F E L
Classical Theory and BRAFEL project

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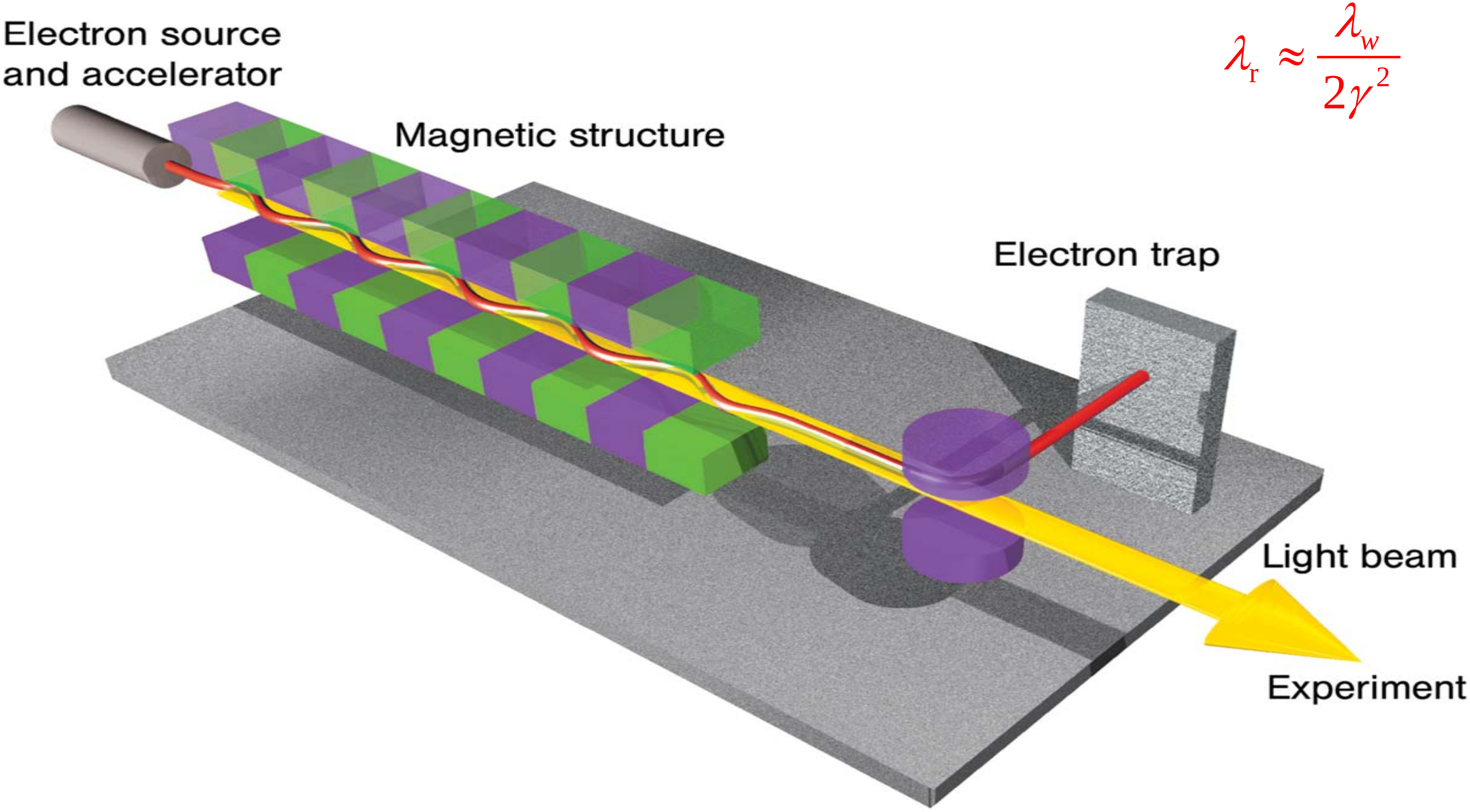
Outline

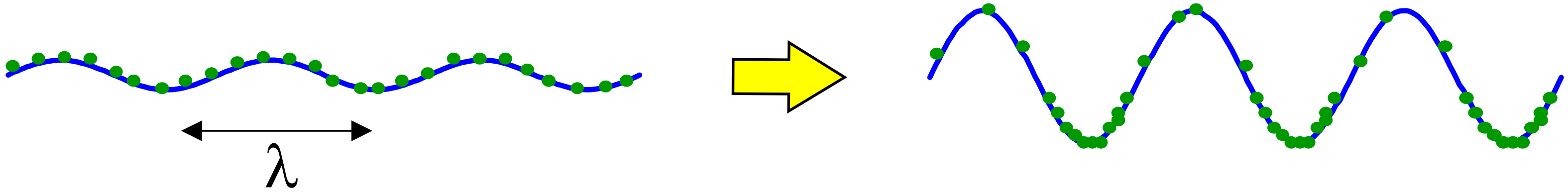
1. Introductory concepts
2. Classical Model: High gain regime (1D)
3. Propagation Effects: Superradiance (SR)
4. SASE
5. 3D effects and 1D Limit
6. BRAFEL project: possible parameters

High Gain Free Electron Laser (FEL)

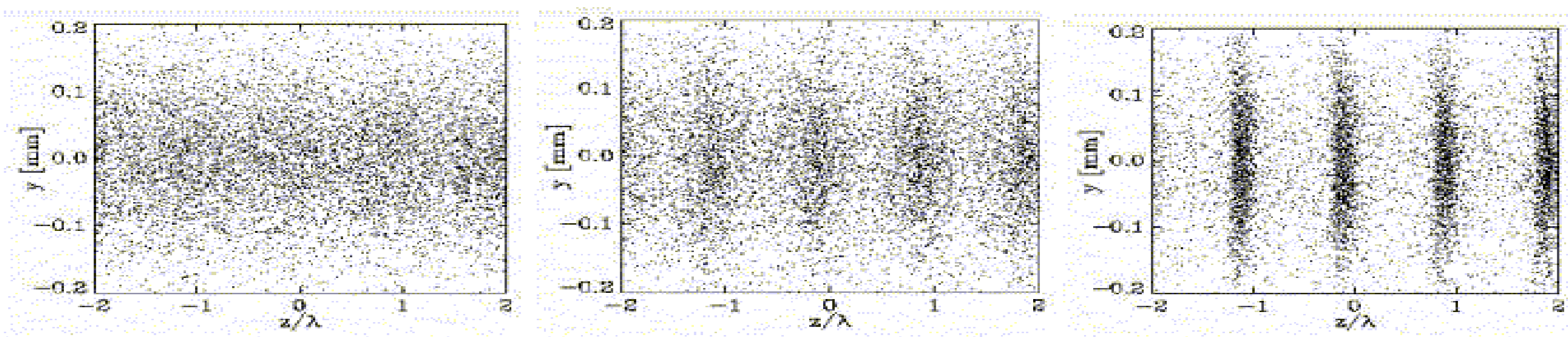
$$\lambda_r \approx \frac{\lambda_w}{2\gamma^2}$$



.. and electrons bunch on the λ scale !!



Self bunching: radiation from incoherent to cooperative spontaneous emission

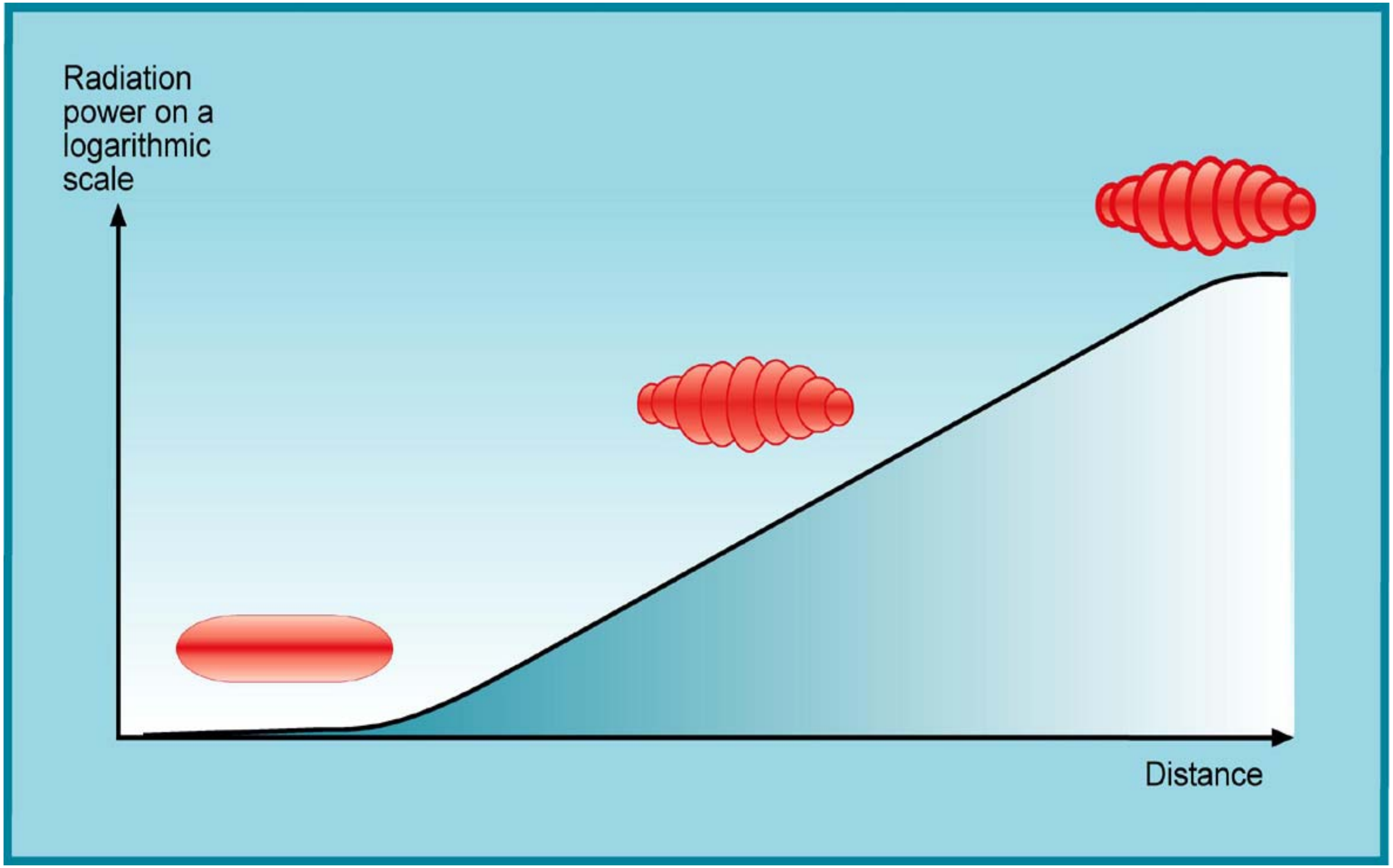


$$(\theta_j = 2\pi z_j / \lambda)$$

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

$$0 \le |b| \le 1$$

High Gain Free Electron Laser:
radiation is amplified exponentially in a single-pass ..



Some references

HIGH-GAIN AND SASE FEL with "UNIVERSAL SCALING"

Classical Theory

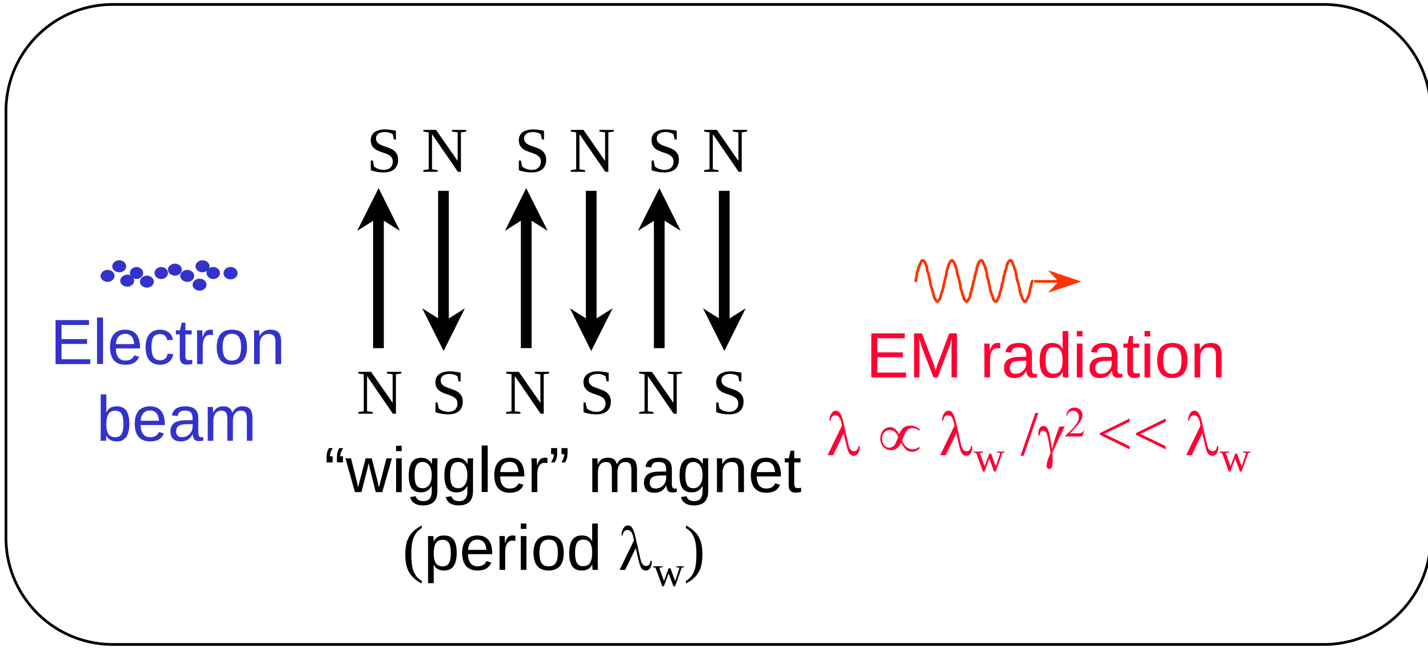
(1) R.B, C. Pellegrini and L. Narducci, Opt. Commun. 50, 373 (1984).
(2) R.B, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)
(3) R.B, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL 73, 70 (1994).
(4, 5) R.B. et al,Physics of High Gain FEL and Superradiance, La Rivista del Nuovo Cimento vol. 13. n. 9 (1990) e vol. 15 n.11 (1992)

QUANTUM THEORY

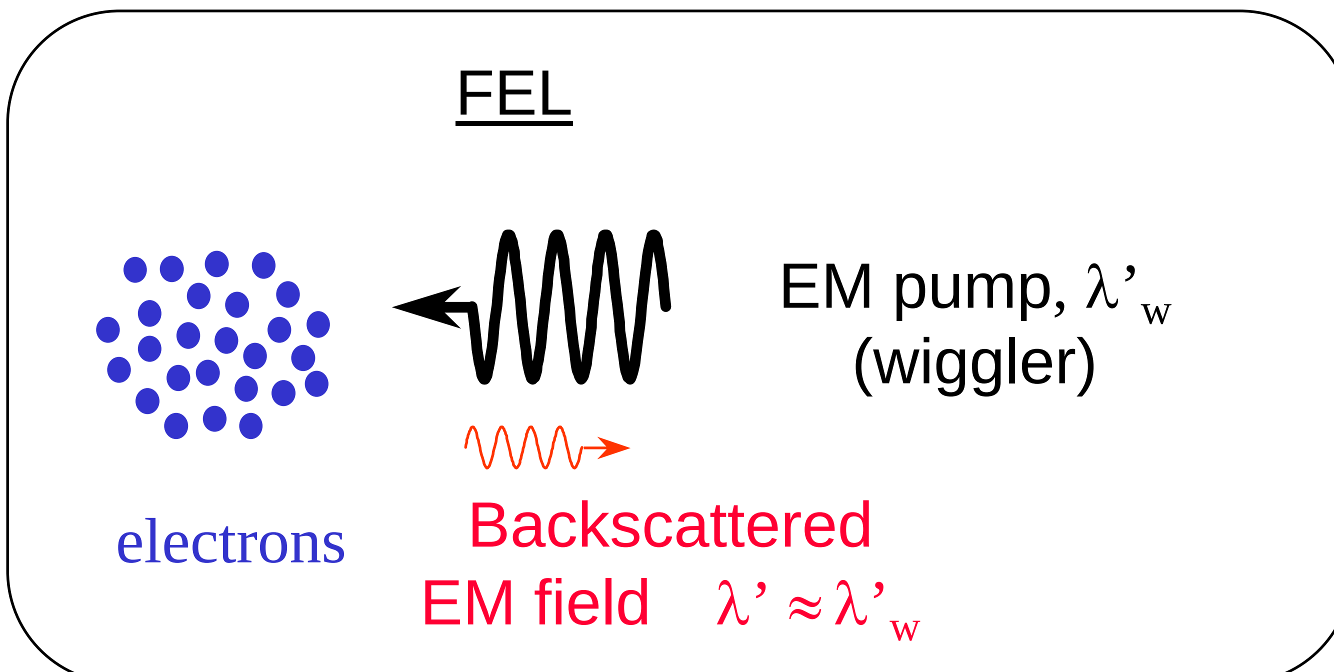
R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, Europhysics Letters, 69, (2005) 55 .
(7) R.B., N. Piovella, G.R.M. Robb, Quantum Theory of SASE-FEL, NIM A 543, 645 (2005), and proc. FEL Conf. 2005
(8) R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, Optics Commun. 252, 381 (2005)

See also

(9) F.T.Arecchi, R. Bonifacio, "MB equation", IEEE Quantum Electron., 1 (1965) 169



transforming to a frame (Λ') moving with electrons



The resonance condition (longitudinal motion)

- Relativistic mirror

$$\lambda_i^{'} = \lambda_i \sqrt{\frac{1-\beta_\parallel}{1+\beta_\parallel}} \qquad \lambda_r^{'} = \lambda_r \sqrt{\frac{1+\beta_\parallel}{1-\beta_\parallel}} \qquad \left(\beta = \frac{v}{c} \approx 1\right)$$

$$\lambda_i^{'} = \lambda_r^{'} \Rightarrow \boxed{\lambda_r = \lambda_i \frac{1-\beta_\parallel}{1+\beta_\parallel} \approx \frac{\lambda_i}{4\gamma_\parallel^2}} \qquad \left(\gamma_\parallel = \frac{1}{\sqrt{1-\beta_\parallel^2}}\right)$$

$$\left(\beta_\parallel \approx 1 \Rightarrow \gamma_\parallel >> 1\right)$$

- One electron : Thomson backscattering $\lambda_i^{'} = \lambda_r^{'}$
- static wiggler λ_w “equivalent” to pseudoradiation field

$$\lambda_i = \lambda_w \frac{1+\beta_\parallel}{\beta_\parallel} \approx 2\lambda_w \Rightarrow \boxed{\lambda_r = \lambda_w \frac{1-\beta_\parallel}{\beta_\parallel} \approx \frac{\lambda_w}{2\gamma_\parallel^2}}$$

$$\beta_\parallel = \beta_r = k/(k+k_w)$$

Full resonance condition

$$\beta^2 = \beta_\parallel^2 + \beta_\perp^2 \qquad \boxed{\beta_\perp^2 = \frac{a_w^2}{\gamma^2}} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Wiggler parameter $a_w \equiv \frac{eA_w}{mc^2} = \frac{eB_w\lambda_w}{2\pi mc^2} \approx \left(B_w(T)\lambda_w(cm)\right)$

$$\frac{1}{\gamma^2} = 1 - \beta_\parallel^2 - \beta_\perp^2 = \frac{1}{\gamma_\parallel^2} - \frac{a_w}{\gamma^2}$$

$$\frac{1}{\gamma_\parallel^2} = \frac{1+a_w^2}{\gamma^2}$$

FEL Resonance

$$\boxed{\lambda_r = \frac{\lambda_w}{2\gamma_\parallel^2} = \frac{\lambda_w\left(1+a_w^2\right)}{2\gamma_r^2}}$$

$$\boxed{\gamma_r = \sqrt{\frac{\lambda_w\left(1+a_w^2\right)}{2\lambda_r}}}$$

$$a_w = 1; \lambda_r = 1\text{A}$$
$$\lambda_w = 2cm; E = 7GeV$$

$$a_w = 1; \lambda_r = 100\,\mu m$$
$$\lambda_w = 4cm; E = 10MeV$$

The Transverse Motion

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 \qquad \gamma m\vec{\mathrm{v}} = \vec{p} - \frac{e}{c}\vec{A} \qquad \vec{A} = \vec{A}_\perp(z)$$

$$\gamma m\vec{\mathrm{v}}_\perp + \frac{e}{c}\vec{A} = \vec{p}_\perp \qquad \frac{\partial p_x}{\partial t} = -\frac{\partial H}{\partial x} = 0 \qquad \vec{p}_\perp = const = 0$$

$$\vec{\beta}_\perp = \frac{\vec{\mathrm{v}}_\perp}{c} = -\frac{\vec{a}}{\gamma} \simeq -\frac{\vec{a}_w}{\gamma}; \quad \left(\vec{a} = \frac{e}{mc^2}\vec{A}\right)$$

$$\vec{a} = \vec{a}_w + \vec{a}_\ell; \quad a_w >> a_\ell$$

The FEL 1-D model

$$\frac{d\gamma mc^2}{dt} = -e\vec{E}\cdot\vec{v}_\perp ; \vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$\vec{a} = \frac{e}{mc^2}\vec{A}; \quad \frac{d\gamma}{dt} = \frac{\partial \vec{a}}{\partial t}\cdot\vec{\beta}_\perp ; \quad \vec{\beta}_\perp = -\frac{\vec{a}}{\gamma} \Rightarrow \boxed{\frac{d\gamma}{dt} = -\frac{1}{2\gamma}\frac{\partial |a|^2}{\partial t}} \quad (1)$$

$$\vec{a} = \vec{a}_w + \vec{a}_l \qquad \vec{a}_w = \frac{a_w}{\sqrt{2}}\left(\hat{e}e^{ik_w z} + cc\right)$$

$$\left(\hat{e} = \hat{x} + i\hat{y}\right)$$

$$\vec{a}_l(z,t) = \frac{-i}{\sqrt{2}}\left(\hat{e}a(z,t)e^{-i(kz-\omega t)} - cc\right)$$

$$\frac{d}{dt} \approx c\frac{d}{dz}$$

$$\theta = (k+k_w)z - ckt = (k+k_w)(z - v_p t) \qquad v_p = ck/(k+k_w) = v_r$$

$$(1) \quad \boxed{\frac{d\gamma_j}{dz} = -\frac{k}{2}\frac{a_w}{\gamma_j}\left(ae^{i\theta_j} + cc\right)}$$

R.B. et al, La Rivista del Nuovo Cimento vol. 13. n. 9 (1990) e vol. 15 n.11 (1992)

The Phase Equation

$$\theta = (k + k_w) z - ckt \qquad \left(\frac{dt}{dz} = \frac{1}{v_\parallel} \right)$$

$$\frac{d\theta}{dz} = (k + k_w) - \frac{k}{\beta_\parallel} = k_w - k\left(\frac{1}{\beta_\parallel} - 1 \right) = k_w \left(1 - \frac{k}{k_w} \left(\frac{1 - \beta_\parallel}{\beta_\parallel} \right) \right)$$

$$= k_w \left[1 - \frac{\gamma_r^2}{\gamma^2} \right] \approx 2k_w \left(\frac{\gamma - \gamma_r}{\gamma_r} \right) \qquad \gamma_r^2 \equiv \frac{\lambda_w}{\lambda} \left(\frac{1 + a_w^2}{2} \right)$$

$$(2) \quad \boxed{\frac{d\theta_j}{dz} = 2k_w \left(\frac{\gamma_j - \gamma_r}{\gamma_r} \right)}$$

Compton limit

$\beta_\parallel \approx 1$ i.e. $(\gamma^2 >> 1 + a_w^2)$; $\gamma_r \approx \gamma$

$$\text{Note: } \left[\left(1 - \beta_\parallel^2 \right) = \frac{1 + a_w^2}{\gamma^2} \Rightarrow 1 - \beta_\parallel \approx \frac{1 + a_w^2}{2\gamma^2} \right]$$

Maxwell Equations

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A(z,t) = -\frac{4\pi}{c} J_{\perp}(z,t)$$

$$J_{\perp}(z,t) = e\sum_{j=1}^{N} v_{\perp j}\,\delta(z - z_j)$$

$$\beta_{\perp j} \approx \frac{a_w}{\gamma_j}$$

(SVEA) $\frac{\partial a}{\partial z} << ka \qquad \frac{\partial a}{\partial t} << \omega a$

$$\omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m}}$$

(3) $$\left(\frac{\partial a}{\partial z} + \frac{1}{c}\frac{\partial a}{\partial t}\right) = \frac{k}{2}\left(\frac{\omega_p}{ck}\right)^2 \frac{a_w}{\gamma_r}\left\langle e^{-i\theta}\right\rangle$$

$$\left\langle e^{-i\theta}\right\rangle = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j} = b$$

(1) $$\frac{d\gamma_j}{dz} = -\frac{k}{2}\frac{a_w}{\gamma_j}\left(ae^{i\theta_j} + cc\right)$$

(2) $$\frac{d\theta_j}{dz} = 2k_w\left(\frac{\gamma_j - \gamma_r}{\gamma_r}\right)$$

in (1) and (2) $\left[\frac{d}{dz} = \frac{\partial}{\partial z} + \frac{1}{v_{\parallel}}\frac{\partial}{\partial t}\right]$ 1-D FEL model with propagation

Classical model with "Universal" Scaling

$$\frac{\partial\theta_j}{\partial\bar{z}} = \bar{p}_j\,;\; \frac{\partial\bar{p}_j}{\partial\bar{z}} = -(Ae^{i\theta_j} + c.c.);\; \frac{\partial A}{\partial\bar{z}} + \frac{\partial A}{\partial z_1} = \left\langle e^{-i\theta}\right\rangle = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

no free parameters

A: scattered field

$$\rho\,|\,A\,|^2 = \frac{\hbar\omega n_{ph}}{n_e\gamma mc^2} = \frac{P_r(MW)}{I(A)E(MeV)} = \eta$$

Momentum

$$\bar{p} = \frac{\gamma-\gamma_r}{\rho\gamma_r}$$

$$\rho = 10^{-3},\; I = 10^2 A,$$

$$E = 10MeV,\; |A|^2 = 1,\; P_r = 1MW\,!$$

"Collective FEL parameter"

R. B, C. Pellegrini and L. Narducci, Opt. Commun. 50, 373 (1984).

$$\boxed{\rho = \frac{1}{\gamma_r}\left(\frac{a_w\omega_p}{4ck_w}\right)^{2/3} \propto n_e^{1/3} = \frac{1}{d_e}}$$

$$\bar{z} = \frac{z}{L_g};\; L_g \equiv \frac{\lambda_w}{4\pi\rho};\qquad z_1 = \frac{z - v_r t}{\beta_r L_c};\; L_c \equiv \frac{\lambda_r}{4\pi\rho}$$

$$\frac{\partial A}{\partial z_1} = 0 \quad\rightarrow$$ Steady State (S.S.) model

S.S. Model : Phase Shift and Gain

$$\dot{\theta}_j = p_j \qquad \dot{p}_j = -(Ae^{i\theta_j} + cc) \qquad \dot{A} = \langle e^{-i\theta} \rangle$$

$$A = ae^{i(\varphi - \pi/2)}$$

$$\ddot{\theta}_j = -2a\sin(\theta_j + \varphi) \qquad V = -2a\cos(\theta_j + \varphi)$$

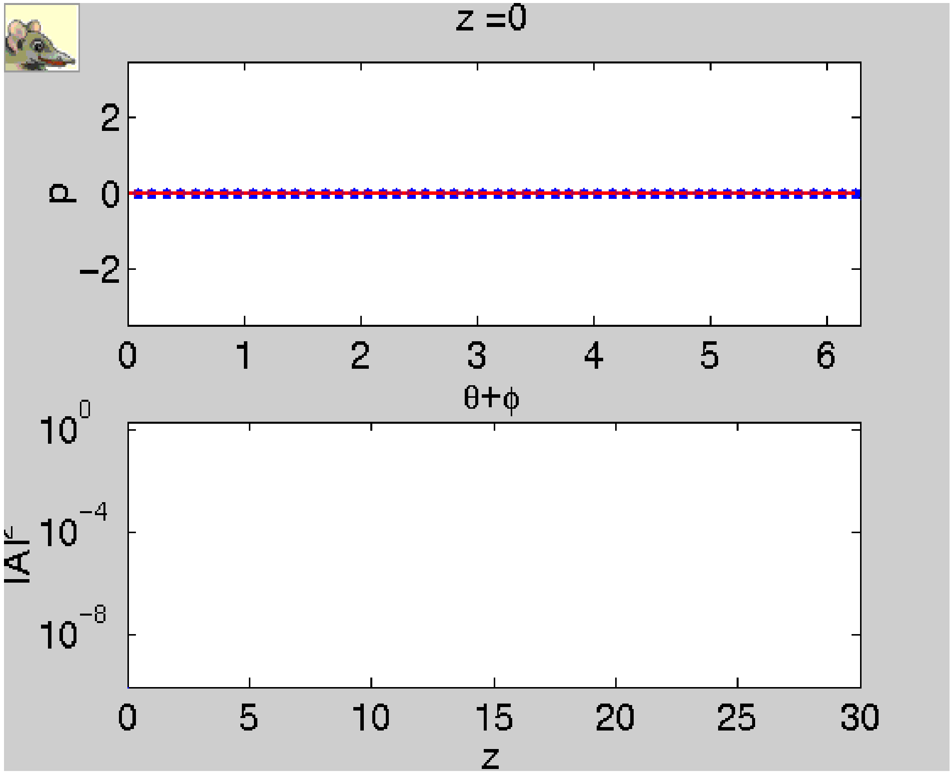
$$\dot{a} = \langle \sin(\theta_j + \varphi) \rangle$$

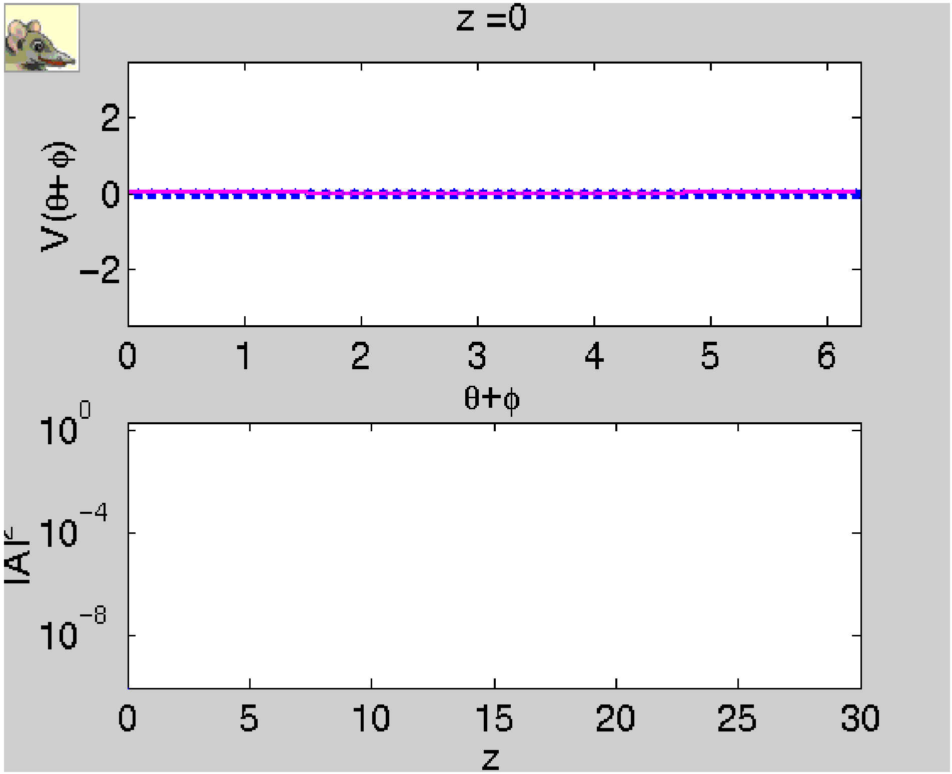
$$\dot{\varphi} = \frac{\langle \cos(\theta_j + \varphi) \rangle}{a}$$

t = 0 : $\varphi = 0$ and small bunching on $\theta = 0 \Rightarrow \dot{a} = 0$ NO GAIN

t > 0 : $\dot{\varphi} > 0 \Rightarrow \dot{a} \approx \sin\varphi > 0$ GAIN

FEL S.S. instability animation





Exponential instability as e^{z/L_g} up to $|A| \approx 1$ (saturation)

independently on initial value

Possibility of start up from noise (SASE)

$\sigma(p) = \sigma(\gamma)/\rho\gamma = 1$ Initial energy spread smaller than ρ

Phase Shift and Optical Guiding (O.G.)

$$\left(\dot{\varphi} > 0 \Rightarrow n > 1\right)$$

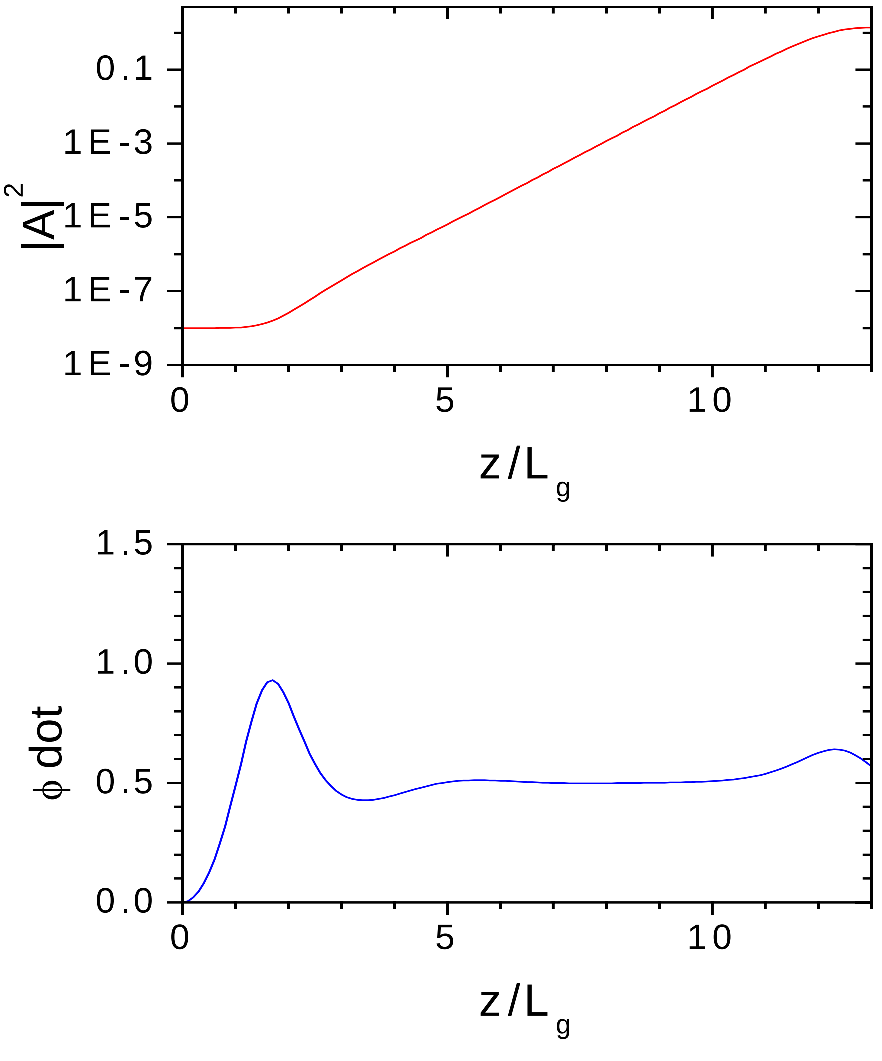
$$A = ae^{i\varphi} \Rightarrow E \propto e^{i\left[kz - \omega t + \varphi(z)\right]}$$

$$\Rightarrow k \to k + \dot{\varphi} \Rightarrow \mathrm{v}_p = \frac{ck}{k + \dot{\varphi}} \qquad \omega = ck$$

$$n = \frac{c}{\mathrm{v}_p} = 1 + \frac{\dot{\varphi}}{ck} > 1$$

e-beam = optical fiber $\Rightarrow$ Optical Guiding

Exponential gain and phase derivative



Linear Theory (S.S.): exponential gain

$$\dot{\theta}_j = p_j \quad \dot{p}_j = -\left(Ae^{i\theta} + cc\right) \quad \dot{A} = \left\langle e^{-i\theta} \right\rangle + i\delta A$$

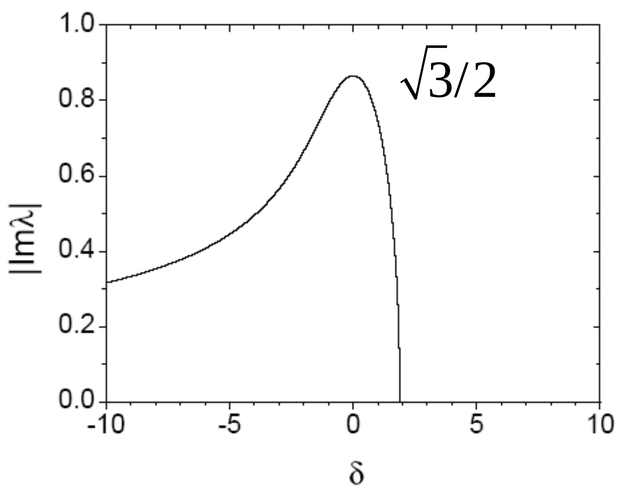
$$p(0) = A(0) = \left\langle e^{-i\theta} \right\rangle = 0; equilibrium \qquad \left\langle p \right\rangle + \left|A\right|^2 = \cos t$$

Linear theory $\dddot{A} - i\delta\ddot{A} - iA = 0$ $\qquad A \propto A_0 e^{i\lambda t}$

$$\boxed{(\lambda - \delta)\lambda^2 + 1 = 0} \qquad \delta = \frac{\gamma_0 - \gamma_r}{\rho\gamma_r} \qquad A \propto e^{(\mathrm{Im}\,\lambda)\bar{t}}$$

runaway solution

Max gain δ=0 $\qquad \mathrm{Im}\lambda = \sqrt{3}/2$



$$\left|A\right|^2 \propto e^{\sqrt{3}\bar{t}} = e^{\sqrt{3}\frac{z}{L_g}} \qquad \left[L_g = \lambda_w/(4\pi\rho)\right]$$

Gain bandwidth $\Delta\gamma / \gamma \,\square\, 2\rho$

End first lecture

Propagation effect: Superradiance and SASE

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Slippage, Resonance and Cooperation Length

Slippage = propagation effects

The slippage length L_s when the electron travel Δz

is $L_s = (c - \mathrm{v}_{\square})\dfrac{\Delta z}{\mathrm{v}_{\square}}$ $\quad \Delta z = \lambda_w \Rightarrow L_s = \lambda_w \left(\dfrac{1}{\beta_{\square}} - 1 \right) = \lambda_r$

Resonance: the slippage is λ for each $\lambda_{w,}$ maintaining the phase.

Total slippage: $L_s = \dfrac{L_w}{\lambda_w} \lambda_r$ negligible if $L_s \ll L_b$

Slippage in a gain length ($L_w = L_g$): $L_c = \lambda_r / (4\pi\rho_F)$ cooperation length

slippage is fundamental when $L_b \le L_c$ and in SASE

$\left(L_g = \lambda_w / (4\pi\rho_F) \right)$

Classical model with "Universal" Scaling

$$\frac{\partial \theta_j}{\partial \bar{z}} = \bar{p}_j \,; \quad \frac{\partial \bar{p}_j}{\partial \bar{z}} = -(Ae^{i\theta_j} + c.c.); \quad \frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \langle e^{-i\theta} \rangle = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

no free parameters

A: scattered field

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R. B, C. Pellegrini and L. Narducci, Opt. Commun. 50, 373 (1984).

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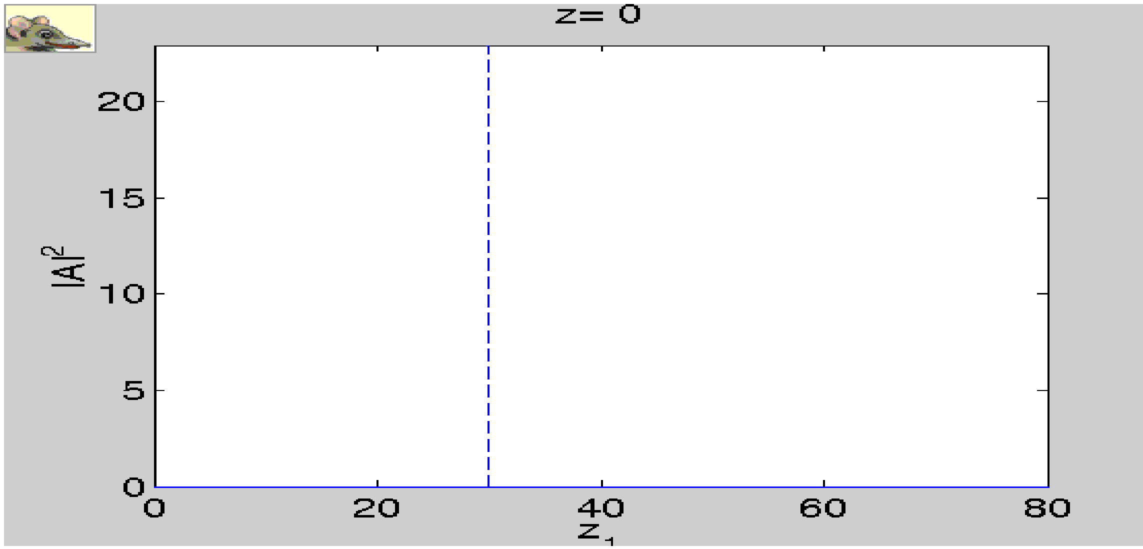
$$\bar{z} = \frac{z}{L_g}; L_g \equiv \frac{\lambda_w}{4\pi\rho}; \qquad z_1 = \frac{z - v_r t}{\beta_r L_c}; L_c \equiv \frac{\lambda_r}{4\pi\rho}$$

$$\frac{\partial A}{\partial z_1} = 0 \quad \rightarrow$$ Steady State (S.S.) model

STEADY STATE AND SUPERRADIANT INSTABILITY,
Long and Short Bunch (uniform seed)

Evolution of radiation time structure in the electron rest frame



$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \left\langle e^{-i\theta} \right\rangle$$

$$\bar{z} = z / L_g$$

$$L = 30 L_c$$

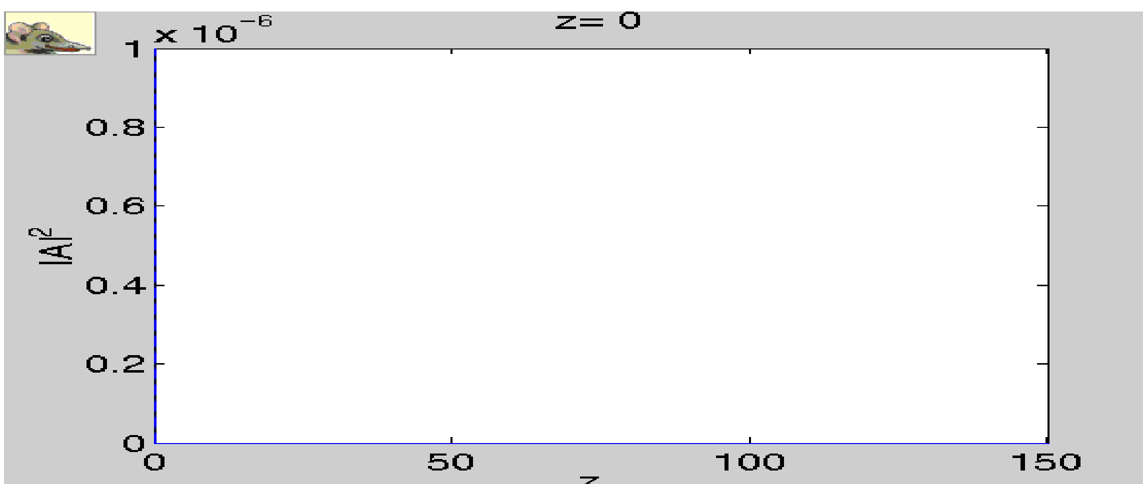
Strong SR

$$\left(L_g = \frac{\lambda_w}{4\pi\rho_F} \right)$$

$$I_{peak} \propto n_e^2$$

$$z_1 = z - vt / L_c$$

$$\left(L_c = \frac{\lambda_r}{4\pi\rho} \right)$$



$$L = 0.1 L_c$$

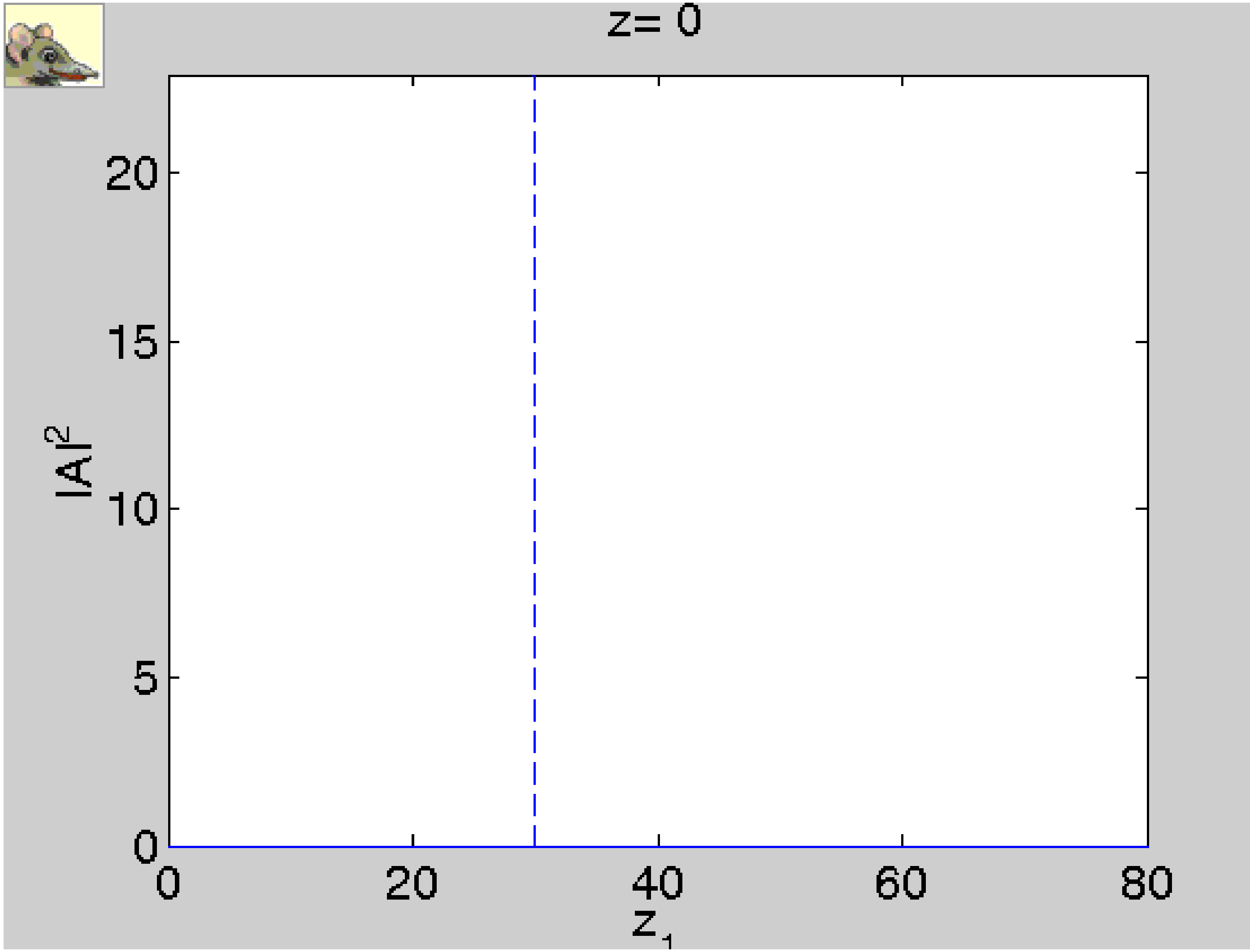
Weak SR

R. Bonifacio, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)

LONG PULSE (uniform excitation)

$L=30L_C$, resonant ($\delta=0$)

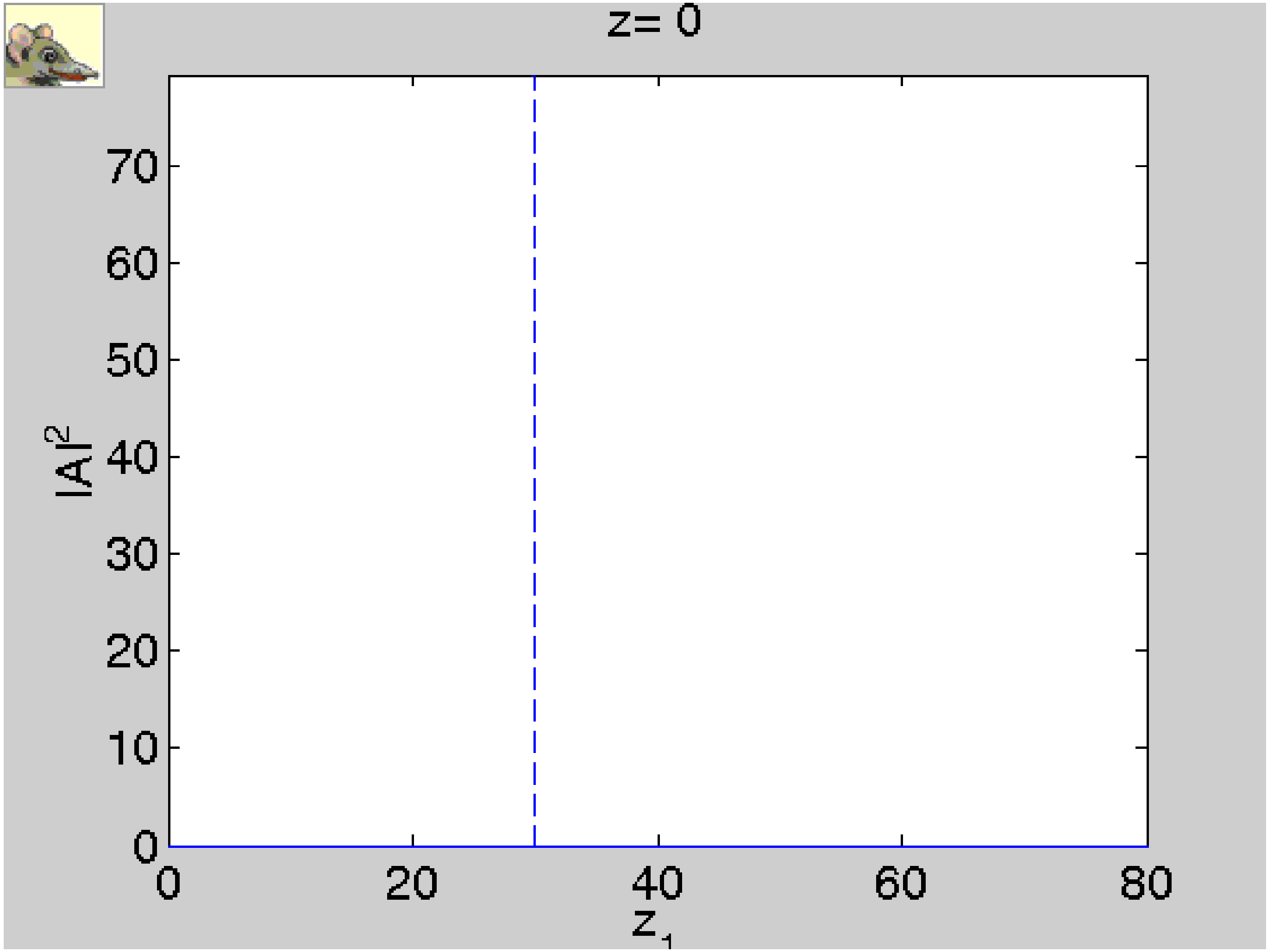
R. Bonifacio, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)



SS and SR instability

LONG PULSE (uniform excitation)
$L=30L_C$, detuned ($\delta=2$)

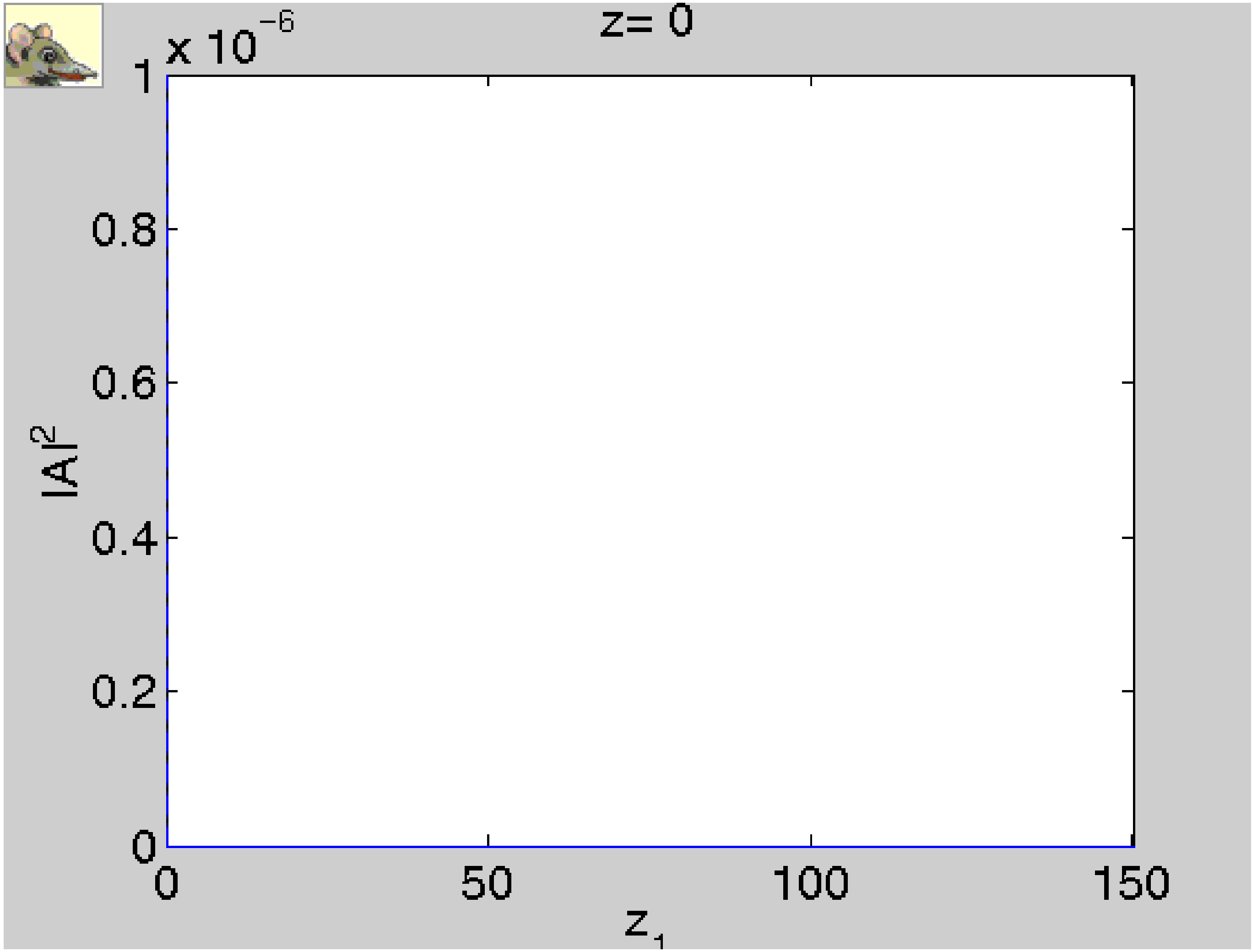
R. Bonifacio, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)



Only SuperRadiant Instability

Short Bunch (uniform seed)

Evolution of radiation time structure in the electron rest frame



$L = 0.1 L_c$

Weak SR

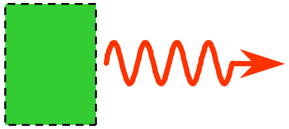
$I_{peak} \propto n_e^2$

Only SuperRadiant Instability

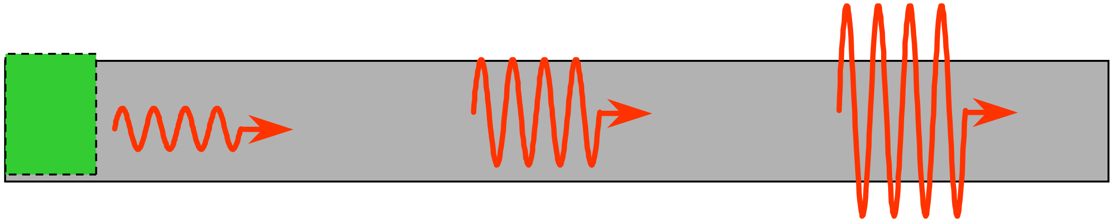
PROPAGATION EFFECTS IN FELs :SUPERRADIANCE

Particles at the trailing edge of the beam never receive radiation from particles behind them: they just radiate in a SUPERRADIANT PULSE or SPIKE which propagates forward.

if $L_b << L_c$ the SR pulse remains small (weak SR).



if $L_b >> L_c$ the weak SR pulse gets amplified (strong SR) as it propagates forward through beam with no saturation.



The SR pulse is a self-similar solution of the propagation equation.

Soliton-Like solution and Superradiant Regime

R.B. et al,Physics of High Gain FEL and Superradiance,

La Rivista del Nuovo Cimento vol. 13. n. 9 (1990) e vol. 15 n.11 (1992)

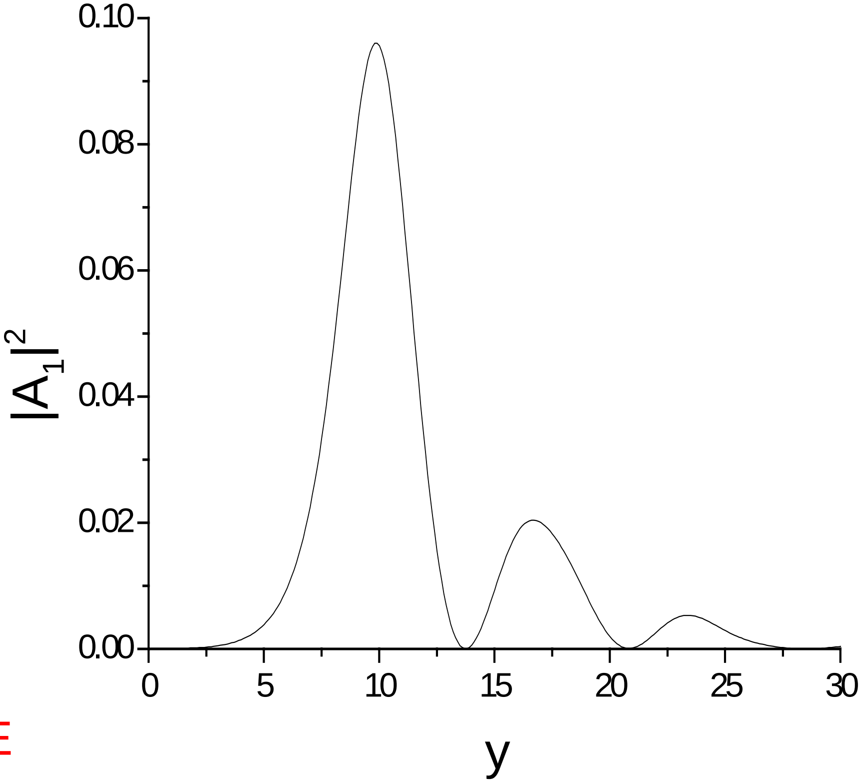
CLASSICAL REGIME:

$$A(z_1, \bar{z}) = z_1 A_1(y)$$

$$y = \sqrt{z_1}(\bar{z} - z_1)$$

width $\propto \dfrac{1}{\sqrt{z_1}}$

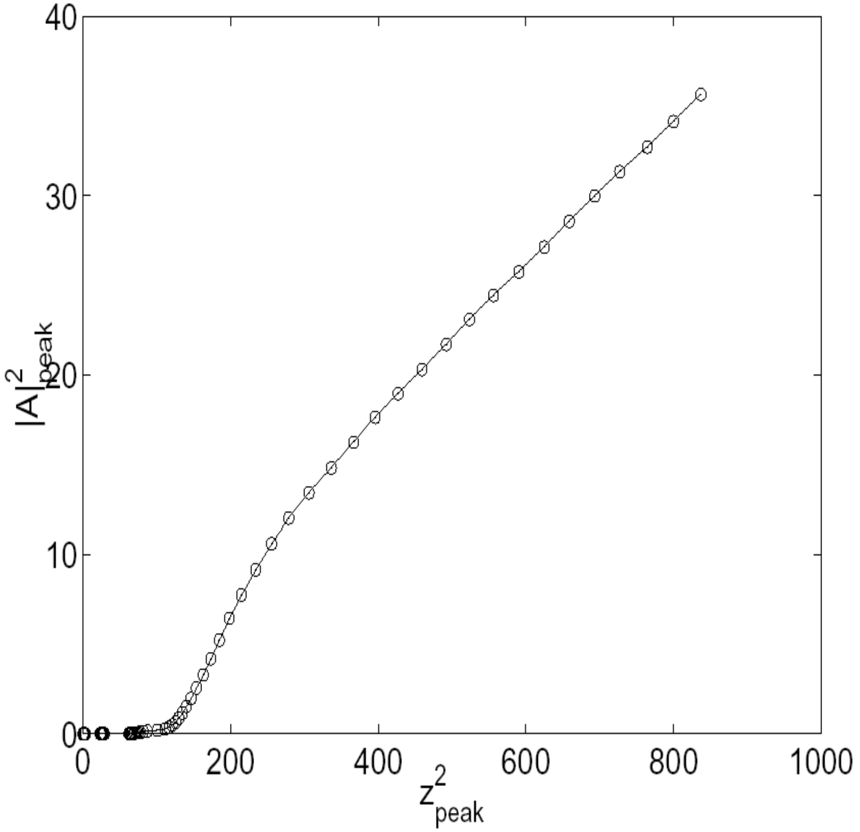
$$|A|^2 \propto z_1^{\,2} \propto N^2$$ SUPERRADIANCE



SELF SIMILAR SOLUTION

Self-similar scaling for long pulses ($L=30\ L_c$)

$$A(z_1, \overline{z}) \propto z_1$$



Propagation and SuperRadiant (SR) Instability

SS instability in the leading edge $\left(z_1 > \overline{z}\right)$
electrons radiates in front what they get from the back
Trailing edge $\left(z_1 < \overline{z}\right)$ particles get nothing from the back:
just radiate in front a SUPERRADIANT PULSE

if $L_b \leq L_c$ the SR pulse remains small (weak SR).
if $L_b >> L_c$ the weak SR pulse gets amplified (strong SR) with
no saturation.

The SR pulse is a soliton like self similar solution of the
propagation equation (ref. 2, 4, 5).
The above description is true for coherent excitation,
NOT FOR SASE, in which SS instability never occur.

Descriptions of SASE as SS instability starting from
noise are wrong.

SASE

Ingredients:

i) Start up from noise

ii) Propagation effects (slippage)

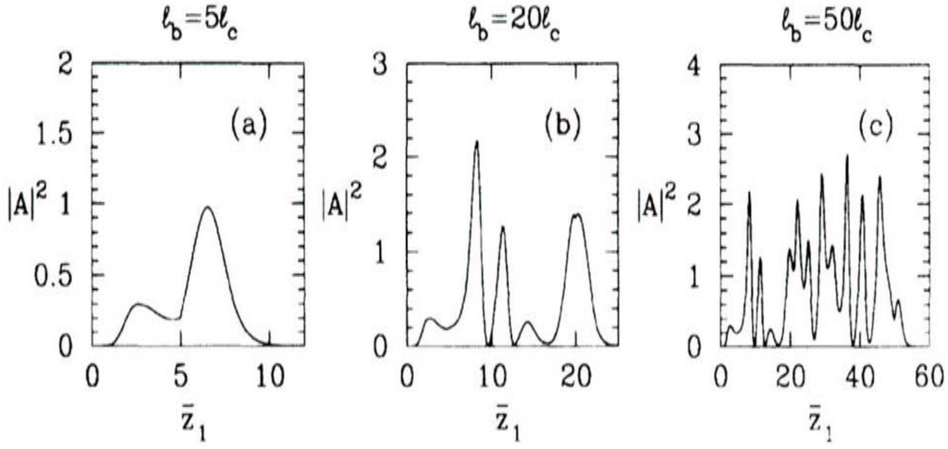
iii) Superradiant instability: (no steady state instability)

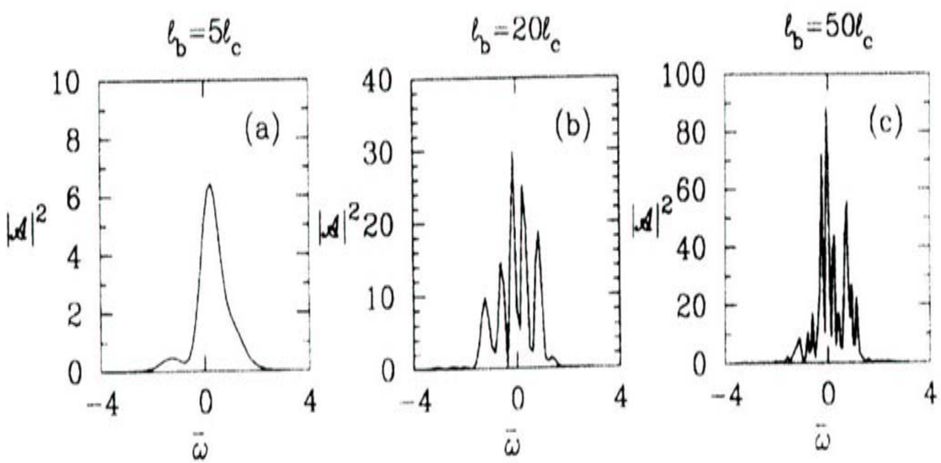
Self Amplified Superradiant Emission

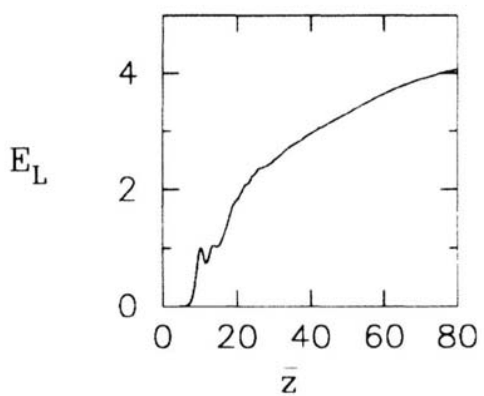
(RB, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL 73 (1994) 70)

$$\Downarrow$$

The electron bunch behaves as if each cooperation length would radiate independently a weak SR spike which gets amplified propagating on the other electrons with no saturation. Spiky time structure and spectrum.







SASE
reprinted from PRL 73 (1994) 70

$$\left(L_c = \lambda / 4\pi\rho\right)$$

Time structure:

Almost chaotic behavior:

number of random spikes goes like L_b / L_c .

Spectrum:

is just the envelope of a series of narrow random spikes

If $L_b \leq L_c$ a single SR spike.

At short wavelengths $L_b >> L_c$ => many random spikes.

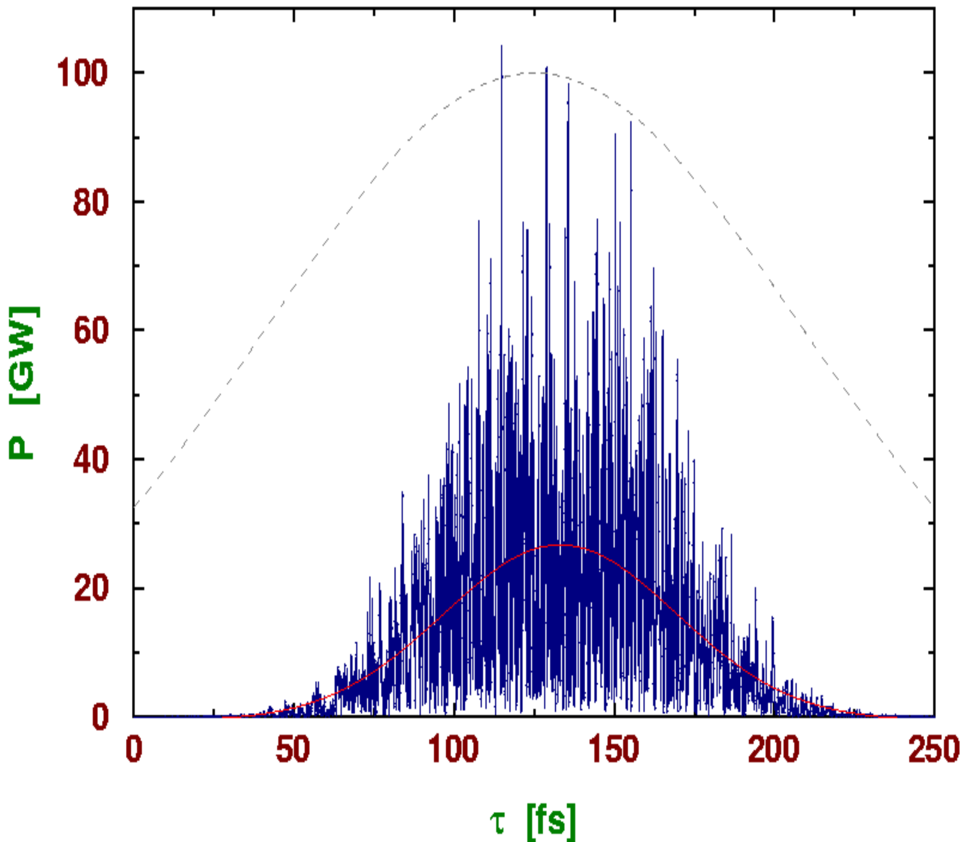
Total energy does not saturate (at 1.4).

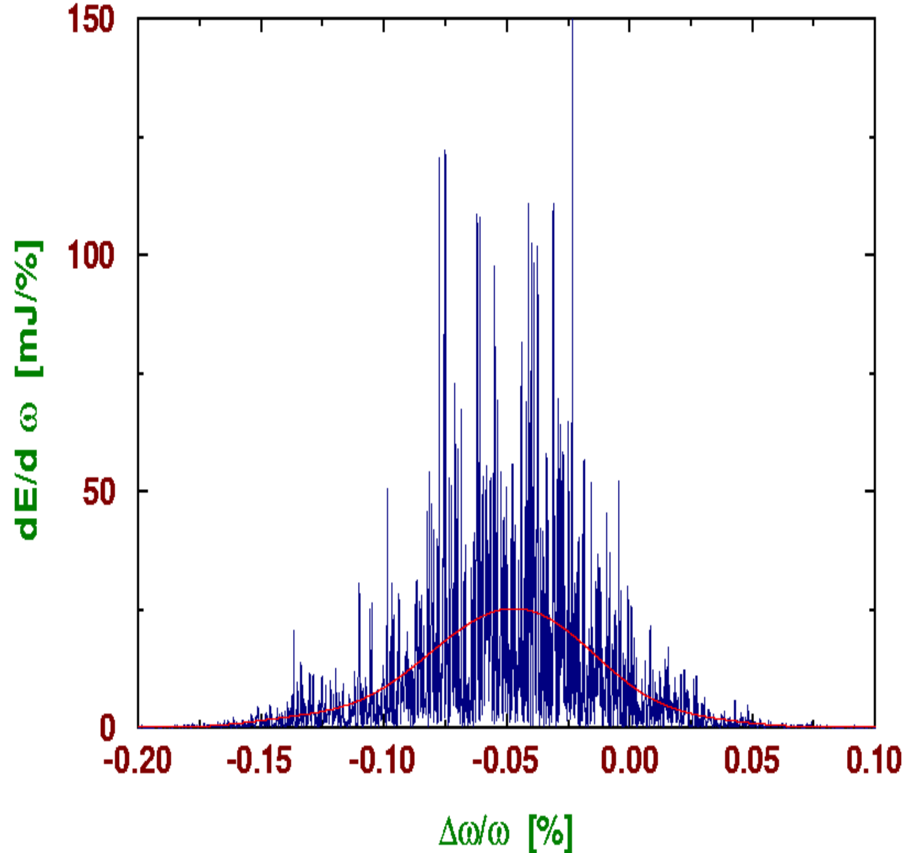
DRAWBACKS OF SASE

Time profile has many random spikes (n= L/L_c)

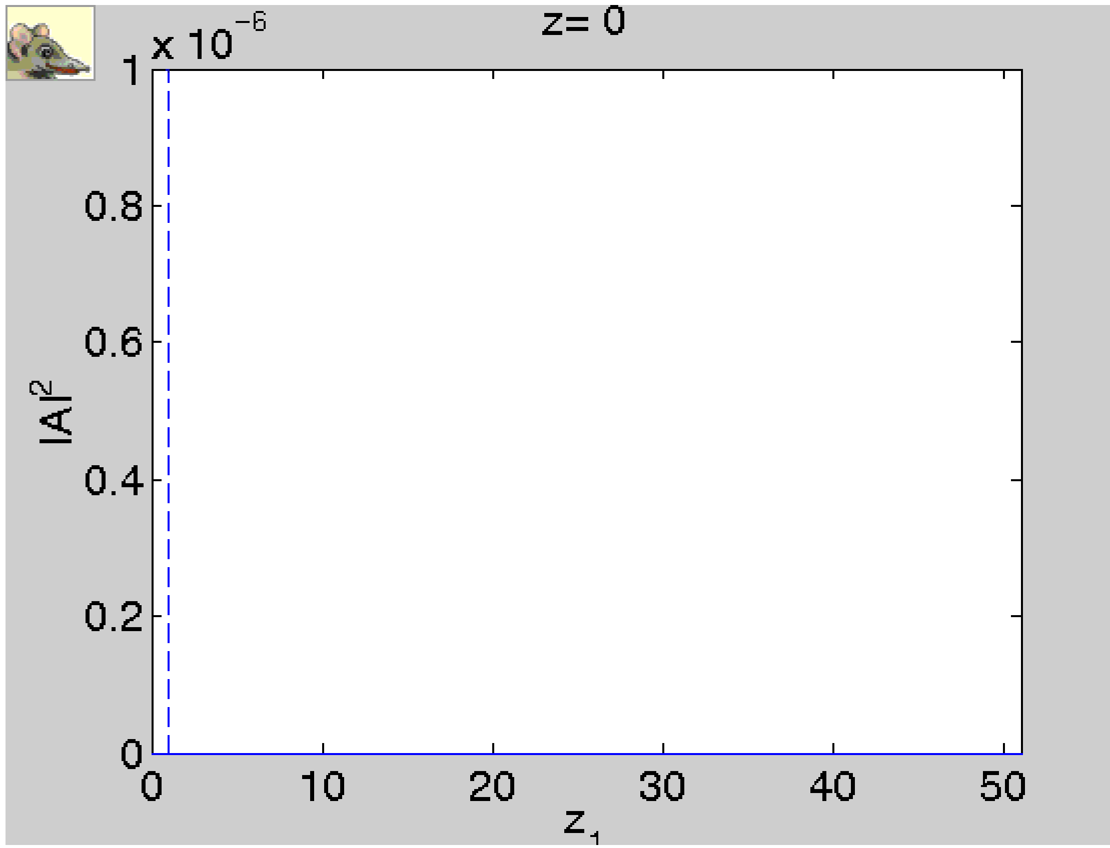
Broad and noisy spectrum at short wavelengths (X-ray FEL)

from DESY (Hamburg) for the SASE experiment (simulation)

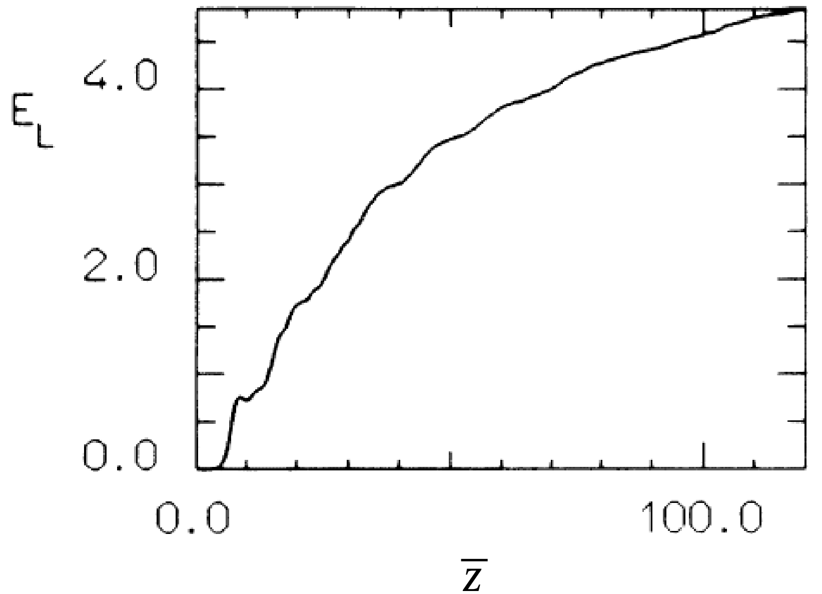




SASE Short bunch Lb = Lc



Short bunch superradiance total energy



$$L_b \approx 2L_c$$

In conclusion SASE gives incoherent spiking unless $L_b \leq 2\pi L_c$

see BRAFEL

In conclusion SASE gives incoherent spiking unless

$$L_b \leq 2\pi L_c$$

see BRAFEL

1D limit

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1D: $\sigma = \infty$, plane wave, zero emittance

In reality: radiation beam and electron beam focused to a cross section σ, diverges on a length Z_r and β:

$$Z_r = \frac{4\pi\sigma^2}{\lambda_r} \qquad \beta = \frac{\gamma\sigma^2}{\varepsilon_n}$$

$$\frac{Z_r}{\beta} = \frac{4\pi}{\gamma\lambda_r\varepsilon_n} \equiv \varepsilon_1 \leq 1 \text{ (Pellegrini criterium)} \quad \varepsilon_{\mathrm{n}} = \frac{\varepsilon_1\gamma\lambda_r}{4\pi}$$

The electron beam does not diverges in $Z_r < \beta$.

1D LIMIT: $L_g \leq Z_r \leq \beta$

In a planar wiggler, for a matched beam(*): $\sigma^2 = \dfrac{\lambda_w\varepsilon_n}{\sqrt{2}\pi a_w}$

(*) Ted Scharlemann, Proc. INFN School on EM Radiation & Particle Beam Acceleration, North Holland, 95 (1989)

We take ε_n maximum ($\varepsilon_1 = 1$), to satisfy the 1D limit at 100 μm

BRAFEL

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The BRAFEL Conceptual design

Rodolfo Bonifacio, Brian McNeil

SASE Classical FEL in Short Bunch Superradiant regime far infrared source with λ_r = 100 μm (tunable)

$$\lambda_r = \frac{\lambda_w (1 + a_w^2)}{2\gamma^2} \qquad a_w \approx B_w(T)\lambda_w(cm)$$

gain length $L_g = \dfrac{\lambda_w}{4\pi\rho}$ cooperation length $L_c = \dfrac{\lambda_r}{4\pi\rho}$

$L_b \leq 2\pi L_c$ SASE Pure Superradiance

$$\rho = \frac{2.6}{\gamma}\left[\frac{I(A)a_w^2\lambda_w^2(cm)}{\sigma^2(\mu m)}\right]^{1/3}$$

Tunability: a_w >1, change resonance changing a_w, changing the gap (B_w)

Given Parameters:

```
lambda_w            = 4 cm
a_w                 = 1
gamma               = 20
current             = 200 A      (75 A)
```

Derived Parameters:

```
lambda_r            = 100 micron
epsilon_n           = 159.155 mm-mrad
sigma_r             = 1.19704 mm
rho                 = 0.0130442  (0.00940648)
l_g                 = 14.0888 cm (19.5372 cm)
Z_R                 = 18.0063 cm
l_b_max             = 3.83313 mm (12.7859  ps)
                      (5.31548 mm (17.7306  ps))
```

Given Parameters:

lambda_w	= 4 cm
a_w	= 1
gamma	= 20
current	= 600 A

Derived Parameters:

lambda_r	= 100 micron
epsilon_n	= 159.155 mm-mrad
sigma_r	= 1.19704 mm
rho	= 0.018813
l_g	= 9.7686 cm
Z_R	= 18.0063 cm
l_b_max	= 2.65774 mm (8.86528 ps)

Given Parameters:

```
lambda_w          = 6.4 cm
a_w               = 1, 2, 3
gamma             = 40
current           = 300 A
```

Derived Parameters:

```
lambda_r     = 40, 100, 200 micron
epsilon_n    = 127.324, 318.31 ,636.62 mm-mrad
sigma_r      = 1.74838 mm
rho          = 0.0094, 0.014, 0.016
l_g          = 31.3, 21.2, 17.8 cm
Z_R          = 57.6, 28.8, 19.2 cm
l_b_max      = 2.1 mm (7.1ps), 3.6 mm (12.0ps)
                  6.1 mm (20.2  ps)
```

Using: $$\rho = \frac{5.6\times 10^{-3}}{\gamma}\left(\frac{I a_w^2 \lambda_w^2}{\sigma_r^2}\right)^{1/3} \quad - \quad SI$$

for a transverse Gaussian beam with RMS radius of σ_r

Now: $$\varepsilon_n = \frac{\gamma\sigma_r^2}{\beta} \quad \text{and} \quad \beta = \frac{f\gamma}{a_w k_w} \quad \Rightarrow \quad \sigma_r^2 = \frac{f\varepsilon_n}{a_w k_w}$$

Let: $$\varepsilon_n = \varepsilon_1 \frac{\gamma\lambda_r}{4\pi} \quad \Rightarrow \quad \sigma_r^2 = \frac{f\varepsilon_1\gamma\lambda_r}{4\pi a_w k_w} = \frac{f\varepsilon_1\gamma\lambda_r\lambda_w}{8\pi^2 a_w} = \frac{f\varepsilon_1\gamma\lambda_w^2\left(1+a_w^2\right)}{16\pi^2 a_w\gamma^2}$$

Where resonance relation is used. $$\lambda_r = \frac{\lambda_w}{2\gamma^2}\left(1+a_w^2\right)$$

Substitute for σ_r^2 into expression for ρ :

$$\rho = \frac{5.6\times10^{-3}}{\gamma}\left(\frac{Ia_w^2\lambda_w^2 16\pi^2 a_w\gamma^2}{f\,\varepsilon_1\gamma\lambda_w^2\left(1+a_w^2\right)}\right)^{1/3} = \frac{5.6\times10^{-3}\left(16\pi^2\right)^{1/3} a_w}{f^{1/3}}\left(\frac{I}{\varepsilon_1\gamma^2\left(1+a_w^2\right)}\right)^{1/3}$$

$$= \frac{3\times10^{-2}\,a_w}{f^{1/3}}\left(\frac{I}{\varepsilon_1\gamma^2\left(1+a_w^2\right)}\right)^{1/3}$$

for the case $f = \sqrt{2}$ **assumed henceforth.**

$$\Rightarrow \quad \rho = 2.7\times10^{-2}\,a_w\left(\frac{I}{\varepsilon_1\gamma^2\left(1+a_w^2\right)}\right)^{1/3}$$

The gain length: $$l_g = \frac{\lambda_w}{4\pi\rho} = 2.95\frac{\lambda_w}{a_w}\left(\frac{\varepsilon_1\gamma^2\left(1+a_w^2\right)}{I}\right)^{1/3}$$

Defining : $$L_g = \frac{l_g}{\sqrt{3}} \Rightarrow \quad L_g = 1.7\frac{\lambda_w}{a_w}\left(\frac{\varepsilon_1\gamma^2\left(1+a_w^2\right)}{I}\right)^{1/3}$$

Define the Rayleigh range: $$Z_R = \frac{4\pi\sigma_r^2}{\lambda_r} = \frac{8\pi\gamma^2\sigma_r^2}{\lambda_w\left(1+a_w^2\right)}$$

The ratio: $$\frac{Z_R}{l_g} = \frac{8\pi\gamma^2\sigma_r^2}{\lambda_w\left(1+a_w^2\right)} \times \frac{a_w}{2.95\lambda_w}\left(\frac{I}{\varepsilon_1\gamma^2\left(1+a_w^2\right)}\right)^{1/3}$$

$$\frac{Z_R}{l_g} = \frac{8\pi\gamma^2}{\lambda_w\left(1+a_w^2\right)} \times \frac{\sqrt{2}\varepsilon_1\gamma\lambda_w^2\left(1+a_w^2\right)}{16\pi^2 a_w\gamma^2} \times \frac{a_w}{2.95\lambda_w}\left(\frac{I}{\varepsilon_1\gamma^2\left(1+a_w^2\right)}\right)^{1/3}$$

$$\frac{Z_R}{l_g} = 7.6\times10^{-2}\,\varepsilon_1^{2/3}\left(\frac{\gamma I}{\left(1+a_w^2\right)}\right)^{1/3}$$

Quantum FEL theory

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Canonical Quantization

$$\dot{\theta} = \frac{p}{\bar{\rho}} = \frac{\partial H}{\partial p}; \quad \dot{p} = -\bar{\rho}\left(Ae^{i\theta} + cc\right) = -\frac{\partial H}{\partial \theta} \quad \left[p = \bar{\rho}\,\bar{p} = \frac{p_z}{\hbar k}\right]$$

$$H = \frac{p^2}{2\bar{\rho}} - i\bar{\rho}\left(Ae^{i\theta} - cc\right) \qquad \theta = kz$$

Quantization $\left[z, p_z\right] = i\hbar$

$$p \rightarrow \hat{p} = -i\frac{\partial}{\partial \theta}; \left[\theta, \hat{p}\right] = i \qquad H \rightarrow \hat{H}$$

The QFEL model for the matter wave Ψ

$$i\frac{\partial \Psi}{\partial \bar{z}} = \hat{H}\Psi = -\frac{1}{2\bar{\rho}}\frac{\partial^2 \Psi}{\partial \theta^2} - i\bar{\rho}\left(Ae^{i\theta} + cc\right)\Psi$$

Derived from Q-field theory by G. Preparata

(Phys. Rev. A, 38 (1988), 233)

QFEL propagation model

$\Psi(\theta, \bar{z}, z_1)$ matter wave

$$i\frac{\partial \Psi}{\partial \bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2}{\partial \theta^2}\Psi - i\bar{\rho}\left[A(\bar{z}, z_1)e^{i\theta} - c.c.\right]\Psi \qquad \left(\hat{p} \equiv -i\frac{\partial}{\partial \theta}\right)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \left\langle e^{-i\theta}\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta \left|\Psi(\theta, z_1, \bar{z})\right|^2 e^{-i\theta}$$

QFEL parameter $\bar{\rho} = \rho_F \frac{\gamma mc}{\hbar k} = \frac{\Delta p}{\hbar k}$ $\qquad \bar{\rho}\left|A\right|^2 = n_{ph} / n_e$

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$\left(\frac{\partial A}{\partial z_1} = 0\right)$; Q. F. T. by G. Preparata† (Phys. Rev. A, 38 (1988), 233)

The multiple scaling method

See ref. (7)

$$\theta = (k + k_w)(z - v_r t) \qquad z_1 = \frac{z - v_r t}{\beta_r L_c}; z_1 = \varepsilon\theta \qquad \varepsilon = 2\rho_F = \frac{\lambda}{2\pi L_c} << 1$$

$$i\frac{\partial \Psi(\theta,\bar{t})}{\partial \bar{t}} = -\frac{1}{\bar{\rho}}\frac{\partial^2 \Psi(\theta,\bar{t})}{\partial \theta^2} - \frac{i\bar{\rho}}{2}\left[A(\theta,\bar{t})\, e^{i\theta} - c.c.\right]\Psi(\theta,\bar{t})$$

$$\left(\frac{\partial}{\partial \bar{t}} + \frac{\partial}{\varepsilon \partial \theta}\right) A(\theta,\bar{t}) = \left|\Psi(\theta,t)\right|^2 e^{-i\theta}$$

$$\frac{\partial}{\partial\theta} \rightarrow \frac{\partial}{\partial\theta} + \varepsilon\frac{\partial}{\partial z_1} \qquad \Psi = \Psi^{(0)} + \varepsilon\Psi^{(1)} + \ldots \qquad A = A^{(0)} + \varepsilon A^{(1)} + \ldots$$

$$i\frac{\partial \Psi^{(0)}(\theta, z_1, \bar{t})}{\partial \bar{t}} = -\frac{1}{\bar{\rho}}\frac{\partial^2}{\partial\theta^2}\Psi^{(0)}(\theta, z_1, \bar{t}) - \frac{i\bar{\rho}}{2}\left(A^{(0)}(z_1,\bar{t})e^{i\theta} - cc\right)\Psi^{(0)}(\theta, z_1, \bar{t})$$

$$\frac{\partial A^{(0)}}{\partial\theta} = 0, \frac{\partial A^{(1)}}{\partial\theta} = \left|\Psi^{(0)}\right|^2 e^{-i\theta} - \left(\frac{\partial A^{(0)}}{\partial \bar{t}} + \frac{\partial A^{(0)}}{\partial z_1}\right)$$

Integrating between 0 and 2π and assuming periodic boundary conditions:

$$\frac{\partial A^{(0)}(z_1,\bar{t})}{\partial \bar{t}} + \frac{\partial A^{(0)}(z_1,\bar{t})}{\partial z_1} = \frac{1}{2\pi}\int_0^{2\pi} d\theta \left|\Psi^{(0)}(\theta, z_1, \bar{t})\right|^2 e^{-i\theta}$$

Classical Limit: $\bar{\rho} \to \infty$

One can prove that the Schroedinger equation for the QFEL model reduces to the classical Vlasov Equation for the Quantum Wigner function in the limit: $\bar{\rho} \to \infty$

In the classical limit, with universal scaling, no dependence on $\bar{\rho}$

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Classical limit when $\bar{\rho} \to \infty$

$$\Psi(\theta, \bar{z}, z_1) \to W(\theta, \bar{p}, \bar{z}, z_1)$$ Wigner function

$$\frac{\partial W}{\partial \bar{z}} + \bar{p}\frac{\partial W}{\partial \theta} - \left(Ae^{i\theta} + A^* e^{-i\theta}\right)\bar{\rho}\left[W\left(\theta, \bar{p} + \frac{1}{2\bar{\rho}}, \bar{z}, z_1\right) - W\left(\theta, \bar{p} - \frac{1}{2\bar{\rho}}, \bar{z}, z_1\right)\right] = 0$$

$$\bar{p} \pm \frac{1}{2\bar{\rho}}; \quad p \pm \frac{\hbar k}{2}$$

$$\frac{\partial A(\bar{z}, z_1)}{\partial \bar{z}} + \frac{\partial A(\bar{z}, z_1)}{\partial z_1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\bar{p} \int_0^{2\pi} d\theta W(\theta, \bar{p}, z_1, \bar{z}) e^{-i\theta} = \left\langle e^{-i\theta} \right\rangle$$

$$\bar{\rho} \to \infty$$

$$\frac{\partial W(\theta, \bar{p}, \bar{z}, z_1)}{\partial \bar{z}} + \bar{p}\frac{\partial W(\theta, \bar{p}, \bar{z}, z_1)}{\partial \theta} - \left(Ae^{i\theta} + A^* e^{-i\theta}\right)\frac{\partial W(\theta, \bar{p}, \bar{z}, z_1)}{\partial \bar{p}} = 0$$

Classical Vlasov Equation

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The Momentum Representation

$$\psi(\theta, z_1, \bar{z}) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} c_n(z_1, \bar{z}) e^{in\theta} \; ; \; (\hat{p} \rightarrow n\hbar k)$$

$|c_n|^2$ is the probability that an electron has a momentum $n\hbar k$

$$\frac{\partial c_n}{\partial \bar{z}} = -iE_n c_n - \bar{\rho}(A c_{n-1} - A^* c_{n+1})$$

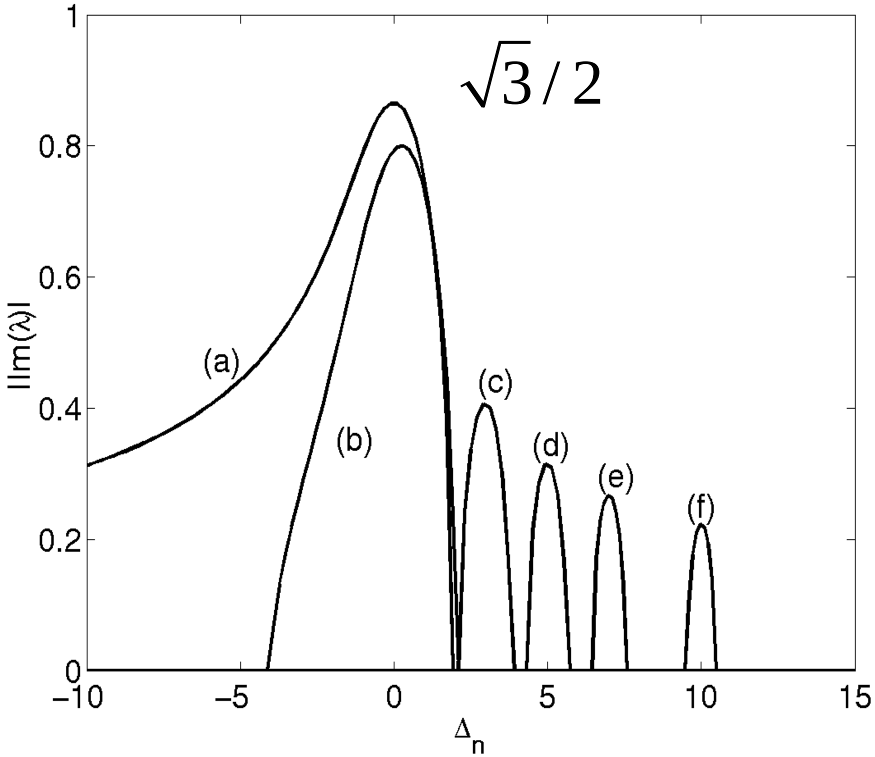
$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \sum_{n=-\infty}^{\infty} c_n c^*_{n-1} = \langle e^{-i\theta} \rangle$$

QFEL "working equations"

$$E_n = \frac{n^2}{2\bar{\rho}} \qquad \left(\frac{p^2}{2m} \right)$$

Linear Theory: QM $(\lambda - \Delta)\left(\lambda^2 - \frac{1}{4\bar{\rho}^2}\right) + 1 = 0$ $\left(e^{i\lambda\bar{z}}\right)$

As if classical rect. dist. $2\sigma_0(\bar{p}) = 1/\bar{\rho}$, i.e., $2\sigma_0(p) = \hbar k$



$\bar{\rho} >> 1$ Classical limit (a)

$\bar{\rho} \leq 1$ Quantum regime

$\Delta_{max} = 1/2\bar{\rho}$; width=$4\sqrt{\bar{\rho}}$

$\mathrm{Im}\,\lambda = \sqrt{\bar{\rho}} \Rightarrow L_g^{'} = \frac{L_g}{\sqrt{\bar{\rho}}},\ L_c^{'} = \frac{L_c}{\sqrt{\bar{\rho}}}$

$1/2\bar{\rho} = 0.$(a), 0.5 (b), 3 (c), 5 (d), 7 (e) and 10 (f).

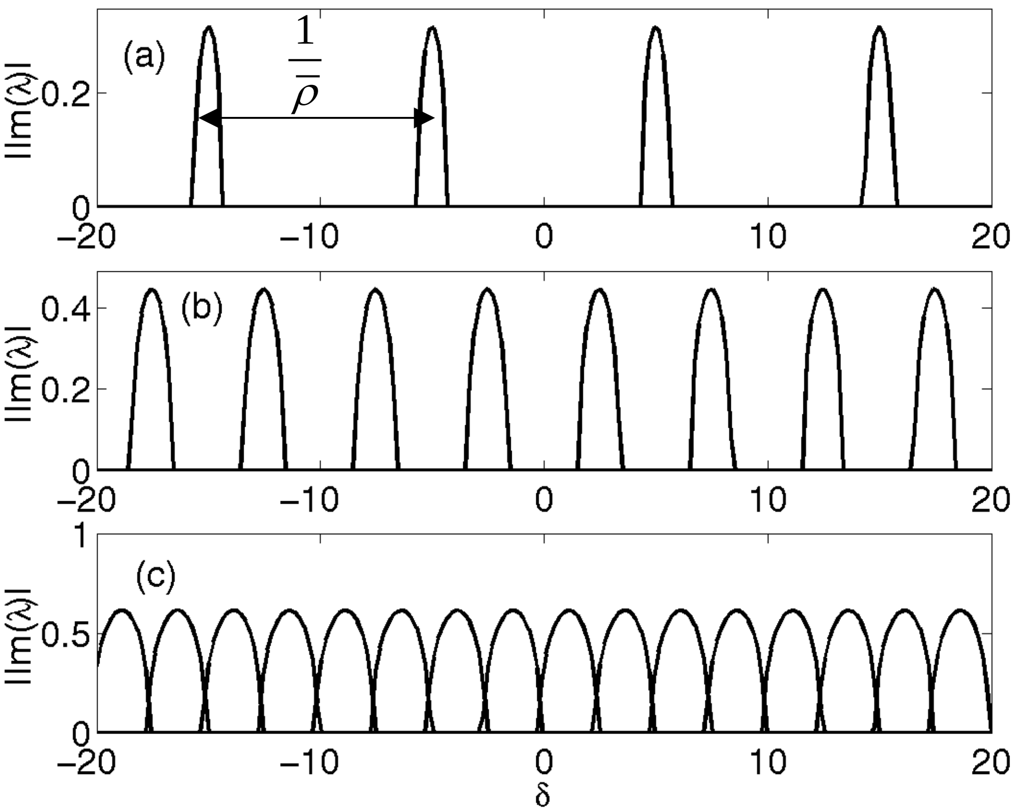
The Discrete frequencies as in a cavity

$$(\lambda-\Delta)\left(\lambda^2-\frac{1}{4\bar{\rho}^2}\right)+1=0 \quad \Delta=\delta+\frac{n}{\bar{\rho}}=\frac{1}{2\bar{\rho}} \rightarrow \delta_n=\frac{1}{2\bar{\rho}}-\frac{n}{\bar{\rho}}\left(\delta \propto \omega-\omega_{sp}\right)$$

$\bar{\rho}=0.1$

$\bar{\rho}=0.2$

$\bar{\rho}=0.4$



δ Frequency separation $\frac{1}{\bar{\rho}}$

$$\left(\omega_r=\frac{\hbar k^2}{\gamma m}\right)$$

Full width $4\sqrt{\bar{\rho}}$

$$\left(\omega_r 4\bar{\rho}^{3/2}\right)$$

Continuous classical limit $4\sqrt{\bar{\rho}}\geq 1/\bar{\rho}\rightarrow\bar{\rho}\geq 0.4\left(4\bar{\rho}^{3/2}\geq 1\right)$

Quantum limit : discrete resonance as in a cavity

$$E = \frac{p^2}{2\bar{\rho}} \qquad p|n\rangle = n|n\rangle \qquad E_n \propto \frac{n^2}{2\bar{\rho}}$$

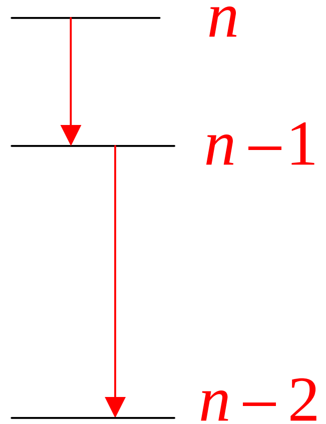
$$\delta_n \propto E_{n-1} - E_n \propto \frac{(n-1)^2 - n^2}{2\bar{\rho}};$$

$$\delta_n = \frac{1}{2\bar{\rho}} - \frac{n}{\bar{\rho}} \qquad \mathrm{n} = 0, -1, ..$$

$$\rho \le 1 \Rightarrow n \to n-1$$

Only recoil : no absorption

$$\bar{\rho} \le 1$$

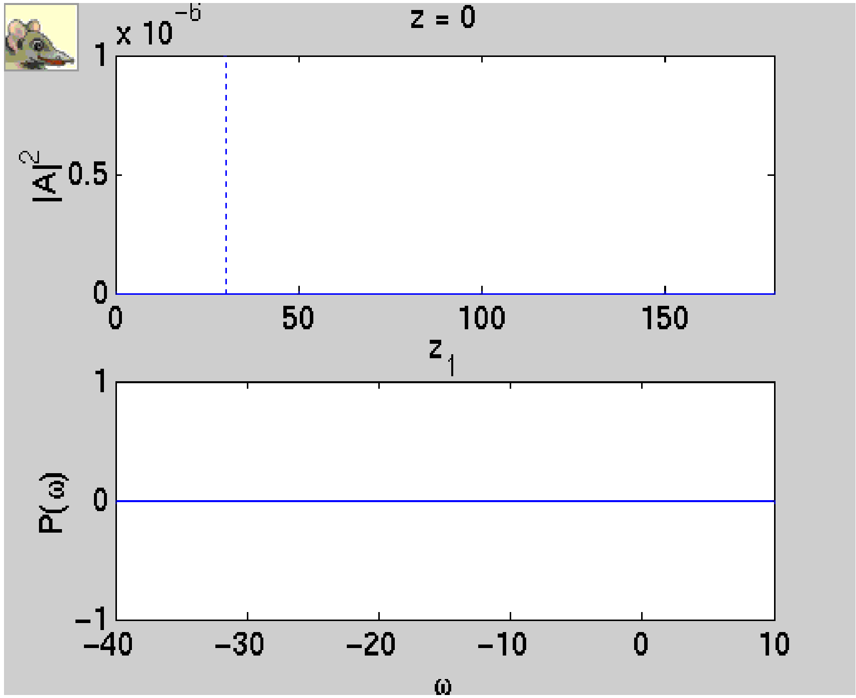


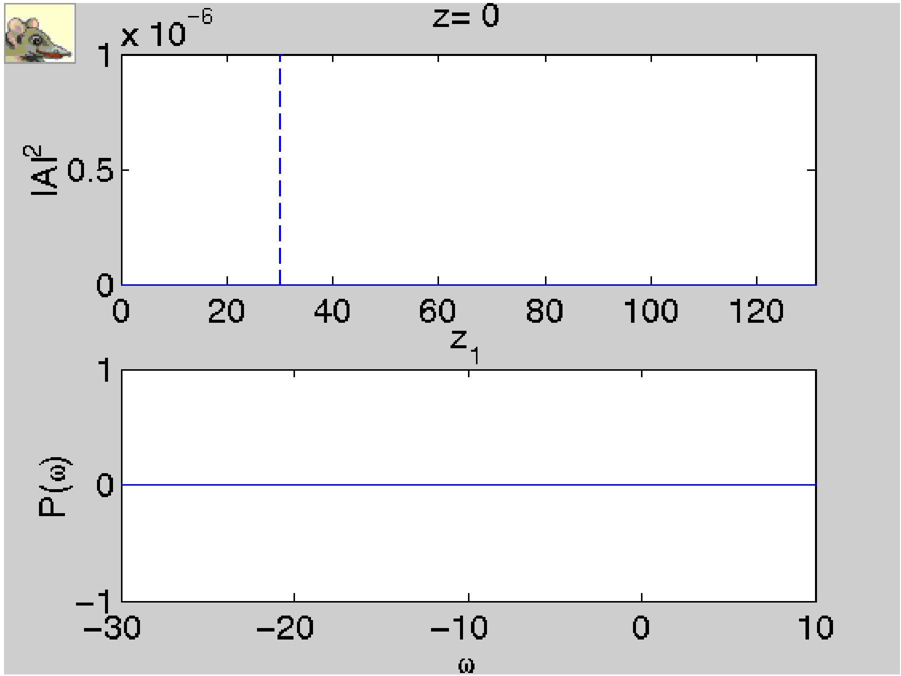
$$\mathrm{d} = \frac{1}{\bar{\rho}}, \text{ width } \sigma = 4\sqrt{\bar{\rho}}; \quad 4\sqrt{\bar{\rho}} \ge \frac{1}{\bar{\rho}} \to \bar{\rho} > 0.4 \text{ (continuous classical limit)}$$

SASE

Quantum $\bar{\rho} = 0.05$ $L/L_c = 30$ Classical $\bar{\rho} = 5$

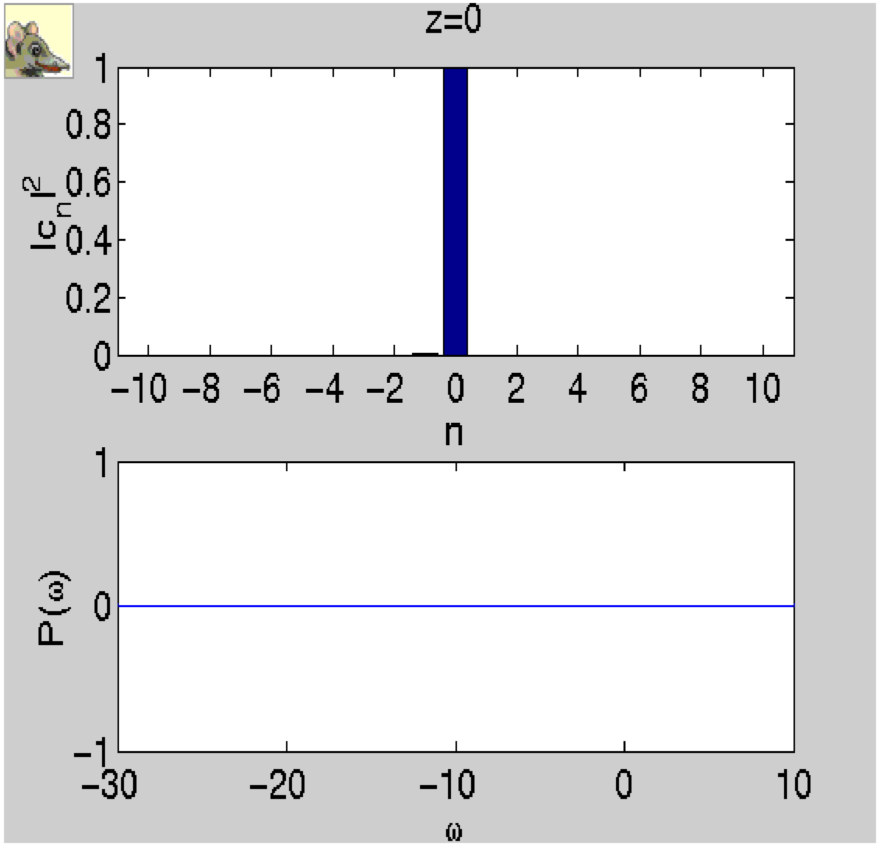
Evolution of radiation time structure in the electron rest frame

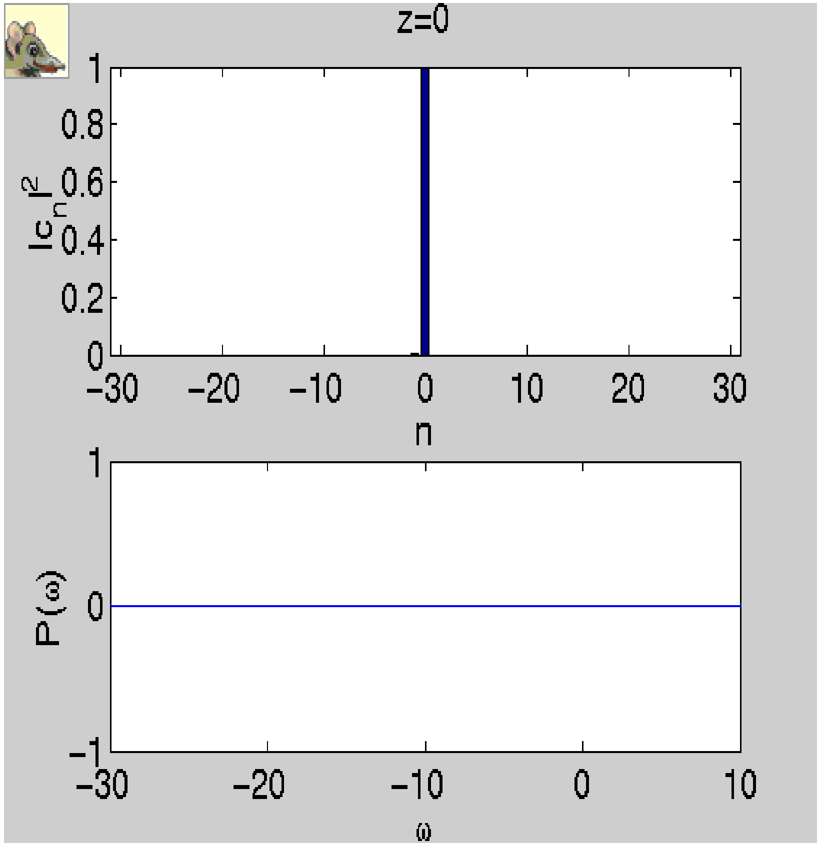




Simulation using QFEL model: Momentum distribution (average)

Quantum regime $\bar{\rho} = 0.1$ $L / L_c = 30$ Classical regime $\bar{\rho} = 5$



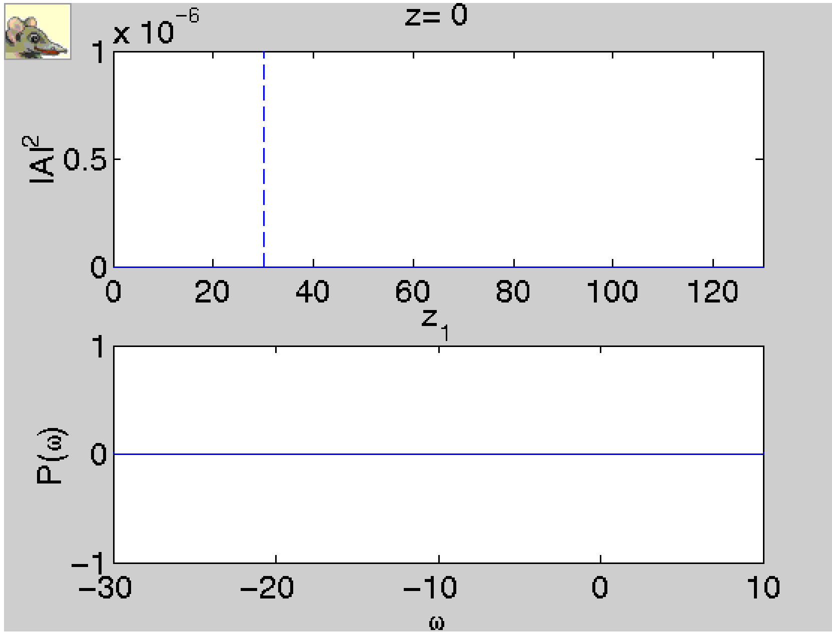


Classical behaviour : both n<0 and n>0 occupied

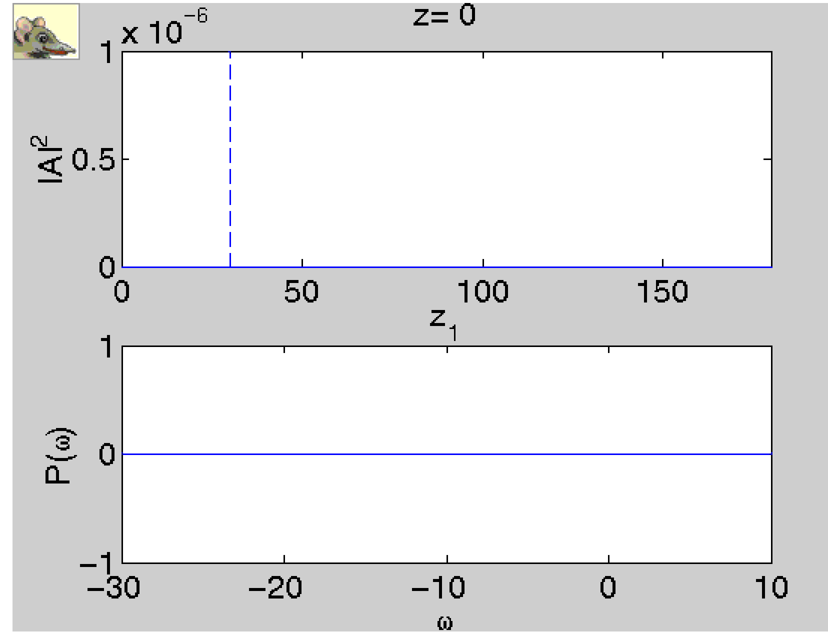
Quantum behaviour : sequential SR decay, only n<0

$(2n-1)/2\bar{\rho}$ $[n = 0, -1, \ldots]$

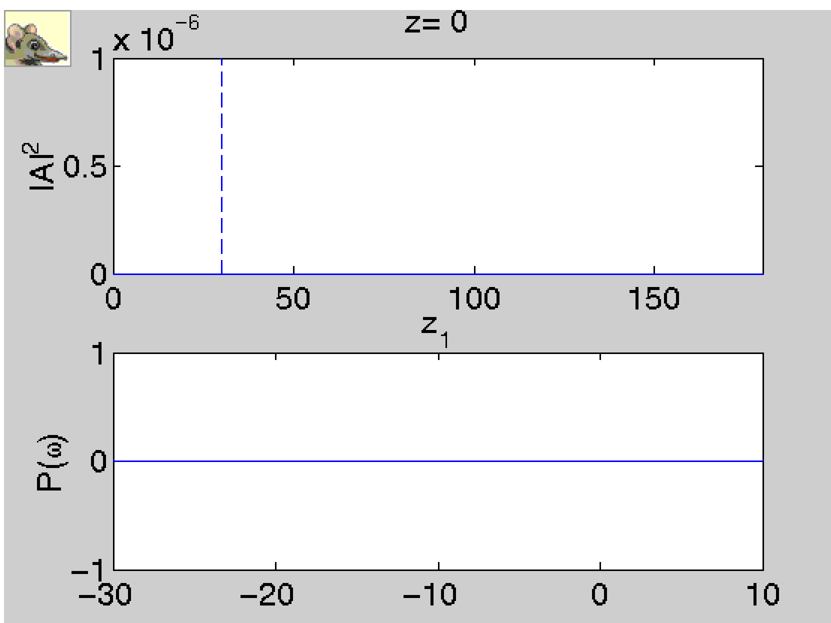
$\bar{\rho} = 0.1$



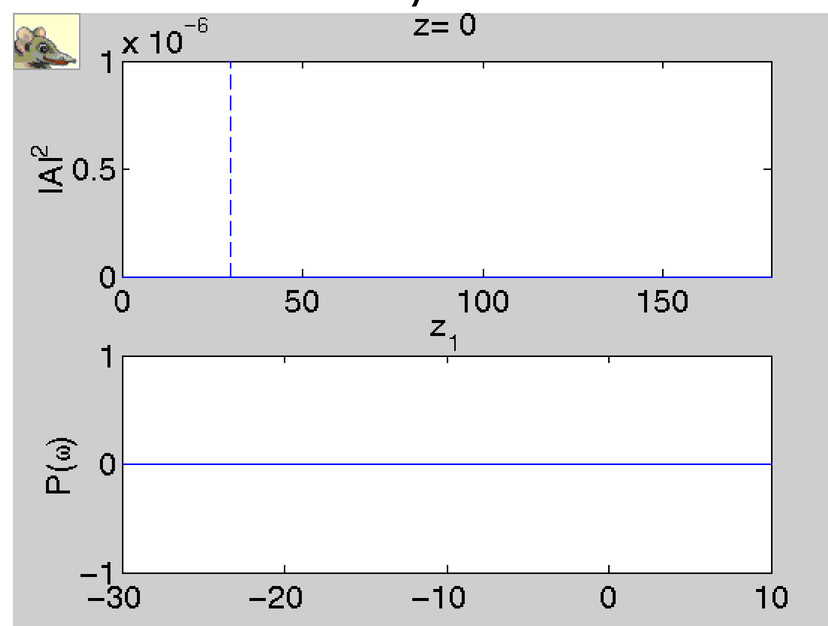
$\bar{\rho} = 0.2$



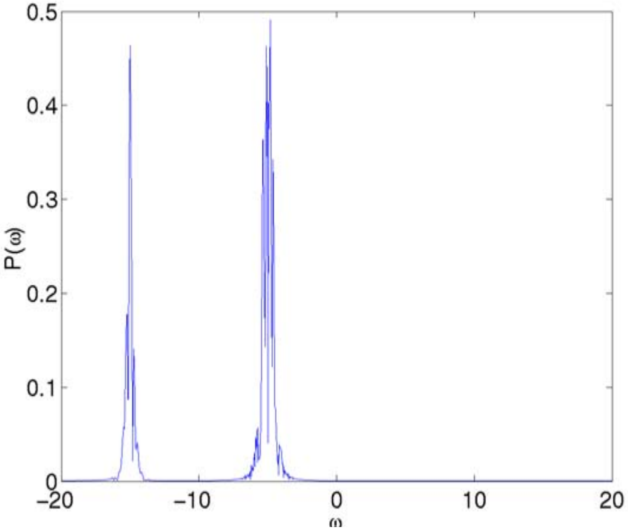
$\bar{\rho} = 0.3$



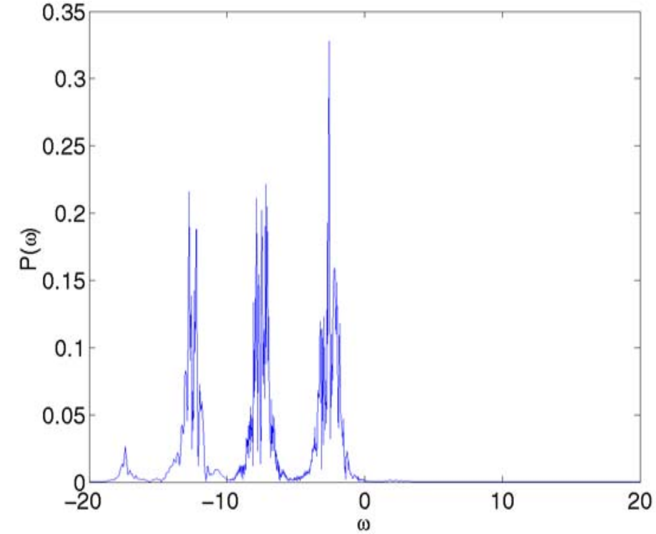
$\bar{\rho} = 0.4$



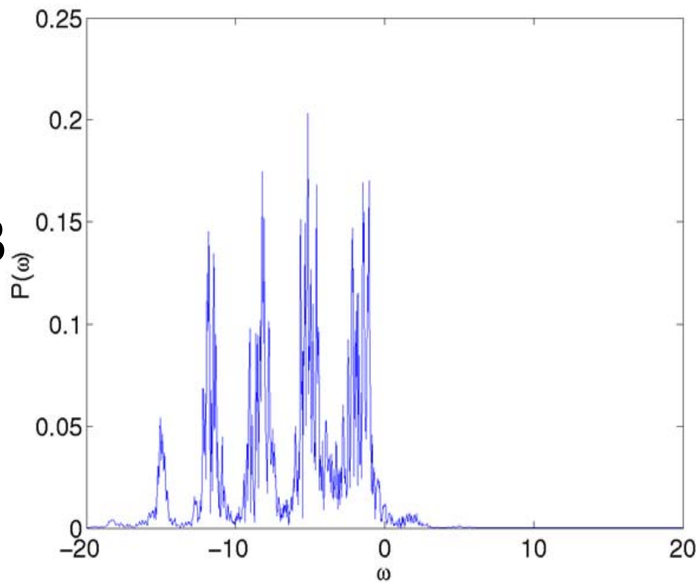
$\bar{\rho} = 0.1$



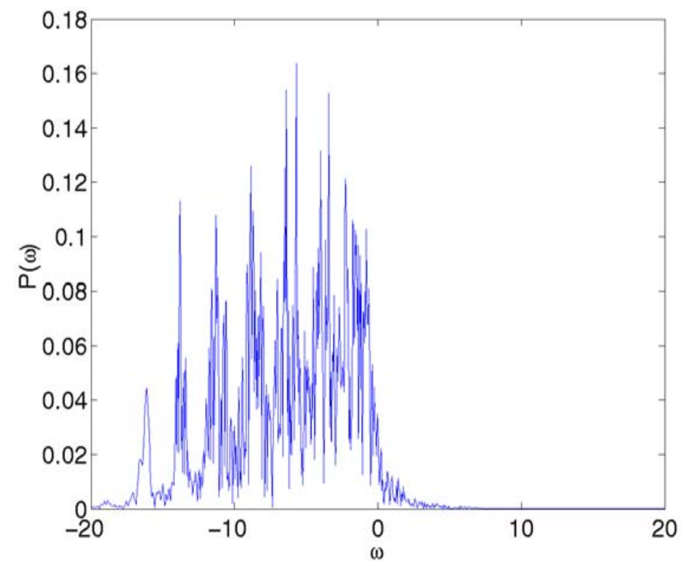
$\bar{\rho} = 0.2$



$\bar{\rho} = 0.3$



$\bar{\rho} = 0.4$

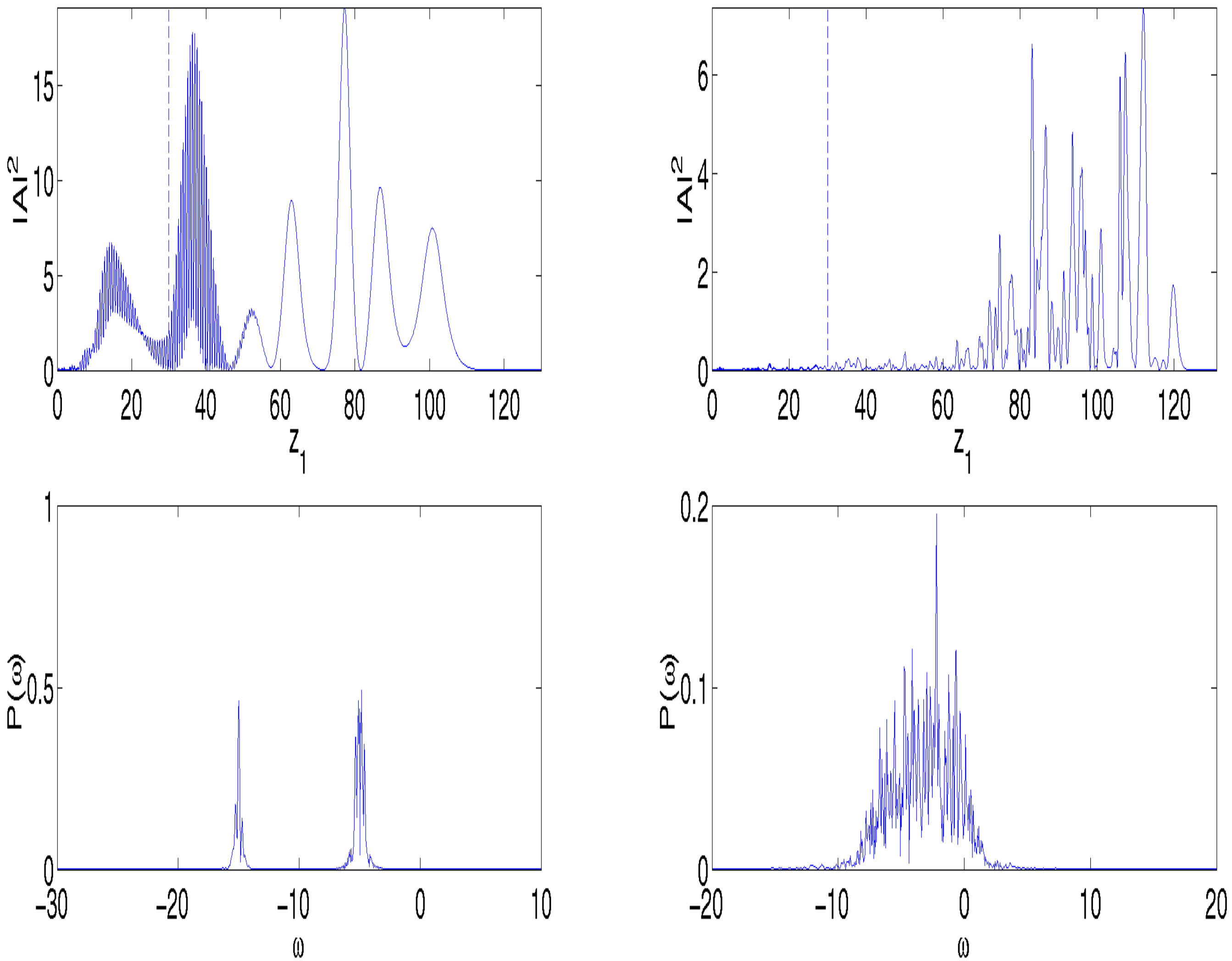


Conclusions

- Classical description of SASE valid IF $\bar{\rho} >> 1$
- IF $\bar{\rho} \leq 1$ one has quantum SASE: the gain bandwidth decreases as $4\sqrt{\bar{\rho}}$ and $L_c \propto 1/\sqrt{\bar{\rho}}$ ⇨ line narrowing, temporal coherence.
- Multiple lines Spectrum:
  - separation $1/\bar{\rho}$, linewidth $4\sqrt{\bar{\rho}}$
- Classical limit: increasing $\bar{\rho}$ separation $\leq$ linewidth $(\bar{\rho} \geq 0.4)$ → continuous spiky classical spectrum.

For experimental setup see R.B., NIM A 546 (2005) 634, and this proceeding

Quantum $\bar{\rho} = 0.1$ Classical $\bar{\rho} = 5$



Quantum Free Electron Laser

QFEL

R. Bonifacio[*](80%), M.M. Cola[+](60%), N. Piovella[+](70%), L. Serafini(10%), L. Volpe[+] (80%) , **INFN-Milano [FTE 3.0]**

D. Babusci(30%), M. Benfatto(20%), S. Di Matteo(30%), **M. Ferrario(20%)**, C. Natoli(20%), L. Palumbo(10%),A. Schiavi^(n.a.), A. Tenore(30%)
INFN-LNF [FTE 1.6]

U. De Angelis(30%), S. De Nicola(20%), **R. Fedele(40%)**, G. Fiore(20%)
INFN-Napoli [FTE 1.1]

G.R.M. Robb, B.W.J. Mc Neil,
University of Strathclyde, Glasgow, UK

V. Shchesnovich
Universidade Federal de Alagoas, Maceio, Brazil

[*] Physics Dep. of Universidade Federal de Alagoas, Maceio', Brazil
[+] Dipartimento di Fisica, Universita' degli Studi di Milano
[^] Dipartimento di Energetica, Universita' di Roma "La Sapienza"

Recenti studi [1] hanno dimostrato l’esistenza di un nuovo

REGIME QUANTISTICO

del FEL SASE (Self Amplified Superradiant Emission)

per la produzione di raggi X coerenti (λ=1 Å)

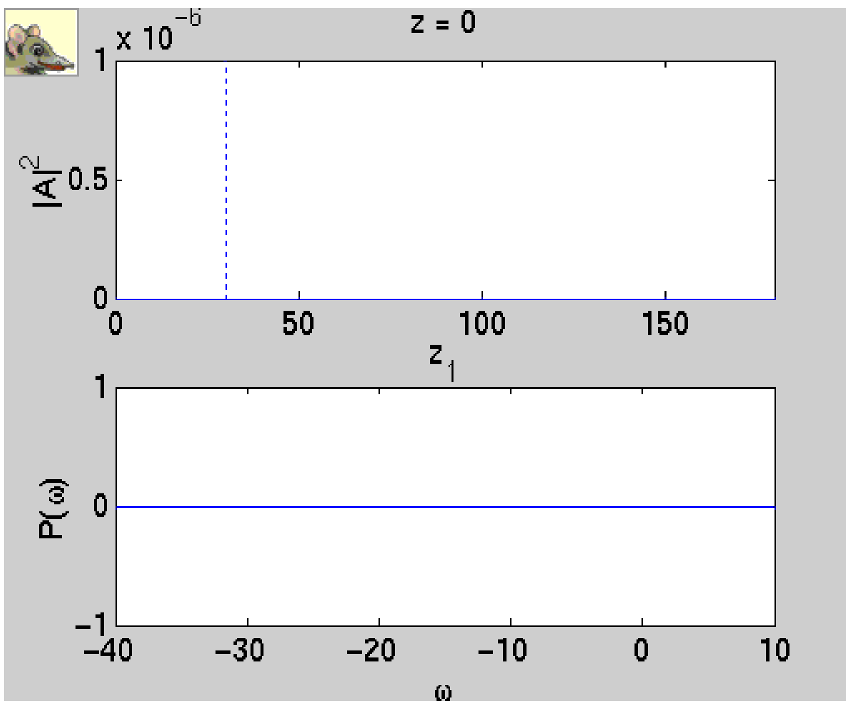
$$\bar{\rho} = \rho_{FEL}\left(\frac{mc^2\gamma}{\hbar\omega}\right)$$

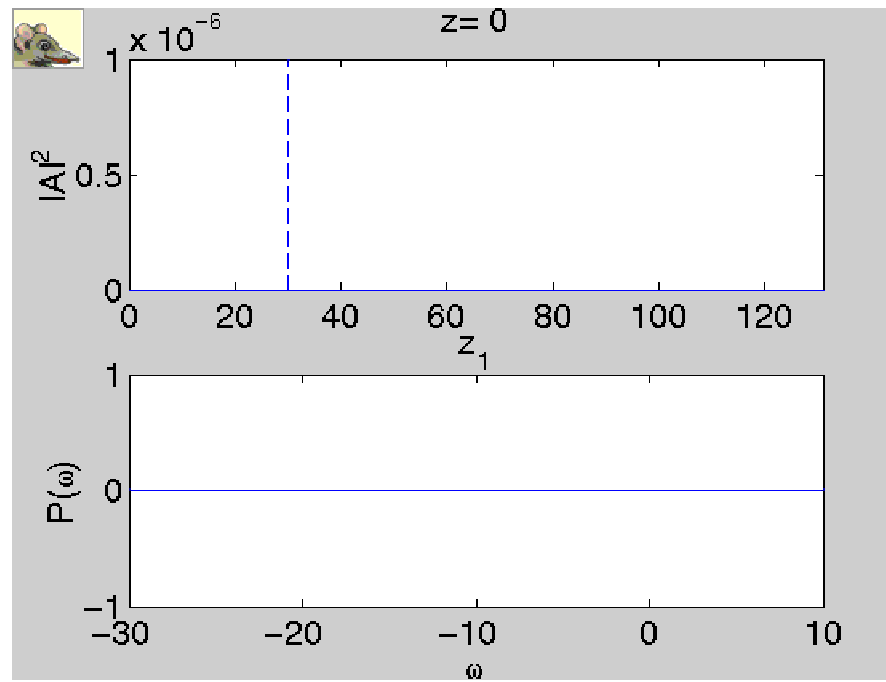
$\bar{\rho}$ = numero medio fotoni emessi per elettrone

$\bar{\rho}$ > 1 classical SASE (spiking incoerente)

$\bar{\rho}$ < 1 quantum SASE (coerente)

$\bar{\rho} = 0.05$ $\quad L / L_c = 30$ $\quad \bar{\rho} = 5$

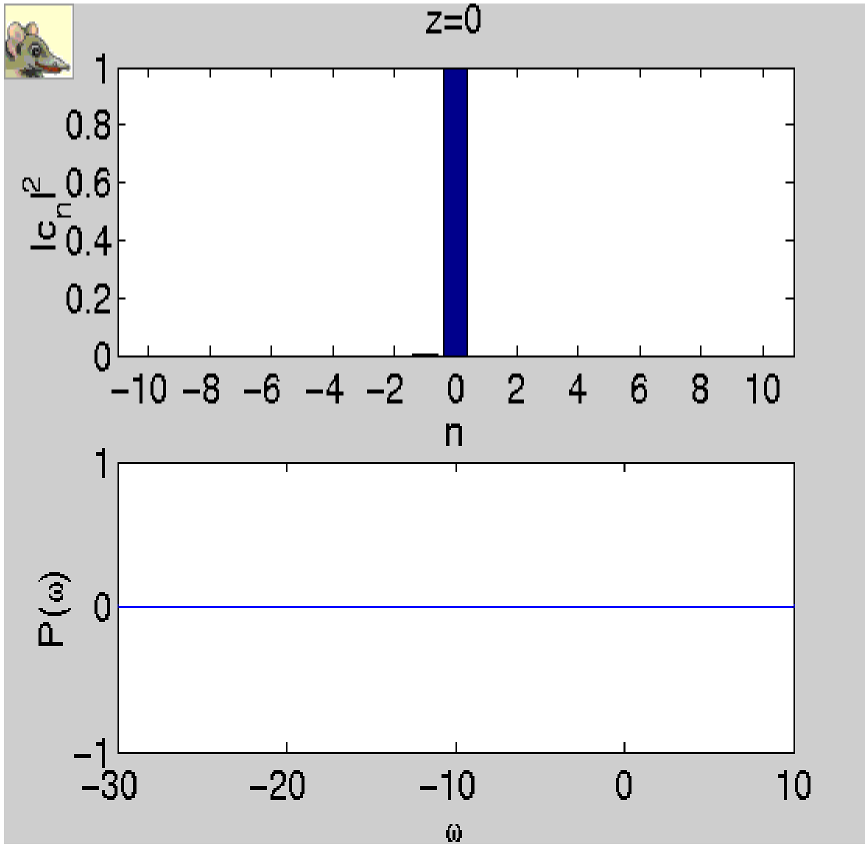


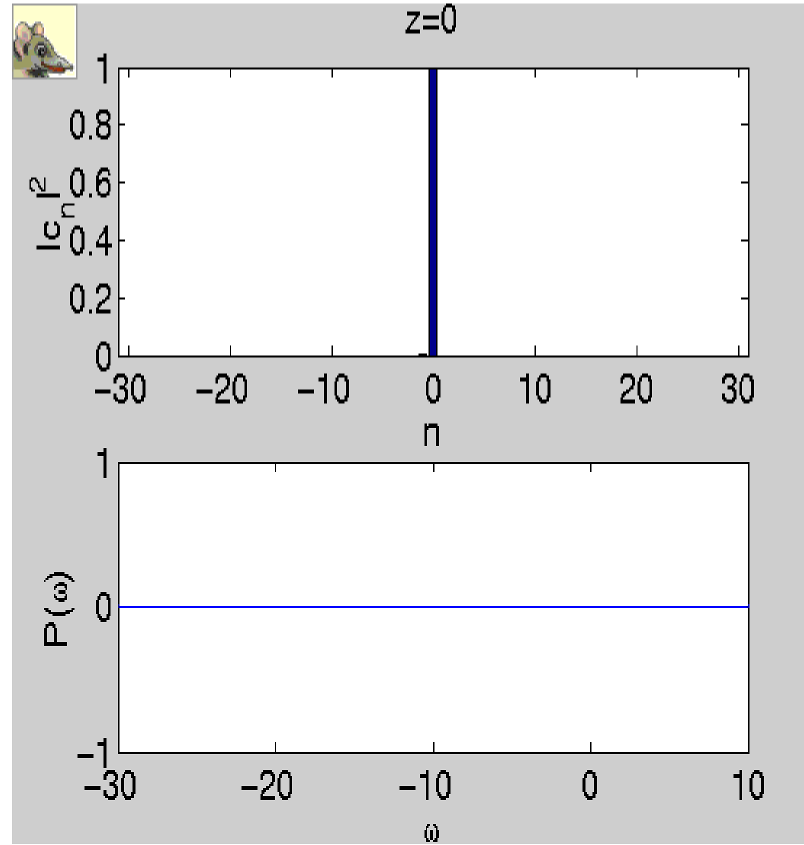


[1] R. Bonifacio, N. Piovella, G.R.M. Robb, “Quantum theory of SASE FEL”, NIMA 543 (2005) 645.

QFEL Model: Momentum distribution and spectrum

Quantum regime $\bar{\rho} = 0.1$ $L / L_c = 30$ Classical regime $\bar{\rho} = 5$



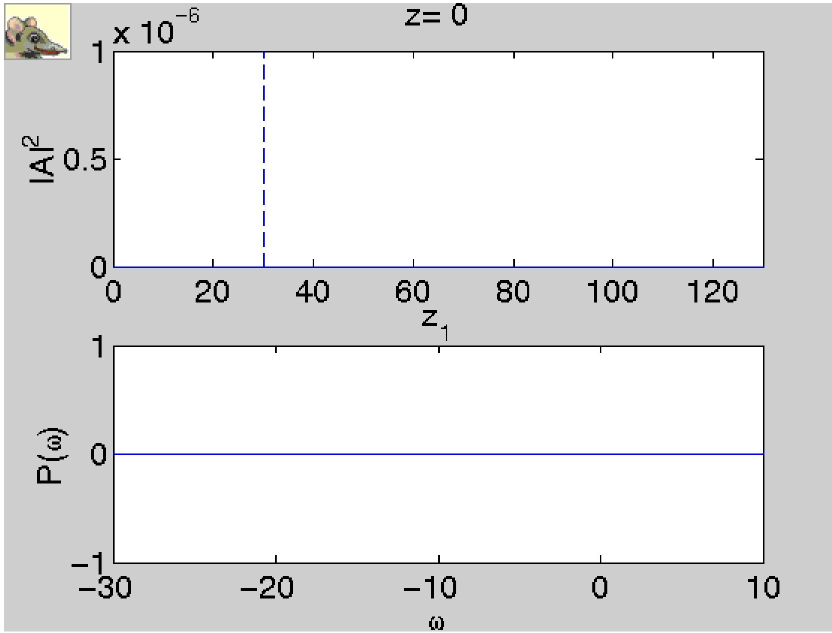


Classical behaviour : both n<0 and n>0 occupied

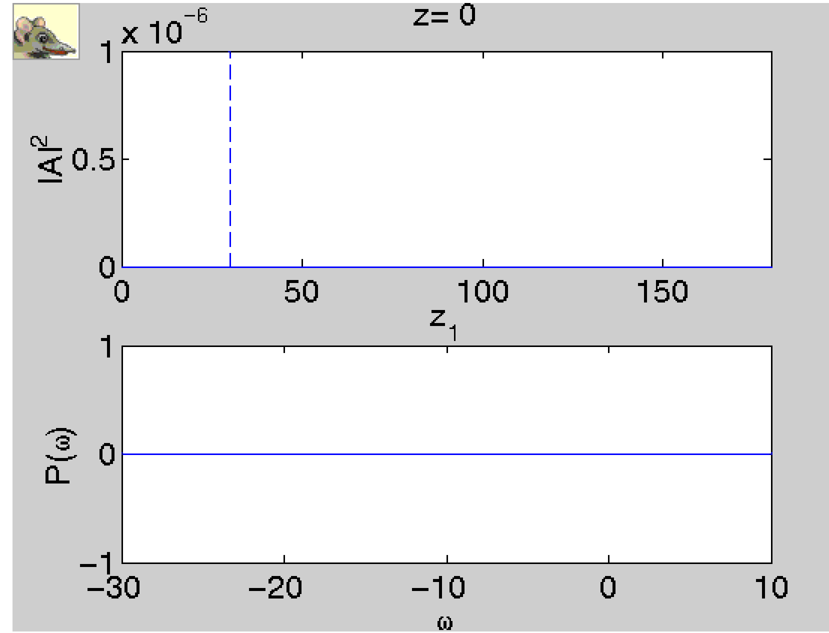
Quantum behaviour : sequential SR decay, only n<0

$(2n-1)/2\bar{\rho}$ $[n = 0, -1, \ldots]$

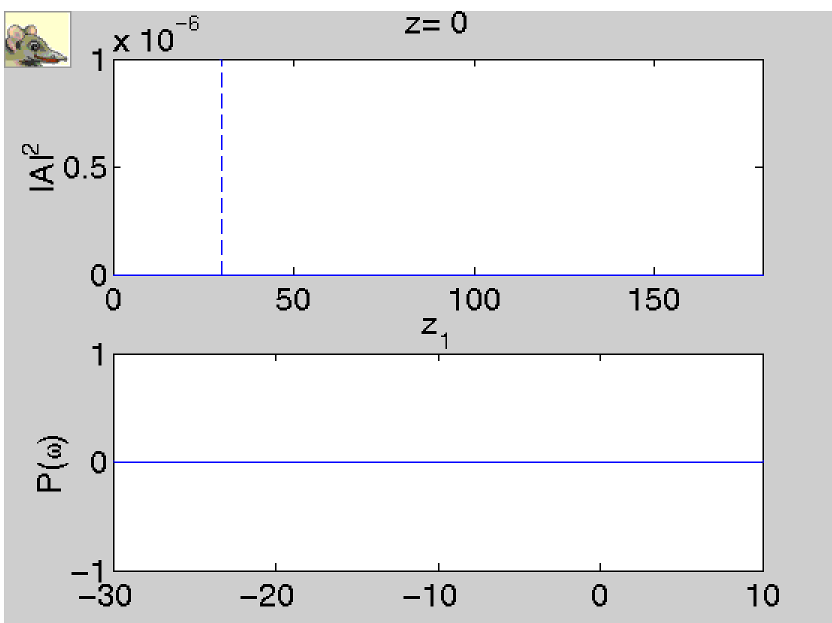
$\bar{\rho} = 0.1$



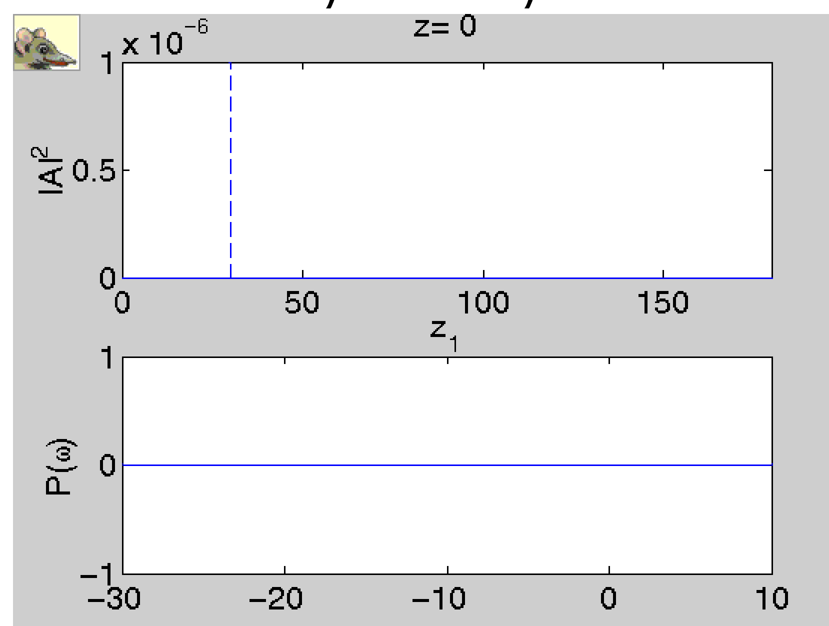
$\bar{\rho} = 0.2$



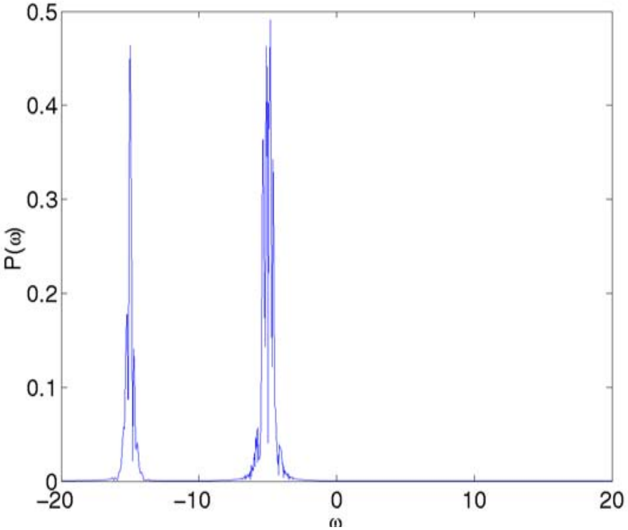
$\bar{\rho} = 0.3 \quad 1/\bar{\rho} = 3.3$



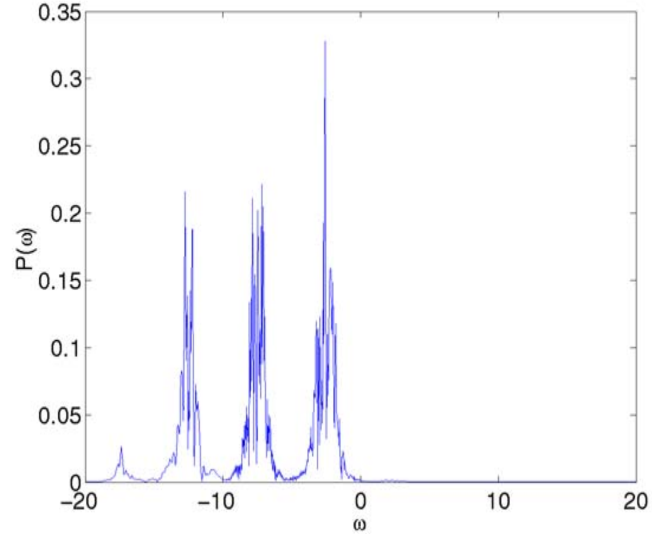
$\bar{\rho} = 0.4 \quad 1/\bar{\rho} = 2.5$



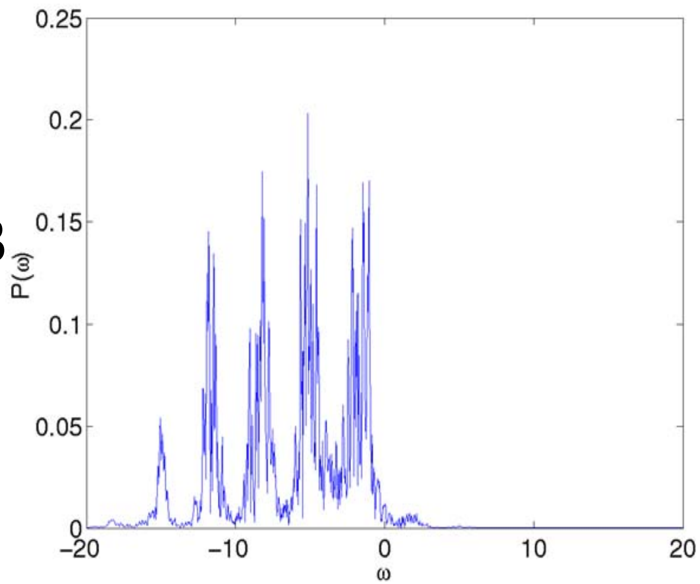
$\bar{\rho} = 0.1$



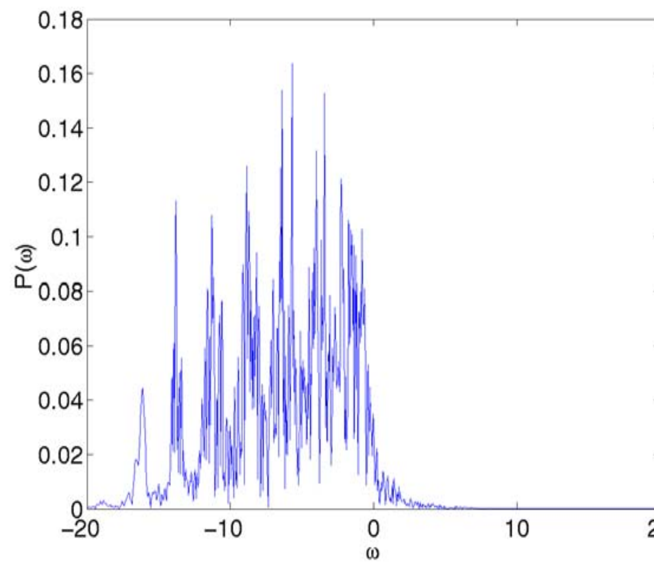
$\bar{\rho} = 0.2$



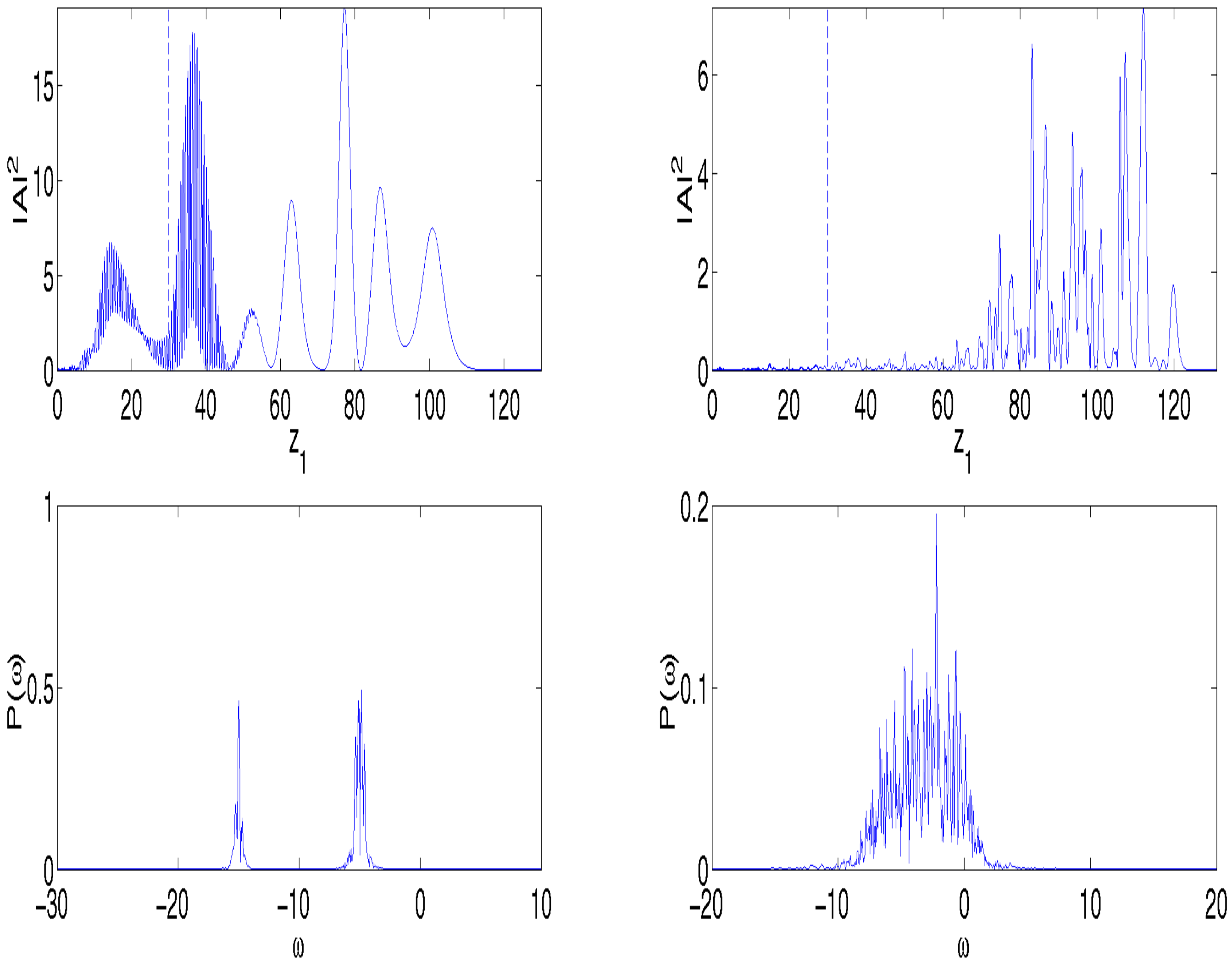
$\bar{\rho} = 0.3$



$\bar{\rho} = 0.4$

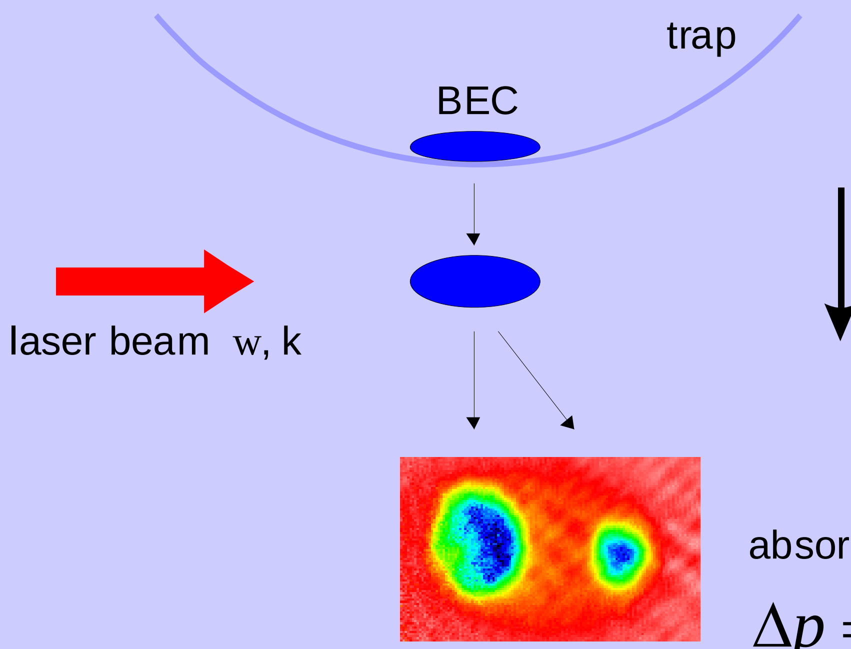


Quantum $\bar{\rho} = 0.1$ Classical $\bar{\rho} = 5$



Experimental Evidence of Quantum Dynamics – The LENS Experiment

- Production of an elongated ^{87}Rb BEC in a magnetic trap
- Laser pulse during first expansion of the condensate
- Absorption imaging of the momentum components of the cloud



absorption imaging

$$\Delta p = 2\hbar k$$

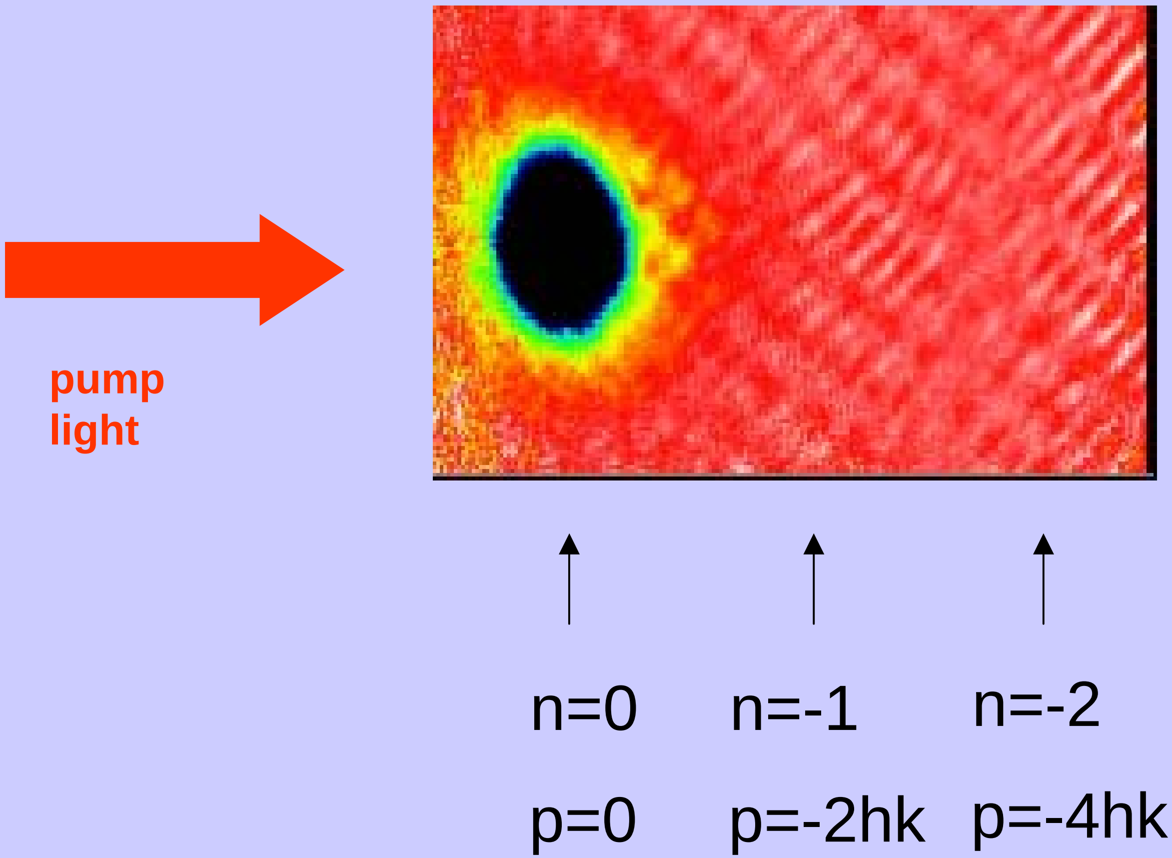
Experimental values:

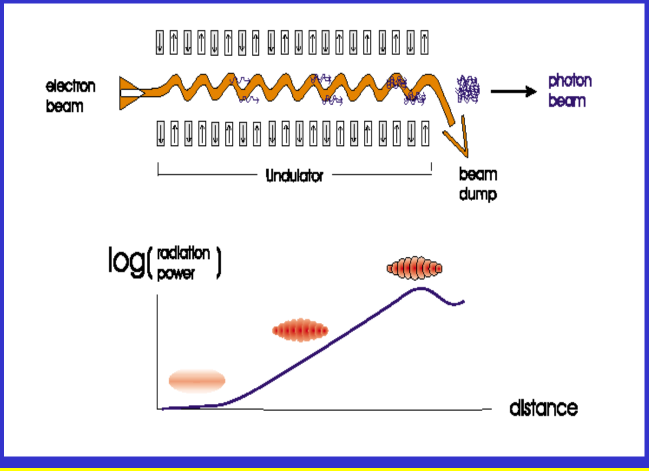
Δ = 13 GHz
w = 750 μm
P = 13 mW

R. B., F.S. Cataliotti, M.M. Cola, L. Fallani, C. Fort, N. Piovella, M. Inguscio J. Mod. Opt. **51**, 785 (2004) and Optics Comm. **233**, 155(2004) and Phys. Rev. A 71, 033612 (2005)

The experiment

Temporal evolution of the population in the first three atomic momentum states during the application of the light pulse.



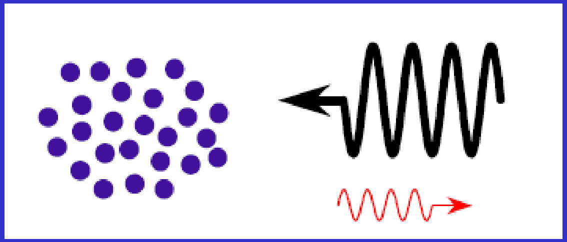


Svantaggi FEL SASE in regime classico (DESY, SLAC):

- richiede Linac ai GeV (Km) e ondulatori molto lunghi (100 m)
- Spettro della radiazione largo e caotico (spikes)
- Costo elevato (10^9 U$) e grandi dimensioni

Vantaggi FEL SASE in regime quantistico:

- quantum purification (spettro monocromatico)
- Possibilità di usare un ondulatore laser
- Costo ridotto (10^6 U$) e Apparato COMPATTO (m)



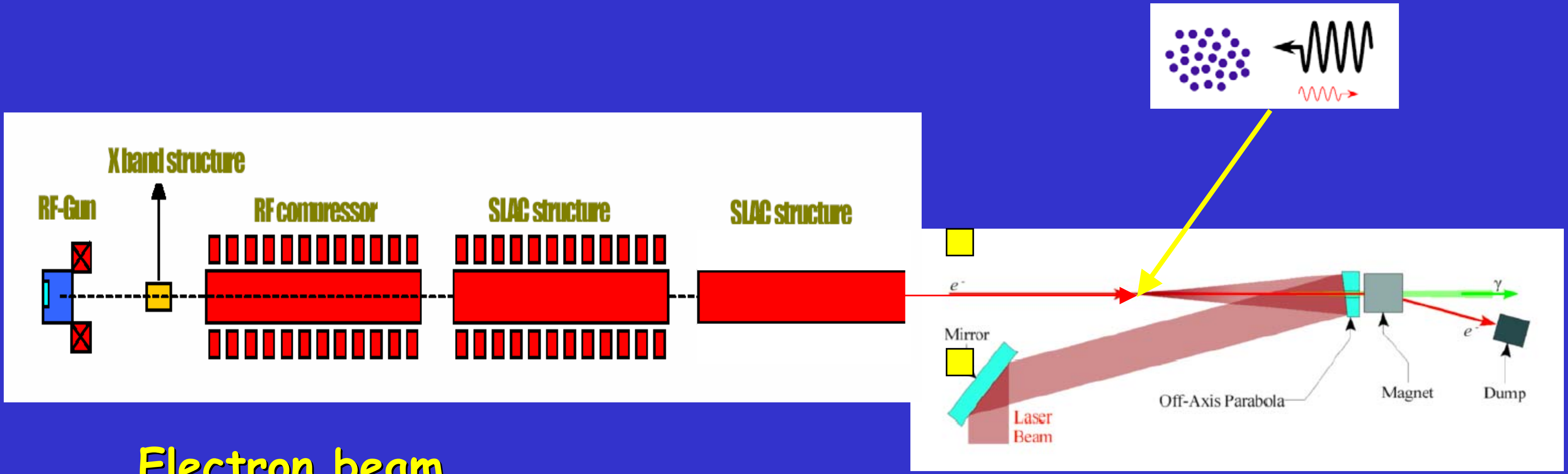
SPARC + FLAME ==> PLASMON X + QFEL

Ingredienti del Quantum FEL SASE:

- Fascio di elettroni 5-100 MeV, 100 A ,
  $\varepsilon_n < 2$ mm mrad
- Laser wiggler a 0.8 micron a 10-100 TW (Ti:Sa)

Entrambi sono in via di realizzazione nel progetto speciale SPARC/PLASMON_X

Preliminary parameters list for QFEL



Electron beam

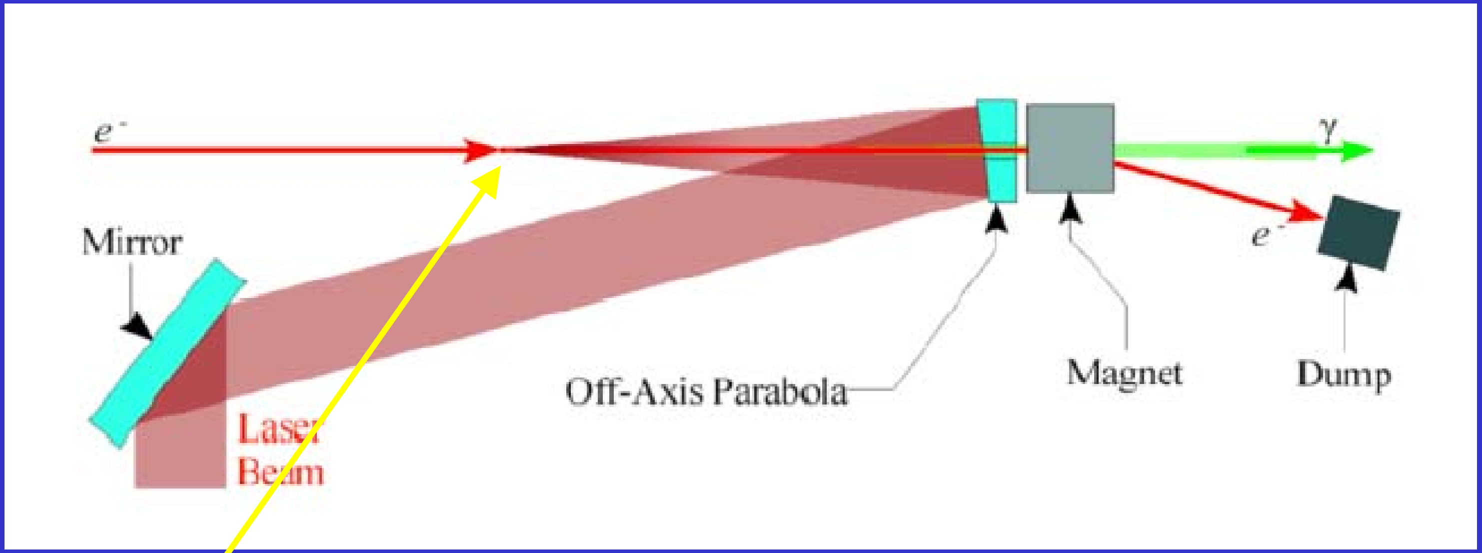
E [MeV]	20
I [A]	40
ε_n [μm]	1
$\delta\gamma/\gamma$ [%]	0.03
β^* [mm]	0.5-1

Laser beam

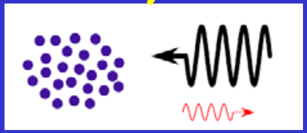
λ [μm]	0.8
P [TW]	1
E [J]	4
w_o [μm]	5-10
Z_r [μm]	80-300

QFEL beam

λ_r [A]	1.7
P_r [MW]	0.3



CDR PLASMONX



Caratteristiche della radiazione QFEL (stime preliminari):

- ~10^{10} fotoni a λ ~1 Å per qualche ps
- monocromaticità ($\Delta\lambda/\lambda<10^{-4}$)

QFEL

I primi studi preliminari sono basati su un
modello quantistico 1D

E' necessario estendere lo studio analitico/numerico del modello 1D a un modello 3D quantistico per dimostrare la fattibilità di un esperimento di Quantum SASE da eseguire ai LNF

Finanziamenti richiesti per MISSIONI e CALCOLO

COMPITI DEI DIVERSI GRUPPI PARTECIPANTI

1) **Sezione di Milano**: studio degli effetti di energy spread del fascio di elettroni sul guadagno FEL in regime quantistico. Estensione del modello quantistico unidimensionale a un modello tridimensionale che includa gli effetti di emittanza trasversa e longitudinale del fascio di elettroni e la variazione trasversa dell'ondulatore laser.

2) **Sezione di Frascati**: ottimizzazione della dinamica del fascio ad alta brillanza del fotoiniettore di SPARC per l'esperimento QFEL. Sviluppo di un codice 3D per la simulazione dell'interazionen FEL in regime quantistico.

3) **Sezione di Napoli**: studio della dinamica trasversale del fascio di elettroni in presenza del campo elettromagnetico totale nel FEL (campo dell'ondulatore + campo generato)tenendo conto di eventuali modulazioni dell'emittanza causate dal damping radiativo e dall'eccitazione quantistica nel framework del TWM.

SPARC

INFN Istituto Nazionale di Fisica Nucleare

ENEA

QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

ULTRAS INFM NATIONAL LABORATORY FOR ULTRAFAST AND ULTRAINTENSE OPTICAL SCIENCE

La Sapienza Università degli Studi di Roma

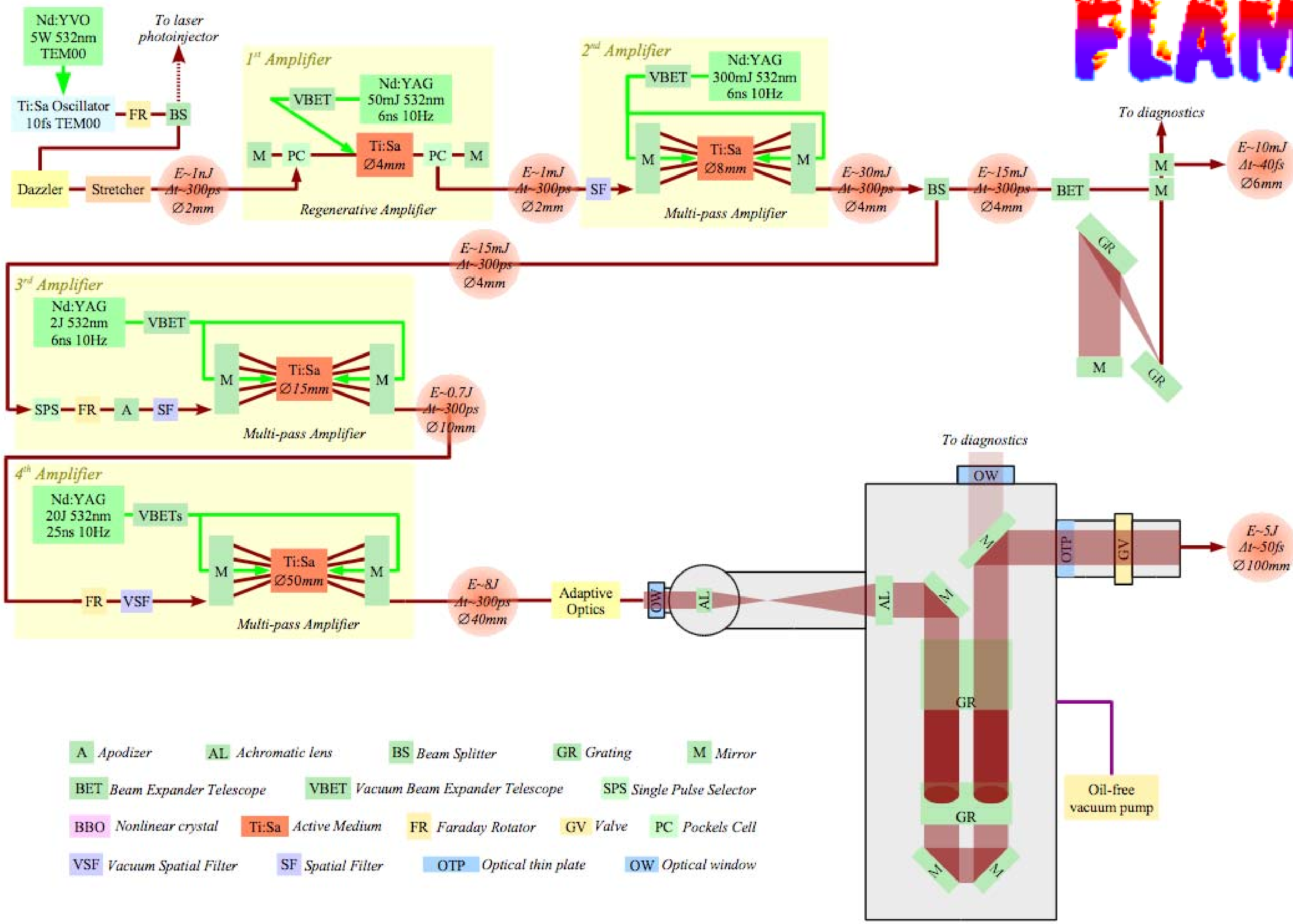
Stanford Linear Accelerator Center

UCLA

elettra Synchrotron Light Laboratory

The Frascati Laser for Acceleration and Multidisciplinary Experiments

FLAME



laser pulses: 50 fs, 800 nm >100 TW @10 Hz

Struttura	MI	ME	Cons	Inv.	Totale
LNF [1.6]	2	6	0.5	4	12.5
MI [3.0] (2.5)	7	14	1	0	22
NA [1.1]	3	2	1	0	6
Tot [5.7]	12	22	2.5	4	40.5


Thank you and see you in my office in Brazil

BRAFEL Possible Experimental Parameters

Beam parameters (Pedro email) from the gun

L_b = 30 μm (τ = 100 fs), I_p = 750 A, σ = 3 mm

ε_n = 4 mm mrad, γ ≈ 3 MeV, Δγ/γ ≈ 10^{-2}

FEL parameters

λ_r = 100 μm, λ_w = 3 cm, γ =17 (E= 8.5 MeV), a_w = 1, B_w = 0.3 T

ρ ≈ 5 10^{-2}, L_g = 7.5 cm, L_c = 200 μm (L_c >> L_b) Superradiance

$L_w \geq 10\, L_g$ = 75 cm

Note: $L_b < \lambda_r$: coherent spontaneous emission (CSE)

Alternative parameters: L_b = 300 μm (τ=1ps) , I_p = 75 A,

ρ ≈ 2.5 10^{-2}, L_g = 15 cm, L_c = 400 μm, (L_c >> L_b) Superradiance

BUT $L_b \gg \lambda_r$, NO CSE

$L_w \geq 10\, L_g$ = 1.5 m

Rayleigh range, Z_r = 1 m >> L_g, OK!

P_r ≈ 1.5 MW in 5 ps (to be checked numerically)

No similar source available at 100 μm.

Large harmonic bunching in a HGFEL

R. B, L. De Salvo, P. Pierini, NIM A293, 627 (1990)

$$\frac{d\theta_j}{d\bar{z}} = p_j$$

$$\frac{dp_j}{d\bar{z}} = -\sum_h F_h(\xi)(A_h e^{ih\theta_j} + c.c.)$$

$$\frac{dA_h}{d\bar{z}} = F_h(\xi)\left\langle e^{-ih\theta_j} \right\rangle$$

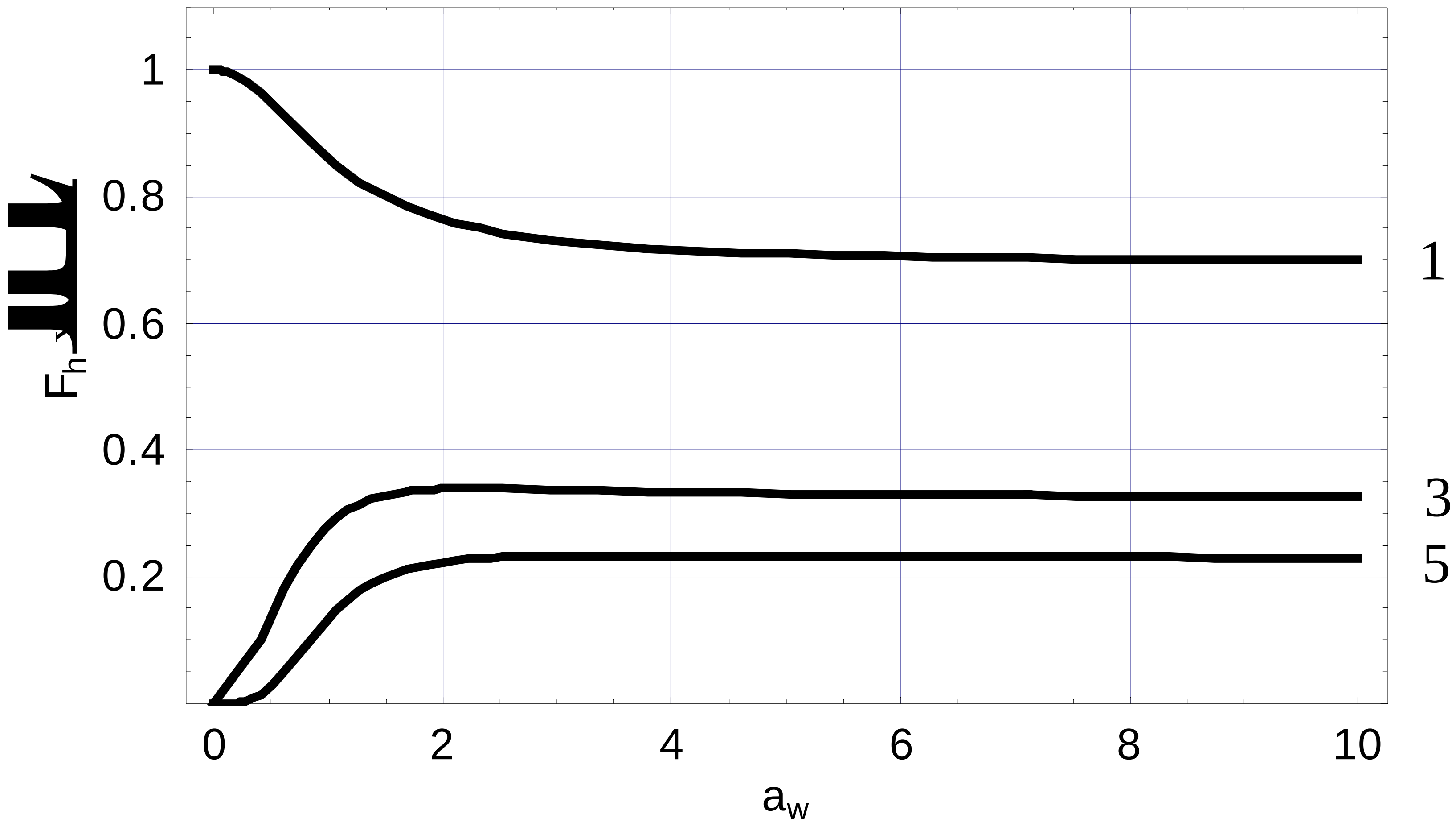
$$F_h(x) = \left(-1\right)^{(h-1)/2}\left[J_{(h-1)/2}(hx) - J_{(h+1)/2}(hx)\right]$$

$$\xi = a_w^2/(1+a_w^2)2$$

$$b_h \equiv \left\langle \exp(-ih\theta) \right\rangle$$

$$F_h(x) = (-1)^{(h-1)/2}\left[J_{(h-1)/2}(hx) - J_{(h+1)/2}(hx)\right]$$

$$\xi = a_w^2/(1+a_w^2)2$$



The driving mechanism for Large Harmonic Bunching

Linearizing

$$\frac{dA_1}{d\bar{z}} = F_1 b_1 \quad (1) \qquad \frac{d^3 b_1}{d\bar{z}^3} = iF_1^2 b_1 \quad (2) \qquad \frac{d^2 b_2}{d\bar{z}^2} = 2iF_1 A_1 b_1 \quad (3) \qquad \frac{dA_3}{d\bar{z}} = F_3 b_3 \quad (4)$$

$$\frac{d^3 b_3}{d\bar{z}^3} = 3iF_3^2 b_3 + 3iF_1 \frac{dA_1 b_2}{d\bar{z}} \qquad (5)$$

$$\downarrow$$

$$\frac{d^3 b_3}{d\bar{z}^3} = 3iF_3^2 b_3 + \frac{9}{2} F_1 A_0^3 e^{3\lambda_1 \bar{z}} \qquad (5)$$

$$\lambda_1 = \frac{\sqrt{3}}{2} F_1^{2/3}$$

$$seed\ A_1(0) \xrightarrow{(1)(2)} A_1, b_1 \propto e^{\lambda_1 \bar{z}}$$

$$A_1 \cdot b_1 \xrightarrow{(3)} b_2 \propto e^{2\lambda_1 \bar{z}} \qquad b_2 \cdot A_1 \xrightarrow{(5)(4)} b_3 \propto e^{3\lambda_1 \bar{z}}$$

In general (see later): $\left| b_n(\bar{z}) \right| \approx \left| b_1(\bar{z}) \right|^n \propto e^{n\lambda_1 \bar{z}}$ large gain

but: larger lethargy $\rightarrow$ noise amplification and energy spread: exponential gain?

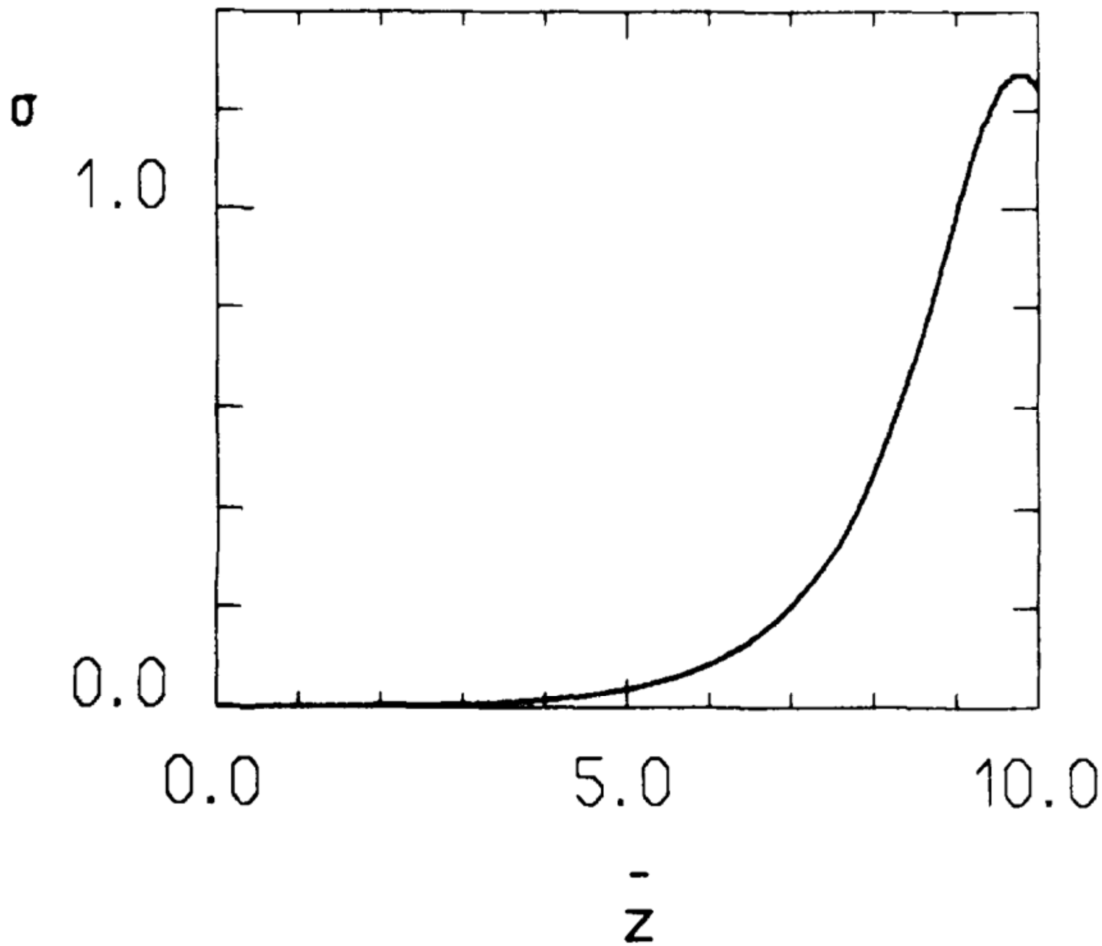
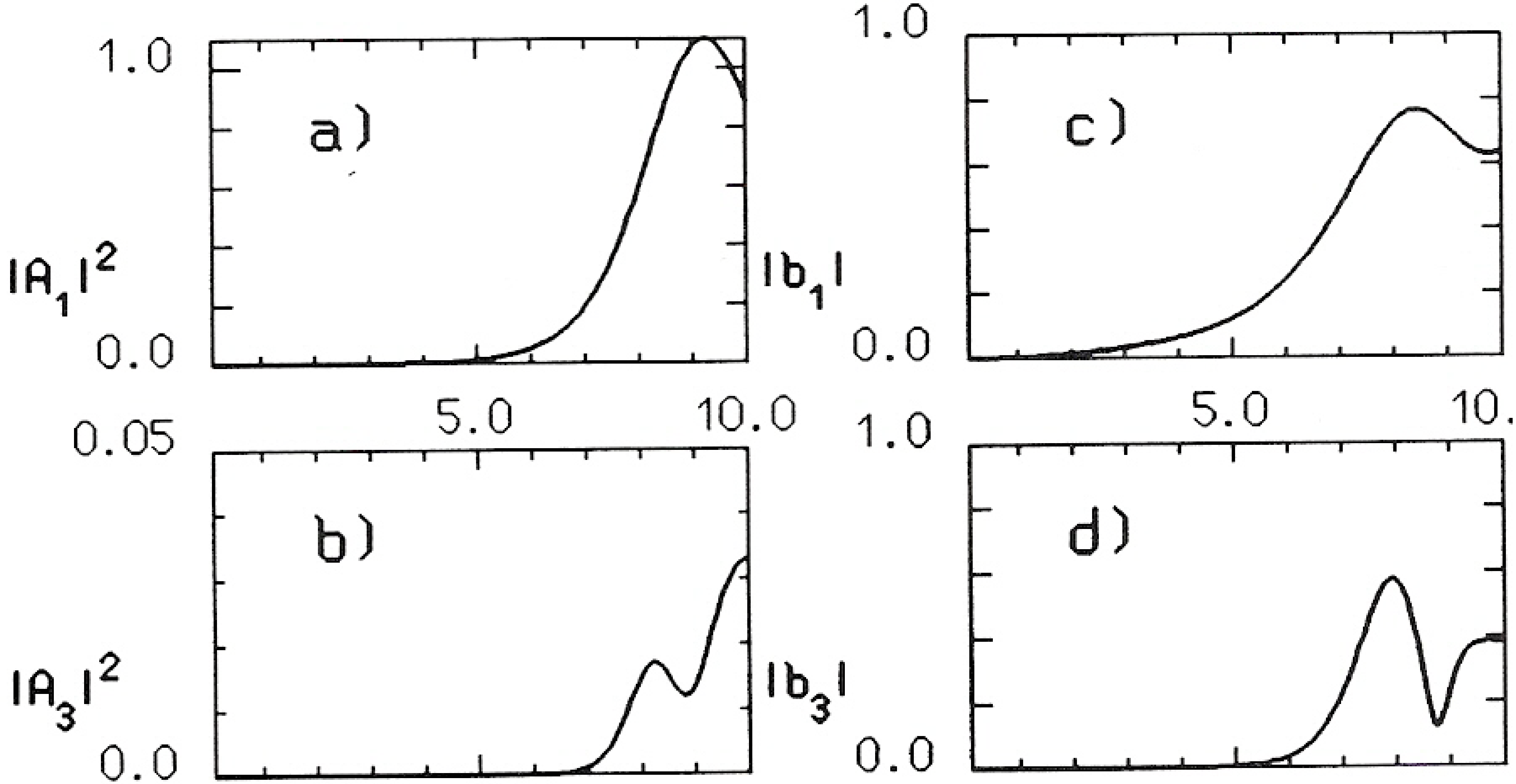
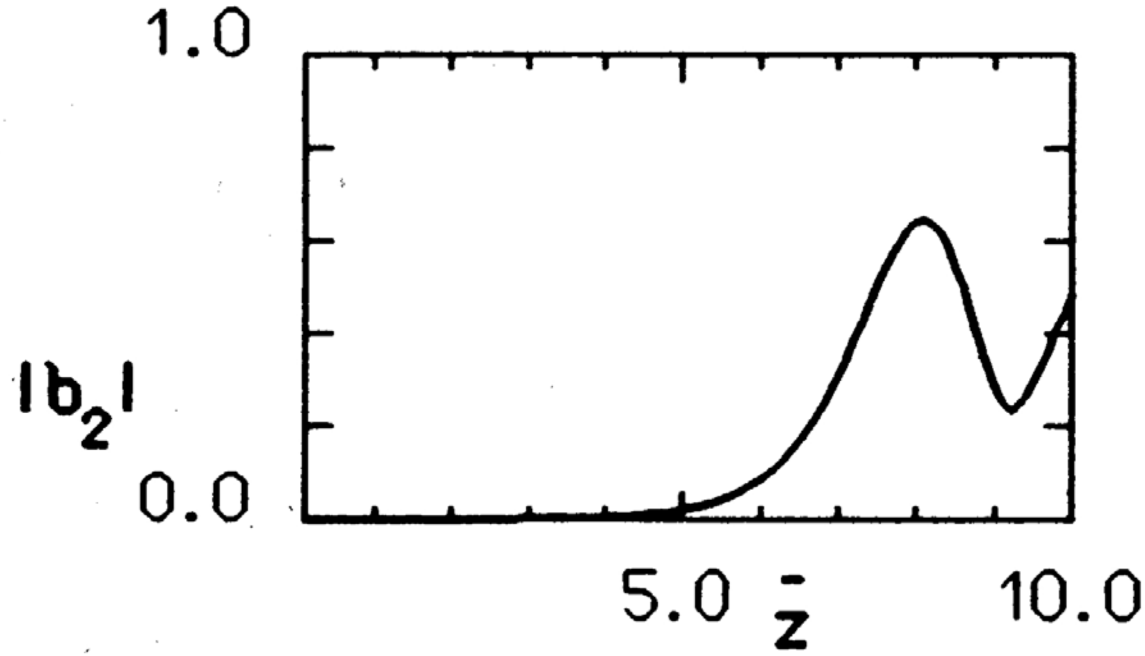
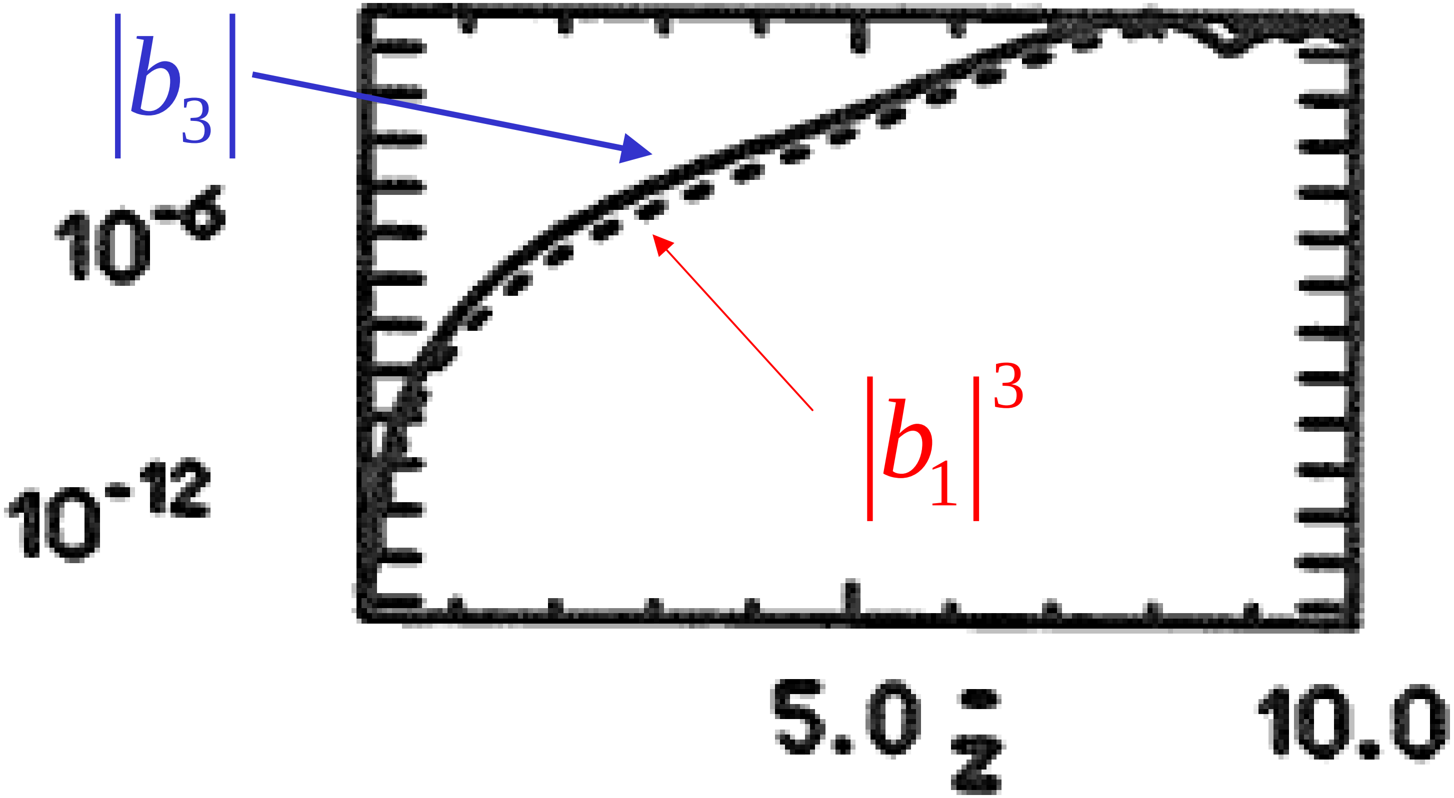


FIG. 4. FEL-induced energy spread, σ as a function of the dimensionless modulator length $\bar{z}$ for $A_0 = 10^{-3}$.



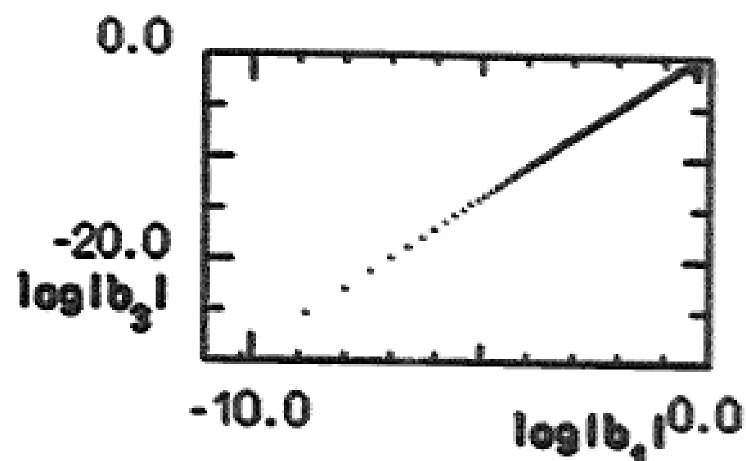
1.0
a)
$|A_1|^2$
0.0
$|b_1|$
1.0
c)
0.0
5.0
10.0
5.0
10.
0.05
b)
$|A_3|^2$
0.0
$|b_3|$
1.0
d)
0.0


1.0
|b2|
0.0
5.0
z
10.0



Proof of the linear driving mechanism deep into the linear regime!

Linear dependence, with a slope of 3!



The multiple wiggler scheme

R. B., L. De Salvo, P. Pierini, E. T. Scharlemann, NIM A 296, 787 (1990)

I. First wiggler: buncher seed at $\lambda_1 = \dfrac{\lambda_w (1 + a_w^2)}{2\gamma^2}$;

II. Second wiggler: $\lambda_n = \dfrac{\lambda_1}{n} = \dfrac{\lambda_w (1 + a_w^2)}{2\gamma^2 n}$;

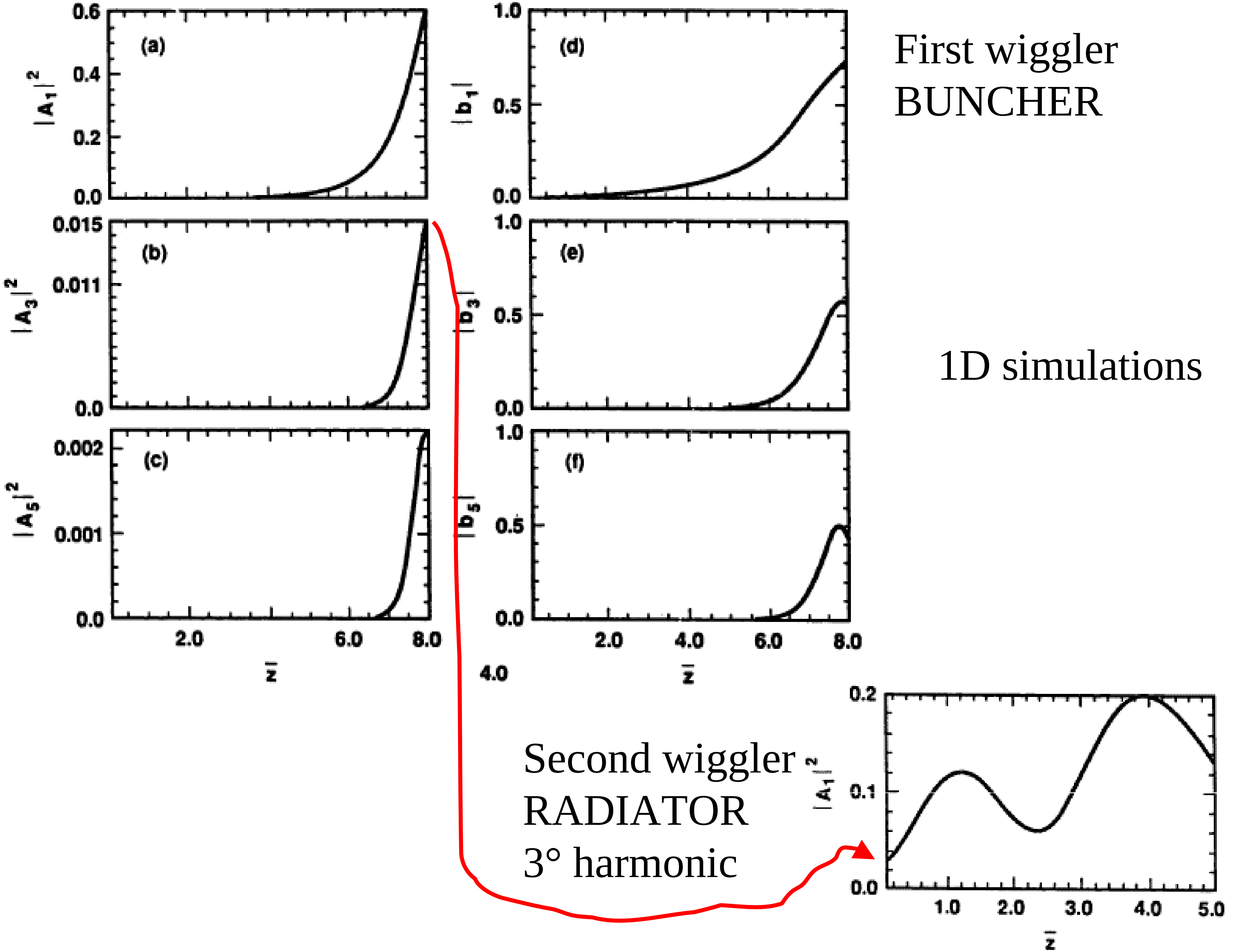
The bunching on the nth harmonic becomes the fundamental.

If $a^2_w >> 1$, $a_w^{II} = \dfrac{a_w}{\sqrt{n}}$

Superradiant emission in second wiggler from prebunched electrons

$I \propto z^2, N^2$ see 3D simulations

Exponential gain? Good luck. Apparently never observed.



First wiggler
BUNCHER
1D simulations
Second wiggler
RADIATOR
3° harmonic
(a)
(b)
(c)
(d)
(e)
(f)
$|A_1|^2$
$|A_3|^2$
$|A_5|^2$
$|b_1|$
$|b_3|$
$|b_5|$
$\bar{z}$

3D simulations (E.T.Scharlemann)

Table 1
Simulation parameters

Electron beam:	
Energy	300 MeV
Current	300 A
Normalized emittance (enclosing 90% of the current)	40π mm mrad
Energy spread (enclosing 90% of the current)	0.5%
Wiggler:	
Period	3 cm
Overall length	< 20 m
Signal:	
Fundamental	240 nm
Input	100 W, focused at wiggler entrance
Third harmonic	80 nm

240 nm FEL buncher

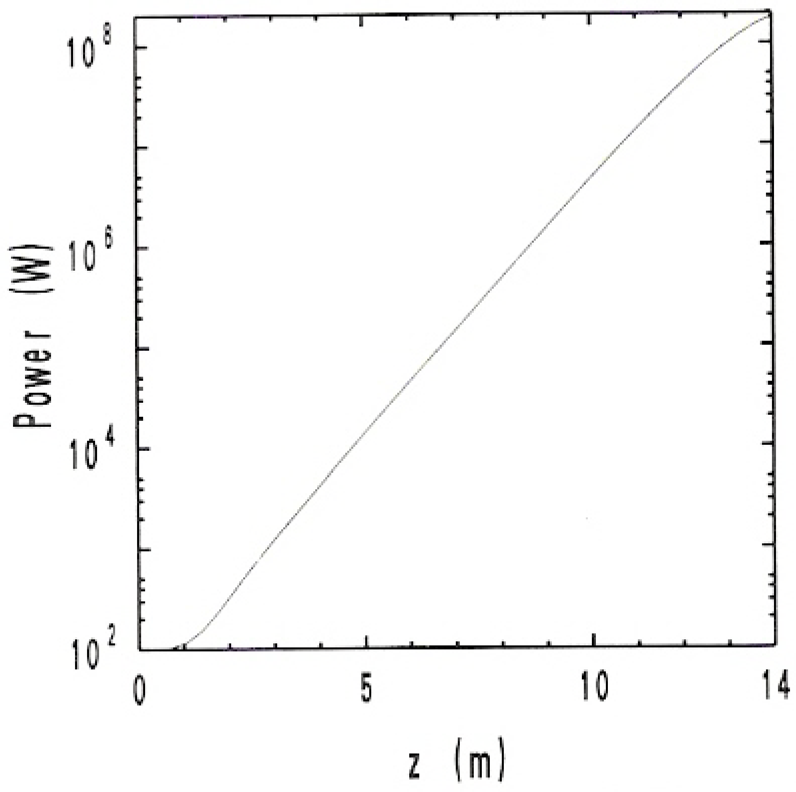
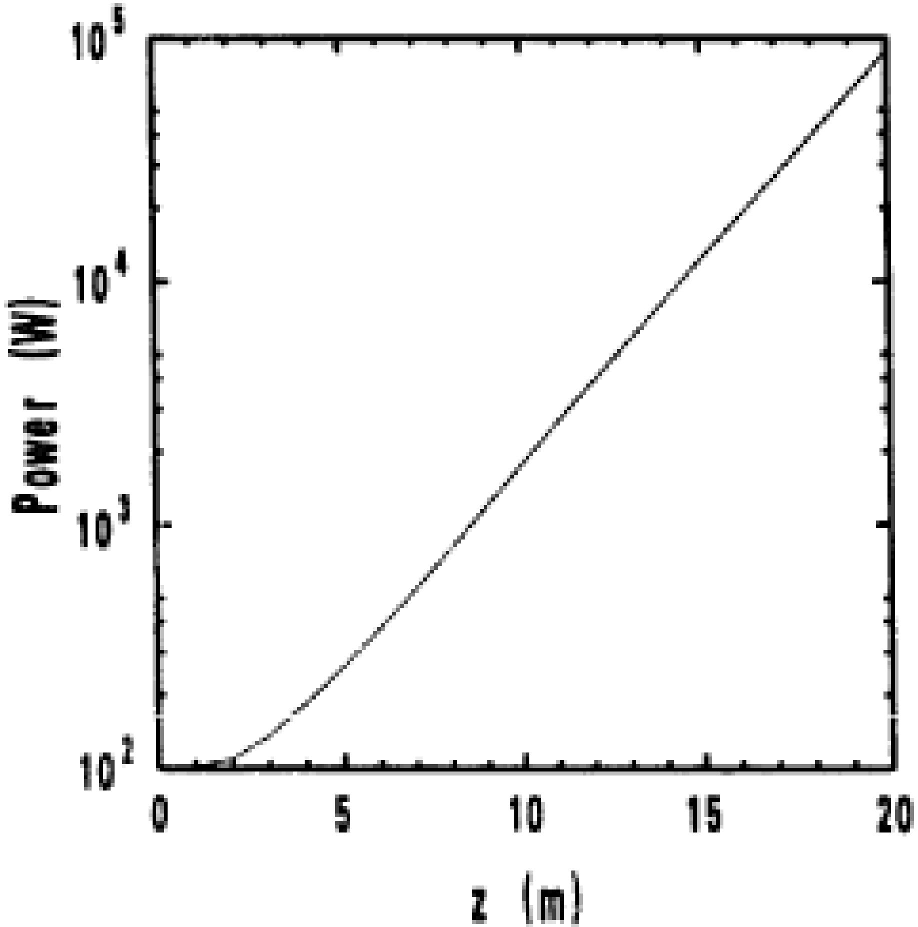


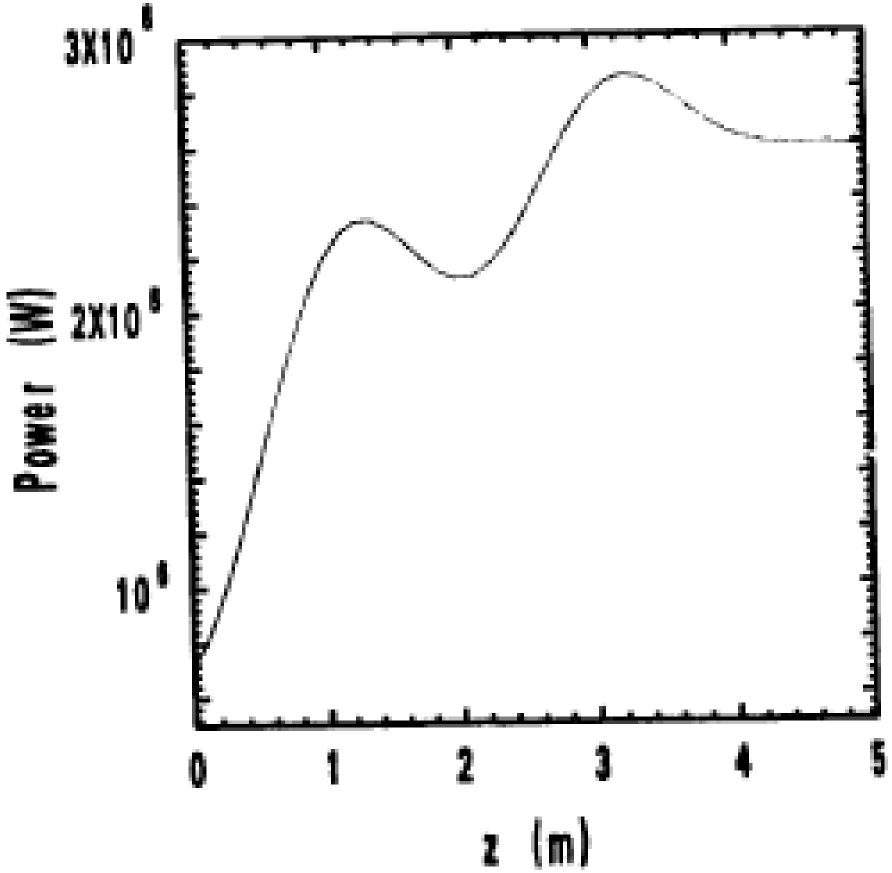
Fig. 4. Power at 240 nm vs z in a 14 m wiggler section resonant at 240 nm, starting from 100 W of 240 nm input power. An exponential gain of 5.2 dB/m is evident.

80 nm FEL



88 kW after 20 m

80 nm FEL RADIATOR (SRHG)



3 MW after 14+3.2 m

Instead of 88 kW in 20 m

Why?

Emittance limitation:

$$\varepsilon_n \leq \frac{\gamma \lambda_r}{4\pi}$$

OK at 240 nm!

NOT OK at 80 nm!

Emittance limitation relaxed in SRHG!

Good agreement with 1D

Exact Theory with Dispersive Section

R. Bonifacio, R. Corsini, P. Pierini, PRA 45, 4091 (1992)

$$\begin{cases} D \quad \frac{S}{4}(-B_0); \quad \frac{S}{2}(B_0); \quad \frac{S}{4}(-B_0) \qquad \text{total } S \\ D = \frac{1}{48}\rho k \left(\frac{eB_0}{mc\gamma}\right)^2 S^3 \end{cases}$$

Gallardo, Pellegrini, NIM A 296, 448 (1990)

Free space $D = \frac{L}{L_g(1+a_w^2)}$

Assuming gaussian momentum spread $\sigma = \frac{\Delta\gamma}{\rho\gamma}$

$$|b_n(\bar{z})| = \left|\left\langle e^{-in(\theta+Dp)}\right\rangle\right| = e^{-n^2D^2\sigma^2/2}\left|J_n\left(\frac{2}{3}nA_0\left(D^2+\sqrt{3}D+1\right)^{1/2} e^{\sqrt{3}\bar{z}/2}\right)\right|$$

i) $|J_n| < 1 \Rightarrow \sigma \le 1/nD$

ii) for $x < 1,\ D = 0,\ \ J_n(x) \propto x^n \Rightarrow |b_n| \propto |b_1|^n$

iii) D=0, J_n decreases very slowly with n $\Rightarrow$ large harmonic bunching

Dispersive section convenient only if $\sigma < 0.1$ ($\Delta\gamma/\gamma < 0.1\ \rho \approx 10^{-5}$)

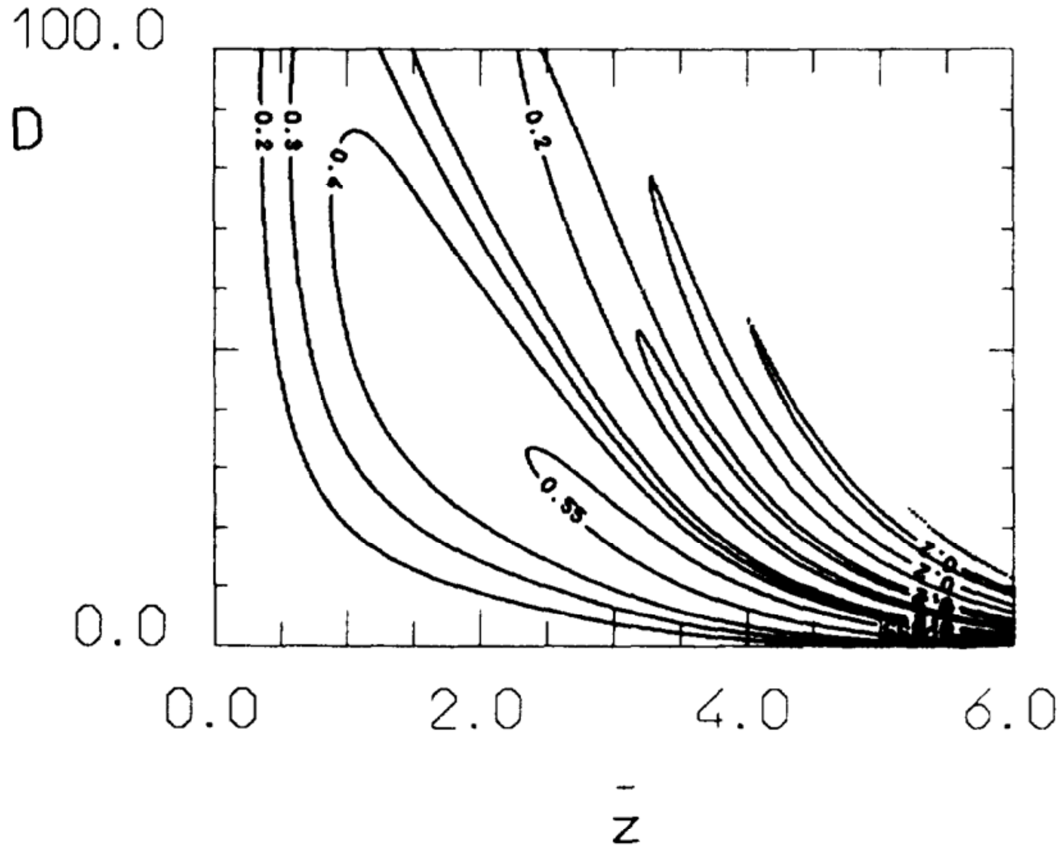


FIG. 1. Level curve of the bunching factor given by expression (28) as a function of the dimensionless length of the modulator $\bar{z}$ (horizontal axis) and the dimensionless dispersive section strength parameter D (vertical axis). The parameters used here are $\sigma = 0.01$ and $A_0 = 0.01$.

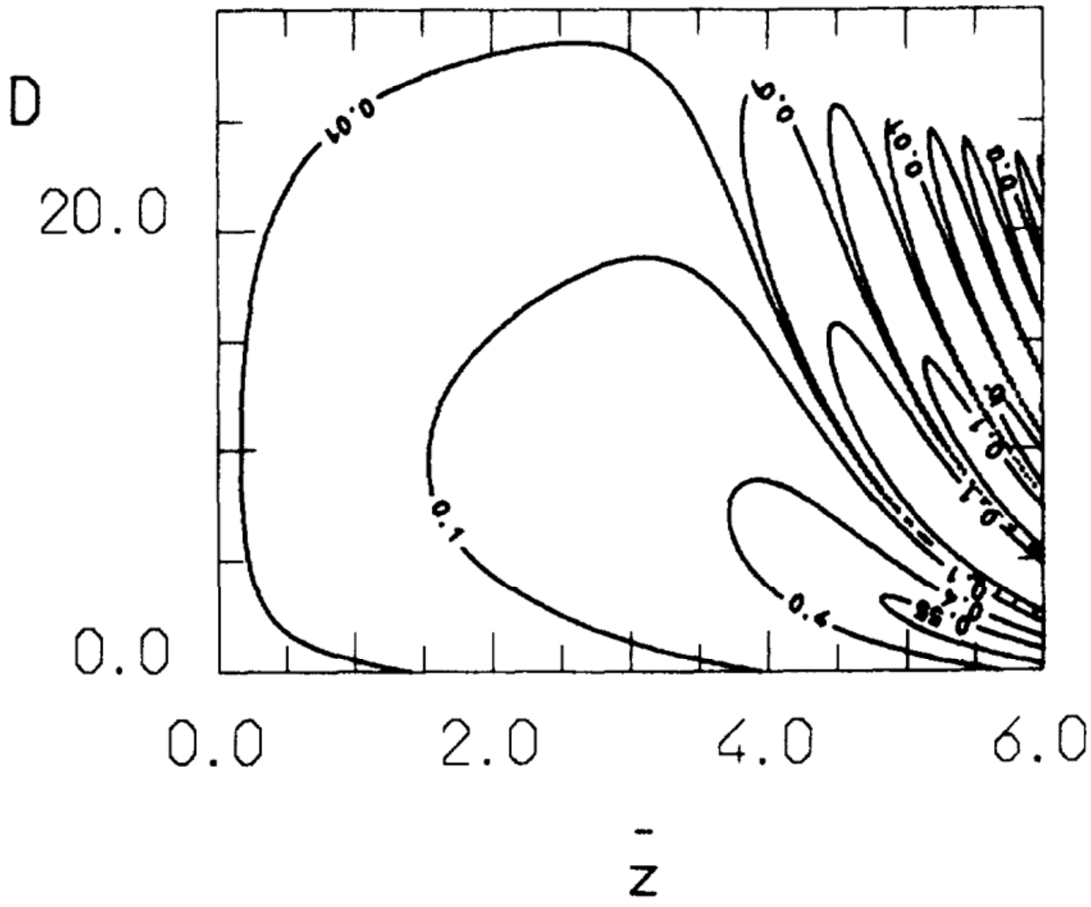


FIG. 2. As Fig. 1, for the parameters $\sigma = 0.1$ and $A_0 =$ 0.01.

Dispersive section ineffective unless $\sigma \leq 0.1$

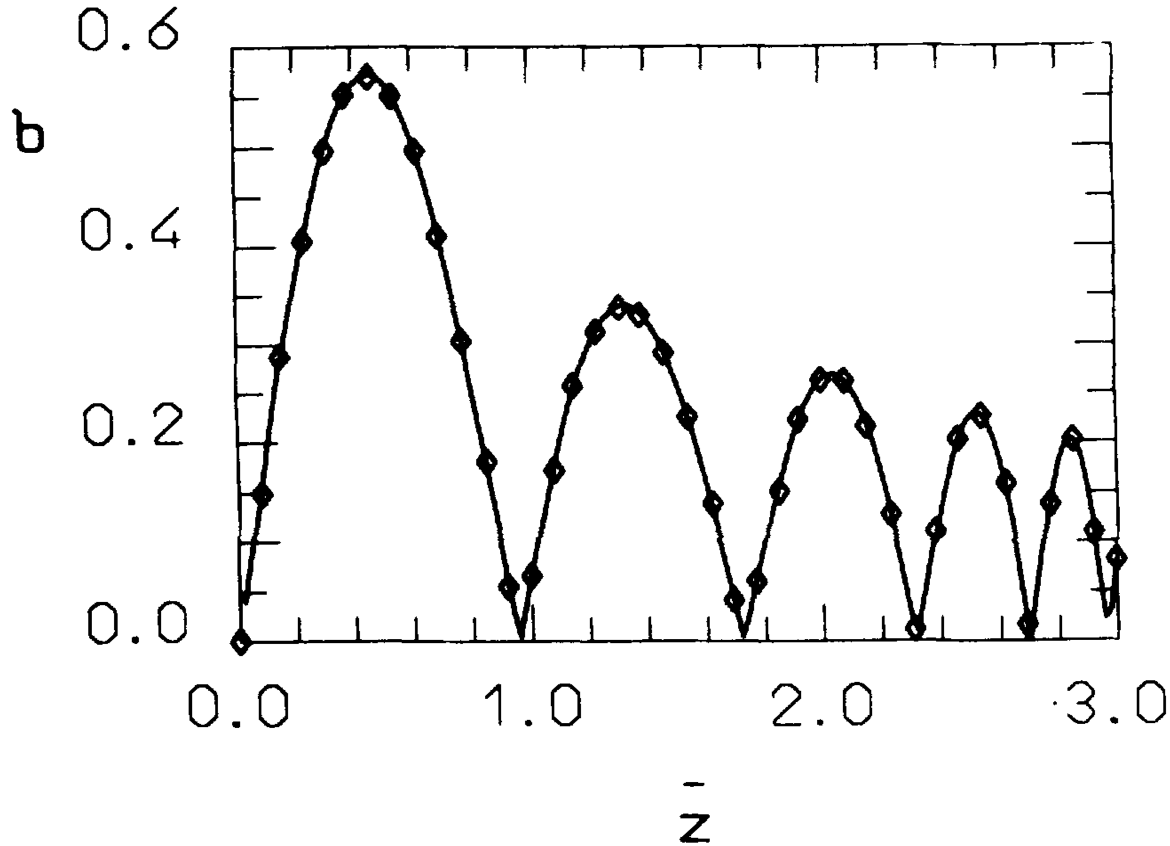


FIG. 3. Bunching at the end of the dispersive section as a function of the dimensionless modulator length. The solid line was produced integrating the system of electron-field equation of Ref. [3]; the symbols are evaluated from the expression (28). The parameters used in this simulation are $D = 2000$, $A_0 = 10^{-3}$, and $\sigma = 10^{-4}$.

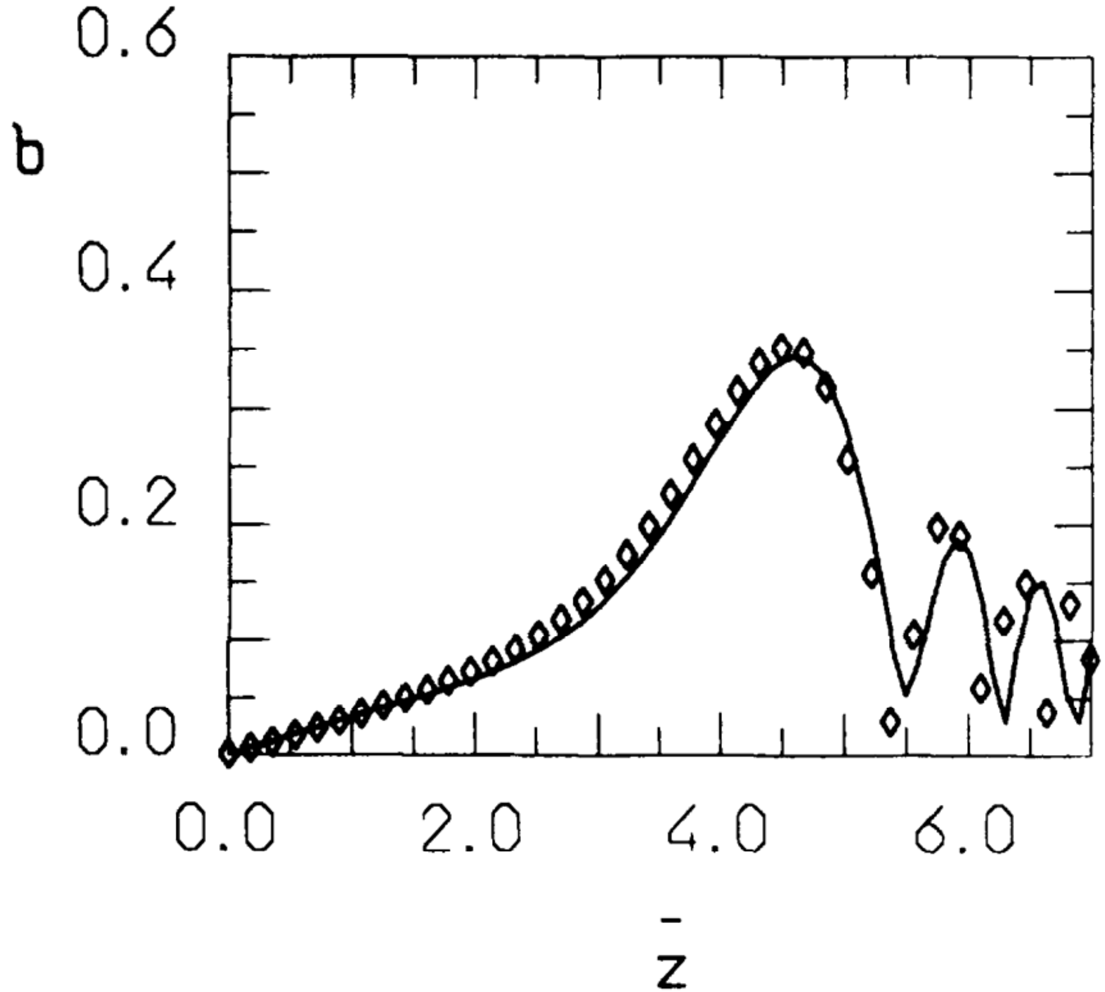


FIG. 5. As Fig. 3, but for the following set of parameters: $D = 5$, $A_0 = 10^{-2}$, and $\sigma = 0.2$.

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