MÉTODO DE ÁLGEBRA DAS CORRENTES PARA FATÔRES DE FORMA E DECAÍMENTOS FORTES COM PIONS E KAONS DUROS

TESE DE DOUTORADO

defendida por

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no Centro Brasileiro de Pesquisas Físicas

em 12 de Setembro de 1969

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AGRADECIMENTOS

É com a máxima satisfação que aproveito esta oportunidade para expressar os meus agradecimentos aos Professores José Leite Lopes e Jayme Tiomno pela orientação recebida em minha formação como pesquisador.

Agradeço também ao Conselho Nacional de Pesquisas pela bôlsa que permitiu minha permanência no C.E.R.N., Genebra, onde foi concluida a maior parte dos trabalhos aqui apresentados.

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INTRODUÇÃO

Nesses últimos anos uma das descobertas importantes na física das partículas elementares tem sido a sintetização esquemática do spin isotópico e a hipercargá pelo grupo de simetria unitária SU(3) [1]. Por outro lado, por exemplo, pelas não igualdade das massas das partículas que pertencem ao mesmo multiplet no SU(3) é evidente que esta simetria é necessàriamente violada, porém, verifica-se, experimentalmente, que os processos de espalhamento e decaimento são pelo menos, qualitativamente, bem descritos dentro desse mesmo esquema de simetria SU(3). Os detalhes da natureza das forças no SU(3), ao contrário do que no caso da simetria do isospin, a qual é violada pelas interações eletromagnéticas, não são bem compreendidas até agora.

A fim de elucidar o conceito da simetria SU(3) aproximada e formular uma descrição quantitativa para o conceito da universalidade digamos, nas interações fracas, Gell-Mann [2] postulou que as correntes hadrônicas obedecem certas relações de comutação, para tempos iguais, e que estas regras continuam prevalecendo, ainda que, haja uma violação da simetria unitária SU(3). Êste postulado, juntamente com a bem conhecida técnica de redução na teoria dos campos, constitui o método da "álgebra das correntes" [3]. Esta técnica tem sido muito utilizada, recentemente, para derivar teoremas de baixas energias, classificação das partículas hadrônicas, formular regras de soma, como por exemplo, de Adler e Weisherger para o cálculo da renormalização da constante de acoplamento axial na desintegração beta, devidas às interações fortes, e ainda para o relacionamento de diversos processos de decaimento, etc.

Os trabalhos apresentados aqui abordam discussões sôbre : a razão F /F entre os acoplamentos [illegible] par de leptons, as regras de soma para funções espectrais de Weinberg, as renormalizações dos fatores de forma de decaimento $K_{\ell 3}$ por causa da violação da simetria SU(3) e ainda os cálculos de decaimentos fortes / das ressonâncias estranhas K^* e K_A. Os últimos três trabalhos sugerem que os fatôres de forma podem satisfazer relações de dispersão com (uma) subtração e que um cálculo auto-consistente dos decaimentos fortes com pions e kaons "duros" é possível sem usar a hipótese da conservação parcial da corrente axial. Além disso, as regras de soma de Weinberg / são obtidas, sem apelar para simetria chiral assintótica, mas saem como condições de auto-consistências.

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3. Ver, por exemplo, as referências citadas no livro "Current Algebras", S.Adler e R. Dashen, Benjamin, N.Y. (1968) e as referências citadas nos trabalhos aqui apresentados.

I. REGRAS DE SOMA DE WEINBERG E A RAZÃO F_K/F_π

I. REGRAS DE SOMA DE WEINBERG E A RAZÃO F_K/F_π

Prem P. Srivastava, Physics Letters 26B,233(1968)

RESUMO:

Aponta-se nêste trabalho que não é admissível usar, concomitantemente, a primeira e segunda regra de soma de Weinberg no grupo SU(3) sem chegar às conclusões inaceitáveis $m_\rho = m_K$ e $m_{A_1} \simeq m_{K_A}$. Usando a segunda regra, sòmente, no sub grupo SU(2), obtemos:

$$\frac{F_K}{F_\pi} = \left(\frac{1 - \frac{m_{K^*}^2}{m_{K_A}^2}}{1 - \frac{m_\rho^2}{m_{A_1}^2}} \right)^{\frac{1}{2}} \simeq 1.07$$

Esclarecemos também, que no decaimento K_{ℓ_3} a experiência mede de fato $(F_+(0)\ \text{sen}\,\theta)$ e temos a seguinte relação:

$$\frac{F_K}{F_+(0)\, F_\pi} \simeq \frac{\sqrt{1 - (F_+(0)\ \text{sen}\,\theta)^2}}{F_+(0)\ \text{sen}\,\theta} \tan\theta_A$$

Tomando o valor experimental de 1.28 ± 0.06 para o lado direito e para F_K/F_π o valor calculado acima, encontramos $F_+(0) \simeq 0.84$, contràriamente, ao valor unitário no caso de simetria SU(3) exata. Finalmente, a razão das larguras $\Gamma(\rho^+ \to \pi^+\pi^0)/\Gamma(K^{*+} \to K\pi)$ é calculada pela expressão:

$$\frac{4}{} \left(\frac{k^\rho}{} \right)^3 \frac{1}{}$$

WEINBERG SUM RULES AND THE RATIO F_K/F_π

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A B S T R A C T

The ratio F_K/F_π is derived using Weinberg's first spectral function sum rule in $SU(3) \times SU(3)$ while the second sum rule is used only in $SU(2) \times SU(2)$.

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*) On leave of absence from Centro Brasileiro de Pesquisas Físicas and Universidade Federal do Rio de Janeiro, Brasil.

380/5 - TH. 848
[N]ovember 1967

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In recent papers [1], the ratio between the decay constants F_K and F_π, the decay constants in the leptonic decays of K and π mesons respectively, has been evaluated using the two spectral function sum rules derived by Weinberg [2] and their SU(3) generalizations given by Das, Mathur and Okubo [3]. The derivations in Ref. [1] make use of both of the sum-rules and thus necessarily imply, in the pole approximation considered, the exact SU(3) × SU(3) result $m_\rho = m_{K^*}$, $m_{A_1} \simeq m_{K_A}$. We derive here the above-mentioned ratio by using the first sum rule [4] in SU(3) × SU(3) while the second sum rule is used only in the SU(2) × SU(2) subgroup. The argument [5] for the restriction on the use of the second sum rule derives from the successful predictions [2, 3] $m_{A_1} \simeq \sqrt{2}\, m_\rho$, $m_{K_A} \simeq \sqrt{2}\, m_{K^*}$ and the disappearance of the unpleasant feature mentioned above.

The first spectral function sum rule gives rise to the following relations [2, 3]:

$$\frac{G_\rho^2}{m_\rho^2} - \frac{G_{A_1}^2}{m_{A_1}^2} = F_\pi^2$$

$$\frac{G_{K^*}^2}{m_{K^*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2} = F_K^2$$

$$\frac{G_\rho^2}{m_\rho^2} = \frac{G_{K^*}^2}{m_{K^*}^2}$$

group SU(2) × SU(2) leads to the relations:

$$G_\rho = G_{A_1} \quad \text{and} \quad G_{K^*} = G_{K_A}$$

These relations then lead to

$$\frac{F_K}{F_\pi} = \frac{m_{A_1}}{m_{K_A}} \sqrt{\frac{m_{K_A}^2 - m_{K^*}^2}{m_{A_1}^2 - m_\rho^2}}$$

which [6] gives $(F_K/F_\pi) \simeq 1.07$ against the value ~ 1.16 found in Ref. [1]. Also it follows:

$$\frac{G_{A_1}^2}{m_{A_1}^2} \simeq \frac{G_{K_A}^2}{m_{K_A}^2}$$

The Cabibbo angles θ_A and θ are related through F_K/F_π by the relation

$$\frac{F_K}{F_\pi} = \frac{\tan\theta_A}{\tan\theta}$$

which can be rewritten [7] as

$$\frac{1}{F_+(0)}\left(\frac{F_K}{F_\pi}\right) \simeq \frac{\sqrt{1 - F_+^2(0)\sin^2\theta}}{F_+(0)\sin\theta} \cdot \tan\theta_A$$

Here $F_+(0)$ is the $K-\pi$ form factor normalized to unity in exact SU(3) limit. Now θ_A can be obtained from the branching ratio of K_{μ_2} and π_{μ_2} decays while $[F_+(0)\sin\theta]$ is determined from K_{ℓ_3} decays. Using for the right-hand side the value [8] ~ 1.28, we obtain for the form factor, $F_+(0) \simeq 0.84$. On the other hand, if we assume $F_+(0) = 1$, the calculated value of the Cabibbo angle turns out to be $\tan\theta \simeq 0.26$ which is about 18% higher than the value quoted in Ref. [8].

In conclusion, it is of interest to note the implications of our results on the branching ratio $\Gamma_{\rho\to\pi\pi}/\Gamma_{K^*\to K\pi}$. Assuming pole dominance in the pion form factor we can derive

$$G_\rho \, g_{\rho\pi\pi} = \sqrt{2}\, m_\rho^2$$

The K_{ℓ_3} form factor F_+ under the assumption of K* pole dominance leads to analogous relation:

$$G_{K^*} \, g_{K^*K\pi} = \frac{F_+(0)}{\sqrt{2}} \, m_{K^*}^2$$

We obtain, thereby, (using $G_{K^*}/m_{K^*} = G_\rho/m_\rho$):

$$\frac{g_{\rho\pi\pi}}{g_{K^*K\pi}} = 2\left(\frac{m_\rho}{m_{K^*}}\right)\frac{1}{F_+(0)}$$

and

$$\frac{\Gamma_{\rho^+\to\pi\pi}}{\Gamma_{K^{*+}\to K\pi}} = \frac{4}{3}\left(\frac{k^{\rho}_{C.M}}{k^{K^*}_{C.M}}\right)^3 \frac{1}{(F_+(0))^2} \simeq \frac{2.455}{(F_+(0))^2}$$

For $F_+(0) \simeq 1$ this implies $\Gamma_{\rho\to\pi\pi} = 122$ MeV for $\Gamma_{K^*\to K\pi} = 49.6$ MeV. For $F_+(0) \simeq 0.84$ it implies $F_{\to\pi\pi} \simeq 173$ MeV. However, due to great uncertainties in the experimental decay widths of the particles involved it is not possible to draw definite information on $F_+(0)$ from this piece of experimental data.

ACKNOWLEDGEMENTS

The author is indebted to Professor D. Amati for several discussions. The author also acknowledges gratefully the hospitality extended to him at CERN and to the "Conselho Nacional de Pesquisas" of Brazil for a fellowship.

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H. T. Nieh - Phys. Rev. Letters 19, 43 (1967).

2. S. Weinberg - Phys. Rev. Letters 18, 507 (1967).

3. T. Das, V. S. Mathur and S. Okubo - Phys. Rev. Letters 18, 761 (1967); Phys. Rev. Letters 19, 470 (1967). We use the notation of their papers.

4. The first spectral function sum rule reads: [i, j being the SU(3) indices] -

$$\int_0^\infty \left[\rho_V^i(\mu^2) - \rho_A^j(\mu^2)\right] \frac{d\mu^2}{\mu^2} = F_j^2$$

while the second sum rule is:

$$\int_0^\infty \left[\rho_V^i(\mu^2) - \rho_A^j(\mu^2)\right] d\mu^2 = 0 \qquad \text{etc.}$$

5. Similar doubts about the applicability of the second sum rule in SU(3) x SU(3) have also been raised by J. Sakurai - Phys. Rev. Letters 19, 803 (1967).

6. For $m_{A_1} = \sqrt{2}\, m_\rho$ and $m_{K_A} = \sqrt{2}\, m_{K*}$ it gives $F_K = F_\pi$ implying no renormalization effect due to SU(3) breaking. We have used $m_{K_A} = 1309$, $m_{K*} = 892.4$, $m_{A_1} = 1058$ and $m_\rho = 774$.

7. We make use of the fact that $F_+(0) \simeq 1$ and θ is a small angle. The author is indebted to Dr. A. Sirlin and Dr. N. Brene for a discussion on this point.

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See, however, the footnote (9) in Ref. 1 (Glashow et al.), where they mention that the value quoted for θ should be $\sim 10\%$ higher.

II. ÁLGEBRA DAS CORRENTES E OS DECAIMENTOS $K_A \rightarrow K^*\pi$ e $K^* \rightarrow K\pi$

II. ALGEBRA DAS CORRENTES E OS DECAIMENTOS $K_A \to K^*\pi$ e $K^* \to K\pi$

Prem P. Srivastava, Physics Letters 26B, 233 (1968)

RESUMO:

Os decaimentos $K_A \to K^*\pi$ e $K^* \to K\pi$ são estudadas usando-se "álgebra das correntes", relações de dispersão e regras de soma de Weinberg. Admite-se a conservação da corrente vetorial e a conservação parcial da corrente axial. A aproximação de dominância pelos polos nos elementos de matriz é também usada. Entretanto é permitido que os fatôres de formas L_1 e K_1 definidos pelos elementos de matrizes invariantes:

$$\sqrt{4k_0p_0V^2}\,\langle \pi^0(k)|V^\mu(0)^1{}_3\,|K_A^+(p)\rangle$$

$$= i\,e_\nu^{K_A}\left[L_1(q^2)g^{\mu\nu} + L_2(q^2)\,k^\nu(p+k)^\mu + L_3(q^2)\,k^\nu(p-k)^\mu\right]$$

$$\sqrt{4k_0p_0V^2}\,\langle \pi^0(k)|A^\mu(0)^1{}_3\,|K^{*+}(p)\rangle$$

$$= i\,e_\nu^{K^*}\left[K_1(q^2)g^{\mu\nu} + K_2(q^2)k^\nu(p+k)^\mu + K_3(q^2)\,k^\nu(p-k)^\mu\right]$$

satisfazem relações de dispersão com uma subtração, enquanto que, os demais fatores de forma satisfazem relações de dispersões sem subtrações. Estas restrições são sugeridas a fim de evitar o comportamento de "superconvergência" para os fatores de forma L_3 e K_3 que nos leva aos resultados que contrariam as experiências. Emprega-se constantes de subtração que decorrem do método da álgebra das correntes. A largura do decaimento do K^* é pouco afetada em relação aos cálculos anteriores usando o antigo método da álgebra das correntes, entretanto, para o decaimento do K [illegible]

perimental, contràriamente, aos cálculos antigos que dão para o decaimento do A_1 uma largura parcial muito maior do que a largura total do A_1. Nosso cálculo prevê, uma mistura apreciável da onda "d" no decaimento do K_A que pode ser testado pelas experiências futuras. Também é sugerido pelos cálculos quantitativos que as relações entre massas $m_{A_1} = \sqrt{2}\, m_\rho$ e $m_{K_A} = \sqrt{2}\, m_{K^*}$ sejam válidas sòmente aproximadamente.

$K_A \rightarrow K^* \pi$ AND $K^* \rightarrow K \pi$ DECAYS

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ABSTRACT

Strong decays $K_A \rightarrow K^* \pi$ and $K^* \rightarrow K \pi$ are studied using current algebra, partial conservation of axial current, dispersion relations and Weinberg sum rules. The pions are treated as "soft" in the sense $k^2 \rightarrow 0$ instead of $k \rightarrow 0$.

*) On leave of absence from Centro Brasileiro de Pesquisas Fisicas and Universidade Federal [illegible]

In recent papers [1] the rates of the strong decays $\rho \to \pi\pi$ and $A_1 \to \rho\pi$ have been related by using the techniques of current algebra and dispersion relations. The predictions are in good agreement with the experimental results. In this paper we relate and calculate the decay rates [2] for $K^*(890) \to K\pi$ and $K_A(1320) \to K^*\pi$ the strangeness carrying vector and axial vector mesons belonging [3] to SU(3) octets containing ρ and A_1, respectively. We follow closely the dispersion relation and current algebra method by Das et al. [1]. The various unknown couplings are determined [4] by making use of Weinberg's spectral function sum rules [5].

We consider the following invariant matrix elements:

$$\langle\pi^0(k)|V'_{\mu\,3}(0)|K_A^+(p)\rangle\sqrt{4p_0\,k_0\,V^2}$$
$$= i\,e_\nu^{K_A}(p)\left[L_1(q^2)\,\delta_{\mu\nu} + L_2(q^2)\,k_\nu(p+k)_\mu + L_3(q^2)\,k_\nu(p-k)_\mu\right] \tag{1}$$

$$\langle\pi^0(k)|A_\mu(0)'_3\,|K^{*+}(p)\rangle\sqrt{4p_0\,k_0\,V^2}$$
$$= i\,e^{K^*}(p)\left[K_1(q^2)\,\delta_{\mu\nu} + K_2(q^2)k_\nu(p+k)_\mu + K_3(q^2)k_\nu(p-k)_\mu\right] \tag{2}$$

where $e_\nu(p)$ denotes the polarization vector and $q^2 = (p-k)^2$.

The conserved vector current hypothesis on Eq. (1) and the condition of partial conservation of axial current (PCAC) on Eq. (2) lead to the following relations for the form f...

$$L_1(q^2) + (m_{K_A}^2 - \mu^2)\, L_2(q^2) - L_3(q^2)q^2 = 0 \qquad (3)$$

$$K_1(q^2) + (m_{K*}^2 - \mu^2)\, K_2(q^2) - K_3(q^2)q^2 = \frac{F_K m_K^2\, G_{K*^+\pi^o K^+}}{(q^2 + m_K^2)} \qquad (4)$$

where F_K is the decay constant appearing in $K \to \mu\nu$ decay and $G_{K*^+\pi^o K^+}$ is the coupling for the decay mode $K*^+ \to \pi^o + K^+$. They are defined as:

$$\sqrt{2k_oV}\, \langle 0 | A_\mu(0)^{1,3} | K^+(k) \rangle = i\, F_K\, k_\mu \qquad (5)$$

$$\sqrt{4p_o q_o V^2}\, \langle K^+(q) | j_{\pi^o}(0) | K*^+(p) \rangle = G_{K*^+\pi^o K^+}\, e^{K*} \cdot q \qquad (6)$$

We shall postulate now the unsubtracted dispersion relations for the form factors $L_2(q^2)$, $L_3(q^2)$, $K_2(q^2)$ and $K_3(q^2)$, and a once-subtracted dispersion relation for $L_1(q^2)$ and $K_1(q^2)$. A justification [6] for the once-subtracted dispersion relation for the latter case may be found by applying PCAC and current algebra to the corresponding invariant matrix elements and neglecting only the k^2 dependent terms [7].

Thus we write:

$$L_1(q^2) = \frac{G_{K_A}}{\sqrt{2}\, F_\pi} + \frac{(q^2 + m_{K_A}^2)}{\pi} \int \frac{\text{Im}\, L_1(q'^2)}{(q'^2 - q^2)(q'^2 + m_{K_A}^2)}\, dq'^2$$

$$L_2(q^2) = \frac{1}{\pi} \int \frac{\text{Im}\, L_2(q'^2)}{(q'^2 - q^2)}\, dq'^2 \qquad (7)$$

$$K_1(q^2) = \frac{G_{K*}}{\sqrt{2}\, F_\pi} + \frac{(q^2 + m_{K*}^2)}{\pi} \int \frac{\text{Im}\, K_1(q'^2)}{(q'^2 - q^2)(q'^2 + m^2\ )}\, dq'^2$$

etc. [8].

If we assume now that the form factors $L_1(q^2)$, $L_2(q^2$ and $L_3(q^2)$ are dominated by the K^* pole, while $K_1(q^2)$ $K_2(q^2)$ and $K_3(q^2)$ by the K and K_A poles, we find:

$$L_1(q^2) = \frac{G_{K_A}}{\sqrt{2}\, F_\pi} - \frac{(q^2+m_{K_A}^2)}{(q^2+m_{K^*}^2)} \frac{G_{K^*} \cdot G_s}{(m_{K_A}^2 - m_{K^*}^2)}$$

$$L_2(q^2) = \frac{G_{K^*}\, G_D}{2(q^2+m_{K^*}^2)} \tag{8}$$

$$L_3(q^2) = \frac{G_{K^*}}{m_{K^*}^2(q^2+m_{K^*}^2)} \left[G_s - \frac{1}{2}(m_{K_A}^2 - \mu^2)\, G_D \right]$$

$$K_1(q^2) = \frac{G_{K^*}}{\sqrt{2}\, F_\pi} + \frac{(q^2+m_{K^*}^2)}{(q^2+m_{K_A}^2)} \frac{G_{K_A}\, G_s}{(m_{K^*}^2 - m_{K_A}^2)}$$

$$K_2(q^2) = -\frac{G_{K_A}\, G_D}{2(q^2+m_{K_A}^2)}$$

$$K_3(q^2) = \frac{F_K\, G_{K^{*+}\pi^0 K^+}}{(q^2+m_K^2)} \tag{9}$$

$$- \frac{G_{K_A}}{m_{K_A}^2(q^2+m_{K_A}^2)} \left[G_s - \frac{1}{2} G_D (m_{K^*}^2 - \mu^2) \right]$$

The various constants appearing here are defined by:

$$\sqrt{2p_0V}\,\langle 0|V_\mu(0)_3|K^{*+}(p)\rangle = G_{K^*}\, e_\mu^{K^*}(p) \qquad (10)$$

$$\sqrt{4p_0q_0V^2}\,\langle K^{*+}(q)|j_{\pi^0}(0)|K_A^+(p)\rangle$$
$$= i\left[G_s\, e^{K_A}\cdot e^{K^*} + G_D\, e^{K_A}\cdot q\; e^{K^*}\cdot p\right] \qquad (11)$$

where G_s and G_d are the s and d wave couplings in $K_A \to K^*\pi$ decay.

Using relations (3) and (4) on Eqs. (8) and (9), we obtain:

$$G_s - \frac{1}{2}\, G_D(m_K^{\;2} - m_{K^*}^2) = \frac{1}{\sqrt{2}\,F_\pi}\;\frac{G_{K_A}}{G_{K^*}}\left(\frac{m_{K_A}^2 - m_{K^*}^2}{m_{K_A}^2}\right) m_{K^*}^2 \qquad (12)$$

$$G_s + \frac{1}{2}\, G_D(m_{K_A}^2 - m_{K^*}^2)$$
$$= \left(\frac{1}{\sqrt{2}\,F_\pi}\;\frac{G_{K^*}}{G_{K_A}} - \frac{F_K\, G_{K^{*+}\pi^0 K^+}}{G_{K_A}}\right)\left(\frac{m_{K_A}^2 - m_{K^*}^2}{m_{K^*}^2}\right) m_{K_A}^2 \qquad (13)$$

where we neglect terms of order $(\mu/m_{K^*})^2$.

For the values of unknown constants appearing on the right-hand side, we make use of the various relations [9] derived in Ref. 4 using the spectral function sum rules [5]. If K-π form factor F_+ in $K_{\ell 3}$ decay is dominated by the K^* pole we obtain by assuming unsubtracted dispersion relation:

$$\frac{G_{K^*}\; G_{K^{*+}\pi^0 K^+}}{2m_{K^*}^{\;2}} = \frac{F_+(0)}{\sqrt{2}} \qquad (14)$$

where $F_+(0) \to 1$ in exact SU(3) limit. This then leads to [4]:

$$G_{K^{*+}\pi^0 K^+} = \left(\frac{m_{K^*}}{F_\pi}\right)\sqrt{1+\delta}\left(F_+(0)\,\frac{F_\pi}{F_K}\right) \qquad (15)$$

where $\delta = (m_{K_A}^2 - 2m_{K^*}^2)/m_{K_A}^2$ and the third factor on the right-hand side can be determined from $K_{\ell 3}$ decays.

From Eqs. (12) and (13) we obtain:

$$G_3 = \frac{(1+\delta)}{2\sqrt{2}\,F_\pi}\,\frac{m_{K^*}^2}{2}\left(1 + \frac{4\Delta}{(1-\delta)^2}\right)$$

$$G_D = \frac{(1-\delta)}{2\sqrt{2}\,F_\pi}\left(-1 + \frac{4\Delta}{(1-\delta)^2}\right)$$

where

$$\Delta = \left(1 - \frac{F_+(0)\,F_\pi}{F_K}(1+\delta)\right)$$

The decay widths are given by:

$$\Gamma_{K^{*+}\to K\pi} = \frac{1}{8\pi}\,k_{CM}^3\,\frac{1}{F_\pi^2}\left(\frac{F_+(0)\,F_\pi}{F_K}\right)^2(1+\delta)$$

$$\Gamma_{K_A^+\to K^*\pi} = \frac{1}{8\pi}\,\frac{k_{CM}^5}{m_{K^*}^2}\,(g_L^2 + 2g_T^2)$$

where[10]:

$$g_T = \sqrt{\frac{1-\delta}{2}}\;\frac{G_s}{k_{CM}^2}$$

$$g_L = G_D - g_T\sqrt{1 + \frac{k_{CM}^2}{m_{K^*}^2}}$$

The presently known values [11] imply $\delta \approx 0.07$. Assuming [12] $(1/F_+(0))(F_K/F_\pi) \approx 1.16$, and for F_π the value obtained from decay, we obtain $\Gamma_{K^{*+} \to K\pi} \sim 40$ MeV and $\Gamma_{K_A^+ \to K^* \pi} \approx 58$ MeV. These predictions are consistent with experimental results available at present. Also, we find

$$\frac{g_L}{g_T} \simeq -\sqrt{1 + \frac{k_{CM}^2}{m_{K^*}^2}} \simeq -1.2$$

implying a large d wave admixture. This can be tested by future experiments.

In conclusion, we find that if we assume $m_{A_1} = \sqrt{2}\, m_\rho$; $m_{K_A} = \sqrt{2}\, m_{K^*}$ and $\Delta = 0$, it is easily seen that the widths of $\rho \to \pi\pi$ and $K^* \to K\pi$ modes are only slightly affected. However, now the partial width for the $A_1 \to \rho\pi$ mode comes out ~ 1.2 times that for $K_A \to K^*\pi$; in fact $\Gamma_{A_1 \to \rho\pi} \simeq 41$ MeV and $\Gamma_{K_A \to K^*\pi} \simeq 34$ MeV. With the small deviations from the exact mass equalities cited above, it seems possible to predict quantitatively the right trend of all of the above strong decays.

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ACKNOWLEDGEMENTS

The author is grateful to CERN for the hospitality extended to him and to the Conselho Nacional de Pesquisas Físicas of Brazil for a fellowship.

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2. The earlier calculations of $\rho \to \pi\pi$ and $K^* \to K\pi$ decays using current algebra, dispersion relations and PCAC [K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966)] neglected the contributions from A_1 and K_A poles respectively. Using Weinberg sum rules [5] to include their effect in the usual pole dominant form of the form factors leads to [4]:

$$g_{\rho\pi\pi} = \frac{m_\rho}{F_\pi} \sqrt{1+\delta_1}$$

$$g_{K^*K\pi} = \frac{m_{K^*}}{F_K} \frac{F_+(0)}{2} \sqrt{1+\delta}$$

where $\delta_1 = (m_{A_1}^2 - 2m_\rho^2)/m_{A_1}^2$ and δ and $F_+(0)$ are defined in Eqs. (14) and (15).

3. We neglect any mixing of this particle with another probable $K^*\pi$ resonance belonging to an SU(3) octet containing a B meson.

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7. We assume $L_1(q^2,\ k^2 = -\mu^2,\ p^2 = -m_{K_A}^2) \approx L_1(q^2,\ k^2 = 0,\ p^2 = -m_{K_A}^2)$. In the soft pion limit with $k \to 0$ we have the usual current algebra result:

$$\lim_{k \to 0} \sqrt{4p_0 k_0 V^2}\, \langle \pi^0(k) | V_\mu(0)_3 | K_A^+(p) \rangle = \frac{i}{\sqrt{2}\, F_\pi}\, G_{K_A}\, e_\mu^{K_A}(p)$$

8. $\sqrt{2p_0 V}\, \langle 0 | A_\mu(0)_3 | K_A^+(p) \rangle = G_{K_A}\, e_\mu^{K_A}(p)$

9. Other relations are [4]:

$$G_{K^*} = G_{K_A} = \frac{m_{K^*}}{m_\rho}\, G_\rho = \frac{\sqrt{2}\, m_{K^*}}{\sqrt{1+\delta}}\, F_K$$

and

$$\frac{F_K}{F_\pi} = \sqrt{\frac{1+\delta}{1+\delta_1}}$$

10. g_L and g_T are the coupling constants if we write the invariant matrix element (11) in the form:

$$i\left[g_L\, e_\lambda^{K_A}(p)\, e_\mu^{K^*}(q) \left(p_\mu - \frac{p\cdot q}{q^2}\, q_\mu\right)\left(q_\lambda - \frac{p\cdot q}{p^2}\, p_\lambda\right)\right.$$

$$\left. + \frac{g_T}{m_{K_A}\, m_{K^*}}\, \epsilon^{\lambda\alpha\beta\gamma}\epsilon^{\mu\alpha'\beta'\gamma}\, p_\alpha\, q_\beta\, p_{\alpha'}\, q_{\beta'}\, \epsilon_\lambda^{K_A} \epsilon_\mu^{K^*}\right]$$

No cross terms appear in this way of writing the interaction.

11. A. H. Rosenfeld et al., UCRL-8030, September 1967.

12. See, for example, footnote 9 in Glashow et al., Phys. Rev. Letters **19**, 139 (1967).
For $\left(\frac{F_K}{F_\pi}\right) \frac{}{F_+(0)} \simeq 1.28$, we obtain $\Gamma_{K^* \to K\pi} \simeq 33$ MeV, and $\Gamma_{K \to K^*\pi} \simeq 90$ MeV. We take $F_\pi \sim 137$ MeV.

Volume 26B, number 4 PHYSICS LETTERS 22 January 1968

WEINBERG SUM RULES, THE RATIO F_K/F_π AND K_A AND K^* DECAYS

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Received 11 December 1967

Using Weinberg's first spectral sum rule in SU(3) × SU(3) while the second sum rule only in SU(2) × SU(2) we calculate the ratio F_K/F_π and study the decays $K_A \to K^*\pi$ and $K^* \to K\pi$ treating pions soft in the sense $k^2 \to 0$.

In recent papers [1], the ratio between the K and π mesons decay constants for decay into lepton pair has been calculated using the two spectral function sum rules derived by Weinberg [2] and their SU(3) generalizations given by Das et al. [3]. The derivations in ref. 1 make use of both the sum rules and thus imply, in the pole approximations considered, the exact SU(3) results $m_\rho = m_{K^*}$ (and $m_{A_1} \approx m_{K_A}$). We derive here the above ratio by using the first sum rule † in SU(3) × SU(3) while the second sum rule †† is used only in the SU(2) × SU(2) subgroup. The unpleasent feature just mentioned is avoided while allowing for the successful predictions [2,3] $m_{A_1} \approx \sqrt{2} m_\rho$ and $m_{K_A} \approx \sqrt{2} m_{K^*}$. The relations thus obtained are used for the study of K^* and K_A decays. The predictions agree fairly well with the experimental data.

The spectral function sum rules lead to [2,3]

$$G_\rho^2/m_\rho^2 - G_{A_1}^2/m_{A_1}^2 = F_\pi^2$$

$$G_{K^*}^2/m_{K^*}^2 - G_{K_A}^2/m_{K_A}^2 = F_K^2 \qquad (1)$$

$$G_\rho^2/m_\rho^2 = G_{K^*}^2/m_{K^*}^2 ,$$

$$G_\rho = G_{A_1} \quad \text{and} \quad G_{K^*} = G_{K_A}$$

These relations then give

$$F_K/F_\pi = [(1+\delta)/(1+\delta_1)]^{\frac{1}{2}} \qquad (2)$$

where

$$\delta_1 = (m_{A_1}^2 - 2m_\rho^2)/m_{A_1}^2$$

and

$$\delta = (m_{K_A}^2 - 2m_{K^*}^2)/m_{K_A}^2$$

This gives $F_K/F_\pi \approx 1.07$ against the value ≈ 1.16 found in ref. 1. Also it follows

$$G_{A_1}^2/m_{A_1}^2 \approx G_{K_A}^2/m_{K_A}^2 \qquad (3)$$

and

$$m_{A_1}/m_\rho \approx m_{K_A}/m_{K^*}$$

The Cabibbo angles θ_A and θ are related through F_K/F_π by #

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† The first spectral function sum rule reads: [i, j being the SU(3) indices]

$$\int_0^\infty [\rho_V^i(\mu^2) - \rho_A^j(\mu^2)] \frac{d\mu^2}{\mu^2} = F_j^2 .$$

while the second sum rule is:

$$\int_0^\infty [\rho_V^i(\mu^2) - \rho_A^j(\mu^2)]\, d\mu^2 = 0 \qquad \text{etc.}$$

†† Similar doubts about the applicability of the second sum rule in SU(3) × SU(3) have also been raised by Sakurai [4].

We make use of the fact that $F_+(0) \approx 1$ and θ is a small angle. The author is indebted to Dr. A. Sirlin and Dr. N. Brene for a discussion on this point.

$$\frac{F_K}{F_\pi} = \frac{\tan\theta_A}{\tan\theta} \approx F_+(0)\,\frac{\sqrt{1-(F_+(0)\sin\theta)^2}}{F_+(0)\sin\theta}\tan\theta_A \qquad (4)$$

where $F_+(0)$ is the K-π form factor in K_{l3} decay normalized to unity in the exact SU(3) limit. θ_A is determined from the leptonic decays of kaon and pion while $F_+(0)\sin\theta$ is determined from K_{l3} decays. Using in eq. (4) for the right-hand side the value [7] $\approx 1.16\ F_+(0)$ we obtain $F_+(0) \approx 0.92$. On the other hand for $F_+(0) = 1$ the calculated value of $\tan\theta$ is found to be ~ 0.26 which is about 18 percent higher than the value quoted in ref. 5.

Next we calculate the $\rho\pi\pi$ and $K^*K\pi$ couplings under the assumption [e.g. 3] that the pion electromagnetic form factor is dominated by ρ pole while K^* dominates the K_{l3} decay form factor. We find ‡

$$g_{\rho\pi\pi} = \left(\frac{m_\rho}{F_\pi}\right)(1+\delta_1)^{\frac{1}{2}}$$

$$g_{K^*K\pi} = \frac{1}{2}\left(\frac{m_{K^*}}{F_\pi}\right)\left(\frac{F_\pi F_+(0)}{F_K}\right)(1+\delta)^{\frac{1}{2}} \qquad (5)$$

and

$$\frac{\Gamma_{\rho\to\pi\pi}}{\Gamma_{K^*\to K\pi}} = \frac{4}{3}\left(\frac{k^\rho}{k^{K^*}}\right)\left(\frac{1}{F_+(0)}\right)^2 \approx \frac{2.64}{(F_+(0))^2} \qquad (6)$$

With F_π value obtained from pion decay and $F_K/F_\pi F_+(0) \approx 1.16$ we find $\Gamma_{\rho\to\pi\pi} \approx 125$ MeV and $\Gamma_{K^*\to K\pi} \approx 40$ MeV which agree fairly well with the experimental results considering the uncertainties in the K_{l3} decay measurements.

The decay $K_A \to K^*\pi$ can be calculated following the method of Das et al. [8] for $A_1 \to \rho\pi$ decay: We consider the following invariant matrix elements:

$$\sqrt{4p_0k_0V^2}\,\langle\pi^0(k)|\,V_\mu(0)^1_3\,|K_A^+(p)\rangle = i e_\nu^{K_A}\,[L_1(q^2)\delta_{\mu\nu} + L_2(q^2)k_\nu(p+k)_\mu + L_3(q^2)k_\nu(p-k)_\mu] \qquad (7)$$

$$\sqrt{4p_0k_0V^2}\,\langle\pi^0(k)|A_\mu(0)^1_3|K^{*+}(p)\rangle = i e^{K^*}\,[K_1(q^2)\delta_{\mu\nu} + L_2(q^2)k_\nu(p+k)_\mu + L_3(q^2)k_\nu(p-k)_\mu] \qquad (8)$$

‡ These result are similar to those derived in refs. 6 and 7, but modified by the inclusion of A_1 and K_A pole contributions via the Weinberg sum rules.

where $e_\nu(p)$ is the polarization vector and $q = p-k$.

The conserved vector current hypothesis on eq. (7) and the condition of partial conservation of axial vector current on eq. (8) give the following relations for the form factors †:

$$L_1(q^2) + (m_{K_A}^2 - \mu^2)L_2(q^2) - L_3(q^2)q^2 = 0\,. \qquad (9)$$

$$K_1(q^2) + (m_{K^*}^2 - \mu^2)K_2(q^2) - K_3(q^2)q^2 = \frac{2F_K m_K^2 g_{K^*K\pi}}{(q^2+m_K^2)}\,. \qquad (10)$$

Assuming now the unsubtracted dispersion relations for the form factors L_2, L_3, K_2 and K_3 while once- subtracted dispersion relation for L_1 and K_1, we can write

$$L_1(q^2) = \frac{G_{K_A}}{\sqrt{2}F_\pi} + \frac{(q^2+m_{K_A}^2)}{\pi}\int\frac{\mathrm{Im}\,L_1(q'^2)}{(q'^2-q^2)(q'^2+m_{K_A}^2)}\,dq'^2 \qquad (11)$$

$$K_2(q^2) = \frac{1}{\pi}\int\frac{\mathrm{Im}\,K_2(q'^2)}{(q'^2-q^2)}\,dq'^2 \qquad \text{etc.}$$

A justification [3,9] for the once-subtracted dispersion relation for the latter case may be found by applying PCAC and current algebra to the corresponding invariant matrix elements and neglecting only the k^2 dependent terms.

In the approximation that $L(q^2)_{1,2,3}$ are dominated by K^* pole while $K(q^2)_{1,2,3}$ are dominated by the K and K_A poles we obtain

† The various constants appearing here and latter are defined as follows:

$$\sqrt{2k_0V}\,\langle 0|A_\mu(0)^1_3|K^+(k)\rangle = iF_K k_\mu$$

$$\sqrt{4p_0q_0V^2}\,\langle K^+(q)|J_{\pi^0}(0)|K^{*+}(p)\rangle = g_{K^*K\pi}\,e^{K^*}\cdot(q-k)$$

$$\sqrt{2p_0V}\,\langle 0|V_\mu(0)^1_3|K^{*+}(p)\rangle = G_{K^*}e_\mu^{K^*}(p)$$

$$\sqrt{2p_0V}\,\langle 0|A_\mu(0)^1_3|K_A^+(p)\rangle = G_{K_A}e_\mu^{K_A}(p)$$

$$\sqrt{4p_0q_0V^2}\,\langle K^{*+}(q)|J_{\pi^0}(0)|K_A^+(p)\rangle = i[G_S e^{K_A}\cdot e^{K^*} + G_D e^{K_A}\cdot q\, e^{K^*}\cdot p]$$

$$L_1(q^2) = \frac{G_{K_A}}{\sqrt{2}F_\pi} - \frac{(q^2+m_{K_A}^2)}{(q^2+m_{K^*}^2)} \frac{G_{K^*}G_S}{(m_{K_A}^2-m_{K^*}^2)}$$

$$L_2(q^2) = \frac{G_{K^*}G_D}{2(q^2+m_{K^*}^2)} \qquad (12)$$

$$L_3(q^2 = \frac{G_{K^*}}{m_{K^*}^2(q^2+m_{K^*}^2)} [G_S - \tfrac{1}{2}(m_{K_A}^2 - \mu^2 G_D)]$$

$$K_1(q^2) = \frac{G_{K^*}}{\sqrt{2}F_\pi} + \frac{(q^2+m_{K^*}^2)}{(q^2+m_{K_A}^2)} \frac{G_{K_A}G_S}{(m_{K^*}^2-m_{K_A}^2)},$$

$$K_2(q^2) = -\frac{G_{K_A}G_D}{2(q^2+m_{K_A}^2)} \qquad (13)$$

$$K_3(q^2) = \frac{2F_K g_{K^*K\pi}}{(q^2+m_K^2)} +$$

$$- \frac{G_{K_A}}{m_{K_A}^2(q^2+m_{K_A}^2)} [G_S - \tfrac{1}{2}(m_{K^*}^2-\mu^2)G_D]$$

The relations (9) and (10) then give:

$$G_S - \tfrac{1}{2}(m_{K_A}^2 - m_{K^*}^2)G_D = \qquad (14)$$

$$= \frac{1}{\sqrt{2}F_\pi} \frac{G_{K_A}}{G_{K^*}} \left(\frac{m_{K_A}^2 - m_{K^*}^2}{m_{K_A}^2}\right) m_{K^*}^2$$

$$G_S + \tfrac{1}{2}(m_{K_A}^2 - m_{K^*}^2)G_D = \qquad (15)$$

$$= \left(\frac{1}{\sqrt{2}F_\pi} \frac{G_{K^*}}{G_{K_A}} - \frac{2F_K g_{K^*K\pi}}{G_{K_A}}\right) \left(\frac{m_{K_A}^2 - m_{K^*}^2}{m_{K^*}^2}\right) m_{K_A}^2$$

where we neglect terms of order $(\mu/m_{K^*})^2$. Making use of eqs. (1), (2) and (5) we find:

$$G_S = \frac{(1+\delta)}{2\sqrt{2}F_\pi} \frac{m_{K^*}^2}{2} \left(1 + \frac{4\Delta}{(1-\delta)^2}\right)$$

$$G_D = \frac{(1-\delta)}{2\sqrt{2}F_\pi} \left(-1 + \frac{4\Delta}{(1-\delta)^2}\right) \qquad (16)$$

where

$$\Delta = [1 - (1+\delta)F_\pi F_+(0)/F_K]$$

The decay width is given by (k being the c.m. momentum):

$$\Gamma_{K_A \to K^*\pi} = \frac{1}{8\pi}\left(\frac{k}{m_{K_A}^2}\right) \left[G_S^2\left(3 + \frac{k^2}{m_{K^*}^2}\right) + \right. \qquad (17)$$

$$\left. + G_D^2 \frac{m_{K_A}^2}{m_{K^*}^2} k^4 - 2G_S G_D \frac{m_{K_A}}{m_{K^*}} k^2 \sqrt{1 + \frac{k^2}{m_{K^*}^2}}\right]$$

For $\delta \approx 0.07$, F_π obtained from pion decay and $F_K/F_\pi F_+(0) \approx 1.16$ we find $\Gamma_{K_A \to K^*\pi} \approx 58$ MeV, consistent with the present experiments. We also find $m_{K_A}^2(G_D/G_S) \approx -1.8$ implying an appreciable admixture of d- wave. This result may be tested in future experiments.

Finally, if we assume $m_A = \sqrt{2}m_\rho$, $m_{K_A} = \sqrt{2}m_{K^*}$ and $\Delta = 0$ the widths of $\rho \to \pi\pi$ and $K^* \to K\pi$ decays are only slightly affected. However, the partial widths for $A_1 \to \rho\pi$ decay now turns out to be ~ 1.2 times that of $K_A \to K^*\pi$ decay ($\Gamma_{A_1 \to \rho\pi} \approx 41$ MeV and $\Gamma_{K_A \to K^*\pi} \approx 34$ MeV).

The author is indebted to Prof. D. Amati for discussions and comments. He also acknowledges gratefully the hospitality extended to him at C.E.R.N. and to the Conselho Nacional de Pesquisas of Brasil for a fellowship.

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III. ÁLGEBRA DAS CORRENTES E FATORES DE FORMA NO DECAIMENTO $K_{\ell 3}$

III. ÁLGEBRA DAS CORRENTES E FATORES DE FORMA NO DECAIMENTO K_{ℓ_3}

Prem P. Srivastava, Nuclear Physics B7, 220 (1968)

RESUMO:

Os fatores de forma no decaimento K_{ℓ_3} são discutidos por um método modificado da álgebra das correntes. Postulamos relações de dispersão sem subtrações para os fatores de forma nos elementos de matrizes entre estados de uma partícula e o vácuo dos comutadores retardados entre as correntes ou entre correntes e divergências das correntes, mantendo fixo uma combinação linear (arbitrária) dos dois momentos transferidos. Os diversos fatores de forma (envolvendo sòmente um momento transferido) na discussão abaixo supõe-se que satisfaçam relações de dispersão com, no máximo, uma constante de subtração e a parte não constante é calculada na aproximação de dominância pelos polos.

A técnica é aplicada, primeiro, ao caso simples do fator de forma no decaimento π_{ℓ_3} e o resultado $f(0) = \sqrt{2}$ é re-obtida como se espera pela hipótese de conservação da corrente vetorial.

No caso dos fatores de forma do K_{ℓ_3} nos permitimos a não conservação da corrente vetorial de estranheza e admitimos a existência de um meson $\varkappa$ sendo êste uma partícula scalar, isospinor, e carregando estranheza.

Os fatores de forma $F_{\pm}$ e $f_{\pm}$ nos elementos de matrizes envolvendo $K^- - \pi^0$ e $\varkappa^+ - \pi^0$, respectivamente, são estudados ao mesmo tempo. Conclui-se que os $f_-(q^2)$ e $F_-(q^2)$ satisfazem relações de dispersão sem subtrações, enquanto que obtemos para $f_+(0)$ e $F_+(0)$ as seguin-

tes expressões:

$$\sqrt{2}\, F_{+}(0) = (F_K^{\ 2} + F_{\pi}^{\ 2} - F_{\varkappa}^{\ 2})/2\, F_K\, F_{\pi}$$

$$\sqrt{2}\, f_{+}(0) = (F_K^{\ 2} - F_{\pi}^{\ 2} - F_{\varkappa}^{\ 2})/2\, F\ \ F_{\pi}$$

Pondo $F_K/(\sqrt{2}\, F_{\pi}\, F_{+}(0)) \simeq 1,28$ o valor obtido pelas experiências sôbre decaimento K_{ℓ_3} e $F_{\varkappa}^2 \simeq 2(F_K^2 - F_{\pi}^2\ )$ decorrendo das relações de soma de Weinberg calculamos $(F_K/F_{\pi}\ )^2 \simeq 1.17$, $(F_{\varkappa}\ /F_{\pi}\ )^2 \simeq 0,34$, $\sqrt{2}\, F_{+}(0) \simeq 0,85$ e $\sqrt{2}\, f_{+}(0) \simeq -1/4\ (F_{\varkappa}\ /F_{\pi}\ ) \simeq -0,15$.

É interessante ressaltar que, para obtermos êstes resultados, não utilizamos a hipótese de conservação da corrente axial. De fato o fator de forma associado ao elemento de matriz

$$\sqrt{4 q_o p_o V^2}\cdot \langle K^-(q)|\, \partial^{\nu}(A_{\nu}(0)_1^1 - A_{\nu}(0)_2^2)\, |\varkappa^-(p)\rangle$$

poderia satisfazer uma relação de dispersão como subtração. Entretanto, se impuzermos que êle satisfaça uma relação de dispersão sem subtração podemos calcular o acoplamento $\varkappa \pi K$ como sendo:

$$G_{\varkappa^+\pi^0 K^+} \simeq \frac{3}{4}\ \frac{F_{\varkappa}}{F_K}\ \frac{(m_{\varkappa}^{\ 2} - m_K^{\ 2})}{\sqrt{2}\, F_{\pi}}$$

e o valor da razão $\xi(0) = F_-(0)/F_+(0)$ como:

$$\xi(0) \simeq 11,58\lambda + 0,28\left(1 - \frac{m_K^{\ 2}}{m_{\varkappa}^{\ 2}}\right)$$

Podemos demonstrar também

$$\xi(0) < \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2}\right)\lambda + \frac{(F_K^2 + F_\kappa^2 - F_\pi^2)}{2\sqrt{2}\, F_\pi F_K F_+(0)}$$

onde

$$F_+(q^2) \simeq F_+(0)\left(1 - \lambda \frac{q^2}{m_\pi^2}\right)$$

As consequências destas relações são discutidas no trabalho.

8.C.4 Nuclear Physics B7 (1968) 220-226, North-Holland Publ. Comp., Amsterdam

$K_{\ell 3}$ DECAY FORM FACTORS AND CURRENT ALGEBRA

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Received 1 July 1968

Abstract: The form factors in $K_{\ell 3}$ decay are discussed in a current algebra approach by writing unsubtracted dispersion relations for the matrix elements of suitable retarded commutators between single particle states and vacuum, at (arbitrary) fixed linear combination of the two momentum transfers. This allows us to include the contributions of all the relevant poles and a consistent solution of the current algebra equations is then possible.

Recently, new techniques [1] of applying consistently current algebra method (with pole dominance approximation) have been suggested in order to remedy the failure of current algebra method, in its earlier form, for strong decays [2]. The calculations on A_1 decay [1, 3] and K_A decay [4], admitting that some of the form factors satisfy a subtracted dispersion relation, predict reasonable widths consistent with experimental data.

In this paper we study the $K_{\ell 3}$ decay form factors (including the contribution of kappa meson) following closely a procedure of Brown and West [1]. They calculated the A_1 decay by assuming dispersion relations for vertex functions with an appropriate fixed invariant, so as to include the poles in all the variables. Their approach gives the same results as those obtained by Schnitzer and Weinberg [1] using Ward identities derived from the current algebras. We will assume in what follows that the form factors are at most once subtracted and the non-constant part is calculated in pole dominance approximation. We illustrate the method by first considering the simple case of $\pi_{\ell 3}$ decay form factor.

The $\pi_{\ell 3}$ decay form factor: We start by considering the matrix element

$$S_\mu = i\sqrt{2k_0 V}\int d^4x\, e^{ip\cdot x}\,\theta(-x_0)\langle\pi^0(k)|[\partial_\nu A_\nu(0)^1_2,\, V_\mu(x)^2_1]|0\rangle$$

$$= (k-q)_\mu F_1(q^2, p^2) + (k+q)_\mu F_2(q^2, p^2) \tag{1}$$

where $p_\mu = k_\mu + q_\mu$ and $k^2 = -m_\pi^2$.

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Then

$$p_\mu S_\mu = C - D, \tag{2}$$

with

$$C = \sqrt{2k_0 V} \int d^4x\, e^{ip\cdot x}\, \delta(x_0) \langle \pi^0(k) | [\partial_\nu A_\nu(0)^1_2, V_0(x)^2_1] | 0 \rangle, \tag{3}$$

and

$$D = \sqrt{2k_0 V} \int d^4x\, e^{ip\cdot x}\, \theta(-x_0) \langle \pi^0(k) | [\partial_\nu A_\nu(0)^1_2, \partial_\mu V_\mu(x)^2_1] | 0 \rangle. \tag{4}$$

Here C is independent of p if the equal-time commutator is assumed to be a local operator.

We postulate now * that $F_{1,2}$ and D satisfy ** an unsubtracted dispersion relation for fixed μ, where

$$\mu = \alpha q^2 + (1 - \alpha) p^2, \tag{5}$$

and α $(0 < \alpha < 1)$ is a fixed constant, and $F_{1,2}$ and D are evaluated in pole dominant approximation. Eq. (2) then leads to the relation:

$$F_\pi m_\pi^2 f(p_1^2) + \tfrac{1}{2} G_\rho\, \beta(q_1^2) \left[-\left(\frac{1-\alpha}{\alpha}\right) + \frac{q_1^2 + m_\pi^2}{m_\rho^2} \right] = -C, \tag{6}$$

where the various form factors and coupling constants are defined as usual by:

$$\begin{aligned}
\sqrt{4k_0 p_0 V^2}\, \langle \pi^0(k) | V_\mu(0)^2_1 | \pi^-(p) \rangle &= f(q^2)(p+k)_\mu, \\
\sqrt{4k_0 p_0 V^2}\, \langle \pi^0(k) | \partial_\nu A_\nu(0)^1_2 | \rho^+(p) \rangle &= \beta(q^2)\, e^\rho \cdot k, \\
\sqrt{2p_0 V}\, \langle 0 | V_\mu(0)^1_2 | \rho^+(p) \rangle &= G_\rho\, e^\rho_\mu(p), \\
\sqrt{2k_0 V}\, \langle 0 | A_\mu(0)^2_1 | \pi^-(k) \rangle &= i F_\pi k_\mu
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
\mu &= \alpha q_1^2 - (1 - \alpha) m_\rho^2 \\
&= -\alpha m_\pi^2 + (1 - \alpha) p_1^2.
\end{aligned} \tag{8}$$

Eq. (6) then implies that

$$\lim_{-\mu \to \infty} [q_1^2 \beta(q_1^2)],$$

* This is equivalent to writing Feynman graphs for the vertex function. The author is indebted to Professor J.S. Bell for this remark.

** In the present case D happens to be vanishing if we assume conserved vector current hypothesis for $V_\mu(0)^1_2$.

is finite so that $\beta(q^2)$ vanishes for infinite momentum transfer, e.g., $\beta(q^2)$ satisfies unsubtracted dispersion relations. However, the subtraction constant in pion form factor $f(q^2)$ is left unspecified. To obtain information about it we consider:

$$S_{\mu\nu} = i\sqrt{2k_0 V} \int d^4x\, e^{-iq\cdot x}\, \theta(x_0) \langle \pi^0(k) | [A_\nu(x)^1_2, V_\mu(0)^2_1] | 0 \rangle . \tag{9}$$

We easily derive:

$$q_\nu S_{\mu\nu} = i\sqrt{2}\, F_\pi\, k_\mu - i S_\mu , \tag{10}$$

where we used the current-algebra commutation relation:

$$\delta(x_0)[A_0(x)^i_j, V_\mu(0)^k_l] = (\delta^i_l A_\mu(0)^k_j - \delta^k_j A_\mu(0)^i_l)\, \delta^4(x) . \tag{11}$$

With the forms of $F_{1,2}$ already derived, we can evaluate the relation (10) at $q_\mu = 0$, $q^2 = 0$, $p^2 = -m_\pi^2$. We obtain the result:

$$f(0) = \sqrt{2} , \tag{12}$$

which is expected because of the assumed charge independence and conserved current hypothesis in the simple case discussed. Next we discuss the $K_{\ell 3}$ decay form factors where now the discussion is more involved due to non-conserved vector current and the appearance of two form factors.

The $K_{\ell 3}$ decay form factors: The $K^-\pi$ form factor describing $K_{\ell 3}$ decay are defined by the matrix element:

$$\sqrt{4k_0 p_0 V^2}\, \langle \pi^0(k) | V_\mu(0)^3_1 | K^-(p) \rangle = F_+(q^2)(k+p)_\mu + F_-(q^2)(p-k)_\mu . \tag{13}$$

In the limit of exact SU(3) symmetry we have $F_K = F_\pi$, $F(q^2) = 0$ and $\sqrt{2}\, F_+(0) = 1$.

As before, we consider now:

$$S_\mu = i\sqrt{2k_0 V} \int d^4x\, e^{ip\cdot x}\, \theta(-x_0) \langle \pi^0(k) | [\partial_\nu A_\nu(0)^1_3, V_\mu(x)^3_1] | 0 \rangle$$
$$= (k-q)_\mu F_1 + (k+q)_\mu F_2 , \tag{14}$$

and obtain

$$p_\mu S_\mu = C - D , \tag{15}$$

where C and D are defined as in eqs. (3) and (4), but with index 3 replacing the index 2. The poles involved here are due to K, K* and an isospinor scalar strangeness carrying meson κ. Proceeding in the same manner as before, we obtain

$$F_\kappa[g_+(q_1^2)(m_\pi^2 - m_\kappa^2) + g_-(q_1^2)q_1^2] - F_K m_K^2[F_+(p_1^2) - \left(\frac{\alpha}{1-\alpha}\right)F_-(p_1^2)]$$

$$- \tfrac{1}{2}G_{K^*}\gamma(q_2^2)\left[-\left(\frac{1-\alpha}{\alpha}\right) + \frac{q_2^2 + m_\pi^2}{m_{K^*}^2}\right] = C. \qquad (16)$$

Here $g_\pm$ are the κ-π form factors introduced by

$$\sqrt{4k_0 p_0 V^2}\,\langle \pi^0(k)|A_\mu(0)_3^1|\kappa^+(p)\rangle = g_+(q^2)(k+p)_\mu + g_-(q^2)(p-k)_\mu, \qquad (17)$$

and

$$\sqrt{4k_0 p_0 V^2}\,\langle \pi^0(k)|\partial_\mu A_\mu(0)_3^1|K^{*+}(p)\rangle = \gamma(q^2)\, e^{K^*}\cdot k,$$

$$\sqrt{2k_0 V}\,\langle 0|A_\mu(0)_1^3|K^-(k)\rangle = iF_K k_\mu,$$

$$\sqrt{2k_0 V}\,\langle 0|V_\mu(0)_3^1|\kappa^+(k)\rangle = iF_\kappa k_\mu$$

$$\sqrt{2p_0 V}\,\langle 0|V_\mu(0)_3^1|K^{*+}(p)\rangle = G_{K^*} e_\mu^{K^*}(p), \qquad (18)$$

while

$$\mu = \alpha q_1^2 - (1-\alpha)m_\kappa^2$$

$$= \alpha q_2^2 - (1-\alpha)m_{K^*}^2$$

$$= -\alpha m_K^2 + (1-\alpha)p_1^2. \qquad (19)$$

From eq. (16) we see,

$$\lim_{-\mu\to\infty}\left[F_\kappa q_1^2 g_-(q_1^2) - \frac{G_{K^*}}{2m_{K^*}^2}q_2^2\gamma(q_2^2)\right], \qquad (20)$$

is a constant independent of α. For eq. (16) then to be satisfied for arbitrary α we must have $F_-(\infty) = 0$ and $\gamma(\infty) = 0$. Eq. (20) then leads to $g(\infty) = 0$. Thus the form factors F_-, g_- and γ satisfy unsubtracted dispersion relations.

To obtain information on the remaining form factors, we consider $S_{\mu\nu}$ defined in eq. (9) with index 2 replaced by 3, and we proceed as before to obtain the sum rule:

$$F_K F_+(0) - F_\kappa g_+(0) = \frac{F_\pi}{\sqrt{2}} \qquad (21)$$

We can obtain more sum rules by setting K or κ on the mass shell and considering retarded products involving the (pionic) current $(A_{\mu 1}^1 - A_{\mu 2}^2)$. The sum rules are:

$$\sqrt{2}\, F_\pi\, F_+(0) + \frac{F_\kappa\, g(0)}{(m_\kappa^2 - m_K^2)} = F_K , \tag{22}$$

$$\sqrt{2}\, F_\pi\, g_+(0) - \frac{F_K\, g(0)}{(m_\kappa^2 - m_K^2)} = -F_\kappa , \tag{23}$$

where

$$\sqrt{4 q_0 p_0 V^2}\, \langle K^-(q) | \partial_\nu (A_\nu(0)_1^1 - A_\nu(0)_2^2) | \kappa^-(p) \rangle = i g(k^2) \tag{24}$$

Solving eqs. (21), (22) and (23):

$$\sqrt{2}\, F_+(0) = \frac{(F_K^2 + F_\pi^2 - F_\kappa^2)}{2 F_K F_\pi} , \tag{25}$$

$$\sqrt{2}\, g_+(0) = \frac{(F_K^2 - F_\pi^2 - F_\kappa^2)}{2 F_\kappa F_\pi} , \tag{26}$$

$$g(0) = \frac{(m_\kappa^2 - m_K^2)(F_K^2 + F_\kappa^2 - F_\pi^2)}{2 F_K F_\kappa} \tag{27}$$

To these we could add the relation

$$F_\kappa^2 \approx 2(F_K^2 - F_\pi^2) , \tag{28}$$

derived from the Weinberg spectral function sum rules *.

From experiments on $K_{\ell 3}$ decays, one knows $(F_K/F_\pi)(1/\sqrt{2}\, F_+(0)) \approx 1.28$. Eqs. (25) and (28) then give ** $(F_K/F_\pi)^2 = 1.17$, $(F_\kappa/F_\pi)^2 \approx 0.34$ and $\sqrt{2} F_+(0) \approx 0.85$. We also obtain

$$\sqrt{2}\, g_+(0) \approx -0.25\, (F_\kappa/F_\pi) \approx -0.15 , \tag{29}$$

and

$$g(0) \approx \frac{3}{4} \left(\frac{F_\kappa}{F_K}\right) (m_\kappa^2 - m_K^2) . \tag{30}$$

If we assume the very successful hypothesis of partially conserved axial current (PCAC) for pion, eqs. (24) and (27) then give for the $\kappa\pi K$ coupling constant

* See ref. [5]. In deriving this sum rule, we assume $m_{A_1} \approx \sqrt{2} m_\rho$ and $m_{K_A} \approx \sqrt{2}\, m_{K^*}$.
** Same results are obtained in ref. [6], using Ward identities.

$$G_{\kappa^+\pi^0K^+} \approx \frac{3}{4}\left(\frac{F_\kappa}{F_K}\right)\frac{(m_\kappa^2 - m_K^2)}{\sqrt{2}\,F_\pi}, \tag{31}$$

which is different from the results obtained from PCVC or PCAC assumptions * for strangeness carrying currents in eqs. (13) or (17) respectively. In fact, the $\kappa\pi K$ coupling vanishes if $m_\kappa = m_K$, as well as in the exact SU(3) symmetry limit.

We can then also calculate $\xi(0)$ for $K_{\ell 3}$ decay and find:

$$\begin{aligned}\xi(0) &= F_-(0)/F_+(0)\\ &= \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2}\right)\lambda + \frac{(m_\kappa^2 - m_K^2)}{m_\kappa^2}\frac{(F_K^2 + F_\kappa^2 - F_\pi^2)}{2\sqrt{2}\,F_\pi F_K F_+(0)}\\ &\approx 11.58\lambda + 0.28\,(1 - m_K^2/m_\kappa^2),\end{aligned} \tag{32}$$

where λ is defined, for small momentum transfers, by:

$$F_+(q^2) \approx F_+(0)\left[1 + \lambda\frac{q^2}{m_\pi^2}\right]. \tag{33}$$

Hence an upper limit on the value of ξ is **

$$\xi(0) < \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2}\right)\lambda + \frac{(F_K^2 + F_\kappa^2 - F_\pi^2)}{2\sqrt{2}F_\pi F_K F_+(0)}. \tag{34}$$

With the experimental value [7] $\lambda \approx -0.023$, we find $\xi < 0.01$. Thus it excludes the possibility of a large positive value for ξ and favours the negative values found in the experiments measuring the polarization of muon in $K_{\mu 3}$ decay ($\xi_{exp} \approx -0.5 \pm 0.3$).

From eq. (32) we find $\xi \approx -0.2$ and -0.24 corresponding to $m_\kappa \approx 560$ MeV and 520 MeV, respectively. The corresponding values for the $\kappa\pi K$ coupling $G_{\kappa^+\pi^0K^+}$ are calculated to be 147 MeV and 56 MeV, respectively ***.

Finally, we remark that the unseen decay mode $K_A \to \kappa\pi$ will be suppressed compared to other modes if g_+ satisfies a subtracted dispersion relation with the subtraction constant close to that given by the right-hand side of eq. (26).

The author wishes to express his gratitude to Professors D. Amati, J. S. Bell, C. Bouchiat, S. L. Glashow, M. Jacob and J. Prentki for construc-

* Note that we do not use any such assumption in deriving the above sum rules.

** If no PCAC assumption is made no upper limit follows and instead we have $G_{\kappa^+\pi^0K^+} \approx (\xi + 0.27)\,m_\kappa^2 F_+(0)/F_\kappa$.

*** In a pole model with only pion pole contributing, the former case gives a 10^{-10} sec for the lifetime of κ^+ decaying into $K^+ + 2\gamma$. The value is very sensitive to the deviation of κ mass from K mass. For $m_\kappa \approx 615$ MeV the lifetime is $\approx 10^{-13}$ sec.

tive discussions and to Professor L. Van Hove and Professor J. Prentki for the hospitality extended to him at the CERN Theoretical Study Division. Acknowledgements are also due to Conselho Nacional de Pesquisas for a fellowship.

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IV. CÁLCULO DOS DECAIMENTOS $K_A \to K\rho$ e $K_A \to K^*\pi$ KAON E PION DUROS

IV. CÁLCULO DOS DECAIMENTOS $K_A \to K\rho$ e $K_A \to K^*\pi$ PARA KAON E PION DUROS

Prem P. Srivastava, Nuclear Physics (1969)

RESUMO:

Um método modificado da álgebra das correntes é discutido para os cálculos dos decaimentos fortes $K_A \to K\rho$ e $K_A \to K^*\pi$. A hipótese da conservação parcial da corrente axial não é usada e os pions e kaons são tratados como partículas "duras" com massas não nulas, contrariamente, à hipótese dos mesons "moles" usada, frequentemente, nos trabalhos que aplicam o método da álgebra das correntes. Postulamos relações de dispersão sem subtrações para os fatores de forma nos elementos de matrizes entre estados de uma partícula e o vácuo dos comutadores retardados entre as correntes ou entre correntes e divergências das correntes, mantendo fixo uma combinação linear (arbitrária) dos dois momentos transferidos. Isto nos permite incluir em nossos cálculos as contribuições de polos em ambos os momentos transferidos. Para os diversos fatores de forma (envolvendo sômente um momento transferido) na discussão, supõe-se que satisfaçam relações de dispersão com, no máximo, uma constante de subtração e a parte não constante é calculada na aproximação da dominância pelos polos. O método da álgebra das correntes é usado para derivar várias regras de soma entre as constantes de acoplamento e as constantes da subtração sem usar a hipótese da conservação parcial da corrente axial ou limite dos mesons "moles". A solução destas regras de soma permite-nos calcular as larguras parciais dos decaimentos e reestabelecer as regras de soma de Weinberg como condições de auto-consistência. As hipóteses feitas no trabalho (II) sô -

bre as constantes de subtração dos fatores de forma $L_i(q^2)$ e $K_i(q^2)$ agora decorrem do nosso método da álgebra das correntes.

A inconveniência do uso da hipótese da conservação parcial da corrente axial correspondente ao meson K no cálculo do decaimento $K_A \rightarrow K\rho$ não se apresenta em nosso método de cálculo.

Notas de Física - Volume XIV - Nº 8

HARD KAON AND PION CALCULATIONS OF THE DECAYS $K_A \to K\rho$ AND $K_A \to K^*\pi$

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INTRODUCTION

The current algebra method [1] along with soft pion hypothesis has been used with considerable success in variety of processes. However, there are cases where this technique does not work so well. Recently, new techniques of applying consistently current algebra method (with pole dominance approximation) have been suggested in these cases. The calculations on strong [2, 3, 4] A_1 and [5, 6] K_A decays, admiting that some of the form factors satisfy a subtracted dispersion relation predict, contrary to the earlier calculations [7] giving very large widths, reasonable widths consistent with experiments. Schnitzer and Weinberg [3] developed a technique of Ward-like identities for the vertex functions to calculate "hard" pion processes $A_1 \to \rho\pi$ and $\rho \to \pi\pi$ successfully. Brown and West [2] on the other hand assume dispersion relations for vertex functions with an appropriate fixed invariant, so as to include the poles in all the variables while admitting that the form factors are at most once subtracted. The results obtained by the two methods are identical. The same holds true in the case of the second order renormalization corrections to the $K_{\ell 3}$ form factors as calculated by [8] Glashow and Weinberg and [9] Srivastava. However, the unsubtracted dispersion relation technique seems to be rather straightforward and amounts to writing Feynman diagrams with form factors at the vertices and establishing appropriate current algebra identities. We will discuss in this paper the strong decays $K_A \rightarrow K\rho$ and $K_A \rightarrow K^*\pi$ following the procedure exposed in references 2 and 9 In what follows, we assume that the form factors are at most once subtracted and the non-constant part is calculated in the pole dominance approximation. Along with the expressions for the relevant decay rates we also re-derive the [10] Weinberg sum rules for SU(3) × SU(3) group and also illustrate that the hypothesis of partial conservation of axial current need not hold necessarily, even for pion, for every form factor. The calculated decay rates are in fair agreement with the experimental results considering the present uncertaintities in experimental data.

* * *

$K_A \rightarrow K^*\pi$ and $K^* \rightarrow K\pi$ decays:

I. K_A and K^* Matrix Elements

We introduce the following matrix elements

$$\sqrt{4k_0 p_0 V^2} < \pi^0(k) | A_\mu(0)_3^1 | K^{*+}(p) >$$
$$= ie_\nu^{K^*}(p) \left[K_1(q^2) g^{\mu\nu} + K_2(q^2)\, k^\nu (p+k)^\mu + K_3(q^2)\, k^\nu\, q^\mu \right] \quad (1)$$

$$\sqrt{4k_o p_o V^2}\,\langle \pi^o(k)|V_\mu(0)^3_1|K_A^-(p)\rangle$$

$$= i\, e_\nu^{K_A}(p)\left[L_1'(q^2)\, g^{\mu\nu} + L_2'(q^2)\, k^\nu (p+k)^\mu + L_3'(q^2)\, k^\nu q^\mu\right] \quad (2)$$

where $q_\mu = p_\mu - k_\mu$ and the form factors $K_{1,2,3}(q^2)$ and $L_{1,2,3}(q^2)$, introduced on the considerations of Lorentz covariance, are calculated in the pole dominance approximation to be:

$$K_1(q^2) = \frac{G_{K_A}\, G_S'}{(m_{K_A}^2 - q^2)} + K_1(\infty)$$

$$K_2(q^2) = -\frac{G_{K_A}\, G_D'}{2(m_{K_A}^2 - q^2)} + K_2(\infty) \quad (3)$$

$$K_3(q^2) = \frac{G_{K_A}}{m_{K_A}^2}\,\frac{\left[G_S' + \frac{1}{2}\, G_D'(m_{K^*}^2 - m_\pi^2)\right]}{(m_{K_A}^2 - q^2)} - \frac{F_K\, G_{K^{*+}\pi^o K^+}}{(m_K^2 - q^2)} + K_3(\infty)$$

and similarly:

$$L_1'(q^2) = -\frac{G_{K^*}\, G_S''}{(m_{K^*}^2 - q^2)} + L_1'(\infty)$$

$$L_2'(q^2) = \frac{G_{K^*}\, G_D''}{2(m_{K^*}^2 - q^2)} + L_2'(\infty) \quad (4)$$

$$L_3'(q^2) = -\frac{G_{K^*}}{m_{K^*}^2}\,\frac{|G_S'' + 1/2\; G_D''(m_{K_A}^2 - m_\pi^2)|}{(m_{K^*}^2 - q^2)} -$$

$$F_\varkappa\, G_{K_A^-\pi^o\varkappa^-}$$

Here K(∞), L(∞) are subtraction constants and the various coupling constants are defined by the following invariant matrix elements:

$$\sqrt{2q_o V}\ \langle 0|A_\mu(0)^1_3|K^+(q)\rangle = i\ F_K\ q_\mu$$

$$\sqrt{2q_o V}\ \langle 0|V_\mu(0)^1_3|\varkappa^+(q)\rangle = i\ F_\varkappa\ q_\mu$$

$$\sqrt{4q_o p_o V^2}\langle K^+(q)|j_{\pi^o}(0)|K^{*+}(p)\rangle = G_{K^{*+}\pi^o K^+}\ e^{K^*}(p)\cdot q$$

$$\sqrt{2q_o V}\ \langle 0|A_\mu(0)^1_3|K_A{}^+(q)\rangle = G_{K_A}\ e_\mu^{K_A}(q)$$

$$\sqrt{4q_o p_o V^2}\langle \varkappa^-(q)|j_{\pi^o}(0)|K_A^-(p)\rangle = G_{K_A^-\pi^o\varkappa^-}\ e^{K_A}(p)\cdot q$$

$$\sqrt{2q_o V}\ \langle 0|V_\mu(0)^1_3|K^{*+}(q)\rangle = G_{K^*}\ e_\mu^{K^{*+}}(q) \qquad (5)$$

$$\sqrt{4q_o p_o V^2}\langle K^{*-}(q)|j_{\pi^o}(0)|K_A^-(p)\rangle = i\left[G_S''\ e^{K_A}\cdot e^{K^{*+}} + G_D''\ e^{K_A}\cdot q\ e^{K^{*+}}\cdot p\right]$$

$$\sqrt{4q_o p_o V^2}\langle K_A^+(q)|j_{\pi^o}(0)|K^{*+}(p)\rangle = -\ i\ \left[G_S'\ e^{K_A^+}\cdot e^{K^*} + G_D'\ e^{K^*}\cdot q\ e^{K_A^+}\cdot p\right]$$

The indices on the currents are the usual SU(3) tensor indices; the coupling constants $G_{s,D}$ determine the decay rate of K_A; and $\varkappa$ is a scalar isospinor strangeness carrying meson. For the discussion below we also need the $K - \pi^o$ and $\varkappa - \pi^o$ form factors. The former are defined by:

$$\sqrt{4k_o p_o V^2}\langle \pi^o(k)|V_\mu(0)^1_3|K^+(p)\rangle = F_+(t)(p+q)_\mu + F_-(t)(p-q)_\mu \qquad (6)$$

where $t = (p-k)^2 = q^2$. In the exact SU(3) limit $F_-(q^2) = 0$ while $F_+(0) = -1/\sqrt{2}$. A similar definition is given for $\varkappa - \pi^o$ form factors $f_\pm(q^2)$, considering the matrix element of axial current.

In the pole dominance approximation we find:

$$F_+(q^2) = -\frac{G_{K^*}\,G_{K^{*+}\pi^0K^+}}{2(m_{K^*}^2 - q^2)} + F_+(\infty)$$
(7)
$$F_-(q^2) = \left(\frac{m_K^2 - m_\pi^2}{2m_{K^*}^2}\right)\frac{G_{K^*}\,G_{K^{*+}\pi^0K^+}}{(m_{K^*}^2 - q^2)} - \frac{F_\varkappa\,G_{K^+\pi^0\varkappa^+}}{(m_\varkappa^2 - q^2)}$$

where we have used the fact that the subtraction constant in $F_-(q^2)$ must be vanishing under the hypothesis of at most once subtracted dispersion relation for the matrix element of the divergence $\partial^\mu(V^1_{\mu 3})$. Here the $K\pi\varkappa$ coupling is defined by:

$$\sqrt{4q_0p_0V^2}\,\langle\varkappa^+(q)|j_{\pi^0}(0)|K^+(p)\rangle = i\,G_{K^+\pi^0\varkappa^+} \qquad (8)$$

For $\varkappa^+-\pi^0$ form factors we find:

$$f_+(q^2) = -\frac{G_{K_A}\,G_{K_A\pi^0\varkappa^+}}{2(m_{K_A}^2 - q^2)} + f_+(\infty)$$
(9)
$$f_-(q^2) = \left(\frac{m_\varkappa^2 - m_\pi^2}{2\,m_{K_A}^2}\right)\frac{G_{K_A}\,G_{K_A^+\pi^0\varkappa^+}}{(m_{K_A}^2 - q^2)} - \frac{F_K\,G_{\varkappa^+\pi^0K^+}}{(m_K^2 - q^2)}$$

* * *

II. Sum Rules from the K_A and K^* Matrix Elements of Two Currents:

We, now, set up a set of self consistent sum rules among the various subtraction constants. The solution of these equations will lead to the Weinberg sum rules and expressions for G_S and G_D to determine the decay width of K_A. We will illustrate the

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procedure by discussing the case of K^* matrix element of two currents introduced below.

Consider the retarded matrix element:

$$W^{K^*}_{\mu\nu} = i\sqrt{2p_oV}\int d^4x\, e^{ik\cdot x}\theta(x_o)\langle 0\left|\left[A_\nu(x)^1_1 - A_\nu(x)^2_2,\ A_\mu(0)^1_3\right]\right|K^{*+}(p)\rangle \tag{10}$$

$$= i\sqrt{2p_oV}\int d^4x\, e^{iq\cdot x}\,\theta(-x_o)\langle 0\left|\left[A_\nu(0)^1_1 - A_\nu(0)^2_2,\ A_\mu(x)^1_3\right]\right. K^{*+}(p)\rangle$$

Then:

$$ik^\nu W^{K^*}_{\mu\nu} = -W^{K^*}_{\mu} + iG_{K^*}\, e^{K^*}_{\mu}(p) \tag{11}$$

where

$$p = k + q\ ,$$

$$W^{K^*}_{\mu} = i\sqrt{2p_oV}\int d^4x\, e^{ik\cdot x}\theta(x_o)\langle 0\left|\left[\partial^\nu (A_\nu(x)^1_1 - A_\nu(x)^2_2),\ A_\mu(0)^1_3\right]\right|K^{*+}(p)\rangle \tag{12}$$

$$= i\sqrt{2p_oV}\int d^4x\, e^{iq\cdot x}\theta(-x_o)\langle 0\left|\left[\partial^\nu (A_\nu(0)^1_1 - A_\nu(0)^2_2),\ A_\mu(x)^1_3\right]\right|K^{*+}(p)\rangle$$

and we use the current algebra equal time commutator relation

$$\delta(x^o)\left[A_o(x)^i_j,\ A_\mu(0)^k_\ell\right] = \delta^4(x)\left[\delta^i_\ell V_\mu(0)^k_j - \delta^k_j\ V_\mu(0)^i_\ell\right] \tag{13}$$

in the second term, on the right hand side, obtained on integration by parts.

We take, now, the limit $k^\nu \rightarrow 0$ so that $k^2 \rightarrow 0$ and $q^2 \rightarrow m^2_{K^*}$. Since the poles involved in $W^{K^*}_{\mu\nu}$ are due to π^o, A^o_1, K_A and K and thus there are no poles due to zero mass in k^2 or corresponding to mass $m^2_{K^*}$ in q^2, it follows that

$$\lim_{k\rightarrow 0} k^\nu W^{K^*}_{\mu\nu} = 0 \tag{14}$$

Consequently:

$$\lim_{k \to 0} W_\mu^{K^*}(k^2 = 0,\ q^2 = m_{K^*}^2) = i\ G_{K^*}\ e_\mu^{K^*}(p) \qquad (15)$$

To apply this result we first calculate the invariant form factors appearing in $W_\mu^{K^*}(k^2, q^2)$ by assuming that they satisfy an unsubtracted dispersion relation for fixed invariant μ, where

$$\mu = \alpha\, q^2 + (1 - \alpha)k^2 \qquad (16)$$

and $\alpha\,(0 < \alpha < 1)$ is a fixed arbitrary constant. We evaluate them in pole dominant approximation. In this way we retain the pole contributions from both the variables k^2 and q^2. We find:

$$W_\mu^{K^*} = i\ e_\mu^{K^*}(p)\left(\frac{\sqrt{2}\ F_\pi\ m_\pi^2\, K_1(q_1^2)}{(m_\pi^2 - k^2)} - \frac{G_{K_A}\beta_1(k_2^2)}{(m_{K_A}^2 - q^2)}\right) \qquad (17)$$

+ terms (†) involving $e^{K^*} \cdot k$.

where

$$\mu = \alpha\, q_1^2 + (1 - \alpha)m_\pi^2 = \alpha\, m_{K_A}^2 + (1-\alpha)k_2^2 = \alpha\ \ m_k^2 + (1-\alpha)k_1^2$$

$$(18)$$

and $\beta_{1,2}(k^2)$ are defined by:

$$\sqrt{4q_o p_o V^2}\,\langle K_A^+(q)|\partial^\mu(A_\mu(0)_1^1 - A_\mu(0)_2^2)|K^{*+}(p)\rangle$$

$$= i\left[\beta_1(k^2)\ e^{K_A^+} \cdot e^{K^*} + \beta_2(k^2)\ e^{K_A^+} \cdot p\ e^{K^*} \cdot q\right] \quad . \qquad (19)$$

The pole dominant expressions for $\beta_{1,2}$ are . . .

(†) See appendix.

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$$\beta_1(k^2) = \frac{-\sqrt{2}\ F_\pi\ m_\pi^2\ G_S'}{(m_\pi^2 - k^2)} + \beta_1\ (\infty)$$

$$\beta_2(k^2) = \frac{-\sqrt{2}\ F_\pi\ m_\pi^2\ G_D'}{(m_\pi^2 - k^2)} + \beta_2\ (\infty)\ . \tag{20}$$

Hence

$$W_\mu^{K^*}(k^2, q^2) = ie_\mu^{K^*}(p)\left(\frac{\sqrt{2}\ F_\pi\ m_\pi^2\ G_{K_A} G_S'}{(m_\pi^2-k^2)(m_{K_A}^2-q^2)} + \right.$$

$$\left. + \frac{\sqrt{2}\ F_\pi\ m_\pi^2\ K_1\ (\infty)}{(m_\pi^2 - k^2)} - \frac{G_{K_A}\ \beta_1(\infty)}{(m_{K_A}^2 - q^2)}\right)$$

$$+ \text{ terms involving } (e^{K^*}\cdot k)\ . \tag{21}$$

From equation (15) we then obtain the sum rule:†

$$G_{K^*} = \sqrt{2}\ F_\pi\ K_1(\infty) + \frac{G_{K_A}(\sqrt{2}\ F_\pi\ G_S' - \beta_1(\infty))}{(m_{K_A}{}^2 - m_{K^*}^2)} \tag{22}$$

A similar sum rule obtained by considering the K_A matrix element is discussed in section (V).

* * *

III. Matrix Elements of Current Divergences

Information on $\beta_{1,2}(\infty)$ can be obtained by considering:

$$iq^\mu W_\mu^{K^*} = - W^{K^*} + C \tag{23}$$

where

$$W^{K^*} = i\sqrt{2p_oV}\int d^4x\ e^{iq\cdot x}\theta(-x_o)\langle 0|\left[\partial^{\nu}(A_{\nu}(0)^1_1 - A_{\nu}(0)^2_2), \partial^{\mu} A_{\mu}(x)^1_3\right] |K^{*+}(p)\rangle \quad (24)$$

$$= i\sqrt{2p_oV}\int d^4x\ e^{ik\cdot x}\ \theta(x_o)\langle 0|\left[\partial^{\nu}(A_{\nu}(x)^1_1 - A_{\nu}(x)^2_2), \partial^{\mu} A_{\mu}(0)^1_3\right] |K^{*+}(p)\rangle$$

and C is the other term obtained on integration by parts and it involves an equal time commutator. If this commutator is assumed to be a local operator, C is a constant. In the present case C is vanishing due to angular momentum considerations. W^{K^*} is expressed (Appendix) in the pole dominant approximation proceeding as in the case of $W^{K^*}_{\mu}$ and, finally, from equation (23) we obtain:

$$- F_{\pi}\ m_{\pi}^2\ K_2(q_1^2) + \frac{G_{K_A}}{2}\ \beta_2\ (k_2^2)\left(\frac{\alpha}{1-\alpha}\right)$$

$$- F_{\pi}\ m_{\pi}^2\ K_3(q_1^2)\left(\frac{1-\alpha}{\alpha}\right) - F_k\ m_k^2\ E_1(k_1^2)$$

$$- \frac{G_{K_A}}{m_{K_A}}\left[\beta_1(k_2^2) + \frac{1}{2}\ (m_{K^*}^2 - k_2^2)\ \beta_2\ (k_2^2)\right] = 0 \qquad (25)$$

where $E_1(k^2)$ is defined by

$$\sqrt{4q_op_oV^2}\ \langle K^+(q)|\partial^{\mu}(A_{\mu}(0)^1_1 - A_{\mu}(0)^2_2)|K^{*+}(p)\rangle = E_1(k^2)e^{K^*}(p)\cdot q \qquad (26)$$

Allowing $\mu \to \infty$ we find from equations (18) and (25), which holds for arbitrary $(0<\alpha<1)$, that

$$\beta_2(\infty) = 0, \qquad K_3(\infty) = 0 \qquad (27)$$

and

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$$- F_\pi m_\pi^2 K_2(\infty) - F_K m_K^2 E_1(\infty) - \frac{G_{K_A}}{m_{K_A}^2}\left[\beta_1(\infty) - \frac{1}{2} \lim_{k^2\to\infty} (k^2 \beta_2(k^2))\right] = 0 \qquad (28)$$

Starting from

$$S_\mu^K = i\sqrt{2q_oV}\int d^4x\, e^{-ip\cdot x}\,\theta(-x_o)\langle K^+(q)\left|\left[\partial^\nu (A_\nu(0)_1^1 - A_\nu(0)_2^2), V_\mu(x)_1^3\right]\right|0\rangle \qquad (29)$$

$$= i\sqrt{2q_oV}\int d^4x\, e^{ik\cdot x}\,\theta(x_o)\langle K^+(q)\left|\left[\partial^\nu (A_\nu(x)_1^1 - A_\nu(x)_2^2), V_\mu(0)_1^3\right]\right|0\rangle$$

and considering

$$- i\, p^\mu S_\mu^K + S^K = \text{constant} \qquad (30)$$

where

$$S^K = i\sqrt{2q_oV}\int d^4x\, e^{-ip\cdot x}\theta(-x_o)\langle K^+(q)\left|\left[\partial^\nu (A_\nu(0)_1^1 - A_\nu(0)_2^2), \partial^\mu V_\mu(x)_1^3\right]\right|0\rangle \qquad (31)$$

we can show, likewise, that

$$E_1(\infty) = 0 \quad . \qquad (32)$$

Considering

$$S_\mu^{\pi^o} = i\sqrt{2k_o V}\int d^4x\, e^{-ip\cdot x}\,\theta(-x_o)\langle \pi^o(k)\left|\left[\partial^\mu A_\mu(0)_3^1, V_\mu(x)_1^3\right]\right|0\rangle \qquad (33)$$

We are lead to:

$$E_2(\infty) = 0 \qquad (34)$$

where

$$\sqrt{4k_op_oV^2}\,\langle\pi^o(k)|\partial^\mu A_\mu(0)_3^1|K^{*+}(p)\rangle = E_2(q^2)\, e^{K^*}(p)\cdot k \qquad (35)$$

that is:

$$E_2(q^2) = - K_1(q^2) + (m_{K^*}^2 - m_\pi^2)K_2(q^2) + q^2 K_3(q^2) \qquad (36)$$

and, similarly, by considering

$$S_\mu^{\pi^0} = i\sqrt{2k_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle\pi^o(k)|\left[\partial^\mu V_\mu(0)^1_3,\ A_\mu(x)^3_1\right]|0\rangle$$

we show that

$$E_3(\infty) = 0 \tag{38}$$

where

$$\sqrt{4k_op_oV^2}\langle\pi^o(k)|\partial^\mu\ V_\mu(0)^1_3|K_A^+(p)\rangle = E_3(q^2)\ e^{K_A}(p)\cdot k \tag{39}$$

that is

$$E_3(q^2) = -L_1(q^2) + (m_{K_A}^2 - m_\pi^2)\ L_2(q^2) + q^2L_3(q^2) \tag{40}$$

We note that $E_2(\ )= 0$ leads to, again, with our assumptions, $K_3(\infty) = 0$.

* * *

IV. Sum Rules from Pion Matrix Elements of Two Currents:

Likewise we consider:

$$S_{\mu\nu}^{\pi^o} = i\sqrt{2k_oV}\int d^4x\ e^{iq\cdot x}\theta(x_o)\langle\pi^o(k)|\left[A_\nu(x)^1_3,\ V_\mu(0)^3_1\right]|0\rangle$$

$$= i\sqrt{2k_oV}\int d^4x\ e^{-ip\cdot x}\theta(-x_o)\langle\pi^o(k)|\left[A_\nu(0)^1_3, V_\mu(x)^3_1\right]|0\rangle \tag{41}$$

$$= A_1\ g_{\mu\nu} + A_2\ k_\mu k_\nu + A_3\ p_\mu p_\nu + A_4\ k_\mu p_\nu + A_5\ k_\nu p_\mu$$

and

$$i\ q^\nu\ S_{\mu\nu}^{\pi^o} + S_\mu^{\pi^o} = \frac{F_\pi}{\sqrt{2}}\cdot k_\mu \tag{42}$$

where

$$S_\mu^{\pi^o} = i\sqrt{2k_oV}\int d^4x\ e^{iq\cdot x}\ \theta(x_o)\langle\pi^o(k)|\left[\partial^\nu A_\nu(x)^1_3,\ V_\mu(0)^3_1\right]|0\rangle$$

$$= i\sqrt{2k_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle\pi^o(k)|\left[\partial^\nu A_\nu(0)^1_3,\ V_\mu(x)^3_1\right]|0\rangle \tag{43}$$

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obtained on integration by parts and using the current algebra commutation relation. We write then unsubtracted dispersion relations for fixed μ for the invariants $A_i(q^2, p^2)$. After a straightforward calculation following the procedure already explained, and using the results already obtained we find:

$$L_2'(\infty) = L_3'(\infty) = K_2(\infty) = 0$$

$$K_1(\infty) = \lim_{q^2 \to \infty} \left[q^2 K_3(q^2) \right] \tag{44}$$

$$L_1'(\infty) = \lim_{q^2 \to \infty} \left[q^2 L_3' (q^2) \right]$$

In addition we derive the following sum rules:

$$\frac{F_\pi}{\sqrt{2}} = 2 F_K F_+'(\infty) - \frac{G_{K_A}}{m_{K_A}^2} L_1'(\infty) + \frac{G_{K_A}}{m_{K_A}^2} \lim_{q^2 \to \infty} \left[q^2 L_2' (q^2) \right] \tag{45}$$

and

$$F_{\mathcal{K}} f_+(\infty) - F_K F_+' (\infty)$$

$$= -\frac{F_\pi}{\sqrt{2}} - \frac{G_{K_A}}{2m_{K_A}^2} \left[L_1'(\infty) - \lim_{q^2 \to \infty} (q^2 L_2' (q^2)) \right]$$

$$+ \frac{G_{K^*}}{2m_{K^*}^2} \left[K_1(\infty) - \lim_{q^2 \to \infty} (q^2 K_2 (q^2)) \right] \tag{46}$$

Here $F_\pm'(q^2)$ denotes the $K^- - \pi^0$ form factors defined by an expression similar to equation (6).

Using the pole dominant forms for the various form factors we can re-cast them as follows (*)

$$F_{\varkappa}\, f_+(0) - F_K\, F'_+(0) = -\frac{F_\pi}{\sqrt{2}} \tag{47}$$

and

$$F_K\, F'_+(0) + F_{\varkappa}\, f_+(0)$$

$$= -\frac{G_{K_A}\, G_{K^*}}{m^2_{K^*}\, m^2_{K_A}}\left[G'_S + \frac{1}{2} G'_D (m^2_{K_A} + m^2_{K^*} - m^2_\pi)\right] +$$

$$+ \frac{F_{\varkappa}\, G_{K_A}\, G_{K_A^- \pi^0 \varkappa^-}}{m^2_{K_A}} + \frac{F_K\, G_{K^*}\, G_{K^{*+} \pi^0 K^+}}{m^2_{K^*}} . \tag{48}$$

From equations (28), (32) and (44) we find:

$$\beta_1(\infty) = \frac{1}{2} \lim_{k^2 \to \infty} \left[k^2\, \beta_2(k^2)\right] \tag{49}$$

and from equations (20) and (27):

$$\beta_1(\infty) = \frac{1}{2} \sqrt{2}\, F_\pi\, G'_D \tag{50}$$

It is interesting to remark that according to the partial conservation of axial pion current hypothesis we should expect both $\beta_1(\infty)=\beta_2(\infty)=0$, contrary to the conclusion arrived by using our procedure.

From equations (3), (22), (44) and (50) we deduce:

(*) In arriving at this result we have been made use of SU(3) symmetric coupling relations $G_{K^{*+}\pi^0 K^+} = -G_{K^{*-}\pi^0 K^-}$, $G_{K_A^+ \pi^0 \varkappa^+} = -G_{K_A^- \pi^0 \varkappa^-}$, $G''_{S,D} = -G'_{S,D}$ etc.

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$$\frac{G_{K*}}{\sqrt{2}\, F_\pi} = F_K\, G_{K*^+\,\pi^0 K^+}$$

$$+ \frac{G_{K_A}\, m_{K*}^2}{m_{K_A}^2\,(m_{K_A}^2 - m_{K*}^2)} \left[G_s' + \frac{1}{2}\, G_D'(m_{K*}^2 - m_{K_A}^2 - m_\pi^2)\right] \qquad (51)$$

* * *

V. Sum Rules From K_A, K and $\varkappa$ Matrix Elements of Two Currents.

Weinberg Sum Rule:

Considering

$$W_{\mu\nu}^{K_A^-} = i\,\sqrt{2p_0 V} \int d^4x\; e^{ik\cdot x}\theta(x_0)\, \langle 0 \left| \left[A_\nu(x)_1^1 - A_\nu(x)_2^2,\; V_\mu(0)_1^3\right] \right| K_A^-(p)\rangle \qquad (52)$$

and proceeding as in section (II) we can obtain:

$$-\frac{G_{K_A}}{\sqrt{2}\, F_\pi} = F_\varkappa\, G_{K_A^-\,\pi^0\,\varkappa^-}$$

$$- \frac{G_{K*}\, m_{K_A}^2}{m_{K*}^2(m_{K_A}^2 - m_{K*}^2)} \left[G_s' + \frac{1}{2}\, G_D'(m_{K_A}^2 - m_{K*}^2 - m_\pi^2)\right] . \qquad (53)$$

From equations (51) and (53) we show:

$$\frac{1}{\sqrt{2}\, F_\pi} \left(\frac{G_{K*}^2}{m_{K*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2} \right)$$

$$= \frac{F_K\, G_{K*} G_{K*^+\pi^0 K^+}}{m_{K*}^2} + \frac{F_\varkappa\, G_{K_A}\, G_{K_A^-\,\pi^0 \varkappa^-}}{m_{K_A}^2} -$$

$$- \frac{G_{K*}\, G_{K_A}}{\ldots} \left[G_s' + \frac{1}{2}\, G_D'\, (m^2 + m^2 - m_\pi^2)\right] \qquad (54)$$

Using equations (4), (7), (44), (47), (48) and (54) we find:

$$\left(\frac{G_{K^*}^{\;2}}{m_{K^*}^{\;2}} - \frac{G_{K_A}^{\;2}}{m_{K_A}^{\;2}} \right)$$

$$= 2\sqrt{2}\, F_\pi F_K F_+'(0) - F_\pi^2 \qquad (55)$$

To obtain information on $F_+'(0)$ we consider

$$S_{\mu\nu}^{K} = i\sqrt{2q_oV}\int d^4x\; e^{+ik\cdot x}\,\theta(+x_o)\langle K^+(q)\big|\left[A_\nu(x)_1^1 - A_\nu(x)_2^2,\; V_\mu(0)_1^3\right]\big|0\rangle \qquad (56)$$

and proceeding as in section (II) we find

$$\lim_{k\to 0} S_\mu^K\,(k^2 = 0 \quad p^2 = m_K^2) = - F_K\, q_\mu \qquad (57)$$

which leads to [9]

$$- F_K = + \sqrt{2}\, F_\pi\; F_+(0) + \frac{F_{\varkappa}\; g(0)}{(m_{\varkappa}^2 - m_K^2)} \qquad (58)$$

where

$$\sqrt{4q_op_oV^2}\,\langle K^+(q)|\partial^\nu(A_\nu(0)_1^1 - A_\nu(0)_2^2)|\varkappa^+(p)\rangle = ig(k^2)\;. \qquad (59)$$

Considering, likewise,

$$S_{\mu\nu}^{\varkappa} = i\sqrt{2q_oV}\int d^4x\; e^{ik\cdot x}\,\theta(x_o)\langle \varkappa^+(q)\big|\left[A_\nu(x)_1^1 - A_\nu(x)_2^2,\; A_\mu(0)_1^3\right]\big|0\rangle \qquad (60)$$

we obtain

$$- F_{\varkappa} = \sqrt{2}\; F_\pi f_+(0) - \frac{F_K\; g(0)}{(m_K^2 - m_{\varkappa}^2)} \qquad (61)$$

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From equations (58) and (61):

$$F_K^2 - F_{\varkappa}^2 = \sqrt{2}\, F_{\pi}\, (F_{\varkappa}\, f_+(0) - F_K F_+(0))$$
$$= \sqrt{2}\, F_{\pi}\, (F_{\varkappa}\, f_+(0) + F_K F'_+(0)) \tag{62}$$

Combining with equation (47) we obtain:

$$\sqrt{2}\, F'_+(0) = \frac{F_K^2 + F_{\pi}^2 - F_{\varkappa}^2}{2\, F_{\pi} F_K} \tag{63}$$

$$\sqrt{2}\, f_+(0) = \frac{F_K^2 - F_{\pi}^2 - F_{\varkappa}^2}{2\, F_{\pi} F_{\varkappa}} \tag{64}$$

and then, from equation (58)

$$g(0) = \frac{(F_K^2 - F_{\pi}^2 + F_{\varkappa}^2)(m_K^2 - m_{\varkappa}^2)}{2\, F_K F_{\varkappa}} \, . \tag{65}$$

These results were already discussed in references (8) and (9) .

From equations (55) and (63) we obtain the Weinberg sum rule

$$\left(\frac{G_{K^*}^2}{m_{K^*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2} \right) = (F_K^2 - F_{\varkappa}^2) \tag{66}$$

as a self consistency constraint.

The results of equations (27) and (44) justify the assumptions made on the subtraction constants of the form factors in reference (5). The equations (51) and (53) are identical to those of the reference mentioned if $F_{\varkappa} = 0$. The decay widths in this case were already discussed and found to be in agreement with experiments. We now discuss $K_A \rightarrow K\rho$ decay made.

* * *

$K_A \rightarrow K\rho$ Decay

I. K_A and ρ Matrix Elements

The decays such as $K_A \rightarrow K\rho$, $K_A \rightarrow K\omega$ and $\varphi \rightarrow K\bar{K}$, with the large mass of kaon, can hardly be treated by soft kaon technique. We will only sketch the calculation since the technique has been already explained above. The partial decay width calculated for $K_A \rightarrow K\rho$ process is in agreement with the present experimental data.

We start by defining the form factors $G_{1,2,3}$ and $H_{1,2,3}$ appearing in the following invariant matrix elements

$$\sqrt{4k_o p_o V^2} \langle \rho^o_{\lambda}(k) | A_\nu(0)_3^1 | K^+(p) \rangle$$

$$= - i(e^\lambda_\rho(k))^+ \left[g_{\lambda\nu}\bar{G}_1(q^2) - \bar{G}_2(q^2)\, q_\lambda (p+k)_\nu - \bar{G}_3(q^2) q_\lambda (k-p)_\nu \right] \tag{67}$$

$$\sqrt{4q_o p_o V^2} \langle K_A^+(q) | V_\mu(0)_1^1 - V_\mu(0)_2^2 | K^+(p) \rangle$$

$$= - i\, (e^\lambda_{K_A}(q))^+ \left[g_{\lambda\mu}\bar{H}_1(k^2) - \bar{H}_2(k^2) k_\lambda (q+p)_\mu - \bar{H}_3(k^2) k_\lambda (q-p)_\mu \right] \tag{68}$$

In the pole dominant approximation we calculate

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$$\bar{G}_1(q^2) = -\frac{G_{K_A} \; G^S}{(m_{K_A}^2 - q^2)} + \bar{G}_1(\infty)$$

$$\bar{G}_2(q^2) = +\frac{G_{K_A} \; G^D}{2(m_{K_A}^2 - q^2)} + \bar{G}_2(\infty) \qquad (69)$$

$$\bar{G}_3(q^2) = +\frac{G_{K_A}}{m_{K_A}^2} \frac{\left[G^S + \frac{1}{2} G^D (m_{K_A}^2 - m_\rho^2)\right]}{(m_{K_A}^2 - q^2)} -$$

$$-\frac{F_K \; G'_{K^+\rho^0 K^+}}{(m_K^2 - q^2)} + \bar{G}_3(\infty)$$

and

$$\bar{H}_1(k^2) = -\frac{\sqrt{2}\; G\rho \; G^S}{(m_\rho^2 - k^2)} + \bar{H}_1(\infty)$$

$$\bar{H}_2(k^2) = \frac{\sqrt{2}\; G\rho \; G^D}{(m_\rho^2 - k^2)} + \bar{H}_2(\infty) \qquad (70)$$

$$\bar{H}_3(k^2) = \frac{\sqrt{2}\; G\rho}{m_\rho^2} \frac{\left[G^S + \frac{1}{2}(m_K^2 - m_{K_A}^2) G^D\right]}{(m_\rho^2 - k^2)} + \bar{H}_3(\infty) .$$

The various coupling constants are defined by:

$$\sqrt{2p_o V} \langle 0 \; V_\mu(0)_1^1 - V_\mu(0)_2^2 | \rho^0(p) \rangle = \sqrt{2}\; G\rho \; e_\mu^\rho(p)$$

$$\sqrt{4q_o p_o V^2} \langle K^+(q) | j_\rho^\mu(0) | K^+(p) \rangle = -\frac{G'_{K^+\rho^0 K^+}}{2} (p+q)^\mu$$

B3 $$\sqrt{4q_o p_o V^2}\,\langle K_A^+(q)|j_\rho^\mu(0)|K^+(p)\rangle = i\left(e^{K_A}(q)_\lambda\right)^\dagger \left[G^S g^{\mu\lambda} + G^D k^\lambda q^\mu\right] \qquad (71)$$

where $k = (p-q)$,

We also introduce the form factor $e(k^2)$ defined by

$$\langle K^+(q)|V_\mu(0)_1^1 - V_\mu(0)_2^2|K^+(p)\rangle = (p+q)_\mu\; e(k^2) \qquad (72)$$

and find easily:

$$e(k^2) = -\frac{\sqrt{2}\, G\rho\; G'_{K^+\rho^o K^+}}{2(m_\rho^2 - k^2)} + e(\infty)\;. \qquad (73)$$

There is only one form factor in this case if we assume that the SU(2) currents $V_{\mu j}^{i}$ $(i,\ j = 1,2)$ are conserved. This assumption also leads to

$$\overline{H}_3(\infty) = 0\;. \qquad (74)$$

* * *

II. Sum Rules from K Matrix Element of Two Currents

Following arguments similar to those in sections (IV) and (V) we can show:

$$\overline{H}_2(\infty) = \overline{G}_2(\infty) = \overline{G}_3(\infty) = 0$$

$$\overline{H}_1(\infty) = -\lim_{k^2\to\infty}\left[k^2\,\overline{H}_3(k^2)\right] \qquad (75)$$

$$\overline{G}_1(\infty) = -\lim_{k^2\to\infty}\left[k^2\,\overline{G}_3(k^2)\right]$$

Consider the retarded matrix element:

$$S_{\mu\nu}^K = i\sqrt{2p_oV}\int d^4x\, e^{ik\cdot x}\,\theta(-x_o)\,\langle 0\left|\left[A_\nu(0)_3^1,\ V_\mu(x)_1^1 - V_\mu(x)_2^2\right]\right|K^+(p)\rangle$$

$$= A_1\, g_{\mu\nu} + A_2 p_\mu p_\nu + A_3 k_\mu k_\nu + A_4 p_\mu k_\nu + A_5\, p_\nu\, k_\mu \qquad (76)$$

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We obtain after integrating by parts and using current algebra commutation relation

$$i k^{\mu} S^{K}_{\mu\nu} = - F_K \, p_{\nu} \tag{77}$$

where we have also used the conserved vector current hypothesis for $V_{\mu\,1,2}^{1,2}$. This relation leads to two sum rules:

$$e(0) = 1 \tag{78}$$

and

$$F_K = - \frac{\sqrt{2}\, G\rho \; G'_{K^+\rho^0 K^+}}{m_\rho^{\,2}}$$

$$+ \frac{\sqrt{2}\, G\rho \; G_{K_A}}{m_{K_A}^{2} \cdot m_\rho^{2}} \left[G^{S} + \frac{1}{2} (m_K^2 - m_{K_A}^2 - m_\rho^2) \; G^{D} \right] . \tag{79}$$

The result in equation (78) is already expected from the C.V.C. hypothesis used above.

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III. K_A and ρ Matrix Elements of Two Currents. Weinberg Sum Rule:

Considering the matrix elements

$$W^{\rho}_{\mu\nu} = i \sqrt{2k_0 V} \int d^4x \; e^{-ip\cdot x} \, \theta(-x_0) \langle \rho^0(k) \left| \left[A_\nu(0)^1_3 , \; A_\mu(x)^3_1 \right] \right| 0 \rangle \tag{80}$$

and

$$W^{K_A}_{\mu\nu} = i \sqrt{2q_0 V} \int d^4x \; e^{-ik\cdot x} \, \theta(-x_0) \langle K_A^+(q) \left| \left[\bar{V}_\nu(0)^1_1 - V^1_1(0)^2_2 , \right.\right.$$

$$\left.\left. A_\mu(x)^3_1 \right] \right| 0 \rangle \tag{81}$$

and the corresponding relations

$$\lim_{p_\nu \to 0} W^{\rho}_{\nu} \, (p^2 = 0 ; \; q^2 = m_\rho^2) = - \frac{iG\rho}{\sqrt{2}} \, e^{\rho}_{\nu}(k) \tag{82}$$

and

$$\lim_{p_\nu \to 0} W_\nu^{K_A}(p^2=0,\ k^2 = m_{K_A}^2) = i\ G_{K_A}\ e_\nu^{K_A}(q) \tag{83}$$

where

$$W_\nu^\rho = i\sqrt{2k_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle\rho^o(k)\left|\left[A_\nu(0)_3^1, \partial^\mu A_\mu(x)_1^3\right]\right|0\rangle \tag{84}$$

and

$$W_\nu^{K_A} = i\sqrt{2q_oV}\int d^4x\ e^{-ip\cdot x}\ \theta(-x_o)\langle K_A^+(q)\left|\left[V_\nu(0)_1^1 - V_\nu(0)_2^2, \partial^\mu A_\mu(x)_1^3\right]\right|0\rangle \tag{85}$$

we obtain the following sum rules:

$$\frac{G\rho}{\sqrt{2}} = -F_K^2\ G'_{K^+\rho^oK^+} - \frac{F_K G_{K_A}\ m_\rho^2}{m_{K_A}^2(m_{K_A}^2 - m_\rho^2)}\left[G^s - \frac{1}{2}G^D(m_\rho^2 - m_{K_A}^2 - m_K^2)\right] \tag{86}$$

and

$$-G_{K_A} = \frac{\sqrt{2}\ G\rho\ F_K\ m_{K_A}^2}{m_\rho^2(m_{K_A}^2 - m_\rho^2)}\left[G^s + \frac{1}{2}\ G^D(m_\rho^2 - m_{K_A}^2 + m_K^2)\right] \tag{87}$$

From equations (78), (86) and (87) we derive the following Weinberg sum rule:

$$\left(\frac{G_\rho^2}{m_\rho^2} - \frac{G_{K_A}^2}{m_{K_A}^2}\right) = F_K^2 \tag{88}$$

Equations (66) and (88) are derived here from entirely different point of view than was used in their original derivation in ref. (10).

We also remark that for the form factors $D_{1,2}$ defined by

$$\sqrt{4k_op_oV^2}\langle\rho^o(k)|\partial^\mu A_\mu(0)_3^1|K_A^+(p)\rangle$$
$$= i\ e_\mu^{K_A}(p)\ e_\nu^{\rho^+}(k)\left[D_1(q^2)\ g^{\mu\nu} + D_2(q^2)p^\nu k^\mu\right] \quad , \tag{89}$$

as in the case of $\beta_{1,2}$, only D_2 is unsubtracted while $D_1(\infty) \neq 0$.

PCAC hypothesis would require both of them to be unsubtracted.

We can use equations (86), (87) and the Weinberg sum rules to determine the couplings G^S and G^D and calculate the partial width for the decay $K_A \to K\rho$, which is given by:

$$\Gamma_{K_A^+ \to K\rho} = \frac{1}{8\pi}\left(\frac{k}{m_{K_A}^2}\right)\left[G_S^2\left(3+\frac{k^2}{m_\rho^2}\right)+G^{D^2}\frac{m_{K_A}^2}{m_\rho^2}k^4 - 2G^S G^D \frac{m_{K_A}}{m_\rho}k^2\sqrt{1+\frac{k^2}{m_\rho^2}}\right] \tag{90}$$

We determine F_π from decay of pion, F_K from the relation [8, 9] $(F_K/F_\pi)^2 \simeq 1.17$ and $G\rho$ and G_{K_A} from Weinberg sum rules assuming $G\rho = G_{A_1}$. If we assume that ρ couples universally so we can make use of $G'_{K^+\rho^0 K^+} = \frac{1}{2} G_{\pi^+\rho^0\pi^+}$ and the experimental decay width of ρ^0 to obtain $G'_{K^+\rho^0 K^+}$. The partial width [11] thus calculated for the decay $K_A^+ \to K\rho$ comes out to be $\simeq$ 8 MeV.

* * *

ACKNOWLEDGEMENTS:

The author is grateful for discussions with Professors M. Jacob, J. S. Bell, G. Furlan and J. Prentki and to Professors L. Van Hove and J. Prentki for the hospitality extended to him at the Theoretical Studies Division of C.E.R.N., Geneva, where major part of this work was done. Acknowledgements are also due to "Conselho Nacional de Pesquisas" of Brazil for a fellowship.

APPENDIX

The complete expressions of $W_\mu^{K^*}$ and W^{K^*} are given by:

$$W_\mu^{K^*} = i\left[e_\mu^{K^*}(p)\left(\frac{\sqrt{2}\,F_\pi\,m_\pi^2\,K_1(q_1^2)}{(m_\pi^2-k^2)} - \frac{G_{K_A}\,\beta_1\,(k_2^2)}{(m_{K_A}^2-q^2)}\right)\right.$$

$$+\, e^{K^*}.k\,(p+k)_\mu\left(\frac{\sqrt{2}\,F_\pi\,m_\pi^2\,K_2(q_1^2)}{(m_\pi^2-k^2)} + \frac{G_{K_A}\,\beta_2(k_2^2)}{2(m_{K_A}^2-q^2)}\right)$$

$$+\, e^{K^*}.k\,q_\mu\left(\frac{\sqrt{2}\,F_\pi\,m_\pi^2\,K_3(q_1^2)}{(m_\pi^2-k^2)} - \frac{F_K\,E_1(k_1^2)}{(m_K^2-q^2)}\right.$$

$$\left.\left.-\frac{G_{K_A}}{m_{K_A}^2(m_{K_A}^2-q^2)}\left\{\beta_1(k_2^2)+\frac{1}{2}\,(m_{K^*}^2-k_2^2)\,\beta_2(k_2^2)\right\}\right)\right]$$

$$W^{K^*} = (e^{K^*}.k)\left[-\frac{F_K\,m_K^2\,E_1(k_1^2)}{(m_K^2-q^2)} + \right.$$

$$\left.+\frac{F_\pi m_\pi^2}{(m_\pi^2-k^2)}\left(-K_1(q_1^2)+(m_{K^*}^2-m_\pi^2)K_2(q_1^2)+q_1^2K_3(q_1^2)\right)\right]$$

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11. In view of the lack of precise information about the two K_A resonances, we ignore mixing and assume in our discussions that K_A is a pure resonance with mass $\simeq 1320$ MeV.

V. SIMETRIA ASSINTÓTICA SU(3) E OS FATORES DE FORMA NO DECAIMENTO $K_{\ell 3}$

V. SIMETRIA ASSINOTÓTICA SU(3) E OS FATORES DE FORMA NO DECAIMENTO K_ℓ

Prem P. Srivastava, Nuovo Cimento 57A, 454 (1968)

RESÚMO:

Os fatores de forma $F_\pm(q^2)$ no decaimento K_{ℓ_3} são estudados admitindo-se que a simetria SU(3) se torna exata quando nas altas energias e momentos transferidos.

Supõe-se que os fatores de forma satisfazem relações de dispersão com, no máximo, uma constante de subtração. Conclui-se então que $F_-(q^2)$ e $(2 F_+(q^2) - f(q^2))$, sendo $f(q^2)$ o fator de forma no decaimento π_{l_3}, devem satisfazer relações de dispersão sem subtrações. Isto nos leva a seguinte relação:

$$\frac{G_{\rho^- \pi^o \pi^-}}{G_{K^{*-} \pi^o K^-}} \simeq 2{,}16 \left(\frac{m_\rho}{m_{K^*}} \right)$$

que está em bom acôrdo com a experiência. Achamos, também que

$$\xi(0) = \frac{F_-(0)}{F_+(0)} = \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2} \right) \lambda + \frac{F_\varkappa \, G_{K^- \pi^o \varkappa^-}}{F_+(0) \, m_\varkappa^2}$$

e para o caso em que $m_K < m_\varkappa < m_K + m_\pi$ o valor estimado da largura para o decaimento $\varkappa \rightarrow K + 2\gamma$ é calculado como $\simeq 10^{-11}$s a 10^{-14}s.

P. P. SRIVASTAVA
21 Settembre 1968
Il Nuovo Cimento
Serie X, Vol. 57 A, pag. 454-456

Asymptotic SU_3 and K_{l3} Decay Form Factors.

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(ricevuto il 12 Agosto 1968)

It has been suggested by GELL-MANN (1) that the SU_3 symmetry, even though badly violated due to observed large mass differences within a multiplet, may still be a good symmetry when high energies and momentum transfers are involved. We study in this paper the application of this idea to the K_{l3} and π_{l3} decay form factors when some of the form factors satisfy (once) subtracted dispersion relations. The results following from this idea are in good agreement with experiments.

The K-π form factors in K_{l3} decay are defined by the matrix element

$$\sqrt{4k_0p_0V^2}\langle\pi^0(k)|V_\mu(0)_1^2|K^-(p)\rangle = (p+k)_\mu F_+(q^2) + (p-k)_\mu F_-(q^2)\,, \tag{1}$$

where $q^2=(p-k)^2$ and in the exact SU_3 limit $F_-(q^2)=0$ while $F_+(0)=1/\sqrt{2}$. The π_{l3} decay form factor is likewise defined by

$$\sqrt{4k_0p_0V^2}\langle\pi^0(k)|V_\mu(0)_1^2|\pi^-(p)\rangle = f(q^2)(p+k)_\mu\,, \tag{2}$$

where the conserved-vector-current hypothesis for $V_{\mu 1}^2$ implies $f(0)=\sqrt{2}$.

The hypothesis of the validity of SU_3 for large momentum transfers implies that

$$2F_+(\infty)-f(\infty)=0$$

and

$$F_-(\infty)=0\,, \tag{3}$$

if $V_{\mu 3}^1$ and $V_{\mu 2}^1$ belong to the same octet. Thus F_- would satisfy an unsubtracted dispersion relation while F_+ and f may satisfy a subtracted dispersion relation. In fact, recently, in an attempt to explain the $A_{1-}\to\rho+\pi$ decay by current algebra technique and to correlate the decay $A_1\to\rho+\pi$, $\rho\to\pi+\pi$ and the $\pi^\pm-\pi^0$ mass difference, it has been suggested (2) that the pion electromagnetic form factor should satisfy a (once)

(1) M. GELL-MANN: *Phys. Rev.*, 125, 1067 (1962).
(2) H. J. SCHNITZER and S. WEINBERG: *Phys. Rev.*, 164, 1828 (1968); R. ARNOWITT, H. M. FRIEDMAN and P. NATH: *Phys. Rev. Lett.*, 19, 1085 (1967); J. SCHWINGER: *Phys. Lett.*, 24 B, 473 (1967); FAYYAZUDDIN and RIAZUDDIN: preprint EFINS 67-94 (1967).

$$(2F_+(0) - f(0)) = (1/\pi)\int_{4m_\pi^2}^{\infty} dq^2 \, \frac{2\,\mathrm{Im}\,F_+(q^2) - \mathrm{Im}\,f(q^2)}{q^2} \,. \tag{4}$$

Now, the experiments at low momentum transfer indicate [4] that the variation of the form factors (with momentum transfer) is very well described by assuming that F_+ is dominated by the K* pole while f by the ρ pole. We may thus calculate the right-hand side in eq. (4) in the pole-dominant approximation to obtain

$$\frac{2G_{K^*}G_{K^{*-}\pi^0K^-}}{2m_{K^*}^2} + \frac{G_\rho G_{\rho^-\pi^0\pi^-}}{2m_\rho^2} = 2F_+(0) - \sqrt{2} \,, \tag{5}$$

where we define

$$\begin{cases} \sqrt{2k_0 V}\,\langle 0|V_\mu(0)_1^3|K^{*-}(k)\rangle = G_{K^*}\varepsilon_\mu^{K^*}(k)\,, \\ \sqrt{4k_0 p_0}\,V^2\langle K^-(k)|j_{\pi^0}(0)|K^{*-}(p)\rangle = G_{K^*} \end{cases} \tag{6}$$

and analogous expressions for other coupling constants.

We can also derive the following relations for K* couplings, assuming once subtracted and is pole-dominated:

$$-\frac{G_{K^*}G_{K^{*-}\pi^0K^-}}{2m_{K^*}^2} \simeq +\lambda\,\frac{m_{K^*}^2}{m_\pi}\,F_+(0)\,. \tag{7}$$

Here λ is the parameter defined by

$$F_+(q^2) = F_+(0)(1 - \lambda q^2/m_\pi^2) \tag{8}$$

It is measured experimentally [4] to be $+0.023$.

We obtain from eq. (5)

$$\frac{G_\rho G_{\rho^-\pi^0\pi^-}}{2m_\rho^2} - \sqrt{2} \quad 2\left[1 - \lambda\left(\frac{m_{K^*}}{m_\pi}\right)^2\right] F_+(0)\,. \tag{9}$$

Using the relation [5] $G_\rho = \sqrt{2}\,m_\rho F_\pi$, we find that corresponding to $\sqrt{2}F_+(0) = 0.85$, the value obtained by current algebra calculation [3], the decay width for ρ comes out

(3) A similar conclusion is also reached in a current algebra approach to the K-π form factors. See, for example: S. L. Glashow and S. Weinberg: *Phys. Rev. Lett.*, 20, 224 (1968); P. P. Srivastava: CERN preprint Th-893 (1968), to be published in *Nucl. Phys.*

(4) See, for example: J. Willis: *Proceedings of the Heidelberg International Conference* (Amsterdam, 1968), p. 273.

(5) K. Kawarabayashi and M. Suzuki: *Phys. Rev. Lett.*, 16, 255 (1966); Riazuddin and Fayyazuddin: *Phys. Rev.*, 147, 1071 (1966)

to be 119 MeV to be compared with the experimental value (110÷140) MeV. The corresponding value for $G_{K^*}G_{K^{*-}\pi^0\pi^-}/2m_{K^*}^2$ is found to be $\simeq -0.57$.

From Weinberg's spectral-function sum rules ([6]) we can derive ([7])

$$G_{K^*}/G_\rho \simeq (m_{K^*}/m_\rho)\sqrt{2-(F_K/F_\pi)^2} \tag{10}$$

The eqs. (7) and (9) then lead (for ([3]) $(F_K/F_\pi)^2 = 1.17$ and $\sqrt{2}F_+(0) = 0.85$) to

$$G_{\rho^-\pi^0\pi^-}/G_{K^{*-}\pi^0K^-} = 2.16(m_\rho/m_{K^*}), \tag{11}$$

which gives a width of 41 MeV for the K* decay, in good agreement with the experimental value of 49 MeV considering the approximations involved in deriving eq. (10).

We may apply similar arguments for $F_-(q^2)$ to obtain

$$\xi(0) = F_-(0)/F_+(0) = (m_K^2 - m_\pi^2)\lambda/m_\pi^2 + F_\varkappa G_{K^-\pi^0\varkappa^-}/F_+(0)\cdot m_\varkappa^2, \tag{12}$$

where we define

$$\left\{\begin{aligned} &\sqrt{2k_0V}\langle 0|V_\mu(0)_1^2|\varkappa^-(k)\rangle = iF_\varkappa k_\mu, \\ &\sqrt{4k_0p_0V^2}\langle K^-(k)|j_{\pi^0}(0)|\varkappa^-(p)\rangle = -iG_{K^-\pi^0\varkappa^-} \end{aligned}\right. \tag{13}$$

for a scalar isospinor strangeness-carrying meson $\varkappa$. We can make an estimate of the $\varkappa$ lifetime from eq. (12). For the interesting case of $m_K < m_\varkappa < m_K + m_\pi$ we find that with ([3]) $\sqrt{2}F_+(0)\sim 0.85$, $(F_\varkappa/F_\pi)^2 \sim 0.34$ and $m_\varkappa = 570$ MeV the lifetime of $\varkappa$ for decay into $K+2\gamma$, calculated in a pole model, is 10^{-11} s, if ([4]) $\xi = -0.5$, and it is $2\cdot 10^{-13}$ s, if the value of ξ is $+0.3$. For $m_\varkappa = 610$ MeV the values in the two cases are $1.5\cdot 10^{-13}$ s and $2.5\cdot 10^{-14}$ s respectively.

* * *

Acknowledgements are due to Conselho Nacional de Pesquisas of Brazil for a fellowship at CERN, Geneva, where part of this work was performed.

We also acknowledge gratefully discussions with Prof. M. JACOB.

([6]) S. WEINBERG: *Phys. Rev. Lett.*, 18, 507 (1967); T. DAS, V. S. MATHUR and S. OKUBO: *Phys. Rev. Lett.*, 18, 761 (1967).

([7]) In this derivation we include the influence of $\varkappa$ and assume $m_{A_1} = \sqrt{2}m_\rho$ and $m_{K_A} = \sqrt{2}m_{K^*}$.