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Neoclassical magnetic microislands in tokamaks

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Possibility of existence of neoclassical magnetic microislands (island width smaller than the ion Larmor radius) in a tokamak in the banana regime is shown. The rotation frequency of such islands is found. It is shown that for the case of positive electron temperature gradient, the bootstrap current destabilizes the microislands while the polarization current leads to their stabilization. Maximally possible neoclassical microisland width is estimated. © *2005 American Institute of Physics*.
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I. INTRODUCTION AND OVERVIEW

Magnetic islands that have a width smaller than the ion Larmor radius are usually referred to as microislands. Their existence was originally indicated by Smolyakov,[1] their properties were further analyzed in detail in Refs. 2 and 3, and the term "microisland" was proposed in Ref. 4. The interest in microislands was in particular stimulated by the idea put forward in Refs. 5 and 6 that overlapping chains of microislands destroy the equilibrium magnetic surfaces and therefore yield the anomalous electron heat transport observed in tokamaks (see also Ref. 7)

The theoretical model developed in Refs. 1–3 was based upon the assumptions that toroidal effects can be neglected and that the collisionality is strong enough so that the fluid model based upon the Braginskii hydrodynamics can be used.[8] The goal of the present paper is to investigate the microislands, taking into account toroidal effects and assuming that electrons are in the neoclassical banana regime.[9] Under these conditions, we call them the "neoclassical magnetic microislands," i.e., the small-scale analogue of nonlinear neoclassical tearing modes investigated in Refs. 10–12, and in many other papers, results of which have been systematized in Ref. 13.

The generalized Rutherford equation for the stationary microisland width in the slab geometry approximation has the form[1–3]

$$\Delta'/4 + \Delta_p = 0, \tag{1}$$

where Δ' is a matching parameter of the linear tearing mode theory and Δ_p is a contribution of the polarization current. By analogy with the large-scale island theory,[10–12] one can suggest, that in the case under consideration, i.e., for toroidal plasma geometry and plasma electrons in the banana regime, the bootstrap current contribution should also be taken into account in this equation. Thus, Eq. (1) should be replaced by

$$\Delta'/4 + \Delta_p + \Delta_{bs} = 0, \tag{2}$$

where Δ_{bs} is the bootstrap current contribution. One of the main goals of the present paper is to obtain the expression for Δ_{bs}.

In accordance with Ref. 3, one of the most important, and up to now unsolved, problems of the microisland theory is the determination of their rotation frequency. The investigation of this problem is also one of main subjects of this paper. The equivalent problem for large-scale islands was recently discussed in Ref. 14, namely, the island rotation in the presence of parallel (neoclassical) viscosity in toroidal geometry. In cylindrical geometry, the island rotation frequency is controlled by the transverse viscosity and taking it into account requires the calculation of a surface integral in the vicinity of the island separatrix, related to the profile function nonstationarity (see for details Ref. 13). Contrary to this, it has been shown in Ref. 14 that the parallel viscosity effect in toroidal geometry is volumetrical, i.e., its contribution to the island rotation frequency equation is characterized by a volume integral. This result is the starting idea of the present paper.

These considerations lead us to focus our treatment on the electron parallel viscosity and its influence on the microisland rotation. The model is based on the kinetic approach to determine the parallel dissipative viscosity in the banana regime. Therefore, it is analogous to that developed in Refs. 11 and 12 but somewhat simpler, since it is not necessary for us to take into account the totality of the kinetic effects.

In the context of the microisland rotation frequency problem, the key effect of toroidicity comes through the particle drift due to curvature and inhomogeneity of the magnetic field, leading to appearance of the magnetodrift current contribution into the electron continuity equation [Eqs. (4) and (5) in Sec. II]. This effect is normally taken into account in the theory of large-scale magnetic islands, where it gives the magnetic-well contribution to the generalized Rutherford equation for the magnetic island width (Refs. 13 and 15–17, and quoted literature in these references). Physically, the difference between our analysis and that of Refs. 15–17 is that the role of the nondissipative part of the magnetodrift current was investigated in the latter while in this paper we investigate the role of its dissipative part.

The question of profile functions is also important for the problem of microislands. As was discussed in Ref. 3, in the framework of this problem one has to deal with two profile functions. One of them, the so-called microisland profile function, designated as Λ, see Eq. (11), characterizes the dependence of the plasma density and electric field on the island magnetic surfaces, and the other one, the electron temperature profile function, designated as h_{T_e}, see Eq. (B4), characterizes the corresponding dependence of the electron temperature. Both profile functions and the island rotation frequency are also determined by dissipative effects. In Ref. 3, such effects provided by plasma diffusion and the transverse electron heat conductivity was calculated in the straight magnetic field line approximation. In our case of curvilinear magnetic field, dissipative effects related to magnetic drift of particles are proved to be more important. Thus the above-mentioned magnetodrift current substitutes the diffusive current and, instead of the standard expression for transverse heat flux,[8] we consider the so-called magnetodrift heat flux [see Eqs. (6) and (7) in the sequel]. Therefore, our main goal is to calculate the mentioned profile functions and the island rotation frequency defined by the magnetodrift effects. We use starting Eqs. (12) and (13) for the case of profile functions, and Eq. (14) for the case of rotation frequency.

The calculation of the magnetodrift effects in the banana regime requires solving the electron drift kinetic equation with allowance for both electron–electron and electron–ion collisions. We follow the approach for solving such kind of equations developed initially for the case of equilibrium plasma rotation,[18,19] and then used in the large-scale magnetic islands problem.[11,12] Presentation of the procedure is a central part of this work.

Following a procedure similar to that used in Refs. 11 and 12, the solution of the drift kinetic equation will be sought by separating the electron distribution function in the physically distinct parts. The first, designated as $\hat{g}$, see Eq. (18), is the θ-dependent part of the distribution function (θ is the poloidal angle); the second, designated as H_e [see Eqs. (17) and (C8)], does not depend on either θ or on the island cyclic variable; and the third, designated as $\tilde{h}$, see Eq. (C8), does not depend on θ but has an oscillatory dependence on the island cyclic variable, with zero average [see explanations following after Eq. (C8)]. The function $\tilde{h}$ is important, in particular, in allowing for the dissipative interaction between the trapped and circulating particles.[11,12] In this paper we neglect this interaction as well as all other effects determined by the function $\tilde{h}$.[11,12] So that of the three parts of distribution function, only two became relevant, $\hat{g}$ and H_e.

We call the function H_e the "profile distribution function." To make distinct the kinetic origin of the function H_e, we will call all the rest "profile functions" (Λ, $h_{T_e}\ldots$) as the hydrodynamic ones. The procedure to obtain the function H_e is well-known.[11,12] The zero order drift kinetic equation is expanded in a series in the frequency of the particle motion around the torus. Then an average over the poloidal angle is applied and the function H_e is obtained by requiring the orthogonality of the higher order terms in the expansion.

We characterize the magnetodrift current effect by the function $\nabla\cdot\mathbf{J}_d$. This function is an integral over particle velocities with the integrand proportional to the electron–ion collision frequency and to the electron distribution function averaged over θ [see Eq. (21)]. In contrast to this, the magnetodrift heat flux effect, characterized by $\nabla\cdot\mathbf{q}_d$, also includes the electron–electron collision contribution; thus $\nabla\cdot\mathbf{q}_d$ is represented as a sum of $\nabla\cdot\mathbf{q}_d^i$ and $\nabla\cdot\mathbf{q}_d^e$ [see Eqs. (22)–(24)].

Owing to the integral (over velocities) character of the magnetodrift effects, for our goals it is sufficient to know not exact distribution profile function H_e, but only its integral over the pitch-angle, designated as H_{e1} [see Eq. (38)] and called the pitch-angle integral of the profile distribution function.

In order to calculate the function H_{e1}, we make expansion in a series in $\varepsilon^{1/2}$, where ε is the local aspect ratio. As in Refs. 11 and 12, it is necessary to take into account both the zeroth and first terms of the series designated as $H_{e1}^{(0)}$ and $H_{e1}^{(1)}$, respectively. Thus not only the terms of the order of $\varepsilon^{1/2}$ are involved, as occurs in the calculation of the bootstrap current in the large-scale islands problem, but also the zero order terms. In this context the question arises about the necessity of allowing for the terms of order of $\varepsilon^{1/2}$, which are formally small. As will be explained later in this paper, this necessity comes from the function $H_{e1}^{(0)}$ entering the equations for $\nabla\cdot\mathbf{J}_d, \nabla\cdot\mathbf{q}_d^i$, and $\nabla\cdot\mathbf{q}_d^e$ in a sum with the pitch-angle integral of θ-dependent part of the distribution function, $\hat{H}_{e1}$. Therefore both zero order terms and first order terms of $\varepsilon^{1/2}$ are important in the expressions for $\nabla\cdot\mathbf{J}_d$ and $\nabla\cdot\mathbf{q}_d$.

A set of two coupled equations for hydrodynamic profile functions, Eqs. (69) and (70), which we call the canonical ones, are obtained from the functions $\nabla\cdot\mathbf{J}_d$ and $\nabla\cdot\mathbf{q}_d$. Solving these equations, we arrive at expressions for the first derivatives of profile functions with respect to island magnetic flux variable [see Eq. (A1)]. However, these expressions are not sufficient to determine the dependence of these functions on the island magnetic flux because the island rotation frequency enters them as a free parameter, which requires separate calculations. Therefore we call these equations the "intermediate" ones.

At this point we have all the ingredients to realize one of the main points of our program–the calculation of the magnetic island rotation frequency. We transform the starting equation for this frequency to a form that is reduced to a double integral over the magnetic surfaces and the island cyclic variable. Then, integrating by parts over the two variables, we arrive at an equation that summarizes the contributions of the separatrix and of the region outside it; the former is called the "separatrix" contribution and the latter the "integral" contribution. The hydrodynamic profile functions depend not only on the island rotation frequency, but also on

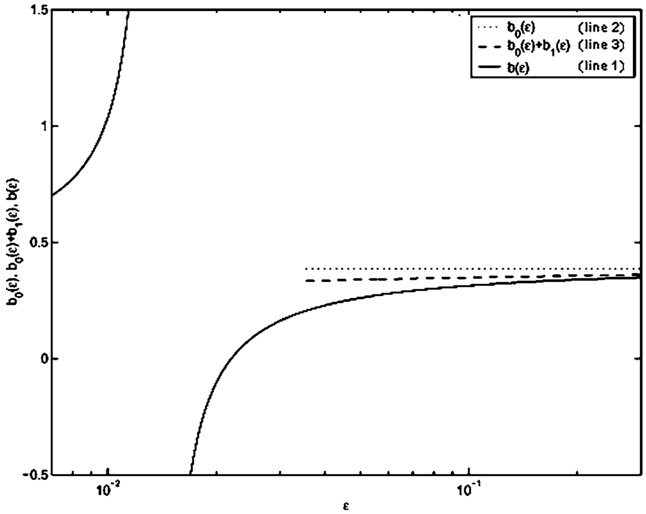

FIG. 1. Numerical and analytical lines of the function $b(\epsilon)$, characterizing the island rotation frequency.

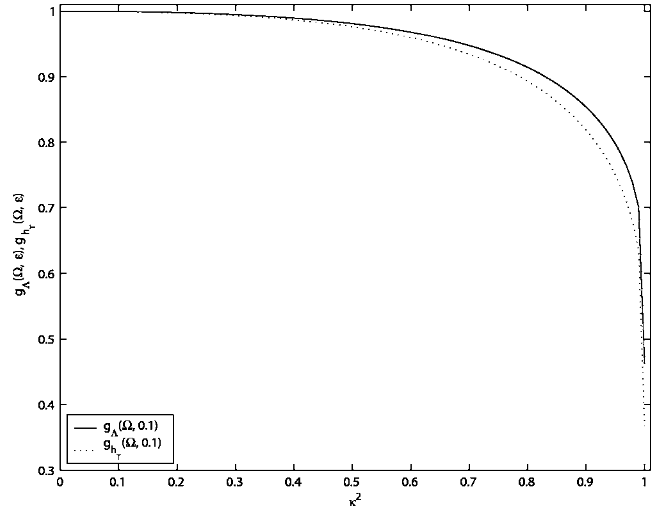

FIG. 2. The form-factors of the hydrodynamic profile functions $g_\Lambda(\Omega,\epsilon)$ and $g_{h_T}(\Omega,\epsilon)$.

the local inverse aspect ratio ε. Therefore, the island rotation frequency turns out to be the function of ε.

The "integral" term of the island rotation frequency equation, Eq. (83), is calculated numerically. The result for the island rotation frequency ω is presented in Fig. 1. The dependence of ω with ε proves to be weak enough for some interval of values of ε. This fact leads to the idea that, for the corresponding interval of the values ε, the island rotation frequency is determined only by the "separatrix" contribution, independent of ε [Eq. (85)]. Following this suggestion, we develop a successive approximation method based on expansion in a series in the ratio of the "integral" to the "separatrix" contributions. Applying this approximation, we obtain an analytical expression for the zero order island rotation frequency [Eq. (88)] which can be useful for qualitative calculations, replacing a full numerical calculation.

Substituting the obtained expression for the island rotation frequency into the aforementioned "intermediate" formulas for these derivatives of hydrodynamic profile functions, we arrive at the "final" expressions for the derivatives, Eqs. (93) and (94). These expressions differ from the ones of Rutherford obtained from the averaged diffusion and heat conduction equations (for details see Refs. 3 and 13). This can be seen in Eqs. (96)–(98) and Fig. 2.

Incorporating the bootstrap current term, Δ_{bs}, to the generalized Rutherford equation, we follow the approach frequently used for the large-scale magnetic island problem.[12,13] To calculate Δ_{bs}, one has to know bootstrap current averaged over the island magnetic surfaces, $\bar{J}_{bs}$. This current is related to the previously discussed part of the distribution profile function $H_{e1}^{(1)}$. Both the bootstrap current effect and the polarization current effect depend essentially on the island rotation frequency and hydrodynamic profile functions.

In Sec. II, we state the problem, represent the starting equations, and give their preliminary transformations. The part of the material necessary for understanding Sec. II and some following sections is placed in Appendices. This concerns relations characterizing the island magnetic field geometry (Appendix A), determination of hydrodynamic profile functions (Appendix B), and transformations and simplification of drift kinetic equation (Appendix C).

In Sec. III, we analyze the drift kinetic equation and derive expressions for profile distribution function H_e, expressions for $H_{e1}^{(0)}$ and $H_{e1}^{(1)}$, and for $\nabla\cdot\mathbf{J}_d, \nabla\cdot\mathbf{q}_d^i$, and $\nabla\cdot\mathbf{q}_d^e$. In Sec. IV, we obtain expressions for $\nabla\cdot\mathbf{J}_d$ and $\nabla\cdot\mathbf{q}_d$ in terms of hydrodynamic profile functions. Section V deals with solution of equations for hydrodynamic profile functions, and Sec. VI is addressed to derivation of the microisland rotation frequency. In Sec. VII, we present a concrete definition of hydrodynamic profile functions, allowing for the microisland rotation frequency found in Sec. VI, and compare them with the Rutherford profile functions. Section VIII deals with analysis of the generalized Rutherford equation. The results are discussed in Sec. IX.

II. THE PROBLEM STATEMENT, STARTING EQUATIONS, AND THEIR PRELIMINARY TRANSFORMATIONS

A. Starting plasmadynamical equations

As in Ref. 3, we proceed with the Boltzmann expression for the ion density

$$N = n_0(r)\exp(-e\Phi/T_{0i}). \tag{3}$$

Here N is the total plasma density, determined by $N=n_0(r)+\hat{n}$; $n_0(r)$ is the equilibrium plasma density; r is the radial coordinate; $\hat{n}$ is the perturbed plasma density; e is the ion charge; T_{0i} is the equilibrium ion temperature; and Φ is the electrostatic potential.

The electron continuity equation with allowing for effect of particle drift due to curvature and inhomogeneity of magnetic field [cf. Eq. (2) of Ref. 3] is one more starting equation

$$ed_0N/dt - (\nabla_\parallel J_\parallel + \boldsymbol{\nabla}_\perp\cdot\mathbf{J}_d) = 0. \tag{4}$$

Here $\mathbf{J}_d$ is the magnetodrift current density characterizing the mentioned effect, $J_\parallel$ is the longitudinal current density; $\nabla_\parallel$ and $\boldsymbol{\nabla}_\perp$ are longitudinal and transverse gradients. The operator d_0/dt is defined by $d_0/dt=\partial/\partial t+\mathbf{V}_E\cdot\boldsymbol{\nabla}_\perp$, where $\mathbf{V}_E=c[\mathbf{E}_\perp\times\mathbf{B}_0]/\mathbf{B}_0^2$ is the cross-field drift velocity, $\mathbf{B}_0$ is the equilibrium magnetic field, $\mathbf{E}_\perp=-\boldsymbol{\nabla}_\perp\Phi$ is the transverse electric field, c is the speed of light. It is assumed that the magnetodrift current $\mathbf{J}_d$ is averaged over the equilibrium magnetic surfaces, i.e., over the poloidal angle θ. We express it in terms of the electron distribution function f, so that

$$\mathbf{J}_d=-e\left\langle\int d^3\mathbf{v}\mathbf{v}_d f\right\rangle_\theta. \tag{5}$$

Here $\langle\ldots\rangle_\theta$ means averaging over $\theta,\mathbf{v}_d$ is the electron magnetodrift velocity, determined by (see Ref. 12) $\mathbf{v}_d=-(v_\parallel^2+\varepsilon_\perp)\mathbf{b}\times\boldsymbol{\nabla}(1/\omega_c)$, where $\varepsilon_\perp=v_\perp^2/2$, $v_\parallel$ and $v_\perp$ are the particle longitudinal velocity and the modulus of transverse velocity, $\mathbf{b}=\mathbf{B}_0/B_0$ is the unit vector along magnetic field direction $\mathbf{B}_0$, $\omega_c=-eB_0/M_ec$ is the electron cyclotron frequency, M_e is the electron mass.

We also take into account the electron magnetic drift effect in the electron heat conductivity equation. Similar to Ref. 8, we represent this equation in the form

$$(3/2)n_0\,\partial T_e/\partial t+\boldsymbol{\nabla}\cdot\mathbf{q}_d=0, \tag{6}$$

where T_e is the electron temperature, $\mathbf{q}_d$ is the electron heat flux,

$$\mathbf{q}_d=\left\langle\int d^3\mathbf{v}\left(\frac{M_ev^2}{2}-\frac{3}{2}T_e\right)\mathbf{v}_d f\right\rangle_\theta. \tag{7}$$

We also use the electron longitudinal motion equation, which in neglecting the dissipative terms and the bootstrap current has the same form as Eq. (3) of Ref. 3:

$$E_\parallel+T_{0e}\nabla_\parallel N/(en_0)=0, \tag{8}$$

where T_{0e} is the equilibrium electron temperature, $E_\parallel$ is the longitudinal electric field. In addition to Eq. (8), in Sec. II B we use a more complicated electron longitudinal motion equation, allowing for the bootstrap current (see in detail Sec. II F).

B. Transformation of magnetodrift terms in continuity and heat conductivity equations

As in Refs. 11 and 12, in addition to the ordinary radial coordinate r, we use the poloidal magnetic flux χ. Thus the equilibrium magnetic field vector $\mathbf{B}_0$ looks as $\mathbf{B}_0=I(\chi)\boldsymbol{\nabla}\phi+\boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\chi$, where $I(\chi)=RB_\phi$ is the toroidal magnetic flux, ϕ is the toroidal angle, $\boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\chi=rB_\theta\boldsymbol{\nabla}\theta$, B_ϕ and B_θ are the toroidal and the poloidal components of the field $\mathbf{B}_0$, R is the radius of the auxiliary cylindrical coordinate system R,ϕ,z, in which the equilibrium magnetic field is axisymmetric (see for details Ref. 20). We also take into account, that $\varepsilon_\perp=\mu_\perp B_0$, where $\mu_\perp$ is the transverse adiabatic invariant of the particle. Then, in accordance with Ref. 12, we obtain from Eqs. (5) and (7)

$$\boldsymbol{\nabla}\cdot\mathbf{J}_d=\frac{eI}{\omega_c}\frac{\partial}{\partial\chi}\int d^3\mathbf{v}v_\parallel\left\langle\frac{v_\parallel}{Rq}\frac{\partial f}{\partial\theta}\right\rangle_\theta, \tag{9}$$

$$\boldsymbol{\nabla}\cdot\mathbf{q}_d=-\frac{I}{\omega_c}\frac{\partial}{\partial\chi}\int d^3\mathbf{v}v_\parallel\left(\frac{M_ev^2}{2}-\frac{3}{2}T_e\right)\left\langle\frac{v_\parallel}{Rq}\frac{\partial f}{\partial\theta}\right\rangle_\theta, \tag{10}$$

where q is the safety factor, $v_\parallel=\sigma_v(v^2-2\mu_\perp B)^{1/2}$, $\sigma_v=sgn\,v_\parallel$. One can show that Eq. (10) can be expressed in terms of the electron parallel viscosity scalar (see in detail Ref. 21).

C. Starting equations for stationary hydrodynamic profile functions and island rotation frequency

Similar to Ref. 3, we introduce the electrostatic potential profile function $h(\Omega)$ and the profile functions of density and temperature, $h_n(\Omega)$ and $h_{T_e}(\Omega)$ (see Appendix B for details). We call all the totality of the mentioned profile functions the hydrodynamic profile functions. We take into account that, for the microislands problem, the profile functions $h_n(\Omega)$ and $h(\Omega)$ are intercoupled by Eq. (B7). In addition to $h(\Omega)$ and $h_n(\Omega)$, we introduce the microisland profile function $\Lambda(\Omega)$, defined by

$$\frac{\tau}{1+\tau}\left(1-\frac{\omega_{*e}}{\omega}\right)\Lambda(\Omega)=h(\Omega)-\frac{1}{1+\tau}\left(1-\frac{\omega_{*i}}{\omega}\right)\langle\hat{\chi}\rangle. \tag{11}$$

Here $\tau=T_{0i}/T_{0e}$, $\omega_{*e}=-mcT_{0e}n_0'/(eqn_0)$, $\omega_{*i}=mcT_{0i}n_0'/(eqn_0)$ are the electron and ion drift frequencies due to the density gradient. The function $\Lambda(\Omega)$ is similar to the function $\lambda(\Omega)$ of Ref. 3. Since we use the coordinate χ instead of the coordinate r, our function $\Lambda(\Omega)$ differs from the function $\lambda(\Omega)$ of Ref. 3 by the factor RB_θ. Consequently, the asymptotic value $\Lambda(\Omega)$ for $\Omega\to\infty$, $\Lambda(\Omega)\to\langle\hat{\chi}\rangle$, is also different.

Similar to Ref. 12, we conclude, that for the case of stationary islands, the profile functions $\Lambda(\Omega)$ and $h_{T_e}(\Omega)$ are defined by

$$\langle\boldsymbol{\nabla}\cdot\mathbf{J}_d\rangle=0, \tag{12}$$

$$\langle\boldsymbol{\nabla}\cdot\mathbf{q}_d\rangle=0. \tag{13}$$

The starting equation for the microisland rotation frequency, in the framework of our previous assumptions, in accordance with Ref. 12, has the following form

$$\sum_{\sigma_v}\int_{-1}^{\infty}d\Omega\oint d\xi\boldsymbol{\nabla}\cdot\mathbf{J}_d=0. \tag{14}$$

Here the lower integration limit over Ω, $\Omega=-1$, corresponds to the center of the magnetic island (see for details Appendix A). In fact, we shall integrate in Eq. (14) only outside the island separatrix, i.e., in the region $\Omega>1$.

D. Transformations of drift kinetic equation and introduction of profile distribution function

We extract from the electron distribution function f the "local-Maxwellian" F, Boltzmann (proportional to Φ) and "shifted" (proportional to $\hat{\chi}$) parts, and also the part related to profile functions h, h_n and h_{T_e} assuming $f=f_0+g$. Here

$$f_0=F+\frac{e}{T}\Phi F-\frac{e}{T}\frac{q\omega}{mc}F\left[\left(1-\frac{\hat{\omega}_{*e}}{\omega}\right)\hat{\chi}-\gamma(v,\Omega)\right], \tag{15}$$

$$\gamma(v,\Omega) = \left(1 - \frac{\omega_{*e}}{\omega}\right)\Lambda(\Omega) - \frac{\omega_{*e}\eta_e}{\omega}\left(\frac{v^2}{v_T^2} - \frac{3}{2}\right)h_{T_e}(\Omega), \tag{16}$$

$\hat{\omega}_{*e}=\omega_{*e}[1+(v^2/v_T^2-3/2)\eta_e]$, $\eta_e=\partial \ln T/\partial \ln n_0$ is the relative electron temperature gradient, $v_T=(2T/M_e)^{1/2}$ is the electron thermal velocity. The subscripts "$0e$" in the equilibrium electron temperature are omitted for simplicity. Then, in accordance with Appendix C, drift kinetic equation for g has the approximate solution

$$g = \hat{g} + \sigma_v H_e(v,\lambda,\Omega). \tag{17}$$

Here $\hat{g}$ is the θ-dependent part of the distribution function defined by

$$\hat{g} = \frac{Iv_\parallel}{\omega_c}\frac{e}{T_e}F\frac{\omega q}{mc}\left(1 - \frac{\partial\gamma}{\partial\chi}\right), \tag{18}$$

while the function $H_e=H_e(v,\lambda,\Omega)$ is an "integration constant" or θ-independent part of distribution function satisfying the equation

$$\left\langle\left\langle \frac{1}{v_\parallel}C_e(\hat{g} + \sigma_v H_e)\right\rangle_\theta\right\rangle = 0. \tag{19}$$

We call the function $H_e(v,\lambda,\Omega)$ the profile distribution function.

E. Magnetodrift effects in terms of profile distribution function

We use Eqs. (10), (11), (16), and (17), and allow for the momentum conservation law for the electron–electron collisions. We take the electron–ion collision term C_{ei} in the form

$$C_{ei}(f) = \frac{2}{B}\nu^{ei}\zeta\frac{\partial}{\partial\lambda}\left(\lambda\zeta\frac{\partial f}{\partial\lambda}\right), \tag{20}$$

where $\nu^{ei}=\nu^{ei}(v)$ is the electron–ion collision frequency, $\zeta=(1-\lambda B)^{1/2}$, $\lambda=2\mu_\perp/v^2$. The subscript "zero" in the equilibrium magnetic field is omitted for simplicity. Then we find

$$\nabla\cdot\mathbf{J}_d = -\frac{eI}{\omega_c}\frac{\partial}{\partial\chi}\int d^3\mathbf{v}v_\parallel\nu^{ei}\langle g\rangle_\theta, \tag{21}$$

$$\nabla\cdot\mathbf{q}_d = \nabla\cdot\mathbf{q}_d^i + \nabla\cdot\mathbf{q}_d^e, \tag{22}$$

where

$$\nabla\cdot\mathbf{q}_d^i = \frac{I}{\omega_c}\frac{\partial}{\partial\chi}\int d^3\mathbf{v}v_\parallel\left(\frac{M_e v^2}{2} - \frac{3}{2}T_e\right)\nu^{ei}\langle g\rangle_\theta, \tag{23}$$

$$\nabla\cdot\mathbf{q}_d^e = -\frac{IT_e}{\omega_c}\frac{\partial}{\partial\chi}\int d^3\mathbf{v}v_\parallel\Psi(v)\langle C_{ee}g\rangle_\theta, \tag{24}$$

$\Psi(v)=M_e v^2/(2T_e)-3/2$, C_{ee} is the electron-electron collision term, which will be discussed in Sec. III A.

Allowing for Eqs. (17) and (18), it is clear that in order to calculate $\nabla\cdot\mathbf{J}_d$ and $\nabla\cdot\mathbf{q}_d$ it is necessary to find H_e. Furthermore, we should know the contribution of the electron–electron collision term in Eq. (24) for the function $\nabla\cdot\mathbf{q}_d^e$.

F. Generalized Rutherford equation for microisland width allowing for bootstrap current

Recalling analysis of Ref. 12, we conclude that allowance for the bootstrap current in the generalized Rutherford equation for microisland width can be performed by the following substitution in Eq. (23) of Ref. 3 [see also Eq. (2)]

$$\Delta' \rightarrow \Delta' + 4\Delta_{bs}. \tag{25}$$

Here the value Δ_{bs} is defined by [see Eq. (5.4) of Ref. 12]

$$\Delta_{bs} = -\frac{2^{3/2}}{cs}\frac{Rq}{wB_0}\sum_{\sigma_c hi}\int_{-1}^{\infty}d\Omega\bar{J}_{bs}\oint\frac{\cos\xi}{\sqrt{\Omega+\cos\xi}}d\xi, \tag{26}$$

where $\bar{J}_{bs}\equiv\bar{J}_{bs}(\Omega)$ is the bootstrap current averaged over island magnetic surfaces and s is a parameter characterizing magnetic shear.

Concrete form of the generalized Rutherford equation, resulting from the modification mentioned above, will be presented in Sec. VIII. Note that in allowing for bootstrap current, "slab version" of electron longitudinal motion equation proves to be insufficient. To obtain "toroidal version" of this equation, taking into account the bootstrap current, one has to turn to the kinetic approach similar to that presented in Refs. 11 and 12.

III. ANALYSIS OF DRIFT KINETIC EQUATION

A. Calculation of distribution profile function

1. Equation for profile distribution function

Now we turn to Eq. (19) for the function H_e. Similar to Ref. 12, we take the electron collision operator $C_e(f)$ in the form

$$C_e(f) = C_{ee}(f) + C_{ei}(f), \tag{27}$$

where C_{ei} is given by Eq. (20) and C_{ee} means

$$C_{ee}(f) = 2\frac{1}{B}\nu^{ee}\zeta\frac{\partial}{\partial\lambda}\lambda\zeta\frac{\partial f}{\partial\lambda} + \zeta[C_e^1(f_1) + \nu^{ee}f_1]. \tag{28}$$

Here $\nu^{ee}=\bar{\nu}^{ee}G(x)/x^3$ is the scattering frequency for the electron collisions, $\bar{\nu}^{ee}$ is the mean scattering frequency for such a collision, $x=v/v_T$,

$$G(x) = \frac{1}{\pi^{1/2}x}e^{-x^2} + \left(1 - \frac{1}{2x^2}\right)\frac{2}{\pi^{1/2}}\int_0^x dte^{-t^2}, \tag{29}$$

$$f_1 = (3/2)\int_0^{\lambda_{\max}} fB_0 d\lambda, \tag{30}$$

$\lambda_{\max}$ is the maximally possible value of λ,

$$C_e^1(f_1) = -\nu_s^e f_1 + 2x\hat{r}(f_1)\nu_s^e F, \tag{31}$$

$\nu_s^e(x)$ is the electron slowing down frequency,

$$\nu_s^e(x) = \frac{2\bar{\nu}^{ee}}{x^3}\left(\frac{2}{\pi^{1/2}}\int_0^x dte^{-t^2} - \frac{2x}{\pi^{1/2}}e^{-x^2}\right), \tag{32}$$

$$\hat{r}(f_1) = \int dx x^3 f_1 \nu_s^e \Bigg/ \left(2 \int dx x^4 F \nu_s^e\right). \tag{33}$$

We call the function f_1 the pitch-angle integral (or the λ-integral) of the function f.

Allowing for Eqs. (17) and (19) we have

$$\langle \hat{g} \rangle = -\sigma_v \zeta \bar{g}, \tag{34}$$

where

$$\bar{g} = xF(A_1 + A_2 x^2), \tag{35}$$

$$A_1 = 2\frac{qR\omega}{mv_T}\left[1 - \left(1 - \frac{\omega_{*e}}{\omega}\right)\left\langle \frac{\partial \Lambda}{\partial \chi}\right\rangle - \frac{3}{2}\eta_e \frac{\omega_{*e}}{\omega}\left\langle \frac{\partial h_{T_e}}{\partial \chi}\right\rangle\right], \tag{36}$$

$$A_2 = 2\frac{qR\omega}{mv_T}\frac{\omega_{*e}\eta_e}{\omega}\left\langle \frac{\partial h_{T_e}}{\partial \chi}\right\rangle. \tag{37}$$

In addition, by analogy with Eq. (30), we introduce the function H_{e1}

$$H_{e1} = (3/2)\int_0^{\lambda_{\max}} H_e B_0 d\lambda. \tag{38}$$

We call the function H_{e1} the pitch-angle integral of the profile distribution function.

Allowing for the aforedescribed relations, we represent Eq. (19) in the form

$$\frac{1}{B}(\nu^{ee} + \nu^{ei})\frac{\partial}{\partial \lambda}\left(\lambda \langle \zeta \rangle_\theta \frac{\partial H_e}{\partial \lambda}\right) = -\frac{1}{2}N_e, \tag{39}$$

where

$$N_e = C_e^1(H_{e1}) + \nu^{ee} H_{e1} + \nu_s^e x F A_2 (x^2 - a) + \nu^{ei} x F (A_1 + A_2 x^2), \tag{40}$$

$$a = \int dx x^6 \exp(-x^2)\nu_s^e \Bigg/ \int dx x^4 \exp(-x^2)\nu_s^e = 1.75. \tag{41}$$

Similarly to Refs. 12 and 19, triple integration over λ leads to the equation for H_{e1}

$$(\nu^{ee} + \nu^{ei})(1 - c_0)H_{e1} = c_0[C_e^1(H_{e1}) + \nu_s^e x F A_2(x^2 - a)] + c_0 \nu^{ei}[-H_{e1} + xF(A_1 + A_2 x^2)], \tag{42}$$

where $c_0 = (3/4)\int_0^{\lambda_{\max}} B^2 \lambda d\lambda / \langle \zeta \rangle_\theta$. Calculating the integral over λ, we obtain (see also Ref. 12) $c_0 = 1 - 3(2\varepsilon/2)^{1/2} I_0$, where $I_0 \approx 0.69$, $\varepsilon = r/R$ is the local inverse aspect ratio.

2. Expansion in a series in $\varepsilon^{1/2}$

We look for H_{e1} as a series in $1 - c_0$ taking

$$H_{e1} = H_{e1}^{(0)} + H_{e1}^{(1)}. \tag{43}$$

In the zero order of the mentioned parameter it follows from Eq. (42) that

$$H_{e1}^{(0)} = xF(A_1 + A_2 x^2). \tag{44}$$

From the same equation we obtain equation for correction $H_{e1}^{(1)}$

$$C_e^1(H_{e1}^{(1)}) - \nu^{ei} H_{e1}^{(1)} = (1 - c_0)(\nu^{ee} + \nu^{ei})H_{e1}^{(0)}. \tag{45}$$

3. Motivation of allowing for the terms of order $\varepsilon^{1/2}$

We introduce the function $\sigma_v \hat{H}_{e1} \equiv \hat{g}_1$ related to the function $\hat{g}$ for circulating particles:

$$\sigma_v \hat{H}_{e1} = (3/2)\int_0^{\lambda_{max}} \hat{g} B_0 d\lambda. \tag{46}$$

We call the function $\hat{H}_{e1}$ the pitch-angle integral (the λ-integral) of the θ-dependent part of distribution function.

Using Eq. (18), we find the function $\hat{H}_{e1}$

$$\hat{H}_{e1} = -2x(qR\omega/mv_T)F(1 - \partial\gamma/\partial\chi), \tag{47}$$

where $\gamma(\Omega)$ is given by Eq. (16). Here we have neglected small term of order $\varepsilon^{3/2}$. It can be seen that in such a neglecting the function $\hat{H}_{e1}$ does not depend on poloidal angle θ.

In terms of the function $\hat{H}_{e1}$ and functions $H_{e1}^{(0)}$ and $H_{e1}^{(1)}$ determined by Eq. (43), Eqs. (21) and (22) mean

$$\nabla \cdot \mathbf{J}_d = -\frac{4}{3}\pi v_T^4 \frac{eI}{\omega_c}\frac{\partial}{\partial \chi}\int_0^\infty dx x^3 \nu^{ei}(\hat{H}_{e1} + H_{e1}^{(0)} + H_{e1}^{(1)}), \tag{48}$$

$$\nabla \cdot \mathbf{q}_d^i = \frac{4}{3}\pi v_T^4 T_e \frac{I}{\omega_c}\frac{\partial}{\partial \chi}\int_0^\infty dx x^3 \left(x^2 - \frac{3}{2}\right)\nu^{ei}(\hat{H}_{e1} + H_{e1}^{(0)} + H_{e1}^{(1)}). \tag{49}$$

Using the formulas of Appendix A, we have

$$\frac{\partial}{\partial \chi} = \frac{\partial \Omega}{\partial \chi}\frac{\partial}{\partial \Omega} = \frac{2^{3/2}}{w_\chi}\sqrt{\Omega + \cos\xi}\frac{\partial}{\partial \Omega}. \tag{50}$$

It follows from Eqs. (16), (44), (47), and (50) that

$$\hat{H}_{e1} + H_{e1}^{(0)} = \frac{2^{5/2}}{w_\chi}\frac{qR\omega}{mv_T}[(\Omega + \cos\xi)^{1/2} - \langle(\Omega + \cos\xi)^{1/2}\rangle] \times xF\left[\left(1 - \frac{\omega_{*e}}{\omega}\right)\frac{\partial \Lambda}{\partial \Omega} + \left(\frac{3}{2} - x^2\right)\eta_e \frac{\omega_{*e}}{\omega}\frac{\partial h_{T_e}}{\partial \Omega}\right]. \tag{51}$$

In accordance with Eq. (51), the sum $\hat{H}_{e1} + H_{e1}^{(0)}$ is proportional to the difference $(\Omega + \cos\xi)^{1/2} - \langle(\Omega + \cos\xi)^{1/2}\rangle$ vanishing for $\Omega \to \infty$. Therefore, one has to allow for the function $H_{e1}^{(1)}$ formally small as $\varepsilon^{1/2}$ in Eqs. (48) and (49).

4. Calculation of function $H_{e1}^{(1)}$

Using Eq. (31), we transform Eq. (45) to

$$H_{e1}^{(1)} = \frac{1}{\nu_s^e + \nu^{ei}}[2x\nu_s^e F\hat{r}(H_{e1}^{(1)}) - (1-c_0)(\nu^{ee}+\nu^{ei})H_{e1}^{(0)}]. \tag{52}$$

Acting on this equation by the operator $\hat{r}$ introduced by Eq. (33), we find

$$\hat{r}(H_{e1}^{(1)}) = -\frac{1-c_0}{2}\frac{\hat{r}[(\nu^{ee}+\nu^{ei})H_{e1}^{(0)}/(\nu_s^e+\nu^{ei})]}{\hat{r}[\nu^{ei}xF/(\nu_s^e+\nu^{ei})]}. \tag{53}$$

Substituting Eq. (53) into Eq. (52), we obtain

$$H_{e1}^{(1)} = -\frac{1-c_0}{\nu_s^e+\nu^{ei}}\left\{(\nu^{ee}+\nu^{ei})H_{e1}^{(0)} + \nu_s^e xF\frac{\hat{r}[(\nu^{ee}+\nu^{ei})H_{e1}^{(0)}/(\nu_s^e+\nu^{ei})]}{\hat{r}[\nu^{ei}xF/(\nu_s^e+\nu^{ei})]}\right\}. \tag{54}$$

For $H_{e1}^{(0)}$ given by Eq. (44) it means

$$H_{e1}^{(1)} = -(1-c_0)xF[a_1(x)A_1 + a_2(x)A_2], \tag{55}$$

where

$$a_1(x) = (\nu^{ee}+\nu^{ei}+\nu_s^e R_1)/(\nu_s^e+\nu^{ei}),$$
$$a_2(x) = [(\nu^{ee}+\nu^{ei})x^2 + \nu_s^e R_2]/(\nu_s^e+\nu^{ei}), \tag{56}$$

$$R_1 = \frac{\hat{r}_e[(\nu^{ee}+\nu^{ei})xF/(\nu_s^e+\nu^{ei})]}{\hat{r}_e[\nu^{ei}xF/(\nu_s^e+\nu^{ei})]} \approx 1.653,$$
$$R_2 = \frac{\hat{r}_e[(\nu^{ee}+\nu^{ei})x^3F/(\nu_s^e+\nu^{ei})]}{\hat{r}_e[\nu^{ei}xF/(\nu_s^e+\nu^{ei})]} \approx 2.609, \tag{57}$$

Using Eq. (50), we reduce Eq. (55) for the function $H_{e1}^{(1)}$ to

$$H_{e1}^{(1)} = \frac{2^{5/2}}{w_\chi}\frac{qR\omega}{mv_T}(1-c_0)\langle(\Omega+\cos\xi)^{1/2}\rangle xF\left[\left(1 - \frac{\omega_{*e}}{\omega}\right)\frac{\partial\Lambda}{\partial\Omega}a_1(x) + \eta_e\frac{\omega_{*e}}{\omega}\frac{\partial h_{T_e}}{\partial\Omega}a_3(x)\right], \tag{58}$$

where

$$a_3(x) = (3/2)a_1(x) - a_2(x). \tag{59}$$

The right-hand sides of Eqs. (48) and (49) include only the derivatives of $H_{e1}^{(1)}$ with respect to Ω, so that in the right-hand side of Eq. (58) we can omit the Ω-independent term related to corresponding term of the right-hand side of Eq. (36) for the function A_1.

Totality of Eqs. (43), (44), and (55) gives the function H_{e1} which, in accordance with Eq. (38), is an integral over λ of profile distribution function $H_e(v,\lambda,\Omega)$. Knowing the value of this integral one can calculate $\nabla\cdot\mathbf{J}_d$ and $\nabla\cdot\mathbf{q}_d^i$ with the help of Eqs. (21) and (23). However, to find $\nabla\cdot\mathbf{q}_d$ it is also necessary to calculate $\nabla\cdot\mathbf{q}_d^e$ defined by Eq. (24).

B. Calculation of $\nabla\cdot\mathbf{q}_d^e$

After integration over λ, averaging over θ, and summation over σ_v, Eq. (24) for $\nabla\cdot\mathbf{q}_d^e$ is transformed to

$$\nabla\cdot\mathbf{q}_d^e = -\frac{4}{3}\pi v_T^4\frac{IT_e}{\omega_c}\frac{\partial}{\partial\chi}\int dx x^3\Psi(x)C_e^1(\hat{H}_{e1}+H_{e1}^{(0)} + H_{e1}^{(1)}). \tag{60}$$

Using the results of Sec. III A, we have

$$C_e^1(\hat{H}_{e1}+H_{e1}^{(0)}) = \frac{2^{5/2}}{w_\chi}\frac{qR\omega_{*e}\eta_e}{mv_T}\frac{\partial h_{T_e}}{\partial\Omega}\nu_s^e xF(x^2-a)[(\Omega + \cos\xi)^{1/2} - \langle(\Omega+\cos\xi)^{1/2}\rangle], \tag{61}$$

$$C_e^1(H_{e1}^{(1)}) = (1-c_0)(\nu^{ee}+\nu^{ei})H_{e1}^{(0)} + \nu^{ei}H_{e1}^{(1)}. \tag{62}$$

Equations (3.34)–(3.36) together with Eqs. (44) and (55) determine $\nabla\cdot\mathbf{q}_d^e$.

IV. MAGNETODRIFT EFFECTS IN TERMS OF HYDRODYNAMIC PROFILE FUNCTIONS

We are interested in the magnetodrift effects only in the region outside the island separatrix. Then, in accordance with Eq. (A2), we have

$$\langle(\Omega+\cos\xi)^{1/2}\rangle = 1/\alpha_{-1}(\Omega), \tag{63}$$

where $\alpha_{-1}(\Omega)=2^{1/2}\kappa K(\kappa)/\pi$, $\kappa=[2/(\Omega+1)]^{1/2}$, $K(\kappa)$ is the complete elliptical integral of the first kind. Note also that $\alpha_{-1}\equiv S(\Omega)$, where the function $S(\Omega)$ has been introduced in Appendix A.

Using Eqs. (51), (58), (61), and (62) we integrate over x in Eqs. (48), (49), and (60). Then we obtain

$$\nabla\cdot\mathbf{J}_d = -\frac{2^6}{3}\pi^{-1/2}i_1^J\frac{eI}{\omega_c}\frac{\bar{\nu}^{ee}qRn_0}{mw_\chi^2}(\omega-\omega_{*e})\sqrt{\Omega+\cos\xi} \times[(\sqrt{\Omega+\cos\xi}-\alpha_{-1}^{-1})(\Lambda'+k_1^J\hat{h}'_{T_e}) + \mu_J\alpha_{-1}^{-1}(\Lambda'+k_2^J\hat{h}'_{T_e})]', \tag{64}$$

$$\nabla\cdot\mathbf{q}_d = \frac{2^6}{3}\pi^{-1/2}i_1^q\frac{I}{\omega_c}T_e\frac{\bar{\nu}^{ee}qRn_0}{mw_\chi^2}(\omega-\omega_{*e})\sqrt{\Omega+\cos\xi} \times[(\sqrt{\Omega+\cos\xi}-\alpha_{-1}^{-1})(\Lambda'+k_1^q\hat{h}'_{T_e}) + \mu_q\alpha_{-1}^{-1}(\Lambda'+k_2^q\hat{h}'_{T_e})]'. \tag{65}$$

Here $(\ldots)'\equiv\partial/\partial\Omega$, $\hat{h}_{T_e}=\omega_{*e}\eta_e h_{T_e}/(\omega-\omega_{*e})$, $k_1^J=-i_2^J/i_1^J=0.5$, $k_1^q=-i_2^q/i_1^q=4.445$, where

$$i_1^J = \int_0^\infty dx x e^{-x^2} = \frac{1}{2}, \quad i_2^J = i_1^q = \int_0^\infty dx\left(x^2-\frac{3}{2}\right)xe^{-x^2} = -\frac{1}{4},$$
$$i_2^q = \int_0^\infty dx\left(x^2-\frac{3}{2}\right)\left[x^2-\frac{3}{2}+\frac{\nu_s^e}{\nu^{ei}}(x^2-a)\right]xe^{-x^2} = 1.111. \tag{66}$$

The coefficients μ_J and μ_q are equal to $\mu_J=(1-c_0)I_1^J/i_1^J \approx 2.24\varepsilon^{1/2}$, $\mu_q=(1-c_0)I_1^q/i_1^q\approx 1.73\varepsilon^{1/2}$, where

$$I_1^J=\int_0^\infty dx a_1(x) x e^{-x^2}\approx 0.766,$$

$$I_1^q=\int_0^\infty dx\left(x^2-\frac{3}{2}\right)\hat{a}_1(x) x e^{-x^2}\approx -0.296, \tag{67}$$

$$\hat{a}_1(x)=(\nu^{ee}+\nu^{ei})/\nu^{ei}.$$

These coefficients characterize the contributions of the function $H_{e1}^{(1)}$. The coefficients k_2^J and k_2^q entering the expressions for these contributions are defined by $k_2^J=-I_2^J/I_1^J\approx 0.386$, $k_2^q=-I_2^q/I_1^q\approx 3.24$, where

$$I_2^J=I_1^q,\quad I_2^q=-\int_0^\infty dx\left(x^2-\frac{3}{2}\right)\hat{a}_3(x) x e^{-x^2}\approx 0.959,$$

$$\hat{a}_3(x)=(3/2-x^2)(\nu^{ee}+\nu^{ei})/\nu^{ei}. \tag{68}$$

V. CALCULATION OF HYDRODYNAMIC PROFILE FUNCTIONS

A. Canonical equations for hydrodynamic profile functions

Substituting Eqs. (64) and (65) into Eqs. (12) and (13), we obtain

$$(D_{11}\Lambda'+D_{12}\hat{h}'_{T_e})'=0, \tag{69}$$

$$(D_{21}\Lambda'+D_{22}\hat{h}'_{T_e})'=0. \tag{70}$$

Here

$$D_{11}=\mu_J\alpha_{-1}^{-1}+(\alpha_1-\alpha_{-1}^{-1}),$$

$$D_{12}=k_2^J\mu_J\alpha_{-1}^{-1}+k_1^J(\alpha_1-\alpha_{-1}^{-1}),$$

$$D_{21}=\mu_q\alpha_{-1}^{-1}+(\alpha_1-\alpha_{-1}^{-1}), \tag{71}$$

$$D_{22}=k_2^q\mu_q\alpha_{-1}^{-1}+k_1^q(\alpha_1-\alpha_{-1}^{-1}),$$

and $\alpha_1(\Omega)=(2\pi)^{-1}\oint(\Omega+\cos\xi)^{1/2}d\xi=2^{3/2}E(\kappa)/(\pi\kappa)$ where $E(\kappa)$ is the complete elliptical integral of the second kind. We call Eqs. (69) and (70) the canonical equations.

B. Solution to canonical equations for hydrodynamic profile functions

We integrate Eqs. (69) and (70) over Ω with the boundary conditions

$$(\Lambda',h'_{T_e})_{\Omega\to\infty}=w_\chi/(2^{3/2}\Omega^{1/2}). \tag{72}$$

We also take into account that the asymptotics of the functions $\alpha_1(\Omega)$ and $\alpha_{-1}(\Omega)$ for $\Omega\to\infty$ are

$$\alpha_1=\frac{2^{1/2}}{\kappa}\left(1-\frac{\kappa^2}{4}-\frac{3\kappa^4}{64}\right),\quad \alpha_{-1}=\frac{\kappa}{2^{1/2}}\left(1+\frac{\kappa^2}{4}+\frac{9\kappa^4}{64}\right). \tag{73}$$

Then, we obtain

$$D_{11}\Lambda'+D_{12}\hat{h}'_{T_e}=C_J, \tag{74}$$

$$D_{21}\Lambda'+D_{22}\hat{h}'_{T_e}=C_q, \tag{75}$$

where

$$C_J=\frac{w_\chi}{2^{3/2}}\left(1+\frac{k_2^J\omega_{*e}}{\omega-\omega_{*e}}\eta_e\right)\mu_J,$$

$$C_q=\frac{w_\chi}{2^{3/2}}\left(1+\frac{k_2^q\omega_{*e}}{\omega-\omega_{*e}}\eta_e\right)\mu_q. \tag{76}$$

It follows from Eqs. (73) and (74) that

$$\Lambda'=(C_JD_{22}-C_qD_{12})/\beta(\Omega,\varepsilon), \tag{77}$$

$$\hat{h}'_{T_e}=(C_qD_{11}-C_JD_{21})/\beta(\Omega,\varepsilon), \tag{78}$$

where $\beta(\Omega,\varepsilon)=D_{11}D_{22}-D_{12}D_{21}$.

We call the expressions for the profile function derivatives, defined by Eqs. (76) and (77), the "intermediate" ones since they contain the yet to be known island rotation frequency. They will be specified after finding this frequency.

VI. CALCULATION OF MICROISLAND ROTATION FREQUENCY

We take into account that Eq. (74) means

$$\mu_J\alpha_{-1}^{-1}(\Lambda'+k_2^J\hat{h}'_{T_e})+(\alpha_1-\alpha_{-1}^{-1})(\Lambda'+k_1^J\hat{h}'_{T_e})=C_J. \tag{79}$$

Using Eq. (79), we exclude the terms with μ_J from Eq. (64). Substituting the resulting expression for $\nabla\cdot\mathbf{J}_d$ into Eq. (15), we obtain the following equation for the island rotation frequency

$$\int_1^\infty d\Omega\oint d\xi\sqrt{\Omega+\cos\xi}[(\sqrt{\Omega+\cos\xi}-\alpha_1)(\Lambda'+k_1^J\hat{h}'_{T_e})]'=0. \tag{80}$$

Integrating here by parts over Ω and ξ, we obtain

$$Z\equiv Z_0+Z_1=0, \tag{81}$$

where

$$Z_0=\mu_J^{-1}(\Omega-\alpha_1^2)(\Lambda'+k_1^J\hat{h}'_{T_e})_{\Omega=1}, \tag{82}$$

$$Z_1=1/(2\mu_J)\int_1^\infty d\Omega(1-\alpha_1\alpha_{-1})(\Lambda'+k_1^J\hat{h}'_{T_e}). \tag{83}$$

We find from Eq. (79)

$$(\Lambda'+k_1^J\hat{h}'_{T_e})_{\Omega=1}=C_J\alpha_1^{-1}{}_{\Omega=1}. \tag{84}$$

Then, taking into account Eq. (75), Eq. (82) is transformed to

$$Z_0=\frac{0.21w_\chi}{2^{3/2}}\left(1+\frac{k_2^J\omega_{*e}\eta_e}{\omega-\omega_{*e}}\right). \tag{85}$$

We look for solution to Eq. (81) in the form

$$\omega = \omega_{*e}[1 - b(\varepsilon)\eta_e], \tag{86}$$

where $b(\varepsilon)$ is a function of ε. Calculating this solution numerically, we obtain the function $b(\varepsilon)$ presented by line 1 in Fig. 1.

One can see from line 1 of Fig. 1 that for $\varepsilon > 0.1$, the island rotation frequency ω has a rather weak dependence on ε. To understand this result, we also calculate $\omega(\varepsilon)$ by the method of successive approximations allowing for Z_0 to be much larger than Z_1. We represent b in the form

$$b = b_0 + b_1, \tag{87}$$

where b_0 is defined from the condition $Z_0=0$, and b_1 is a small correction to b_0 due to Z_1. We find from Eq. (85)

$$b_0 = k_2^J. \tag{88}$$

Taking into account Z_1, we have

$$b_1 = 0.21^{-1}\mu_J(k_1^J - k_2^J)(k_2^J - k_2^q)Y(\varepsilon), \tag{89}$$

where

$$Y(\varepsilon) = (1/2)\int_1^\infty d\Omega(\alpha_1 - \alpha_{-1}^{-1})/\beta(\Omega,\varepsilon). \tag{90}$$

The function b_0 and the sum b_0+b_1 are presented by the lines 2 and 3 in Fig. 1. One can see, that the function b defined by Eq. (87) is close to b_0 for mentioned above $\varepsilon > 0.1$. However, for a smaller value ε the difference between these functions becomes essential, so that for the case of such ε our method of successive approximation is invalid.

VII. CONCRETE DEFINITION OF HYDRODYNAMIC PROFILE FUNCTIONS AND THEIR COMPARISON WITH THE RUTHERFORD PROFILE FUNCTIONS

For ω defined by Eq. (86) the values C_J and C_q take the form

$$C_J = w_\chi \mu_J (b - k_2^J)/(2^{3/2}b), \quad C_q = w_\chi \mu_q (b - k_2^q)/(2^{3/2}b). \tag{91}$$

Equation (71) means for the mentioned value of ω we have

$$h_{T_e} = - b\hat{h}_{T_e}. \tag{92}$$

Using Eqs. (76), (77), (91), and (92), we express the functions Λ' and h'_{T_e} in the "final" form

$$\Lambda'(\Omega,\varepsilon) = w_\chi[\mu_J(b - k_2^J)D_{22} - \mu_q(b - k_2^q)D_{12}]/(2^{3/2}\beta b), \tag{93}$$

$$h'_{T_e}(\Omega,\varepsilon) = w_\chi[\mu_J(b - k_2^J)D_{21} - \mu_q(b - k_2^q)D_{11}]/(2^{3/2}\beta). \tag{94}$$

It is of interest to compare profile functions defined by Eqs. (93) and (94) with the "Rutherford's" expressions for them, $\Lambda'_R, (h'_{T_e})_R$ (see Ref. 3 for details)

$$\Lambda'_R = (h'_{T_e})_R = w_\chi/2^{3/2}\alpha_1(\Omega). \tag{95}$$

For such a comparison we introduce dimensionless functions g_Λ and g_{h_T}, meaning $g_\Lambda = \Lambda'/\Lambda'_R$, $g_{h_T} = h'_{T_e}/(h'_{T_e})_R$ called the form-factors of the hydrodynamic profile functions. Using Eqs. (93)–(95), we find

$$g_\Lambda(\Omega,\varepsilon) = \alpha_1[\mu_J(b - k_2^J)D_{22} - \mu_q(b - k_2^q)D_{12}]/(\beta b), \tag{96}$$

$$g_{h_T}(\Omega,\varepsilon) = \alpha_1[\mu_J(b - k_2^J)D_{21} - \mu_q(b - k_2^q)D_{11}]/\beta. \tag{97}$$

Far from the island separatrix, $\Omega \to \infty$, and for arbitrary values of ε the standard result follows from Eqs. (96) and (97): $(g_\Lambda, g_{h_T})_{\Omega\to\infty}=1$. For the separatrix, $\Omega=1$, we have

$$\{g_\Lambda(1,\varepsilon), g_{h_T}(1,\varepsilon)\} = \left\{ \frac{\mu_J(b - k_2^J)k_1^q - \mu_q(b - k_2^q)k_1^J}{b(k_1^q - k_1^J)}, \frac{\mu_J(b - k_2^J) - \mu_q(b - k_2^q)}{k_1^q - k_1^J} \right\}. \tag{98}$$

The examples of the form-factors $g_\Lambda(\Omega,\varepsilon)$ and $g_{h_T}(\Omega,\varepsilon)$ for $\varepsilon=0.1$ are presented in Fig. 2.

VIII. GENERALIZED RUTHERFORD EQUATION FOR NEOCLASSICAL MICROISLAND WIDTH

A. Starting expressions for polarization current and bootstrap current contributions

In accordance with Ref. 3, polarization current contribution into Eqs. (1) and (2), Δ_p, has the form

$$\Delta_p = \frac{2G_2}{1 + T_e/T_i}\frac{L_s^2}{v_A^2 k_y^2 \rho_i^2 w}(\omega - \omega_{*i})(\omega - \omega_{*e}). \tag{99}$$

Here G_2 is a parameter, defined by

$$G_2 = \frac{4}{wRB_\theta}\sum_{\sigma_\chi}\sigma_\chi\int_0^1 \frac{d\kappa}{\kappa^3}\left\{\frac{2}{\kappa^2}\left[1 - \frac{E(\kappa)}{K(\kappa)}\right] - 1\right\}\frac{\partial\Lambda}{\partial\Omega}, \tag{100}$$

$k_y=m/r$, $L_s=qR/s$ is the shear length, ρ_i is the ion Larmor radius, v_A is the Alfvèn velocity. The value G_2 is sensitive to neoclassical effects for two reasons. First, these effects determine the island rotation frequency entering Eq. (100). Second, profile function Λ depends on these effects.

Note also that for arbitrary ratio w and ρ_i we have qualitatively

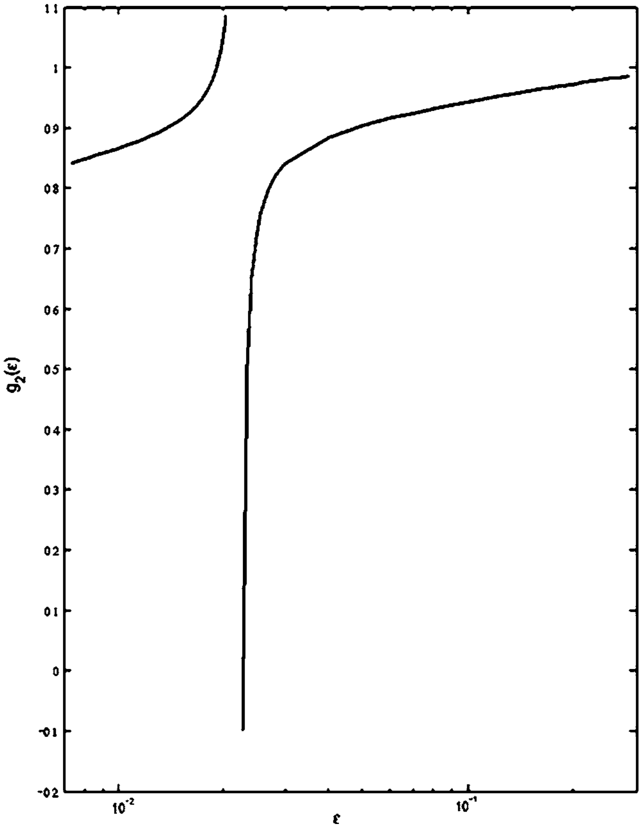

FIG. 3. Dependence of the geometric parameter g_2 on the local inverse aspect ratio ϵ.

$$\Delta_p \sim \frac{1}{w(w^2+\rho_i^2)}. \tag{101}$$

For the case of large-scale islands, $w>\rho_i$, it yields $\Delta_p \sim 1/w^3$,[1,2,13] while for the case of microislands, $w<\rho_i$, instead of Eq. (101), we have, in accordance with Eq. (100), $\Delta_p \sim 1/(w\rho_i^2)$.

In accordance with Eq. (26) and Eqs. (5.5) and (5.41) of Ref. 12,

$$\bar{J}_{bs} = -(4/3)\pi e \int_0^\infty v^3 dv H_{e1}^{(1)}. \tag{102}$$

B. The role of polarization current

For $\partial\Lambda/\partial\Omega$ given by Eq. (93), Eq. (100) means

$$G_2(\varepsilon) = \frac{2^{3/2}}{b}\int_0^1 \frac{d\kappa}{\kappa^3\beta}\left\{\frac{2}{\kappa^2}\left[1-\frac{E(\kappa)}{K(\kappa)}\right]-1\right\}[\mu_J(b - k_2^J)D_{22} - \mu_q(b-k_2^q)D_{12}]. \tag{103}$$

We introduce G_2^R coincident with G_2 for the Rutherford's expression for $\partial\Lambda/\partial\Omega$ defined by Eq. (95). In accordance with Ref. 3, G_2^R=0.396. We characterize the difference between $G_2(\varepsilon)$ and G_2^R by a geometric parameter $g_2(\varepsilon) = G_2(\varepsilon)/G_2^R$. Behavior of the function $g_2(\varepsilon)$ is presented in Fig. 3.

Using Eq. (86), we have

$$(\omega-\omega_{*i})(\omega-\omega_{*e}) = -\eta_e\omega_{*e}^2 b(1+\tau-\eta_e b). \tag{104}$$

Assuming ε to be not too small, so that $G_2>0$, allowing for Eq. (104), we find that the polarization current is destabilizing, $\Delta_p>0$, only for

$$-\eta_e b(1+\tau-\eta_e b) > 0. \tag{105}$$

This means that the electron temperature gradient must be either negative, $\eta_e<0$, or large enough positive, so that

$$\eta_e > (1+\tau)/b. \tag{106}$$

In the contrary case, i.e., for

$$0 < \eta_e < (1+\tau)/b \tag{107}$$

the polarization current is stabilizing, $\Delta_p<0$.

C. The role of bootstrap current

Using Eq. (58) for $H_{e1}^{(1)}$ and integrating over velocities, we reduce Eq. (102) to

$$\bar{J}_{bs} = -\frac{16\sqrt{2}}{3}\pi^{-1/2}I_1^{bs}(1-c_0)\frac{eqRn_0(\omega-\omega_{*e})}{mw_\chi}\alpha_{-1}^{-1}(\Lambda' - k_{bs}\hat{h}'_{T_e}). \tag{108}$$

Here k_{bs}=$-I_2^{bs}/I_1^{bs}\approx 0.401$,

$$I_1^{bs} = \int_0^\infty dx a_1(x) x^4 e^{-x^2} \approx 1.12,$$
$$I_2^{bs} = \int_0^\infty dx a_3(x) x^4 e^{-x^2} \approx -0.449. \tag{109}$$

For Λ' and h'_{T_e} defined by Eqs. (93) and (94), Eq. (108) means

$$\bar{J}_{bs} = \frac{8}{3}\pi^{-1/2}I_1^{bs}(1-c_0)\frac{eqRn_0\eta_e\omega_{*e}}{m\beta}\alpha_{-1}^{-1}[\mu_J(b-k_2^J)(D_{22} + k_{bs}D_{21}) - \mu_q(b-k_2^q)(D_{12}+k_{bs}D_{11})]. \tag{110}$$

Substituting Eq. (110) into Eq. (26) and integrating over Ω, we arrive at

$$\Delta_{bs} = \frac{8}{3}\pi^{-1/2}I_1^{bs}(1-c_0)\beta_e\frac{r}{sw}\frac{\eta_e}{L_n}c_{bs}(\varepsilon), \tag{111}$$

where $1/L_n$=$-d\ln n_0/dr$, β_e is the "electron beta,"

$$c_{bs}(\varepsilon) = 4\sqrt{2}\int_0^1 \frac{d\kappa}{\kappa^3\beta}\left\{\frac{2}{\kappa^2}\left[1-\frac{E(\kappa)}{K(\kappa)}\right]-1\right\}\times[\mu_J(b-k_2^J)\times(D_{22}+k_{bs}D_{21}) - \mu_q(b-k_2^q)(D_{12}+k_{bs}D_{11})]. \tag{112}$$

Approximately, Eq. (111) reduces to

$$\Delta_{bs} = \frac{2^7}{3}\pi^{-1/2} I_1^{bs}(1-c_0)\frac{1}{cs}\frac{eq^2R^2n_0\omega_{*e}\eta_e}{mwB_0}(b+k_{bs})G_2. \tag{113}$$

It hence follows that for $\eta_e > 0$ the bootstrap current is destabilizing.

D. Relative role of polarization current and bootstrap current effects

Taking into account the given aforedescribed relations, we find that the total effect of the polarization current and the bootstrap current is destabilizing, $\Delta_p + \Delta_{bs} > 0$, for

$$\eta_e\left[\frac{2^5}{3}\pi^{-1/2} I_1^{bs}(1-c_0)(b+k_{bs})(1+\tau)s\frac{L_n}{r} - b(1+\tau - b\eta_e)\right] > 0. \tag{114}$$

Assuming $b = b_0$ [see Eqs. (87) and (88)], we find that for $\tau \sim 1$ and $\eta_e \sim 1$ Eq. (114) means

$$\varepsilon^{1/2} s L_n / r > 0.04. \tag{115}$$

Since the right-hand side of Eq. (115) contains a small numerical coefficient, this inequality can be fulfilled in experimental conditions.

E. Critical width of neoclassical microislands

Assuming $\Delta_{bs} > \Delta_p$, we find from $\Delta_{bs} > \Delta'$ and $\Delta' = -2m/r$ that excitation of the microislands is possible only for $w < w_{\max}$, where

$$w_{\max} = 4.93(r/m)(r/L_n)\varepsilon^{1/2}\beta_p \eta_e c_{bs}/s, \tag{116}$$

where β_p is the electron poloidal beta.

The value $w_{\max}$ can be used as an estimate for the characteristic width of the islands considered. Note that we have neglected the electron inertia in the electron longitudinal motion equation, Eq. (8). The neglect is valid if the island width is larger than the electron skin depth c/ω_{pe}, $w > c/\omega_{pe}$, where ω_{pe} is the electron plasma frequency. Stability, microisland formation, and consequent anomalous transport for finite c/ω_{pe} in neglecting the neoclassical effects have been considered in Refs. 22–24. Appearance of the scale length of order $w_{\max}$ given by Eq. (116) is due to allowing for these effects.

IX. DISCUSSION

The main result of this paper is the demonstration of the possible existence of neoclassical magnetic islands in a tokamak plasma with electrons in the banana regime. In order to obtain this result, we had to analyze a rather wide circle of problems, a detailed list of which was presented in Sec. I. In the framework of this analysis, the problem of calculating the microisland rotation frequency, which was the "stumbling-block" for Ref. 3, and the problem of the bootstrap current incorporation into the microisland theory, as in the case of large-scale islands (see in detail Ref. 13), have been solved.

It follows from our analysis that neoclassical microislands are rather sensitive to the presence of electron temperature gradient. An important role of this gradient also has been noted in the theory of "slab" microislands.[1–3] In our toroidal problem the presence of the temperature gradient is a necessary condition for revealing the effects of both bootstrap current and polarization current and, at the same time, the reason of the magnetic island existence. Such a picture is in accordance with the general concept[7] that excitation of microislands should result in vanishing of the cause of their excitation.

In order to estimate the anomalous electron diffusivity χ_e based on the theory presented we will follow, similarly to Ref. 23, the approach of Ref. 25 (see also Ref. 26) using the formula $\chi_e \approx (v_{Te}/L_s)(mw^3/r)$ and taking $w \approx w_{\max}$ given by Eq. (116). Then we arrive at

$$\chi_e \approx [v_{Te}/(qRs^2)][r^5/(m^2L_n^2)]\varepsilon^{3/2}\beta_p^3. \tag{117}$$

Since in our case $w \gg c/\omega_{pe}$, this anomalous electron diffusivity is larger than that obtained in Ref. 23.

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APPENDIX A: RELATIONS CHARACTERIZING MAGNETIC ISLAND GEOMETRY

Similarly to Ref. 12, we represent the perturbed electric and magnetic fields $\mathbf{E}$ and $\hat{\mathbf{B}}$ in the form $\mathbf{E} = -\nabla\Phi - (\mathbf{b}/c)\,\partial A_\parallel/\partial t$, $\hat{\mathbf{B}} = \mathbf{b}\times\nabla A_\parallel$, where $A_\parallel$ is the longitudinal component of the vector potential. We introduce the magnetic flux perturbation $\psi = -RA_\parallel$ taken in the form $\psi = \tilde{\psi}\cos\xi$, where $\tilde{\psi}$ is a constant or a function weakly dependent on time and $\xi = m\theta - n\phi - \omega t$ is the island cyclic variable, m and n are the poloidal and toroidal mode numbers, and ω is the island rotation frequency. We assume that the islands are localized in the vicinity of radial coordinate $\chi = \chi_s$ and introduce the island magnetic flux function Ω defined by

$$\Omega = [\psi_0(\chi) - \psi]/\tilde{\psi}, \tag{A1}$$

where $\psi_0(\chi) = \tilde{\psi}(\chi - \chi_s)^2/w_\chi^2$, $w_\chi^2 = 4\tilde{\psi}q_s/q_s'$, q_s is the safety factor for $\chi = \chi_s$, the prime denotes the derivative with respect to χ. The island halfwidth w is related to w_χ by $w = |w_\chi|/RB_\theta$. The intervals $\Omega > 1$ and $-1 < \Omega < 1$ correspond to the regions outside and inside the island separatrix.

We designate the averaging over the island magnetic surface by the symbol $\langle\ldots\rangle$. Outside the island separatrix we have

$$\langle(\ldots)\rangle = \oint (\Omega + \cos\xi)^{-1/2}(\ldots)d\xi/[2\pi S(\Omega)], \tag{A2}$$

where $S(\Omega)=(2\pi)^{-1}\oint(\Omega+\cos\xi)^{-1/2}d\xi$. Here we integrate over ξ within the limits $0\leqslant\xi\leqslant 2\pi$. Inside the separatrix we substitute Eq. (A1) by

$$\langle(\ldots)\rangle = \sum_{\sigma_\chi} \oint (\Omega + \cos\xi)^{-1/2}(\ldots)d\xi/[2\pi S(\Omega)], \tag{A3}$$

where $\sigma_\chi = sgn(\chi-\chi_s)=\pm 1$, and the integration interval is defined from the condition $\cos\xi \geqslant -\Omega$.

APPENDIX B: HYDRODYNAMIC PROFILE FUNCTIONS

We use the poloidal magnetic flux χ introduced in Sec. II B. We assume that the magnetic island chain is localized in the vicinity of the rational magnetic surface $\chi=\chi_s$. Then, the total plasma density $N(\chi)$ can be presented in the form

$$N(\chi) = n_0(\chi_s) + n_0'(\chi_s)\hat{\chi} + \hat{n}, \tag{B1}$$

where $\hat{n}$ is the perturbed plasma density and $\hat{\chi}=\chi-\chi_s$. The density profile function $h_n(\Omega)$ is introduced by

$$h_n(\Omega) = \langle\hat{\chi}\rangle + \langle\hat{n}\rangle/n_0'. \tag{B2}$$

Similarly to Eq. (B1), we represent the total electron temperature $T_e(\chi)$:

$$T_e(\chi) = T_{0e}(\chi_s) + T_{0e}'(\chi_s)\hat{\chi} + \hat{T}_e, \tag{B3}$$

where $\hat{T}_e$ is the perturbed electron temperature. The electron temperature profile function $h_{T_e}(\Omega)$ is introduced similarly to Eq. (B2):

$$h_{T_e}(\Omega) = \langle\hat{\chi}\rangle + \langle\hat{T}_e\rangle/T_{0e}'. \tag{B4}$$

The electrostatic potential profile function $h(\Omega)$ is introduced by the same way as in Ref. 12:

$$\langle\Phi\rangle = (\omega q/mc)[\langle\hat{\chi}\rangle - h(\Omega)]. \tag{B5}$$

Expanding in a series in $\hat{\chi}$ and Φ we find from Eq. (3)

$$N(\chi) = n_0(\chi_s) + n_0'(\chi_s)\hat{\chi} - e\Phi n_0(\chi_s)/T_{0i}. \tag{B6}$$

It follows from Eqs. (B1), (B5), and (B6) that

$$h_n(\Omega) = (1-\omega/\omega_{*i})\langle\hat{\chi}\rangle + (\omega/\omega_{*i})h(\Omega). \tag{B7}$$

APPENDIX C: TRANSFORMATIONS AND SIMPLIFICATION OF DRIFT KINETIC EQUATION

Similarly to Ref. 12, we take the electron drift kinetic equation in the form

$$\frac{\partial f}{\partial t} + v_\parallel\nabla_\parallel f + \mathbf{v}_E\,\boldsymbol{\nabla} f + \mathbf{v}_d\,\boldsymbol{\nabla} f - \frac{e}{M_e}\frac{v_\parallel}{v}E_\parallel\frac{\partial f}{\partial v} + \frac{e}{M_e}\frac{\mathbf{v}_d\cdot\boldsymbol{\nabla}\Phi}{v}\frac{\partial f}{\partial v} = C_e. \tag{C1}$$

Here f is the total electron distribution function, e is the ion charge (so that "$-e$" is the electron charge), $v=(v_\parallel^2+2\varepsilon_\perp)^{1/2}$ is the total particle velocity. We take the electrostatic potential Φ in the form [cf. Eq. (B5)]

$$\Phi = (\omega q/mc)[\chi - h(\Omega)] + \alpha, \tag{C2}$$

where α is a function satisfying the condition $\langle\alpha\rangle=0$. In Ref. 3 in addition to α the function δ has been used related to α by $\alpha=\alpha^0+\delta$, where

$$\alpha^0 = -\frac{1}{1+\tau}\frac{q\omega}{mc}\left(1-\frac{\omega_{*i}}{\omega}\right)[\chi - \langle\chi\rangle_\xi]. \tag{C3}$$

In the approximation, when the parallel electron motion equation, Eq. (8), is valid, we have $\delta=0$. In a series of following equations we will assume the function δ to be finite, to illustrate the physical effects related to it. In deriving the equations of interest for the hydrodynamic profile functions and the island rotation frequency we neglect such effects in this paper. Then, similarly to Ref. 12, we find

$$-\left(\frac{\partial h}{\partial\chi}\omega - k_\parallel v_\parallel\right)\frac{\partial g}{\partial\xi} + \frac{mc}{q}\frac{\partial\Omega}{\partial\chi}\left(\frac{\partial g}{\partial\xi}\frac{\partial\alpha}{\partial\Omega} - \frac{\partial g}{\partial\Omega}\frac{\partial\alpha}{\partial\xi}\right) + \frac{v_\parallel}{Rq}\frac{\partial g}{\partial\theta} - \frac{e}{T}F\omega\frac{\partial\gamma}{\partial\chi}\frac{\partial\alpha}{\partial\xi} - \frac{v_\parallel}{Rq}I\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{e}{T}F\frac{\omega q}{mc}\left(1-\frac{\partial\gamma}{\partial\chi}\right) = C_e, \tag{C4}$$

where the value $k_\parallel=-m\hat{\chi}q_s'/(Rq_s^2)$ is a longitudinal wave number.

Similarly to Ref. 12, we represent g as a series in $Rq/v_\parallel$, $g=g^{(0)}+g^{(1)}+\ldots$, where the function $g^{(0)}$ satisfies the equation

$$\frac{v_\parallel}{Rq}\frac{\partial g^{(0)}}{\partial\theta} - \frac{v_\parallel}{Rq}I\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{e}{T_e}F\frac{\omega q}{mc}\left(1-\frac{\partial\gamma}{\partial\chi}\right) = 0, \tag{C5}$$

and the function $g^{(1)}$ is determined by

$$\frac{v_\parallel}{Rq}\frac{\partial g^{(1)}}{\partial\theta} - \frac{\partial\lambda}{\partial\chi}\left[\frac{\tau}{1+\tau}(\omega-\omega_{*e}) - \bar{k}_\parallel v_\parallel\right]\frac{\partial g^{(0)}}{\partial\xi} - \frac{e}{T_e}F\left[\omega\frac{\partial\gamma}{\partial\chi}\frac{\partial\delta}{\partial\xi} + \frac{q\omega}{mc}\frac{\omega-\omega_{*i}}{1+\tau}\frac{\partial\gamma}{\partial\Omega}\frac{\partial\Omega}{\partial\xi}\right] = C. \tag{C6}$$

Similarly to Ref. 12, it follows from Eq. (C7) that

$$g^{(0)} = \hat{h} + \hat{g}, \tag{C7}$$

where $\hat{g}$ is given by Eq. (18), and $\hat{h}$ is an arbitrary θ-independent function. Then, as done in Ref. 12, we take different forms of Eq. (C8) for circulating and trapped particles. In the case of circulating particles we assume

$$\hat{h} = \widetilde{h} + \sigma_v H_e, \tag{C8}$$

where $\langle \widetilde{h} \rangle = 0$, and $\sigma_v H_e$ is the value of $\hat{h}$ averaged over the magnetic island surface, i.e., $\sigma_v H_e = \langle \hat{h} \rangle$. Below we neglect the function $\widetilde{h}$, so that Eq. (C7) reduces to Eq. (17). Then, turning to Eq. (C6), similar to Ref. 12, we find, that the function H_e satisfies Eq. (19).

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