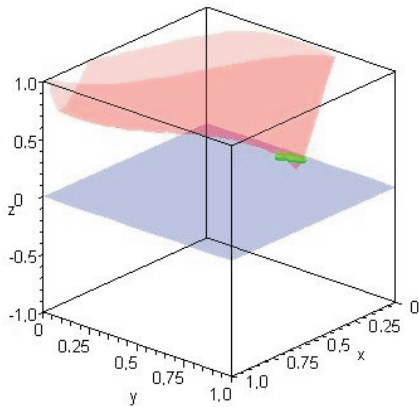
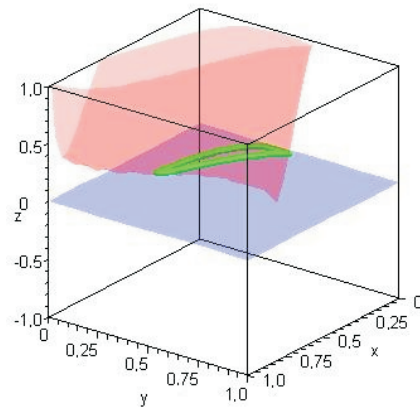**Figure 5.** $\kappa = 2.5$ and $\tau = 13$.

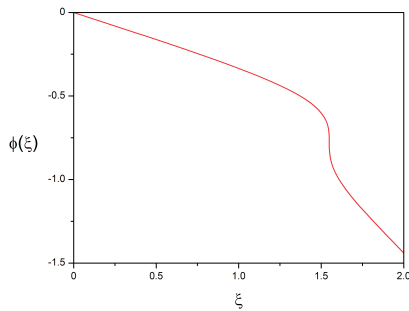**Figure 6.** $\kappa = 2.5$ and $\tau = 16$.

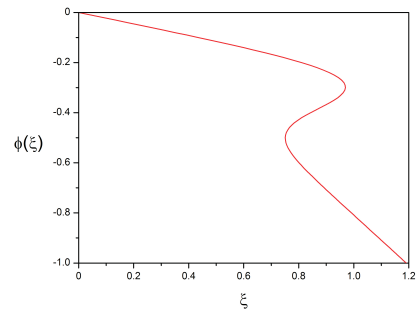**Figure 7.** $\kappa = 500$, $\alpha = 0.01$ and $\tau = 9.9$.

Figure 8. $\kappa = 500$, $\alpha = 0.01$ and $\tau = 20$.
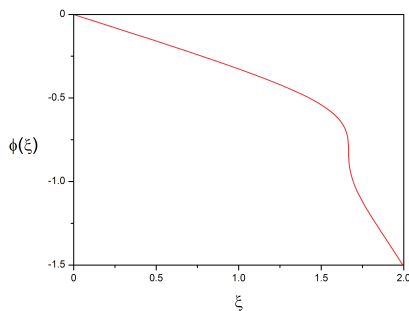

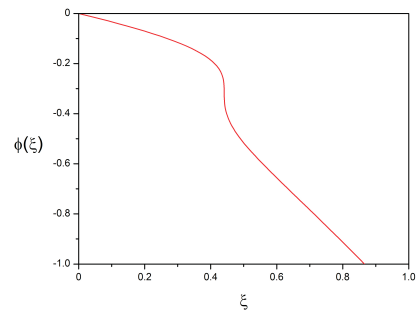**Figure 9.** $\kappa = 2.5$, $\alpha = 0.01$ and $\tau = 10.2$.

Figure 10. $\kappa = 2.5$, $\alpha = 0.2$ and $\tau = 14.2$.

appear for all the values of α (figures 4 and 6).

Solving (5) numerically, we can analyze the different profiles obtained for the electrostatic potential ϕ. In figures 7 and 8 we present the results for $\kappa = 500$, $\alpha = 0.01$ and $\tau = 9.9$ and 20, respectively. In figure 7 we notice the eminent formation of the shock (singularity), while

in figure 8 it is observed that ϕ is not a single valued function of ξ. This discontinuity in the profile of the electrostatic potential represents the shock formation: in a reference frame moving with velocity $\xi \approx 0.9$ we observe two different values for ϕ. For such a small value of α ($= 0.01$) and $\kappa = 2.5$, the onset of the singularity appears for $\tau = 10.2$ (figure 9). As α grows ($\alpha = 0.2$, figure 10), the shock becomes eminent only for $\tau = 14.2$. Is is also noticed that the shock starts to appear for smaller values of ξ as α becomes larger.

4. Conclusions

The presented results indicate that electron nonthermality (represented by the parameter κ) influences the onset of the singularity in the rarefaction wave and the formation of the shock. For distributions with longer tails (small κ) the formation of the shock becomes eminent only for larger values of τ when compared to the Maxwellian case. It is also noticed that, for a fixed τ, a decrease in κ implies the disappearance of the shock for larger values of α.

Acknowledgments

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