Probing the Effects of Lorentz-Symmetry Violating Chern-Simons and Ricci-Cotton Terms in Higher Derivative Gravity

B. Pereira-Dias,* C. A. Hernaski,† and J. A. Helayël-Neto‡
Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud 150, Urca,
Rio de Janeiro, Brazil, CEP 22290-180

The combined effects of the Lorentz-symmetry violating Chern-Simons and Ricci-Cotton actions are investigated for the Einstein-Hilbert gravity in the second order formalism modified by higher derivative terms, and their consequences on the spectrum of excitations are analyzed. We follow the lines of previous works and build up an orthonormal basis of operators that splits the fundamental fields according to their individual degrees of freedom. With this new basis, the attainment of the propagators is remarkably simplified and the identification of the physical and unphysical modes gets a new insight. Our conclusion is that the only tachyon- and ghost-free model is the Einstein-Hilbert action added up by the Chern-Simons term with a time-like vector of the type $v^{\mu} = \left(\mu, \vec{0}\right)$. Spectral consistency imposes taht the Ricci-Cotton term must be switched off. We then infer that gravity with Lorentz-symmetry violation imposes a drastically different constraint on the background if compared to usual gauge theories whenever conditions for suppression of tachyons and ghosts are required.

PACS numbers: 04.50.Kd, 04.20.Cv

I. INTRODUCTION

In spite of the sacred role that Lorentz symmetry plays in fundamental Particle Physics, over the last two decades there has been a remarkable activity in considering models where this symmetry is violated. One reason for this excitement is that the main candidates for a consistent quantum theory of gravitation, such as loop quantum gravity [1, 2] and Horava-Lifshitz gravity [3], exhibits some phase where Lorentz symmetry is broken, whereas string theory may spontaneously break Lorentz invariance by the vacuum condensation of non-trivial Lorentz tensors [4]. Most interestingly, the relics of the Lorentz-symmetry violation (LV) could be detectable at low-energy experimental measurements, yielding physical constraints on these fundamental theories.

The rise of these quantum gravity theories outside the scope of the canonical quantum field theory approach grew mainly from the difficulty of obtaining, simultaneously, a renormalizable and unitary quantum field theory for gravity. We have learned that modifications of Einstein-Hilbert gravity by addition of R^2-type terms could, on the one hand, improve the ultraviolet divergences but, on the other hand, could jeopardize unitarity. With this in mind, we judge that a first and important test for the quantum consistency of any modified gravity theory is to require unitarity in the sense that its particle spectrum does not propagate tachyon and ghost modes.

The hallmark of most part of gravity theories with LV is the presence of a constant vector, v^{μ}, that spoils the isotropy of space-time. In [5], this vector is coupled to gravitation via a Chern-Simons-like term. A great motivation for considering the 4D C-S-like term comes from the striking effect of the Chern-Simons (C-S) term in 3-D that yields a consistent description of a massive graviton. In this paper we intend to verify whether this consistent mass generating mechanism survives in 4D. Furthermore, along with the Einstein-Hilbert and C-S term, we also consider the effects of (curvature)^2-terms and an extra LV-term, built up in analogy with the Ricci-Cotton (R-C) term in 3-D.

Invoking spatial isotropy, the authors of [5] restrict their discussion to $v^{\mu} = \left(\mu, \vec{0}\right)$. For the sake of generality, and motivated by recent works that point to a possible spatial anisotropy at cosmological scales [6, 7], we shall not restrict our discussion solely to particular choices of v^{μ}. In fact, it is known that the nature of the LV background vector may drastically change the spectrum of the model. For example, in the electromagnetic C-S LV model, it has been argued that a space-like Lorentz background vector of the type $v^{\mu} = (0, \vec{\mu})$ renders the theory free from ghosts and tachyons, whereas a time-like vector of the type $v^{\mu} = \left(\mu, \vec{0}\right)$ yields an inconsistent quantum theory [8, 9]. In

*Electronic address: bpdias@cbpf.br
†Electronic address: carlos@cbpf.br
‡Electronic address: helayel@cbpf.br

this paper, surprisingly, we conclude just the opposite for the gravity model extended by a C-S LV term: the only tachyon- and ghost-free model is the one with a time-like vector of the type $v^\mu = \left(\mu, \vec{0}\right)$.

In order to fulfill the task of analyzing the spectral properties of these modified gravities, we follow the lines of previous works [10–12] and build up an orthonormal basis of operators that splits the fundamental fields into their individual degrees of freedom. With this basis, the attainment of the propagator is feasible and it is possible to identify the excitation modes of the model. Also, we may suitably interpret them as unitary representation of the subgroup that survives from the Lorentz breakdown and, in this manner, as propagating particles. Furthermore, we obtain conditions on the parameters of the Lagrangian so that we may ensure propagation of non-tachyonic and non-ghost modes.

This paper is organized as follows: In Sec. II, we introduce our notations, conventions, and we start off with a general Lagrangian including the Einstein-Hilbert term, (curvature)^2-terms and the LV extensions given by the C-S and R-C terms. We pursue the attainment of the propagators in Sec. III. The particle spectrum of the model is obtained in Sec. IV, where we also discuss the tachyon- and ghost-free conditions. In Sec. V, we present our Concluding Remarks. The operator basis, suitable for the attainment of the propagators, is presented in the Appendix A. In the Appendix B, we ca an alternative method to obtain the propagator so that we can confirm the results found out in Sec. III.

II. HIGHER DERIVATIVE GRAVITY MODIFIED BY LV TERMS

Let us start off our analysis by considering the Einstein-Hilbert Lagrangian modified by higher derivative terms and C-S and R-C Lorentz-violating actions,

$$\mathcal{L} = \sqrt{-g}\left(\alpha R + \beta R_{ab}R^{ab} + \gamma R^2\right) + \mu \mathcal{L}_{CS} + \lambda \mathcal{L}_{RC}, \tag{1}$$

where α, β, γ, μ and λ are arbitrary parameters. The Chern-Simons term is given by

$$\mathcal{L}_{CS} = -\frac{1}{2}\epsilon^{abcd} v_a \Gamma^e_{cf}\left(\partial_b \Gamma^f_{ed} + \frac{2}{3}\Gamma^f_{bg}\Gamma^g_{de}\right), \tag{2}$$

whereas the Ricci-Cotton Lagrangian reads

$$\mathcal{L}_{RC} = \varepsilon^{abcd} v_d R_{ae} D_b R_c{}^e. \tag{3}$$

The background vector v^μ is assumed to be invariant under particle (or active) transformations (for further discussion see [13]). We shall use Latin letters $(a, b, c, \cdots)$ for space-time index and adopt the plus-minus convention for the Minkowski metric $\eta_{ab} = \mathrm{diag}\,(1, -1-1-1)$. Also, we shall follow the conventions: $R^a{}_{bcd} = \partial_c \Gamma^a_{bd} + \Gamma^a_{ce}\Gamma^e_{bd} - (c \leftrightarrow d)$, $\Gamma^a_{bc} = \frac{1}{2}g^{ad}\left(\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{bc}\right)$, $R_{bd} = R^a{}_{bad}$, $R = g^{bd}R_{bd}$. We stress that we are working with the second order formalism and torsion effects are not included in our study.

The R-C term is, in the linearized limit, a sort of a higher derivative of the C-S term, as it shall soon become evident. Some properties of the electromagnetic couterpart of this R-C term in 3D are discussed in [14], where it is argued that a higher derivative term no longer preserves the topological properties of the C-S term. It is not known to the authors of this paper an extensive study of the R-C term in 3D gravity. In principle, one could choose different background vectors for the C-S and R-C terms. We shall, however, adopt a minimalist point of view, where the LV arises from just a background vector, v_μ. A conclusive answer to this question can only be settled with a more fundamental approach where the mechanism for spontaneous symmetry breaking is clearly defined.

By means of the weak-field approximation, $g_{ab} = \eta_{ab} + h_{ab}$, we are able to write the quadratic Lagrangian, up to total derivatives, as

$$\begin{aligned}
\mathcal{L}_{(2)} &= \frac{\alpha}{2}\left(-\frac{1}{2}h^{ab}\Box h_{ab} + \frac{1}{2}h\Box h - h\partial_a\partial_b h^{ab} + h^{ab}\partial_a\partial_c h^c{}_b\right) \\
&+ \frac{\beta}{4}\left(h^{ab}\Box^2 h_{ab} + h\Box^2 h - 2h\Box\partial_a\partial_b h^{ab} - 2h^{ab}\Box\partial_a\partial_c h_b{}^c + 2h^{ab}\partial_a\partial_b\partial_c\partial_d h^{cd}\right) \\
&+ \gamma\left(h\Box^2 h - 2h\Box\partial_a\partial_b h^{ab} + h^{ab}\partial_a\partial_b\partial_c\partial_d h^{cd}\right) \\
&+ \frac{\mu}{4}\epsilon^{abcd} v_a h_d{}^e \partial_b\left(\partial_e\partial_f h^f{}_c - \Box h_{ec}\right) \\
&+ \frac{\lambda}{4}\epsilon^{abcd} v_a h_d{}^e \Box\partial_b\left(\partial_e\partial_f h^f{}_c - \Box h_{ec}\right).
\end{aligned} \tag{4}$$

For a non-trivial v^{μ}, the non-linearized model (1) is not invariant under active diffeomorphism transformations. However, the authors of [5] show that diffeomorphism symmetry is recovered dynamically by means of the equations of motion. This fact is reflected in a gauge symmetry of the action. The model is invariant under the field transformation:

$$h'_{ab} = h_{ab} + \partial_a \xi_b + \partial_b \xi_a. \tag{5}$$

Other fundamental symmetry lost is CPT-invariance due to the fact that we have a LV tensor (vector) with an odd number (namely one) of indices [15].

One readily realizes that, besides derivatives and η's, there is the emergence of the Levi-Civita tensor and the Lorentz-breaking background vector, which cannot be accommodated into the well-known Barnes-Rivers operators. An analogous situation occurs in parity-breaking theories in (1+2)-D, where there is the need of an extension of the spin-operators basis in order to handle the Levi-Civita tensor [10]. Such an extension is also necessary in the present problem, and this is the content of the Appendix A.

III. WRITING DOWN THE PROPAGATORS

A first step for the attainment of the propagator is to cast (up to total derivatives) the linearized Lagrangian (4) under the form

$$\mathcal{L}_{(2)} = \frac{1}{2} h^{ab} \mathcal{O}_{ab,cd} h^{cd}, \tag{6}$$

where the wave-operator, $\mathcal{O}_{ab,cd}$, in momentum space reads as below:

$$\mathcal{O}_{ab,cd} \;=\; \frac{1}{2} p^2 \left(\alpha + \beta p^2\right) P\left(2\right)_{ab,cd} + \frac{1}{4}\left(\mu p^2 + \lambda p^4\right) S_{ab,cd} + p^2 \left[\left(6\beta + 2\gamma\right) p^2 - \alpha\right] P_{11}\left(0\right)_{ab,cd}. \tag{7}$$

With the degree-of-freedom basis of operators discussed in the Appendix A, the wave operator can be expanded as

$$\mathcal{O}_{ab,cd} = \sum_{J,ij} a_{ij}\left(J\right) P_{ij}\left(J\right)_{ab,cd}. \tag{8}$$

Let us clarify the notation. The $a_{ij}\left(J\right)$ are the coefficients in the wave operator expansion. The diagonal operators, $P_{ii}\left(J\right)$, are projectors for each of the degrees of freedom of the spin (J)-sectors of the field h_{ab}, while the $P_{ij}(J)$, with $i \neq j$, are mappings between the projectors $P_{ii}\left(J\right)$ and $P_{jj}\left(J\right)$. The attribution of projectors and mapping operators comes from the relation that these operators satisfy:

$$\sum_{cd} P_{ij}\left(I\right)_{ab,cd} P_{kl}\left(J\right)^{cd}{}_{,ef} = \delta_{jk} \delta^{IJ} P_{il}\left(I\right)_{ab,ef}. \tag{9}$$

The completeness property of the basis is expressed by:

$$\sum_{i,J} P_{ii}\left(J\right)_{ab,cd} = \delta_{ab,cd}. \tag{10}$$

The coefficients $a_{ij}\left(J\right)$ can be arranged as matrices that represent the contribution to the particular spin (J). When these matrices are non-singular, the saturated propagator is given by:

$$\Pi = i \sum_{J,i,j} a^{-1}_{ij}\left(J\right) \mathcal{J}^{*ab} P_{ij}\left(J\right)_{ab,cd} \mathcal{J}^{cd}, \tag{11}$$

where $\mathcal{J}^{ab}$ are physical sources that couple to the propagator under consideration.

However, Lagrangian (1) in its linearized form is invariant under some local transformations of the fields (5). Gauge invariance renders the coefficient matrices degenerate. In Ref. [17], it is shown that the correct gauge invariant propagator is obtained by taking the inverse of any largest non-degenerate sub-matrix, which we shall denote by $A_{ij}\left(J\right)$, which is next saturated with sources.

For the model (1), the coefficients $a_{ij}(J)$ form the (2×2) spin-0, (3×3) spin-1 and (5×5) spin-2 matrices corresponding to the 10 degrees of freedom contained in the metric field:

$$a(0) = \begin{pmatrix} 2\left(3\beta+\gamma\right)p^4 - \alpha p^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{12}$$

$$a\left(2\right) \;=\; \frac{p^2}{2}\begin{pmatrix} \beta p^2+\alpha & 0 & 0 & -i\sqrt{p_*^2}\left(\mu+\lambda p^2\right) & 0 \\ 0 & \beta p^2+\alpha & -i\frac{\sqrt{p_*^2}}{2}\left(\mu+\lambda p^2\right) & 0 & 0 \\ 0 & i\frac{\sqrt{p_*^2}}{2}\left(\mu+\lambda p^2\right) & \beta p^2+\alpha & 0 & 0 \\ i\sqrt{p_*^2}\left(\mu+\lambda p^2\right) & 0 & 0 & \beta p^2+\alpha & 0 \\ 0 & 0 & 0 & 0 & \beta p^2+\alpha \end{pmatrix}. \tag{13}$$

The spin-0 matrix is degenerate, while the spin-1 matrix vanishes identically, manifesting the gauge symmetry of the model. For the spin-2, we have sort of a block-diagonal structure of the (1-4)-sector, (2-3)-sector and the 5-sector. This allows an inversion of each sector separately. Their inverses, needed for the attainment of the propagators, with the degeneracies duly extracted, are given by

$$A\left(0\right) = \frac{1}{p^2\left(2\left(3\beta+\gamma\right)p^2-\alpha\right)}, \tag{14}$$

$$a_{14}^{-1}\left(2\right) = \frac{2}{p^2\Delta_{14}}\begin{pmatrix} \beta p^2+\alpha & i\sqrt{p_*^2}\left(\mu+\lambda p^2\right) \\ -i\sqrt{p_*^2}\left(\mu+\lambda p^2\right) & \beta p^2+\alpha \end{pmatrix}, \tag{15}$$

$$a_{23}^{-1}\left(2\right) = \frac{2}{p^2\Delta_{23}}\begin{pmatrix} \beta p^2+\alpha & i\frac{\sqrt{p_*^2}}{2}\left(\mu+\lambda p^2\right) \\ -i\frac{\sqrt{p_*^2}}{2}\left(\mu+\lambda p^2\right) & \beta p^2+\alpha \end{pmatrix}, \tag{16}$$

$$a_5^{-1}\left(2\right) = \frac{2}{p^2\left(\alpha+\beta p^2\right)}, \tag{17}$$

where $\Delta_{14} = \left(\beta p^2+\alpha\right)^2 - p_*^2\left(\mu+\lambda p^2\right)^2$, $\Delta_{23} = \left(\beta p^2+\alpha\right)^2 - \frac{p_*^2}{4}\left(\mu+\lambda p^2\right)^2$. An alternative method to obtain the propagator is given in the Appendix B. This extra method is somewhat algebraically simpler, so that it may be used to check our calculations; but, unfortunately, it provides less physical insights on the properties of the propagating modes.

At a first glance, one observes that the spin-0 sector is unaffected by the LV term. An explanation for this fact comes from the observation that the LV operator, $S_{ab,cd}$, lives entirely in the spin-2 sector, as pointed out in the Appendix A.

However, the spin-2 sector is completely split for this LV theory. In relativistic field theory, the spin of the particles is defined in connection with the unitary representations of the little group defined by a representative momentum of the class which the particle belongs to. A background vector that breaks the isotropy of space-time may therefore modify the spin structure of the theory. This is seen explicitly in the spin-2 sector, where some degrees of freedom may propagate independently.

An analogous phenomenon occurs in (1+2)-D in gravity theories with parity-breaking terms. Since spin is represented by a pseudo-scalar operator in 3-D, there must be a doublet of spins with the same absolute value for the mass, $|m|$, so that an irreducible representation of the Lorentz group extended by time-inversion and parity transformations be set up. One the other hand, in a parity-breaking theory, such as gravity theories added to a Chern-Simons parity-breaking term, this doublet structure is lost and each spin component acquires a different mass, which we interpret as degrees of freedom propagating independently. For more details, see [12].

The close link between the four-dimensional LV theory and the three-dimensional parity breaking theory is established when one considers the LV vector with spatial components $v^\mu = (0,\vec{v})$. In this case, there is a symmetry breaking

$$SO\left(1,3\right) \to SO\left(1,2\right); \tag{18}$$

therefore, the massive particles should not be any longer defined by the embedding of $SU(2) \hookrightarrow SO(1,3)$, but rather by $U(1) \hookrightarrow SO(1,2)$. Furthermore, if the symmetry of the model considered is extended by parity conservation, the particles can be defined as doublets of spins $(s,-s)$, since, by parity operation, the spin s is mapped onto $-s$. This is the reason why the 5-dimensional matrix (13) is split into the direct sum of the $U(1)$ fundamental representations and their complex conjugate (or equivalently by $SO(2)$ representations). In this case, the new emerging degrees of freedom can be enumerated by the usual $U(1)$ multiplication rule coming from group theory:

$$(1 \oplus -1 \oplus 0) \otimes (1 \oplus -1 \oplus 0) = (3 \times \underline{0} \oplus 2 \times \underline{1} \oplus 2 \times -\underline{1} \oplus \underline{2} \oplus -\underline{2}) \,. \tag{19}$$

The three blocks appearing in the spin-2 sector may be viewed as coming from a sort of dimensional reduction. If it is assigned the 0-helicity for the τ-operator, the spin-2 operator decomposition (A16a)-(A16e) can be compared with the three-dimensional operators defined in [10]. For example: the $P_{11}(2)$ and $P_{44}(2)$ are clearly $P(2^{++})$ and $P(2^{--})$ of the paper [10], respectively. In this case, the spin-2 is broken to a "three-dimensional" spin-2, which corresponds to the (1-4)-sector ($P(2) = P_{11}(2) + P_{44}(2)$); spin-1, which corresponds to the (2-3)-sector ($P(1) = P_{22}(2) + P_{33}(2)$) and spin-0, which corresponds to the 5-sector ($P(0) = P_{55}(2)$).

IV. ANALYSIS OF THE SPECTRAL CONSISTENCY

In this Section, we analyze the spectral consistency of the model. As a result of our study, we shall be able to impose conditions on the parameters of Lagrangian (1) in order to inhibit the propagation of unphysical modes, that is, ghosts and tachyons.

In a quantum field theory with relativistic invariance, it is known that the condition for absence of tachyon is $m^2 \geq 0$, where $p^2 = m^2$ appears as a pole of a given propagator. Also, the statement for absence of ghosts reads

$$\Im \mathrm{Res}(\Pi|_{p^2=m^2}) > 0. \tag{20}$$

In the projection operators formalism, we can take advantage from the general decomposition of the spin projection operator,

$$P_{ij}(J) = (-1)^P \, \psi^{(i)} \psi^{(j)}, \tag{21}$$

where P is the parity related to the spin operator, to rewrite the propagator (11) as

$$\Pi \;=\; i\,(-1)^P \sum_{J,i,j,m^2} J_i^* A_{ij}\left(J, m^2\right) J_j \left(p^2 - m^2\right)^{-1}, \tag{22}$$

where $J_j = \psi^{(j)}_{cd} J^{cd}$ and $A_{ij}\left(J, m^2\right)$ are the inverse sub-matrices with the pole extracted. Therefore, the positiveness condition (20), for arbitrary sources, is ensured by the positiveness of the eigenvalues of the $A_{ij}\left(J, m^2\right)$ matrix.

Nevertheless, it can be shown that these matrices, for massive poles, have only one non-vanishing eigenvalue at the pole, which is equal to the trace of $A(J, m^2)$. Therefore, the condition for absence of ghosts for each spin is reduced to:

$$(-1)^P \mathrm{tr} A(J, m^2)|_{p^2=m^2} > 0. \tag{23}$$

On the other hand, in a LV theory, these conditions must be carefully reassessed, since, for example, the dispersion relation is modified and can be more general than simply $p^2 = m^2$. In this case, it is interesting to characterize tachyon and ghost excitations even if Lorentz symmetry is no longer present. Due to subtleties that appear for different choices of the background vector, we postpone the discussion of the characterization of tachyon and ghost excitation modes in each considered case.

The spin-0 sector is unaffected by the LV terms. So, the usual constraints remain in order. The spin-0 massive pole keeps unchanged the expression $m^2 = \frac{\alpha}{6\beta + 2\gamma}$, and the ghost- and tachyon-free conditions are given by

$$\text{Spin-}\mathbf{0}: \quad \alpha > 0; \;\; 3\beta + \gamma > 0. \tag{24}$$

Interestingly, the 5-sector of the spin-2 shows up the massive graviton yielded by the $R_{ab}R^{ab}$ term. However, the massive pole, $m^2 = -\frac{\alpha}{\beta}$, results in contradictory conditions with positiveness of the Newton's constant, if one requires ghost and tachyon absence:

$$\text{Sector-5}: \quad \alpha < 0; \;\; \beta > 0. \tag{25}$$

Being inconsistent with unitarity requirements, we shall henceforth assume that $\beta = 0$, that is, the absence of the Ricci squared term. Before analyzing the general tachyon- and ghost-free conditions for the general case of the theory with the R-C and the C-S term, we think it is instructive to study the simplest case, where the Ricci-Cotton term is switched off ($\lambda = 0$). In this situation, the matrix corresponding to the 1-4-sector is cast as

$$a_{14}^{-1}(2) = \frac{2}{\mu^2 p^2 \left(\frac{\alpha^2}{\mu^2} - p_*^2\right)} \begin{pmatrix} \alpha & i\mu\sqrt{p_*^2} \\ -i\mu\sqrt{p_*^2} & \alpha \end{pmatrix}. \tag{26}$$

The particle content of the model is set up by the poles of the propagator. For the 1-4-sector of the spin-2 matrix, the poles are given by the roots of

$$\frac{\alpha^2}{\mu^2} - \left((v\cdot p)^2 - v^2 p^2\right) = 0. \tag{27}$$

So, the solution to this equation for the energies of the modes reads:

$$p_0 = \frac{|\vec{p}|}{|\vec{v}|}\left[v_0\cos\theta \pm \sqrt{\frac{\alpha^2}{\mu^2\vec{p}^2} - v^2\sin^2\theta}\right], \tag{28}$$

where θ is the angle between $\vec{v}$ and $\vec{p}$. One should notice that it is necessary that $|\vec{v}| \neq 0$ for the presence of massive poles. At this stage, it is interesting to open up the discussion of the interpretation of the modes with negative energy, so as to ensure that no tachyons are present.

In field theories with Lorentz invariance, the dispersion relations read like $p_0 = \pm E(\vec{p})$, with $E(\vec{p}) = \sqrt{\vec{p}^2 + m^2}$. The negative energy solutions are naturally incorporated into the quantum description with the definition of the causal Feynman propagator that takes into account the contribution of a negative energy mode as being the annihilation of a positive energy mode. Also, the CPT-invariance of the model suggests the interpretation of a negative energy mode propagating to the past as being a positive energy mode propagating to the future.

However, at the present case, CPT-invariance is lost and the dispersion relations can be in general of the form $p_0 = E_\pm$ with $|E_+| \neq |E_-|$ (e.g. (28)). These different masses for the particle and antiparticle can be problematic to the locality of the quantum theory, as discussed in [19].

By analysing (28), we realize that, in the case $v_0 \neq 0$, there are different masses for the particle and anti-particle and it happens that the positivity cannot be ensured to arbitrary directions of propagation, by virtue of the explicit dependence on $\cos\theta$. We therefore conclude that it is incompatible to assume $v_0 \neq 0$ and $|\vec{v}| \neq 0$ simultaneously, if we wish to ensure a consistent quantum theory.

In the particular case $v_0 = 0$, the poles are given by:

$$p_0 = \pm\frac{1}{|\vec{v}|}\sqrt{\frac{\alpha^2}{\mu^2} + \vec{v}^2\vec{p}^2\sin^2\theta}. \tag{29}$$

In this case, the dispersion relation is like $p_0 = \pm E$ and, potentially, the interpretation of the negative energy modes could be consistently associated to the anti-particles. In fact, in spite of the lack of CPT-invariance of the model, as discussed in Sec. II, in the case of a background vector with spatial components, the breaking of Lorentz symmetry leaves a residual $SO(1,2)$ symmetry. This can also be seen by the strict relation between this massive spin-2 mode and the massive graviton that appears in the topological massive gravity (TMG) in 3D. This forces an interpretation of the negative energy modes, that are consistently discussed in the three-dimensional TMG. We know that, in the four-dimensional model, a $C\tilde{P}T$ symmetry remains, where C and T are the usual charge conjugation and time reversal operation, whereas $\tilde{P}$ is a discrete improper Lorentz transformation that reverses only one spatial component in the orthogonal plane defined by the LV vector, $\vec{v}$. The $\tilde{P}$ is the usual parity operation in $(1+2)$-D. We therefore conclude that the negative energy mode could be consistently interpreted as a positive energy mode with opposite spin polarization. The fact that this excitation is non-tachyonic is a result that is in agreement with previous discussion in the literature [20].

To conclude the spectral analysis, one must ensure the positivity of the residues at the poles of the propagators. First of all, one should observe that, off-shell, the residue matrix $a_{14}^{-1}(2)$ (eq. (26)) has two distinct eigenvalues, namely $C\left(\alpha + \mu\sqrt{p_*^2}\right)$ and $C\left(\alpha - \mu\sqrt{p_*^2}\right)$, with $C = 2\left[\mu^2 p^2\left(\frac{\alpha^2}{\mu^2} - p_*^2\right)\right]^{-1}$. On-shell, that is, at $p_*^2 = \frac{\alpha^2}{\mu^2}$, only one of these two eigenvalues survives depending on the sign of $\frac{\alpha}{\mu}$. Physically, this corresponds to the fact that just one spin polarization propagates: $+2$ for $\frac{\alpha}{\mu} > 0$, or -2 for $\frac{\alpha}{\mu} < 0$. This is of great resemblance with topologically massive gravity in 3D, where there is a propagation of single massive mode of helicity ± 2, depending on the sign of the Chern-Simons term [21].

As previously discussed, the condition for the positivity of the eigenvalues at the pole is simplified to the positivity of the trace (23). In this case,

$$\text{Res } \text{tr} a_{14}^{-1}(2)\big|_{p_0^2=E^2} = -\frac{2\alpha}{\mu^2 \vec{v}^2 p^2}, \tag{30}$$

where, at the pole,

$$p^2 = \frac{\alpha^2}{\mu^2 \vec{v}^2} - \vec{p}^2 \cos^2\theta. \tag{31}$$

For low momenta, $\vec{p}^2 < \frac{\alpha^2}{\mu^2 \vec{v}^2}$, the condition for the propagation of a non-ghost mode, expressed in eq. (30), dictates:

$$\alpha < 0. \tag{32}$$

One could wonder if the model would be consistent with $\alpha > 0$ for specific choices of $\vec{p}^2 \cos^2\theta$. We understand that an explicit dependence of the consistency of the model on the direction of propagation is an odd situation and so we discard this possibility. In fact, for the case where the dynamics is restricted to the plane orthogonal to $\vec{v}$, $\cos\theta$ vanishes identically and p^2 is positive definite. Again, the requirement of a negative Newton's constant is in agreement with previous consideration of the relation of this model with TMG.

In the more general case, with the Ricci-Cotton term present ($\lambda \neq 0$), the massive poles of the propagator of the spin (1-4)-sector are given by the roots of

$$\Delta_{14} \;=\; \alpha^2 - p_*^2 \left(\mu + \lambda p^2\right)^2. \tag{33}$$

The condition for absence of ghost reads

$$\text{Res } \left(\text{tr} a_{14}^{-1}(2)\right)\big|_{p_0^2=E^2} = \frac{2\alpha\left(p_0^2 - E^2\right)}{p^2 \Delta_{14}} > 0. \tag{34}$$

However, condition (34) is doomed to propagate ghost modes. The reason is simple: the denominator (33) brings three massive poles, $\Delta_{14} = C\left(p_0^2 - E_1^2\right)\left(p_0^2 - E_2^2\right)\left(p_0^2 - E_3^2\right)$, with $E_1^2 < E_2^2 < E_3^2$ or two massive poles (depending on the choice of v^μ), $\Delta_{14} = C'\left(p_0^2 - E_1^2\right)\left(p_0^2 - E_2^2\right)$, with $E_1^2 < E_2^2$. In either cases, if tr $a_{14}^{-1}(2)\big|_{p_0^2=E_1^2}$ is positive (negative), then tr $a_{14}^{-1}(2)\big|_{p_0^2=E_2^2}$ is negative (positive). So, the positivity of the residue in all poles cannot be ensured simultaneously. Such a phenomenon is ubiquitous in theories with higher derivatives, when more than one massive pole in the same spin sector usually brings ghosts. In view of this problem, we are bound to take $\lambda = 0$, eliminating then the R-C term.

The spectral consistency analysis for the (2-3)-sector follows the same reasoning of the (1-4)-sector, and the main results remains unchanged. This finishes the discussion of quantum consistency for the massive poles. Let us tackle now the spectrum consistency analysis for the massless pole.

A. The Massless Graviton Pole

The massless poles must be handled with extra care. At a first sight, one sees that the basis of operators is ill-defined for light-like momenta. However, one can show that the physical sources satisfy constraints ($p_a \mathcal{J}^{ab} = 0$) due to gauge symmetries of the model. These constraints make the saturated propagator a well-defined expression even for light-like momenta. In fact, the surviving structures when the projectors are saturated by the sources at light-like momenta ($p^2 = 0$) are given by:

$$P_{11}\left(0\right)_{ab,cd} = \frac{1}{2}\eta_{ac}\eta_{cd} + \text{t.d.n.c.res.}, \tag{35a}$$

$$P_{11}\left(2\right)_{ab,cd} = \frac{1}{2}\left(\rho_{ac}\sigma_{bd} + \rho_{ad}\sigma_{bc} + \sigma_{ac}\rho_{bd} + \sigma_{ad}\rho_{bc}\right) + \text{t.d.n.c.res.}, \tag{35b}$$

$$P_{22}\left(2\right)_{ab,cd} = \text{t.d.n.c.res.}, \tag{35c}$$

$$P_{33}\left(2\right)_{ab,cd} = \text{t.d.n.c.res.}, \tag{35d}$$

$$P_{44}\left(2\right)_{ab,cd} = \frac{1}{2}\left(\rho_{ab}\rho_{cd} + \sigma_{ab}\sigma_{cd}\right) - \frac{1}{2}\left(\rho_{ab}\sigma_{cd} + \sigma_{ab}\rho_{cd}\right) + \text{t.d.n.c.res.}, \tag{35e}$$

$$P_{55}\left(2\right)_{ab,cd} = \frac{1}{2}\eta_{ac}\eta_{cd} + \text{t.d.n.c.res.}, \tag{35f}$$

where t.d.n.c.res. is an acronym for "terms that do not contribute to the residue". From the expression of τ_{ab} in terms of v_a and p_a (equation (A5d)), one can show that $\tau_{ab}\mathcal{J}^{bc} = 0$. This is why $P_{22}\left(2\right)$ and $P_{33}\left(2\right)$ do not contribute to the residues, and only the ρ's σ's and η's appear in the final expression.

Another remark that should be made is that, for non-vanishing mapping operators, the projection contribution can be diagonalized $\left(\tilde{P}_{ij} = A_{ik}P_{kl}A^T_{lj}\right)$ in such a way that the rotated projectors contribute with the corresponding eigenvalue. In this way, the propagators can be written as

$$\Im\text{Res}(\Pi|_{p^2=0}) = \tilde{\mathcal{J}}^{*ab}\left[\frac{2}{\alpha - \mu\left|v\cdot p\right|}\tilde{P}_{11}\left(2\right)_{ab,cd} + \frac{2}{\alpha + \mu\left|v\cdot p\right|}\tilde{P}_{44}\left(2\right)_{ab,cd}\right]\tilde{\mathcal{J}}^{cd}. \tag{36}$$

It is interesting to notice that the contribution of the mapping operators is responsible for breaking up the two degrees of freedom of the usual graviton. In the case where the LV parameter, μ, vanishes, we recover the well-known graviton propagator in 4D.

The origin of the splitting in the dynamics of these independent degrees of freedom of the massless particle is analogous to the phenomenon that occurs for massive particles in 3D, when there is a parity-breaking term. There is indeed a great resemblance between massive particles in 3D and massless particles in 4D, since they essentially share the same representation structure of the Poincaré group. As in 3D, parity-breaking terms yield different masses for particles that would propagate as doublet of spins, as discussed in [12]. This same role of the parity-breaking shows up in the massless graviton propagator (36), where the helicities modes are no more related by CPT transformations.

To analyze unitarity, one should recall that the sources are arbitrary and $\tilde{P}_{11}\left(2\right)$ and $\tilde{P}_{44}\left(2\right)$ are independent operators. In order to ensure the positivity of the expression (36), we must impose that the eigenvalues are positive-definite, independently. This implies that

$$\left|\mu\left(v\cdot p\right)\right| < \alpha. \tag{37}$$

Condition (37) means, in particular, that the unitarity constraint of our LV theory is in accordance with the usual requirement of the positiveness of the Newton's constant in 4D. However, such a condition is conflicting with the requirement of (32). This is not a surprise for us in the light of the close link between parity-breaking theories in 3D and LV theories in 4D with a breaking vector with spatial components. In fact, it is known that Chern-Simons massive gravities are consistent solely if $\alpha < 0$. In 3D, the massless graviton does not propagate and there is no conflict with respect to unitarity. In a Chern-Simons LV gravity in 4D, we conclude, therefore, that the propagation of the massless graviton and the massive modes coming from the violation of the Lorentz symmetry are not compatible if one is enforced to respect the requirement of propagating only non-ghost modes.

V. CONCLUDING REMARKS

We have considered a general gravity Lagrangian with higher derivatives and CPT/Lorentz-violating Chern-Simons and Ricci-Cotton term in the second-order formalism. It was our interest to investigate the implication of the CPT/Lorentz-symmetry violating terms in the unitary properties of the model and, also, determine the particle spectrum of the theory.

With this aim, we have developed a basis of degree-of-freedom operators. The proposed basis turns out to be algebraically convenient and still exhibits a clear physical interpretation. The principle of defining degree-of-freedom operators renders a great flexibility to identify the physical content modes excitation. This generality is specially interesting if the background vector is not fixed a-priori, since, in this case, there is no definition, in advance, for the particles. In the case where Lorentz symmetry remains unbroken, these degree-of-freedom operators will always rearrange to the usual spin projectors, as expected. However, if there is a breaking of the Lorentz symmetry by a background vector with only spatial components, $SO(1,2)$ symmetry is residual. In this case, the degrees-of-freedom operators rearrange in such a way that the planar-like excitations can be readily identified. Or, loosely speaking, this represents a dynamical generation of the spin-particle by using the degrees of freedom of the fields.

Extensions to include more general background tensors, as discussed in the Standard Model Extension (SME), considered by Kostelecky and collaborators [15], could be addressed too. In the case of two linear independent background vectors, there will be a Lorentz symmetry breaking, $SO(1,3) \to U(1)$, and we expect that the excitations shall not be arranged in planar-like modes. This situation may be relevant whenever LV is triggered by a spontaneous supersymmetry breaking mechanism [22, 23], since another background vector may appears by means of a background fermion condensation. Furthermore, we suppose that the degree-of-freedom operators could be defined and the other Lorentz breaking vector should be split into three orthogonal contributions: p_μ, $e_{3\mu}$ and $e_{2\mu}$, according to the Gram-Schmidt decomposition.

The breaking of the CPT-symmetry threatens the interpretation of the negative energy modes that are common in quantum field theory. This is the matter of a detailed discussion. We conclude that a background vector, with $v_0 \neq 0$ and $\vec{v} \neq 0$, brings up an inconsistent quantum model, since the propagating modes does not have a positive-definite energy for arbitrary directions of propagation. The situation where $v_0 = 0$ and $\vec{v} \neq 0$, revealed non-tachyonic excitations after a revision of the role of the discrete symmetries. In spite of this known fact, we showed, however, that these modes are negative-normed states (ghosts) and with mass comparable to the Planck mass. These fact warns for possible inconsistencies in the quantum version of the model. Nevertheless, in the search for small deviations in low-energy processes, where the problematic mode should not be excited, one may obtain consistent results if the effective model is viewed as a relic of a more fundamental (and consistent) theory. The last case, where $v_0 \neq 0$ and $\vec{v} = 0$, which was the subject of an investigation in [5], revealed, in the limiting case of Einstein-Hilbert Chern-Simons gravity, no other propagating modes besides the massless graviton. It is in agreement with the results by Jackiw-Pi that the two polarizations of the graviton propagates independently. The general behavior of (curvature)^2-terms is not altered by the LV Chern-Simons term, in the sense that R^2 propagates a massive spin-0 and $(R_{ab})^2$ a massive spin-2 ghost. The higher derivative Ricci-Cotton term brings unavoidable ghosts modes.

Another interesting pursuit is to tackle an analogous problem, but in the first order formalism, where the vielbein and the spin connection are understood to be independent fields. In such a situation, we expect that the difficulty in the calculations shall be greatly increased, since in the analogous problem with dynamical torsion there is a plethora of propagating modes, 2^+, 2^-, 1^+, 1^-, 0^+, 0^- [11, 18] in which, with LV, the spin modes would be split into spin polarization modes. In spite of this difficulty, this problem would be rather interesting, since, in this formalism, there is the possibility of considering, besides the usual Chern-Simons term in the first order formalism $\epsilon^{abcd} v_d (R_{abef} \omega_c{}^{ef} + \frac{2}{3} \omega_{af}{}^{g} \omega_{bg}{}^{e} \omega_{ce}{}^{f})$, other combinations that have not been considered previously: $\epsilon^{abcd} v_a T_{bc}{}^{e} R_{de}$, $\epsilon^{abcd} v_a R T_{bcd}$ and $\epsilon^{abcd} v_a T_{be}{}^{e} R_{cd}$. Also, torsion effects could reveal interesting consequences in this scenario.

Acknowledgments

The authors are very grateful to Professors S. A. Dias and H. Belich for the supporting discussions. Thanks are also due to Professor J. A. Accioly for the encouragement in pursuing a deeper investigation of our work. The authors B. P. D. and C. A. H. express their gratitude to CNPq-Brazil and FAPERJ-Rio de Janeiro for the Graduate fellowships. J. A. H.-N. acknowledges CNPq for the invaluable financial support.

Appendix A: Degree-of-Freedom Projection Operators

In order to analyze the particle spectrum of the model and the nature of its associated particles, the attainment of the propagator becomes a primary goal. With the propagator at hand, one reads off the masses of the particles and may verify the positiveness of the residues of the propagators at the correspond poles, so as to determine conditions for the absence of ghosts. A suitable method to obtain the propagator and identify the particle content of the models is the one based on the spin projector operators [11, 16–18]. In four dimensions, in the second order formalism, one has the Barnes-Rivers operators that form a complete orthonormal basis of operators for models with Lorentz and

CPT invariances:

$$P\left(2\right)_{ab,cd} = \frac{1}{2}\left(\theta_{ac}\theta_{bd}+\theta_{ad}\theta_{bc}\right)-\frac{1}{3}\theta_{ab}\theta_{cd}; \tag{A1a}$$

$$P\left(1\right)_{ab,cd} = \frac{1}{2}\left(\theta_{ac}\omega_{bd}+\theta_{ad}\omega_{bc}+\theta_{bc}\omega_{ad}+\theta_{bd}\omega_{ac}\right); \tag{A1b}$$

$$P_{11}\left(0\right)_{ab,cd} = \frac{1}{3}\theta_{ab}\theta_{cd}; \quad P_{12}\left(0\right)_{ab,cd}=\frac{1}{\sqrt{3}}\theta_{ab}\omega_{cd}; \tag{A1c}$$

$$P_{21}\left(0\right)_{ab,cd} = \frac{1}{\sqrt{3}}\omega_{ab}\theta_{cd}; \quad P_{22}\left(0\right)_{ab,cd}=\omega_{ab}\omega_{cd}. \tag{A1d}$$

However, when one adds terms that spoil these symmetries, one must enlarge the basis in order to accommodate the new operators that appear in the linearized Lagrangian. Since we wish to keep the background vector arbitrary, we define a complete basis of operators that splits the fundamental fields according to their individual degrees of freedom. This is more convenient because different choices for the background vector may lead to different particle species, once the symmetry group that survives from the breaking of Lorentz symmetry depends on the explicit form of the background vector.

As a starting point to set up a suitable basis of degree-of-freedom operators, we can decompose the Barnes-Rivers operators in order to accommodate LV operators, such as

$$S_{ab,cd}=\frac{1}{2}\left(\theta_{ac}S_{bd}+\theta_{ad}S_{bc}+\theta_{bc}S_{ad}+\theta_{bd}S_{ac}\right), \tag{A2}$$

where, $S_{ab}=\epsilon_{abcd}v^c\partial^d$, that comes from the C-S and R-C LV terms (in the linearized version of the theory).

A first hint to solve this task is to notice that the $S_{ab,cd}$-operator "lives" entirely in the spin-2 sector. This is seen from the fact that $S_{ab,cd}$ is annihilated by the spin-1 and spin-0 operators, but is remains unchanged by the spin-2 projection operator, as shown in the following relations:

$$P\left(2\right)_{ab,cd}S^{cd}{}_{,ef} = S_{ab,ef} \tag{A3a}$$

$$P\left(1\right)_{ab,cd}S^{cd}{}_{,ef} = 0 \tag{A3b}$$

$$P_{ij}\left(0\right)_{ab,cd}S^{cd}{}_{,ef} = 0, \quad i,j=1,2. \tag{A3c}$$

With these remarks, we shall pursue the task of the attainment of the basis of degree-of-freedom operators for gravity models, but first it is necessary to classify the building blocks.

1. Building Blocks

In a context of LV, there is the need of redefining the building blocks of the projections operators. This can be motivated by the decomposition of the Lorentz-breaking vector, v^a, into a term proportional to the momentum and an orthogonal component,

$$v^a=\frac{(v\cdot p)}{p^2}p^a+\sqrt{\frac{p_*^2}{p^2}}e_3^a, \tag{A4}$$

where p^a is the relativistic four-momentum assumed to be time-like and $p_*^2=(v\cdot p)^2-v^2p^2$. Nevertheless, the whole discussion can take place in the case of a light-like momentum, necessary for the analysis of the massless poles (Sec. IV A).

The building blocks for the operators can be built up from p_a and other three orthonormal space-like vectors. Without loss of generality, we may choose one of them as $e_3^a=\sqrt{\frac{p^2}{p_*^2}}\left[v^a-\frac{(v\cdot p)}{p^2}p^a\right]$ and another two vectors, e_1^a and e_2^a, orthogonal to each other and orthogonal to p^a and e_3^a. We assume that $p_*^2\neq 0$, since if $p_*^2=0$ implies that $v^a\parallel p^a$ and so the C-S and R-C terms necessarily vanish.

With these vectors, one may define the following projection operators:

$$\omega^{ab} = \frac{p^a p^b}{p^2}, \tag{A5a}$$

$$\rho^{ab} = -e_1^a e_1^b, \tag{A5b}$$

$$\sigma^{ab} = -e_2^a e_2^b, \tag{A5c}$$

$$\tau^{ab} = -e_3^a e_3^b = -\frac{1}{p_*^2}\left[p^2 v^a v^b - (v\cdot p)\left(p^a v^b + v^a p^b\right) + (v\cdot p)^2\,\omega^{ab}\right]. \tag{A5d}$$

One should notice that the transverse operator, θ^{ab}, can be related to these operators by,

$$\theta^{ab} \equiv \eta^{ab} - \omega^{ab} = \rho^{ab} + \sigma^{ab} + \tau^{ab}. \tag{A6}$$

The operator for the LV Chern-Simons term, $S^{ab} = i\epsilon^{abcd} v_c p_d$, may also be written in terms of the building blocks. Using (A4) and (A6), one actually shows that

$$S^{ab} = i\sqrt{p_*^2}\left(e_1^a e_2^b - e_2^a e_1^b\right). \tag{A7}$$

With the basis (A5a)-(A5d) at hand, one could carry out the task of getting the propagator of the Maxwell-Chern-Simons LV theory, with eventually a massive term

$$\mathcal{L} = -\frac{1}{4}F_{ab}F^{ab} - \frac{M^2}{2}A_a A^a + \frac{1}{4}\epsilon^{abcd} v_a A_b F_{cd}. \tag{A8}$$

This problem has been considered in [8, 9] using different approaches. Instead, let us move on to deeper waters and tackle a related problem in gravity.

2. Degree-of-freedom Operators for Gravity Models

The spin-2 sector operators result from the decomposition of the operator $P\,(2)_{ab,cd} = \frac{1}{2}\left(\theta_{ac}\theta_{bd} + \theta_{ad}\theta_{bc}\right) - \frac{1}{3}\theta_{ab}\theta_{cd}$. A more efficient method to decompose this spin projection operator into orthogonal sub-components can be achieved by its decomposition in normalized eigenvectors, $\psi^{(i)}_{ab}$:

$$P\,(2)_{ab,cd} = \sum_i \psi^{(i)}_{ab}\psi^{(i)}_{cd}, \tag{A9}$$

(for arbitrary spin, this decomposition must include a factor coming from the parity P of the spin sector: $P_{ij}\,(J) = (-1)^P \sum_i \psi^{(i)}\psi^{(i)}$), where

$$P\,(2)_{ab,cd}\,\psi^{(i)cd} = \psi^{(i)}_{ab}. \tag{A10}$$

With these eigenvectors at hand, one can define the full set of degree-of-freedom projectors and mapping operators:

$$P_{ij} = \psi^{(i)}\psi^{(j)}. \tag{A11}$$

A remark that has already been made in [18] is that the projection-operator basis is defined up to a rotation transformation. In fact, a rotation of the eigenvector basis,

$$\tilde{\psi}^{(i)} = \sum_j U_{ij}\psi^{(j)}, \tag{A12}$$

with U_{ij} orthogonal ($UU^T = 1$), redefine rotated projectors

$$\tilde{P}_{ij} = \tilde{\psi}^{(i)}\tilde{\psi}^{(j)} = U_{ik}\psi^{(k)}U_{jl}\psi^{(l)} = U_{ik}P_{kl}U^T_{lj}. \tag{A13}$$

But, the completeness relation may be used and, therefore, the spin-projectors remain unaltered:

$$\tilde{P} = \sum_i \tilde{\psi}^{(i)}\tilde{\psi}^{(i)} = \sum_{i,k,l} U_{ik}U_{il}\psi^{(k)}\psi^{(l)} = \sum_i \psi^{(i)}\psi^{(i)} = P. \tag{A14}$$

A convenient choice of eigenvectors helpful in the comparison with the 3-dimensional analog theory is given below:

$$\psi^{(1)}_{ab} = \frac{1}{\sqrt{2}}\left(e_{1a}e_{2b}+e_{2a}e_{1b}\right), \tag{A15a}$$

$$\psi^{(2)}_{ab} = \frac{1}{\sqrt{2}}\left(e_{1a}e_{3b}+e_{3a}e_{1b}\right), \tag{A15b}$$

$$\psi^{(3)}_{ab} = \frac{1}{\sqrt{2}}\left(e_{2a}e_{3b}+e_{3a}e_{2b}\right), \tag{A15c}$$

$$\psi^{(4)}_{ab} = \frac{1}{\sqrt{2}}\left(\rho_{ab}-\sigma_{ab}\right), \tag{A15d}$$

$$\psi^{(5)}_{ab} = \frac{1}{\sqrt{6}}\left(\rho_{ab}+\sigma_{ab}-2\tau_{ab}\right). \tag{A15e}$$

With the eigenvectors (A15a)-(A15e), the degree-of-freedom operators are built up by the relation (A11). The projection operators are then cast as:

$$P_{11}\left(2\right)_{ab,cd} = \psi^{(1)}_{ab}\psi^{(1)}_{cd} = \frac{1}{2}\left(\rho_{ac}\sigma_{bd}+\rho_{ad}\sigma_{bc}+\sigma_{ac}\rho_{bd}+\sigma_{ad}\rho_{bc}\right), \tag{A16a}$$

$$P_{22}\left(2\right)_{ab,cd} = \psi^{(2)}_{ab}\psi^{(2)}_{cd} = \frac{1}{2}\left(\rho_{ac}\tau_{bd}+\rho_{ad}\tau_{bc}+\tau_{ac}\rho_{bd}+\tau_{ad}\rho_{bc}\right), \tag{A16b}$$

$$P_{33}\left(2\right)_{ab,cd} = \psi^{(3)}_{ab}\psi^{(3)}_{cd} = \frac{1}{2}\left(\tau_{ac}\sigma_{bd}+\tau_{ad}\sigma_{bc}+\sigma_{ac}\tau_{bd}+\sigma_{ad}\tau_{bc}\right), \tag{A16c}$$

$$P_{44}\left(2\right)_{ab,cd} = \psi^{(4)}_{ab}\psi^{(4)}_{cd} = \frac{1}{2}\left(\rho_{ab}\rho_{cd}+\sigma_{ab}\sigma_{cd}\right)-\frac{1}{2}\left(\rho_{ab}\sigma_{cd}+\sigma_{ab}\rho_{cd}\right), \tag{A16d}$$

$$P_{55}\left(2\right)_{ab,cd} = \psi^{(5)}_{ab}\psi^{(5)}_{cd} = \frac{1}{6}\left[\rho_{ab}\rho_{cd}+\sigma_{ab}\sigma_{cd}+4\tau_{ab}\tau_{cd}+\rho_{ab}\sigma_{cd}+\sigma_{ab}\rho_{cd}\right. \tag{A16e}$$

$$\left.-2\left(\rho_{ab}\tau_{cd}+\tau_{ab}\rho_{cd}\right)-2\left(\sigma_{ab}\tau_{cd}+\tau_{ab}\sigma_{cd}\right)\right]. \tag{A16f}$$

Accordingly, the mapping operators are given by $P_{ij}\left(2\right)_{ab,cd}=\psi^{(i)}_{ab}\psi^{(j)}_{cd}$ $(i\neq j)$. For example,

$$P_{14}\left(2\right)_{ab,cd} = \psi^{(1)}_{ab}\psi^{(4)}_{cd} = \frac{1}{2}\left(e_{1a}e_{2b}+e_{2a}e_{1b}\right)\left(\rho_{cd}-\sigma_{cd}\right). \tag{A17}$$

However, the mapping operators may be expressed in terms of ϵ, v^a, ρ and σ so as to facilitate the expansion of the wave operator in terms of the degree-of-freedom operators (8). For $P_{14}\left(2\right)$, one can show that

$$P_{14}\left(2\right)_{ab,cd} = \frac{1}{2}\epsilon^{efgh}\left(\rho_{ac}\sigma_{bf}\rho_{de}+\rho_{bd}\sigma_{af}\rho_{ce}-\sigma_{bd}\rho_{ae}\sigma_{cf}-\sigma_{ac}\rho_{be}\sigma_{df}\right)\frac{v_g p_h}{\sqrt{p_*^2}}. \tag{A18}$$

The other mapping operators can be expressed, if desired, in an analogous manner.

As already remarked, the linearized model is invariant under the gauge transformation (5). For this reason, the spin-1 sector was fully suppressed from our discussion. In spite of this, the spin-1 sector may be important for a gravitational theory without gauge symmetry or with more propagating fields. For completeness, we present in this appendix the spin-1 sector degree-of-freedom operators. They are built up along the same lines as the spin-2 sector; the projectors are cast as:

$$P_{11}\left(1\right)_{ab,cd} = \frac{1}{2}\left(\rho_{ac}\omega_{bd}+\rho_{bc}\omega_{ad}+\rho_{ad}\omega_{bc}+\rho_{bd}\omega_{ac}\right), \tag{A19a}$$

$$P_{22}\left(1\right)_{ab,cd} = \frac{1}{2}\left(\sigma_{ac}\omega_{bd}+\sigma_{bc}\omega_{ad}+\sigma_{ad}\omega_{bc}+\sigma_{bd}\omega_{ac}\right), \tag{A19b}$$

$$P_{33}\left(1\right)_{ab,cd} = \frac{1}{2}\left(\tau_{ac}\omega_{bd}+\tau_{bc}\omega_{ad}+\tau_{ad}\omega_{bc}+\tau_{bd}\omega_{ac}\right). \tag{A19c}$$

Appendix B: An Alternative Method for the Attainment of Propagator

In previous works, the derivation of the propagators for gravity models with LV was also considered [20], but using the algebraic method without decomposition into degree-of-freedom operators. Therefore, it is worthy to verify the consistency of our resulting propagators. This comparison can be made by noting that the C-S operator can be cast in the following form:

$$S_{ab,cd} = i\sqrt{p_*^2}\left[-2P_{14}\left(2\right)_{ab,cd} + 2P_{41}\left(2\right)_{ab,cd} - P_{23}\left(2\right)_{ab,cd} + P_{32}\left(2\right)_{ab,cd}\right].$$

Such a decomposition clears up a fact, not obvious at a first sight, that the operator $S_{ab,cd}$ is made up of two reducible components:

$$\begin{aligned}(\tau S)_{ab,cd} &= \frac{1}{2}\left(\tau_{ac}S_{bd} + \tau_{ad}S_{bc} + \tau_{bc}S_{ad} + \tau_{bd}S_{ac}\right) \quad \text{(B1)}\\ &= -i\sqrt{p_*^2}\left[P_{23}\left(2\right)_{ab,cd} - P_{32}\left(2\right)_{ab,cd}\right],\\ (\theta\tau S)_{ab,cd} &= \frac{1}{2}\left(\left(\theta_{ac}-\tau_{ac}\right)S_{bd} + \left(\theta_{ad}-\tau_{ad}\right)S_{bc} + \left(\theta_{bc}-\tau_{bc}\right)S_{ad} + \left(\theta_{bd}-\tau_{bd}\right)S_{ac}\right) \quad \text{(B2)}\\ &= -2i\sqrt{p_*^2}\left[P_{14}\left(2\right)_{ab,cd} - P_{41}\left(2\right)_{ab,cd}\right].\end{aligned}$$

One can also show that other two key operators, $(\tau S)^2_{ab,cd}$ and $(\theta\tau S)^2_{ab,cd}$, can be written either in terms of θ, τ and S or $P_{ij}\left(2\right)$, according to the convenience:

$$\begin{aligned}(\tau S)^2_{ab,cd} &= p_*^2\left[\frac{1}{2}\left(\theta_{ac}\tau_{bd} + \theta_{ad}\tau_{bc} + \tau_{ac}\theta_{bd} + \tau_{ad}\theta_{bc}\right) - \left(\tau_{ac}\tau_{bd} + \tau_{ad}\tau_{bc}\right)\right] \quad \text{(B3)}\\ &= p_*^2\left[P_{22}\left(2\right)_{ab,cd} + P_{33}\left(2\right)_{ab,cd}\right],\\ (\theta\tau S)^2_{ab,cd} &= p_*^2\left[\left(\theta_{ac}-\tau_{ac}\right)\left(\theta_{bd}-\tau_{bd}\right) + \left(\theta_{ad}-\tau_{ad}\right)\left(\theta_{bc}-\tau_{bc}\right)\right] - \left(S_{ac}S_{bd} + S_{ad}S_{bc}\right) \quad \text{(B4)}\\ &= 4p_*^2\left[P_{11}\left(2\right)_{ab,cd} + P_{44}\left(2\right)_{ab,cd}\right].\end{aligned}$$

With the operators (B1)-(B4), one can show, by direct inspection, that the following multiplication table is fulfilled:

	$P\left(2\right)_{cd,ef}$	$(\tau S)_{cd,ef}$	$(\tau S)^2_{cd,ef}$	$(\theta\tau S)_{cd,ef}$	$(\theta\tau S)^2_{cd,ef}$
$P\left(2\right)_{ab,cd}$	$P\left(2\right)$	τS	$(\tau S)^2$	$\theta\tau S$	$(\theta\tau S)^2$
$(\tau S)_{ab,cd}$	τS	$(\tau S)^2$	$p_*^2\left(\tau S\right)$	0	0
$(\tau S)^2_{ab,cd}$	$(\tau S)^2$	$p_*^2\left(\tau S\right)$	$p_*^2\left(\tau S\right)^2$	0	0
$(\theta\tau S)_{ab,cd}$	$\theta\tau S$	0	0	$(\theta\tau S)^2$	$4p_*^2\left(\theta\tau S\right)$
$(\theta\tau S)^2_{ab,cd}$	$(\theta\tau S)^2$	0	0	$4p_*^2\left(\theta\tau S\right)$	$4p_*^2\left(\theta\tau S\right)^2$

Table I: Multiplication table of spin-2 operators

Restricting the discussion for the spin-2 sector, the problem of calculating the propagator is reduced to the problem of solving the linear problem

$$\left(xP\left(2\right) + y\tau S + z\left(\tau S\right)^2 + u\theta\tau S + v\left(\theta\tau S\right)^2\right)\left(aP\left(2\right) + bS\left(2\right)\right) \equiv P\left(2\right), \quad \text{(B5)}$$

for which the table above is very helpful. The general solution is given by:

$$x = \frac{1}{a},\ y = \frac{b}{b^2p_*^2 - a^2},\ z = -\frac{1}{a}\frac{b^2}{b^2p_*^2 - a^2},\ u = \frac{b}{4b^2p_*^2 - a^2},\ v = -\frac{1}{a}\frac{b^2}{4b^2p_*^2 - a^2}. \quad \text{(B6)}$$

For the wave operator (7), $a = \frac{1}{2}p^2\left(\alpha + \beta p^2\right)$ and $b = \frac{1}{4}\left(\mu p^2 + \lambda p^4\right)$, it can be verified that it coincides with the result obtained with the orthogonal basis of operators as done in Sec. III.

A first remark that must be made is that, without defining an orthonormal basis of operators, there are some ambiguities for choosing the fundamental blocks that close the algebra of operators, such as the one in table I. In this way, such a procedure to obtain the propagator may yield redundant operators that harm the task of inverting the wave operator. Another remark is that, even with the propagator at hand, the use of a non-orthonormal basis also renders difficult the physical interpretation of the propagating modes, since, as discussed in the Sec. IV, the degree-of-freedom basis of operators allows the splitting of the propagator into independent sectors that result in a direct identification of the spins of the propagating particles.

---

[1] A. Ashtekar, Phys. Rev. Lett. **57**, 2244 (1986).
[2] C. Rovelli, Living Rev. Rel. **1**, 1 (1998) [arXiv:gr-qc/9710008].
[3] P. Horava, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[4] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989).
[5] R. Jackiw and S. Y. Pi, Phys. Rev. D **68**, 104012 (2003) [arXiv:gr-qc/0308071].
[6] J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell and M. B. Bainbridge, arXiv:1008.3907 [astro-ph.CO].
[7] V. A. Kostelecky and M. Mewes, Astrophys. J. **689**, L1 (2008) [arXiv:0809.2846 [astro-ph]].
[8] A. P. Baeta Scarpelli, H. Belich, J. L. Boldo and J. A. Helayel-Neto, Phys. Rev. D **67**, 085021 (2003).
[9] C. Adam and F. R. Klinkhamer, Nucl. Phys. B **607**, 247 (2001) [arXiv:hep-ph/0101087].
[10] C. A. Hernaski, B. Pereira-Dias and A. A. Vargas-Paredes, Phys. Lett. A **374**, 3410 (2010) [arXiv:1001.2784 [hep-th]].
[11] C. A. Hernaski, A. A. Vargas-Paredes and J. A. Helayel-Neto, Phys. Rev. D **80** (2009) 124012, arXiv:0905.1068 [hep-th].
[12] J. A. Helayel-Neto, C. A. Hernaski, B. Pereira-Dias, A. A. Vargas-Paredes and V. J. Vasquez-Otoya, Phys. Rev. D **82**, 064014 (2010), arXiv:1005.3831 [hep-th].
[13] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997) [arXiv:hep-ph/9703464].
[14] S. Deser and R. Jackiw, Phys. Lett. B **451**, 73 (1999) [arXiv:hep-th/9901125].
[15] V. A. Kostelecky, Phys. Rev. D **69**, 105009 (2004) [arXiv:hep-th/0312310].
[16] R. J. Rivers Hagiwara, Nuovo Cimento 34 (1964) 387.
[17] D. E. Neville, Phys. Rev. D **18**, 3535 (1978).
[18] E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D **21**, 3269 (1980).
[19] O. W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002) [arXiv:hep-ph/0201258].
[20] J. L. Boldo, J. A. Helayel-Neto, L. M. de Moraes, C. A. G. Sasaki and V. J. Vasquez Otoya, Phys. Lett. B **689**, 112 (2010) [arXiv:0903.5207 [hep-th]].
[21] S. Deser, R. Jackiw and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); **185**, 406(E) 1988; **281**, 409 1 (2000).
[22] H. Belich, J. L. Boldo, L. P. Colatto, J. A. Helayel-Neto and A. L. M. Nogueira, Phys. Rev. D **68**, 065030 (2003) [arXiv:hep-th/0304166].
[23] A. P. Baeta Scarpelli, H. Belich, J. L. Boldo, L. P. Colatto, J. A. Helayel-Neto and A. L. M. Nogueira, Nucl. Phys. Proc. Suppl. **127**, 105 (2004) [arXiv:hep-th/0305089].