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Pavlovian Prisoner's Dilemma—Analytical results, the *quasi*-regular phase and spatio-temporal patterns

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ABSTRACT

The Prisoner's Dilemma (PD) game is applied in several research fields due to the emergence of cooperation among selfish players. In this work the PD is studied in a one-dimensional lattice, where each cell represents a player, which in turn can interact with the neighbors playing the PD (cooperate or defect). The update of states adopts the Pavlovian Evolutionary Strategy (PES) or Darwinian Evolutionary Strategy (DES). Adopting PES, if a player receives a positive payoff greater than his/her aspiration level, he/she keeps the current state, and switches otherwise. Adopting DES, player compares his/her payoff with payoff of opponents. If it is not the highest, he/she copies the state of fittest player, switching the state if it is different of his/her current state. The critical temptation values obtained analytically are reported, and the cluster patterns that emerge from the interactions among the players are shown. Also we defined analytical functions that calculate the maximum/minimum size of defective/cooperative clusters. Also, the parameter space is explored with exhaustive computational simulations, which confirm the analytical results and reinforce that Pavlovian strategy foments cooperation among players. In steady state, system can reach the cooperative or *quasi*-regular phases, when adopting the PES, and cooperative, defective or chaotic phases, adopting the DES. The new *quasi*-regular phase occurs when several players switch their states in each round, but the proportion of cooperators does not show significant variation. Additionally, the present work shows that the lowest temptation level ($T=1$) may be considered a trivial case only for the particular case where the players interact with only one neighbor, otherwise system presents the same features that for higher temptation values.

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1. Introduction

Prisoner's Dilemma (PD) is a game where two players confront each other and they can either cooperate or defect. Players receive a payoff R (reward) in the case of mutual cooperation and a payoff P (punishment) if they are both defectors. If one player cooperates and the other defects, they receive S (sucker) and T (temptation), respectively. These payoff values must satisfy the inequalities $T > R > P > S$ and $T + S < 2R$ to create the dilemma (Axelrod, 1984). In a single round game, the best choice is the defection, since it assures a larger payoff compared with cooperation, independent of the opponent's decision (Nash equilibrium). However, a local minimum occurs under mutual defection, thereby generating the dilemma.

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When the PD is played repeatedly, it is called Iterated Prisoner Dilemma (IPD). In the computer tournament proposed by Axelrod and Hamilton (1981) and Axelrod (1984) to compare different strategies employed when playing the IPD, a simple strategy called *tit-for-tat* (TFT), with only one time step memory, was by far the most stable one. The player using TFT cooperates in the first round and subsequently copies the action of the opponent in the last round. In this game, the cooperation emerged as a profitable action among selfish agents. The dilemma and the cooperation make the PD the most prominent game in the Game Theory. It is used to model problems in several research fields such as Politics (Sociophysics) (Stauffer, 2004), Economics (Econophysics) (Anteneodo et al., 2002; Bouchaud, 2002), and Biology (Nowak and Sigmund, 1998; Cooper and Wallace, 1998; Turner and Chao, 1999).

Consider a one-dimensional automaton, where each cell is a player that can play the IPD with z neighbors. All players play against their respective neighbors and update their states. This process is called *round* and is the system time unit. After long enough time, the system may reach a steady state, and the

asymptotic proportion of cooperators— ρ_∞ —becomes time independent.

Consider a lattice model being discrete, where each lattice site represents an individual, which can take several states. This lattice can be separated into domains (subpopulation) that maintain topological structure. Some models depend on this structure, such as interacting particle systems, but others do not, for instance the voter model (Durrett and Levin, 1994b) or the model we consider in this study. To avoid the model dependence on the spatial structure, consider only the number of occurrence of a given state, not its specific by their localization in the lattice. In this case, the populations are divided into local homogeneous domains, which are called patches, very different one from another (Chesson, 1981). The patch modeling takes into account the importance of space at local scales, nevertheless the union of patches does not present a spatial structure (Durrett and Levin, 1994a, b). Physically, a lattice model with asynchronous individual state update breaks the topological structure such as a stochastic process would do. The mathematical proof is given in Durrett and Restrepo (2008) and Thomson and Ellner (2003). In the present model each patch is equivalent to one player and his/her neighborhood, similarly to voter model, so the one dimensional modeling proposed here can be an useful surrogate for real lattice systems of finite dimension to derive a condition of the evolution of cooperation.

Concerning the biological applications and the one-dimensional approach, think of migratory groups sharing resource spots, where the “interface” members, agents who interact simultaneously with inner and outer members of their groups. Also, one can think in hunter groups that interact with competitors in a common hunting area. In these situations, the group members are not constrained to have a specific spatial distribution, the only constraint that we impose is that each agent should to interact with the same members, wherever they are located, i.e. keep their links during time.

The player state update process varies according to the adopted evolutionary strategy (Pincus, 1970), such as Darwinian Evolutionary Strategy (DES) (Nowak and May, 1992) or Pavlovian Evolutionary Strategy (PES) (Fort and Viola, 2005). These are non-stochastic strategies, i.e., players interact according to deterministic rules during dynamics. In DES, the update process consists in copying the state of the best adapted player (known as the fittest player, who receives the greatest payoff). This is also known as the “survival of the fittest” and is equivalent to the Darwin natural selection principle (Beyer and Schwefel, 2002). In our case, each player compares his/her own payoff to the neighboring ones, and then copies the state of the fittest neighbor.

Let us consider the following learning techniques for PES:

- (i) Win-stay, lose-shift (WSLS) is a general learning method used for iterated decision problems of all kinds. It was proposed by Thorndike (1911), assuming that actions that yield satisfaction will be reinforced and actions leading to discomfort will be weakened. This strategy is also called *Pavlov*. Kraines and Kraines (1989) have used positive and negative reinforcement to teach an individual to respond. In the PD context, an individual is a player. For example, in the first round, a player randomly chooses the action *C* or *D* (to cooperate or to defect). He/she plays the game and evaluates the outcome. If he/she receives a reward due to action *C*, this player will be more prone to keep doing action *C*. Otherwise, if he/she is punished due to action *C*, it will be more probable that the player changes his/her action to *D*. This process can be thought as the strategy “never change a winning team”. If it is desirable that a player acts like *C*, then he/she must be rewarded, or

punished, repeatedly according to his/her choice, to reinforce action *C*. In the PD, under these conditions, a player keeps a given action when he/she receives a payoff *R* or *T* and switches actions if his/her payoff is *S* or *P*. Namely, a player keeps his/her action when playing against a cooperator and switches it when he/she confronts a defector.

- (ii) Setting up an aspiration level (AL) for the IPD player (Posch, 1999; Fort and Viola, 2004). The payoff can be lower, equal to, or greater than the AL. If they receive a payoff greater than AL, they do not change their states, otherwise they switch them. In PES all players generally have the same aspiration level.

The Pavlov-based strategy is very robust in situations such as (i) the presence of noise, i.e., a player can switch his/her state at any moment, with probability $p > 0$, regardless of the strategy adopted by this player (mutation) (Kraives and Kraives, 1993b); (ii) playing against deceiving or profiteer strategies (Kraives and Kraives, 1993a); (iii) competition for surviving in coevolutionary games (Lorberbaum et al., 2002; Nowak and Sigmund, 1993). The most important features of Pavlov-based strategy are that (i) it does not forgive a defection; (ii) it exploits altruistic strategies while it is not punished with a defection; (iii) it can correct occasional mistakes (noisy environment), which is not possible with the *tit-for-tat* strategy. Nevertheless, if the Pavlovian strategy is used as WSLS with an aspiration level, so that a player interacts with more than one neighbor, then the strategy presents a weakness: it can be exploited by defective strategies. This happens because each player is concerned with his/her own payoff only, thus ignoring the opponent’s payoff. So the player can achieve their aspiration level despite being exploited by the neighbors.

The PD order parameter is the *proportion of cooperators*, and the main variable is the *temptation*. The system dynamics shows a transient regime and eventually reaches a steady state, which defines the phase of the system. Some temptation values yield a total payoffs that force players to switch their states, thereby generating a phase transition, these are the critical temptations values. They depend on the adopted strategy and system connectivity.

Concerning the one-dimensional structure, it can map higher dimensions, if the agent spatially distribution can be neglected and only the number of interacting agents is important (as it is the case in Darwinian and Pavlovian strategies). It is necessary only that they keep the relationships among the agents (interaction links) during time and disregard the constrained geometrical spatial distribution. In the migratory and hunter groups examples, the clusters interactions (external) occur at the resource spot and the inner interactions, occur in the group itself, which can be in a different location.

In this paper we present an analytical treatment and a new steady state regime, called *quasi-regular* phase. The analytical results are the general payoff function for the PD (for any strategy considered), and the new functions for the critical temptation values and the maximum/minimum sizes of the defective/cooperative clusters (for the Pavlovian strategy).

For the numerical results we adopted the PES for the one-dimensional cellular automaton with variable number of interacting neighbors as considered in Pereira et al. (2008a, b). In these works players adopt the Darwinian strategy. In first work, the system evolution concerning the local interactions have been described. The one-dimensional geometry allowed to follow the dynamics evolution (history in a static two-dimensional image) and to explain the mechanism of cluster invasions and the temporal evolution of the proportion of cooperators during transient regime and their oscillation at the steady state. In later

work, an exhaustive study of the same system has been made and verified that the phase transition values are the same as for system with higher dimensionality. Also have been verified that the steady state phases are the same of those obtained in square-lattices (Nowak and May, 1992), i.e. the system can reach the cooperative, defective and chaotic phases. The asymptotic values for one-dimensional and square-lattices are similar.

Comparing the present work with these results one can see that for the Pavlovian strategy only the cooperative and *quasi*-regular phases may occur. Chaotic and defective phases are absent and appeared the new *quasi*-regular phase. The present work is presented as follows. In Section 2 the model is introduced. In Section 3 the critical temptation values for PES are analytically derived. In Section 4 the *quasi*-regular phase, which is a new phase emerging from our numerical results, is presented. Further, the cluster patterns that emerge during dynamics evolution and the exploration of parameter space (temptation T to defect and initial proportion ρ_0 of cooperators) for some connectivity values— z are shown. Final remarks are presented in Section 5. The pattern formation from the *quasi*-regular phase is presented in a cater detail in Appendix A.

2. The one-dimensional model

Consider a one-dimensional cellular automaton with L cells. Each cell represents a *player*, who has two possible states: $\theta = 0$ (defector) or $\theta = 1$ (cooperator) (see Fig. 1). The automaton has no empty cells, so that $\rho_c(t) + \rho_d(t) = 1$, where $\rho_c(t) = (1/L) \sum_{i=1}^L \theta_i(t)$ is the proportion of cooperators at time t , and $\rho_d(t)$ is the proportion of defectors. The initial proportion of cooperators, $0 \leq \rho_c(0) \equiv \rho_0 \leq 1$, is a parameter in this problem. Therefore, the total number of cooperators is $L\rho_0$, and they are positioned randomly in the automaton, following a uniform deviate. The initial configuration is the only stochasticity in the model.

Consider the i -th player, his/her neighborhood (or connectivity) is given by $z = \{1, 2, \dots, L\}$. If z is even, there are $\alpha = z/2$ adjacent players in the right-hand side and $\alpha = z/2$ in the left-hand side. If z is odd, each side has $\alpha = (z-1)/2$ adjacent players and player i interacts with himself/herself (*self-interaction*) (Pereira et al., 2008a, b, Soares and Martinez, 2006). Nowak and Sigmund (1993) argue that self-interaction makes sense, for example, if several animals (a family) or molecules can occupy a single cell. The *self-interaction* is considered an *intra*-group interaction.

In the one-dimensional topology, it is possible to vary the lattice connectivity z (coordination number) to any integer value in the range $1 \leq z \leq L$. This is not possible, for instance, in a square lattice, because it is limited to von Neumann ($z=4$, see Fig. 2a) or Moore ($z=8$, see Fig. 2b) neighborhoods. In a square lattice, if z is different from $z = \{4; 8; 24\}$, the neighborhood is asymmetric. For example, to obtain $z=6$, one must consider the honeycomb lattice. Since the critical temptation depends only on the coordination number, this neighborhood may be considered in a one-dimensional lattice, where $z = \{4; 5\}$ corresponds to the von Neumann neighborhood, $z = \{8; 9\}$ matches the Moore one, and

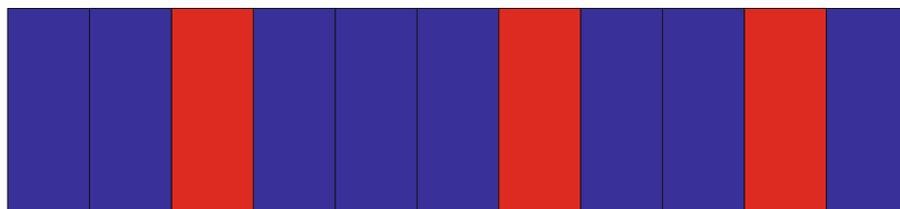


Fig. 1. Cellular automaton in the one-dimensional lattice with $L=11$ players and open boundary condition. Blue cell (dark gray): cooperator, red cell (light gray): defector. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$z = \{6; 7\}$ is related to the honeycomb case, with and without self-interaction, respectively. Periodic boundary conditions (PBC) were employed, so every player has the same connectivity. Once the lattice is one-dimensional, the boundary effect is smaller than that observed in d dimensional lattices (Pereira et al. 2008a, b).

3. Analytical calculation of critical temptation

Let us start with a brief review of the results obtained by Durán and Mulet (2005) for the Darwinian strategy. Consider the parameters T , R , P , and S for the payoff evaluation and two players i and j playing PD in a cellular automaton. The player i payoff with respect to player j is

$$g_{i,j} = T[(1-\theta_i)\theta_j] + R[\theta_i\theta_j] + P[(1-\theta_i)(1-\theta_j)] + S[(1-\theta_j)\theta_i], \tag{1}$$

where θ_k is the player k state, with $k = \{1; 2; \dots; L\}$. The total payoff of player i is $G_i = \sum_{j=1}^z g_{i,j}$. For odd values of z , it is noteworthy that there is an extra payoff component $g_{i,i}$, due to the self-interaction. From Eq. (1), the payoff of player i due to the interaction with a single defector ($\theta_j = 0$) and a single cooperator ($\theta_j = 1$) is

$$g_{i,j} = \begin{cases} P(1-\theta_i) + S\theta_i & \text{if } \theta_j = 0, \\ T(1-\theta_i) + R\theta_i & \text{if } \theta_j = 1. \end{cases} \tag{2}$$

Due to interactions with c_i cooperators within the z neighbors, the payoff of player i is $G_i^{(c)}(\theta_i) = [T(1-\theta_i) + R\theta_i]c_i$, and in the case of d_i defectors, the payoff is $G_i^{(d)}(\theta_i) = [P(1-\theta_i) + S\theta_i]d_i$. Due to the interactions with all the z neighbors, the sum of payoffs leads to the i -th player total payoff: $G_i(\theta_i) = [T(1-\theta_i) + R\theta_i]c_i + [P(1-\theta_i) + S\theta_i]d_i$. Since the number of cooperators and defectors in a given

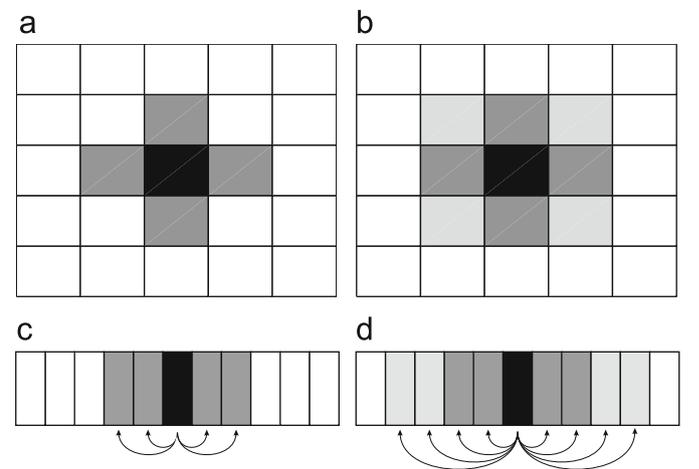


Fig. 2. Neighborhood representation in square lattice: (a) von Neumann, $z=4$ and (b) Moore, $z=8$. And their representations in the one-dimensional lattice to: (c) $z=4$ and (d) $z=8$. Black: central player; dark gray: first neighbors; and light gray: second neighbors. Remember that for even z there is no self-interaction.

neighborhood are complementary, $d = z - c$:

$$G_i(\theta_i) = Tc_i + P(z - c_i) + [(R - T)c_i + (S - P)(z - c_i)]\theta_i. \quad (3)$$

Therefore, the total payoff is

$$G_i(\theta_i) = \begin{cases} Tc_i + P(z - c_i) & \text{if } \theta_i = 0, \\ Rc_i + S(z - c_i) & \text{if } \theta_i = 1. \end{cases} \quad (4)$$

In the following, we show the payoffs for DES and PES and the critical temptation values, which depend on the adopted strategy.

3.1. Darwinian evolutionary strategy (DES)

Nowak and May (1992) used the set parameters $R = 1, P = S = 0$, the only free parameter was temptation $1 \leq T \leq 2$, which ensures the conflict conditions. These values are different from those originally defined by Tucker (Dresher, 1961) ($T = 5, R = 3, P = 1, S = 0$). The conditions $T > R > P > S$ and $T + S < 2R$ were relaxed ($P = S$; for $T = 1, T = R$; and for $T = 2, T + S = 2R$) without any harm to the DP conflict features. This modification is known as Weak Prisoner Dilemma. Placing the values adopted by Nowak and May in Eq. (1), the payoff becomes $g_{ij} = T(1 - \theta_i)\theta_j + \theta_i\theta_j$. A similar result has been obtained by Durán and Mulet (2005) $g_{ij} = T(1 - \theta_i\theta_j)\theta_j + \theta_i\theta_j$. The difference between our and the result of Durán and Mulet is the presence of θ_j multiplying θ_i inside the parenthesis, which is unnecessary and does not alter the result, in this case.

However, in the case where the state of the players can assume rational values, the result of Durán and Mulet is not valid. This situation occurs in the Continuous Prisoner's Dilemma (CPD) (Ifti et al., 2004; Wahl and Nowak, 1999a, b), where a player has a cooperation level (CL) with $0 \leq CL \leq 1$, instead of only defecting or cooperating. For the CPD, our results give the correct payoff values, considering the linear interpolation for intermediate values.

For $R = 1, P = S = 0$ in Eq. (3), we have $G_i(\theta_i) = [T - (T - 1)\theta_i]c_i$. Notice that (i) the payoff for a cooperator who plays with c_i cooperators is $G_i^{(c_i)}(\theta_i = 1) = c_i$, while (ii) a defector who plays with c_i cooperators has a payoff equal to $G_i^{(c_i)}(\theta_i = 0) = c_i T$. For $T > 1$: (i) $G_i^{(c_i)}(\theta_i = 0) > G_i^{(c_i)}(\theta_i = 1)$; and (ii) $G_i^{(c_i)}(\theta) \geq G_i^{(c-1)}(\theta)$. In DES, the payoff of each player is always non-negative, $G_i \geq 0$. After all players calculate their payoffs, they update their states. During this process, each player i compares his/her payoff G_i with G_k , where G_k is the payoff of his/her k -th neighbor, with $k = \{1, 2, \dots, z\}$. If $G_i < G_k$ and $G_k = \max\{G \in z\}$, player i replicates the player k state, otherwise he/she maintains his/her current state.

The system evolves till it eventually reaches the steady state, where the proportion of cooperators ρ_∞ is stationary. The ρ_∞ phase transitions occur when the temptation value passes through critical values T_c . In the conflict region, $1 < T < 2$, these transitions have been calculated (Durán and Mulet, 2005): $T_c(n, m) = (z - n) / (z - n - m)$, where $0 \leq n < z$ and $1 \leq m \leq \text{int}[(z - n - 1) / 2]$ are integers.¹ For example, for $z = 8$, these values are $T_c = \{8/7, 8/6, 8/5, 8/4\}$.

3.2. Pavlovian evolutionary strategy (PES)

The parameters used are $P = -R$ and $S = -T$, which are placed in Eq. (4):

$$G_i(\theta_i) = \begin{cases} Tc_i - R(z - c_i) & \text{if } \theta_i = 0, \\ Rc_i - T(z - c_i) & \text{if } \theta_i = 1. \end{cases} \quad (5)$$

For a system using PES, each player payoff can be either positive or negative in the range: $-zT < G_i < zT$ (Eq. (5) extreme

cases are: $c_i = 0$ and $c_i = z$). Each player i evaluates his/her payoff G_i . If the payoff is greater than the aspiration level ($G_i > AL$, with $AL = 0$), the player maintains his/her current state, otherwise, he/she switches the current state. The aspiration level was defined as a null payoff here, but any other value can be chosen.

Player i switches his/her state only if his/her payoff is null or negative, that is $G_i(\theta_i) \leq 0$. Applying this condition to the null payoff situation ($G_i(\theta_i) = 0$) in Eq. (5), one has

$$G_i(\theta_i) = \begin{cases} c_i T - (z - c_i) R \leq 0 & \text{if } \theta_i = 0, \\ c_i R - (z - c_i) T \leq 0 & \text{if } \theta_i = 1. \end{cases} \quad (6)$$

For a defector to maintain his/her current state, T must provide a null gain $c_i T - (z - c_i) R = 0$, which leads to the critical temptation value $T_c = [(z - c_i) / c_i] R$. In the case of a cooperator, the null payoff occurs when $c_i R - (z - c_i) T = 0$ and $T_c = [c_i / (z - c_i)] R$. These two cases can be written by a simple equation:

$$T_c(z, c_i) = [(z - c_i) / c_i]^{(-1)^{\theta_i}} R. \quad (7)$$

The relevant variable is $[(z - c_i) / c_i]^{\theta_i}$, which is directly related to the problem variables, in contrast with the DES relevant variable $(z - n) / (z - n - m)$. However, as in DES, notice that T_c does not depend on the spatial configuration of the c_i cooperators within the z neighbors; it depends only on the absolute values of c_i and z . For this reason, we can use the one-dimensional geometry in the following.

An interesting feature observed for T_c in PES was its dependence on the player state. Critical temptation values are the same for defectors and cooperators, but they appear in reverse order. For example, consider a cooperator playing against $z = 4$ neighbors, if there is no cooperator in the neighborhood, then $T_c(4, 0) = 0$; if there is one cooperator, $T_c(4, 1) = 1/3R$ and so on. Then $T_c(4, c_i) = \{0; 1/3R; R; 3R; \infty\}$, for $c_i = \{0, 1, 2, 3, 4\}$. Now consider a defector in the same situation, then $T_c(4, c_i) = \{\infty; 3R; R; 1/3R; 0\}$, for $c_i = \{0, 1, 2, 3, 4\}$.

4. Numerical results: emergence of a new quasi-regular phase

PD has been simulated by adopting PES in a one-dimensional cellular automaton. The system has $L = 1000$ cells²; $L\rho_0$ are set as cooperators and $L(1 - \rho_0)$ as defectors. We have used $t = 1000$ steps because the steady state is reached rapidly if compared to number of steps (approximately 100 steps in slower systems, see figures in Appendix) and it is faster for higher connectivity.³ The asymptotic proportion of cooperators, ρ_∞ , is averaged over 1000 realizations. The quantity T varies in fixed steps of $\Delta T = 0.01$ in the range $1 \leq T \leq 2$, and ρ_0 varies in steps of $\Delta\rho_0 = 0.1$ in the range $0 < \rho_0 < 1$. We explored the connectivity in the range $2 \leq z \leq 30$. For the spatio-temporal patterns generated by the cooperative/defective clusters, smaller systems were employed ($L = 500$).

Despite the equivalence with d dimensional lattices for the determination of the critical temptation values, the one-dimensional case has several advantages (Pereira et al., 2008a, b): it facilitates explanation of the process of invasion of cooperative/defective clusters as well as the oscillations of ρ_∞ during the steady regime, as observed in the pioneer work of Nowak and May (1992) and Nowak and Sigmund (1993). In addition to explaining these phenomena, it is also possible to save the system history in a single static image (see the spatio-temporal patterns).

² We have performed numerical studies of the finite size effects. Due to the one-dimensional geometry, they do not significantly alter the results for $L \gg z$.

³ Each numerical simulation requires $time = 10z$ (seconds) in a 3 GHz processor. Simulations are basically processor demanding and low memory requiring. Our computational codes were done using C++ language.

¹ For x positive, the function $\text{int}(x)$ gives the largest integer less than or equal to x .

After a transient regime, the system reaches the steady state with the asymptotic proportion ρ_∞ . In the steady state, the system can present the cooperative, chaotic or defective phases by adopting DES, and the cooperative or *quasi*-regular phases (which was not characterized in the literature before) by adopting PES. The cooperative phase is characterized by the majority of players being cooperators. If the majority of players are defectors, the system is in defective phase. These two phases are not sensible to the initial configuration. In these cases the fluctuations of ρ_∞ , given by the value of the standard deviation (SD), are almost null. In contrast, the chaotic phase is highly sensitive to small changes in the initial configuration (larger ρ_∞ fluctuations— $SD \sim 0.5$). In the *quasi*-regular phase, ρ_∞ displays small oscillations around $\rho_\infty \sim 0.5$. Although, there is a very large number of players switching their states, they balance themselves, and the system is not sensitive to the initial configuration ($SD \sim 0$).

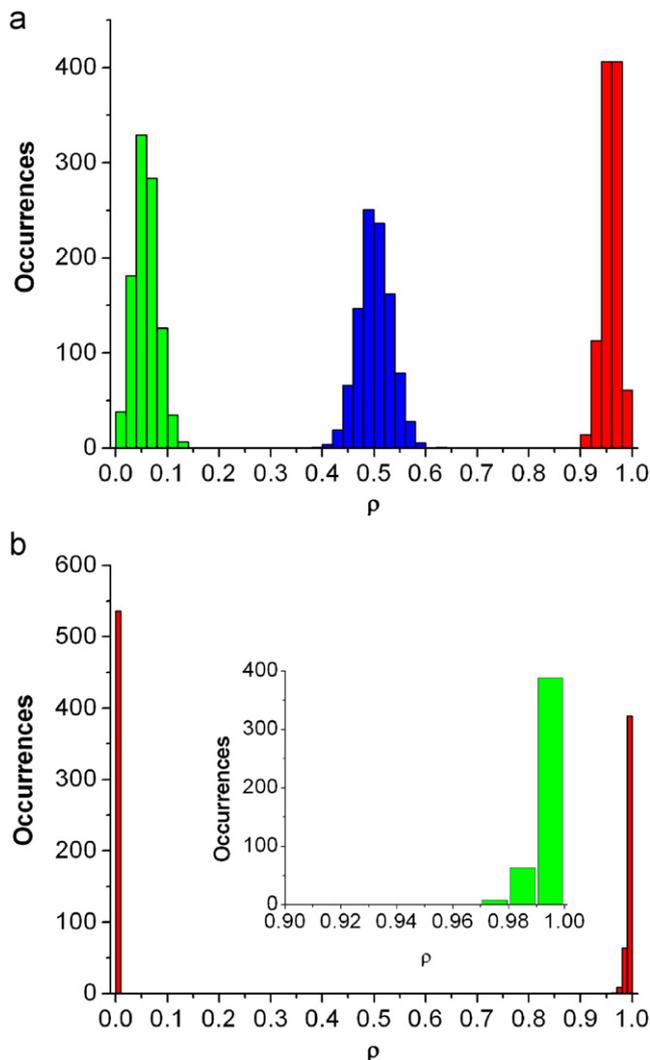


Fig. 3. Histograms of 1000 simulations of systems that reach the cooperative, defective, chaotic, and *quasi*-regular phases. Each color gives the distribution of ρ_∞ for the different steady state in the system that reached: (a) blue: *quasi*-regular phase— $\rho_\infty \sim 0.5$ (system adopting the EEP; $z=8$; $\rho_0=0.5$; $T=1.67$); red: cooperative phase— $\rho_\infty \sim 1.0$ (EED; $z=8$; $\rho_0=0.8$; $T=1.45$); green: defective phase— $\rho_\infty \sim 0$ (EED; $z=8$; $\rho_0=0.8$; $T=1.75$). (b) chaotic phase— $\rho_\infty \sim 0.5$, but a large standard deviation (EED; $z=8$; $\rho_0=0.5$; $T=1.35$). Inset: magnification of region $\rho_\infty \sim 1.0$, notice the existence of more than two possible stationary states for the system. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 3 depicts the histograms generated by these phases. The pattern of the distribution is a normal deviate centered in ρ_∞ , with small standard deviation compared with the mean value. Chaotic phase is an exception because its mean value is $\rho_\infty \sim 0.5$, but the histogram reveals a distribution with two main poles: one where the cooperation emerged and another where the defection dominated the system. Notice that this result is not an oscillation with period two.

In the following figures we show the phase-diagram of systems adopting PES, which consist of cooperative and *quasi*-regular phases, and the patterns that emerge during the transient time and the ones which persist in the steady state. The patterns are a visual way to understand the phases. Simulation results reveal that the defective and chaotic phases are absent when players adopt PES. The defective phase does not occur because a defective cluster yields negative payoff to its members, and players change their states. The absence of chaotic phase is demonstrated by the small standard deviation ($SD \sim 0$) over all the parameter space.

4.1. Transient and steady regimes: exploration of the parameter space

To depict the asymptotic proportion of cooperators in the steady state, we have used surfaces to show ρ_∞ as a function of T and ρ_0 . Figs. 4 and 5 display these ρ_∞ surfaces for even (without self-interaction) and odd z values (with self-interaction), respectively. This phase-diagram presents abrupt variations after T_c and, eventually, it may go from cooperative to *quasi*-regular phase or decrease the proportion of cooperators in the system. The standard deviation of ρ_∞ is almost null for all these systems.

An interesting aspect is the ρ_∞ symmetry with respect to the $\rho_0 = 1/2$, that is, $\rho_\infty(\rho_0 = 1/2 - \phi) = \rho_\infty(\rho_0 = 1/2 + \phi)$ with $0 \leq \phi \leq 1/2$. In a given region of the parameter space, the presence or absence of the self-interaction changes T_c values and may change the phase (cooperative or *quasi*-regular).

In PES, if all players are cooperators ($\rho_0 = 1$), they always receive a positive payoff, and no player changes his/her state, so $\rho_\infty(T; \rho_0 = 1; z) = 1$. Otherwise, if all players are defectors ($\rho_0 = 0$), in the first round all players receive a negative payoff and all of them switch their states, returning to the previous mentioned situation. Consequently, $\rho_\infty(T; \rho_0 = 0; z) = 1$. Thus, the ρ_∞ symmetry around $\rho_0 = 1/2$ is a consequence of PES. For $\rho_0 = \beta$ (any value in the range $[0, 1]$), rN players receive a positive payoff and $(1-r)N$ players a negative payoff, whereas when $\rho_0 = 1-\beta$, $(1-r)N$ players receive positive payoff and rN players obtain negative payoff, thereby generating the symmetry.

The surface projection $\rho_\infty(T, \rho_0, z)$ at plane $\rho_\infty T$ shows ρ_∞ as a function of T for different ρ_0 values. In Fig. 6, one sees the plots for some even and odd z values. The transitions in ρ_∞ can be seen when the parameter T passes through critical temptation thresholds, T_c , given by Eq. (7). In these plots, the T_c values are marked by dashed vertical lines. For example, in Fig. 6c ($z=8$ —without self-interaction) $T_c=5/3$. Meanwhile, in Fig. 6d ($z=9$ —with self-interaction) $T_c=\{5/4; 2\}$.

The results show that the cooperative phase is more prominent than the *quasi*-regular phase. Increasing the z values, the quantity of T_c values rises. When the system goes through T_c , ρ_∞ varies. The non-dependence on a group provides more freedom for each player to seek the best action, so that his/her aspiration level is reached. When the *quasi*-regular phase emerges, mean total payoff is an average between cooperation and defection payoffs, which is worse than cooperation but better than defection.

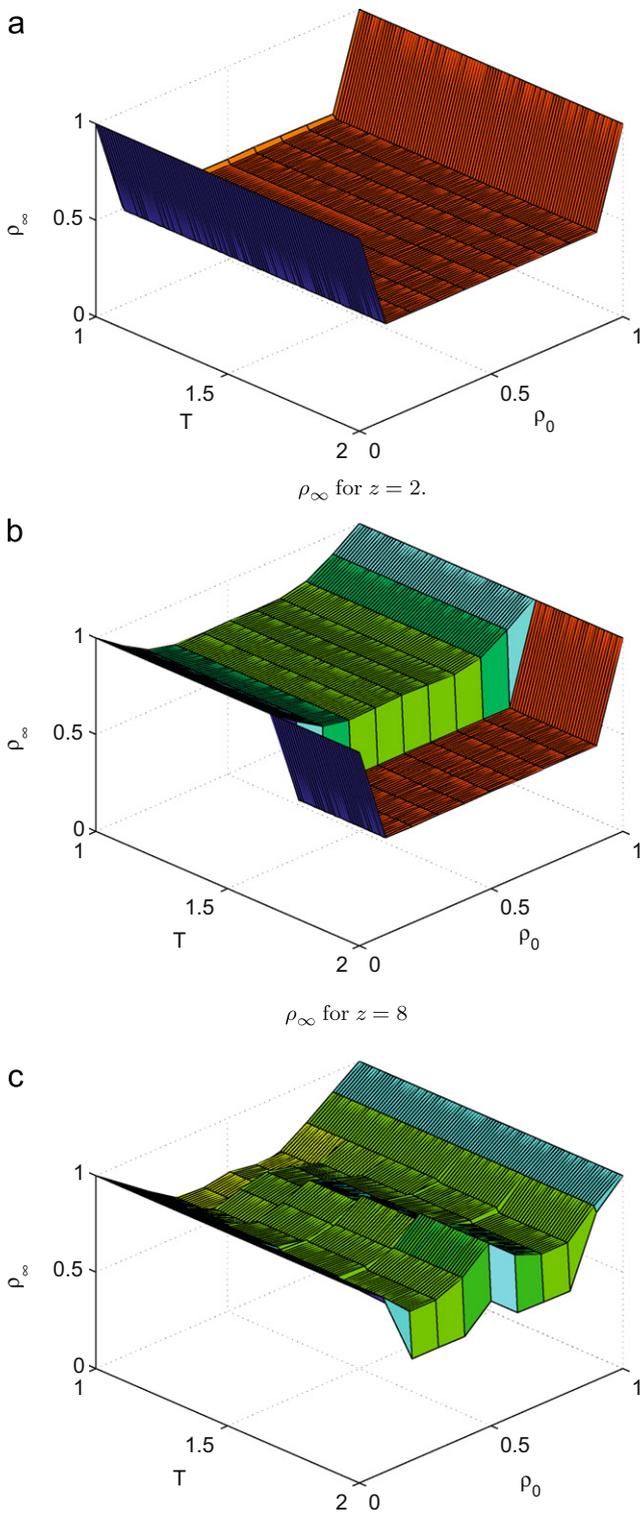


Fig. 4. Asymptotic proportion of cooperators (ρ_∞) as a function of temptation value— T —and initial proportion of cooperators— ρ_0 —for $z=\{2; 8; 30\}$.

4.2. Lowest temptation level

Consider the particular case $T=1$, the lowest temptation level to defect. We adopted the following payoffs: $R=1$ and $P=S=0$ for DES; and $R=1$, $P=-R=-1$, and $S=-T=-1$ for PES. For $T=1$ and $R=T$, when a cooperator plays against a defector both receive the same payoff. This result led Duran and Mulet to explicitly

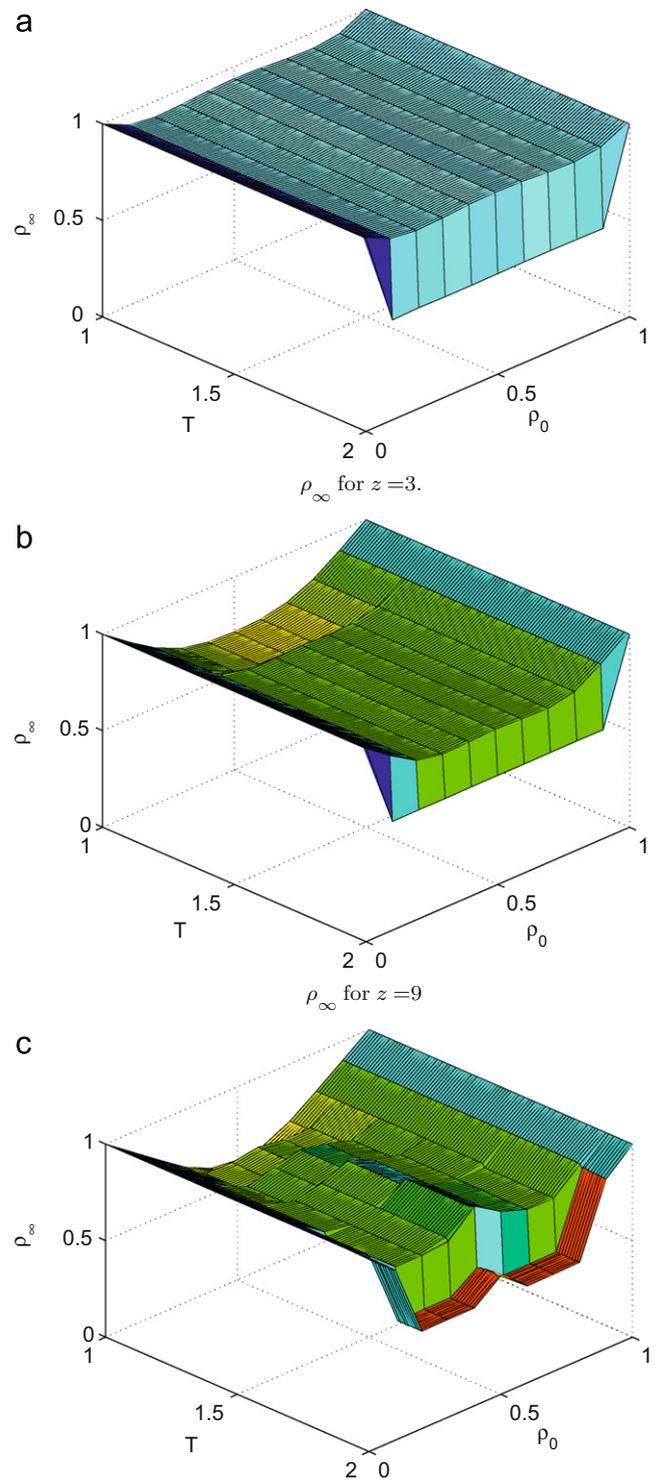


Fig. 5. Asymptotic proportion of cooperators (ρ_∞) as a function of temptation value— T —and the initial proportion of cooperators— ρ_0 —for $z=\{3;9;29\}$.

consider, $T=1$ as a trivial case (other authors do not even mention this case), since players do not switch their states during dynamics. Meanwhile, this statement is true only for players using DES in the case of each player interacting with only one neighbor. In this case, all players do not switch their states during dynamics and the system keeps its initial configuration ($\rho_\infty = \rho_0$) for every set parameters. Then, for $T=1$, the ρ_∞ plot is a flat surface as a function of ρ_0 and z , as depicted in Fig. 7. If players

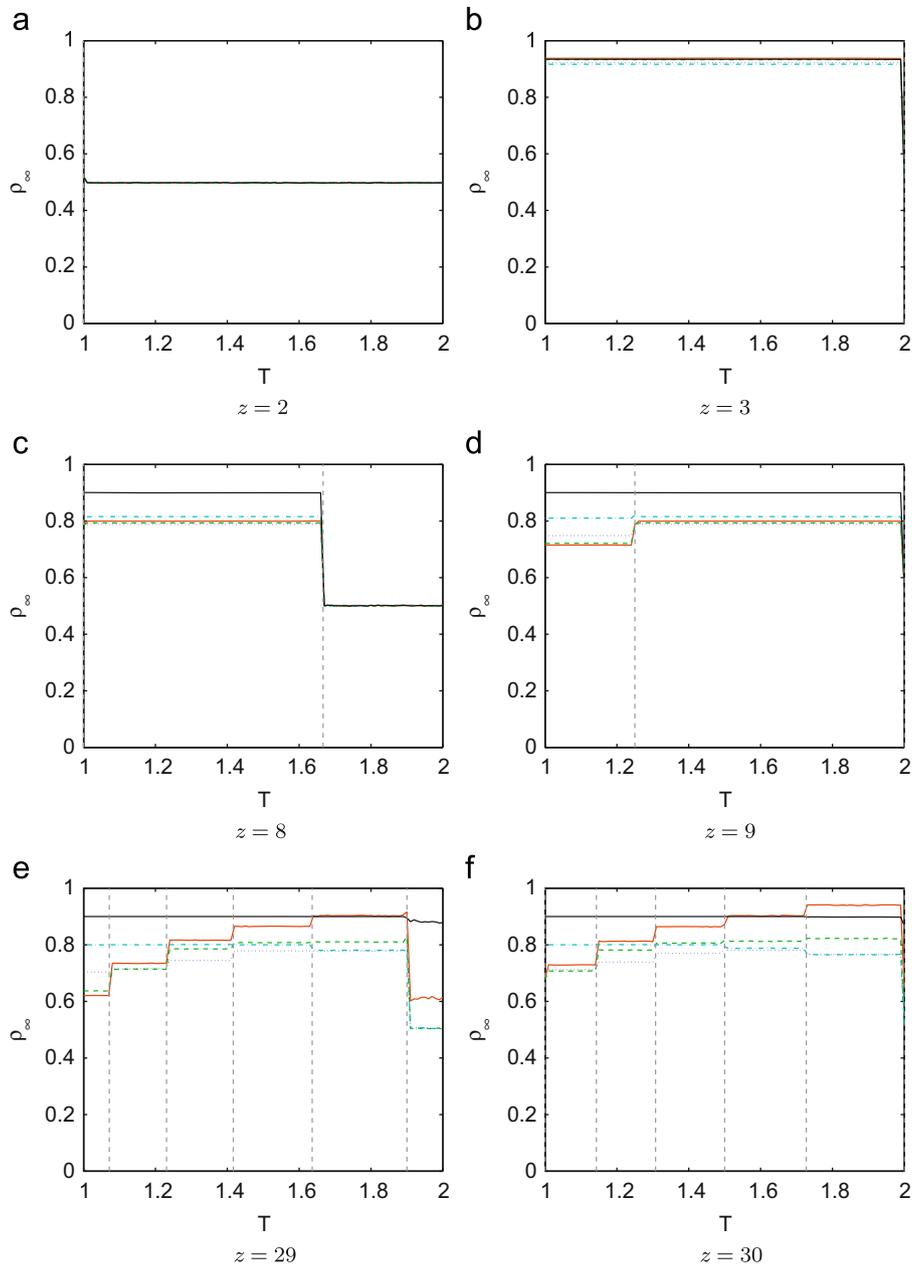


Fig. 6. Asymptotic proportion of cooperators (ρ_∞) as a function of temptation to defect— T —for $z=(2; 3; 8; 9; 29; 30)$. The vertical dashed lines sign T_c in plots and these values are given by Eq. (7). Blue: $\rho_0 = 0.5$; green: $\rho_0 = 0.6$; red: $\rho_0 = 0.7$; cyan: $\rho_0 = 0.8$; magenta: $\rho_0 = 0.9$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

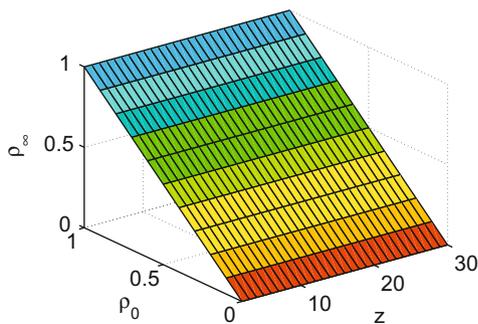


Fig. 7. If $T=1$ was a trivial case, ρ_∞ surface as a function of ρ_0 and z is not altered from the initial configuration due to dynamics.

play with more than one neighbor, they can switch their states and in fact they do. It is shown below that $T=1$ is indeed a non-trivial case.

4.2.1. *Players adopting Darwinian evolutionary strategy*

The $\rho_\infty(T=1; \rho_0; z)$ plots and their standard deviation are depicted in Fig. 8 with and without self-interaction ((a) odd and (b) even z values, respectively). They show that players do not maintain their initial states during system evolution. Players switch their states because ρ_∞ is not a straight line as displayed in Fig. 7. This occurs because players compare their total payoff instead of the payoff per play. In these systems the cooperative phase is more prominent than the defective one. Self-interaction

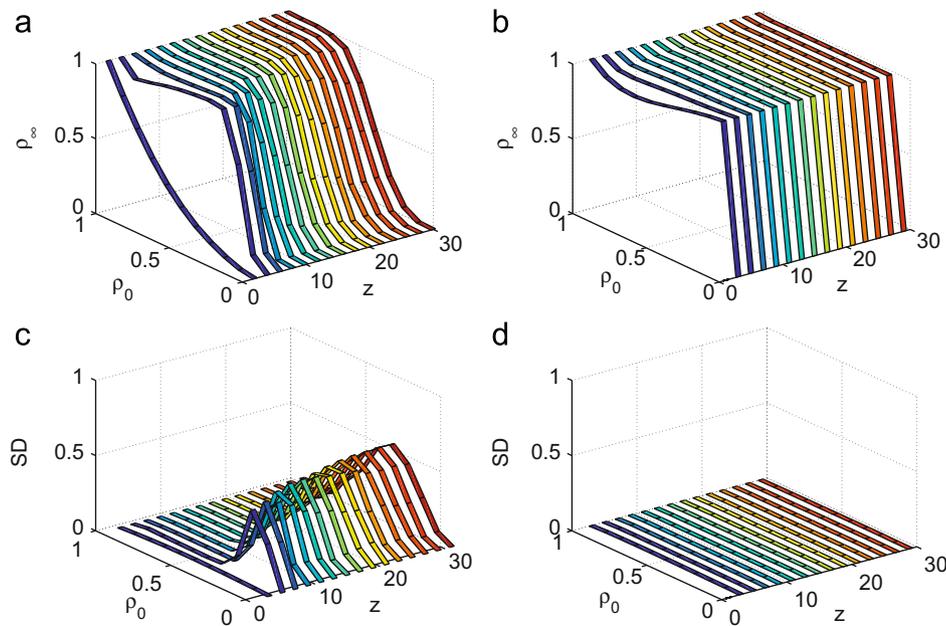


Fig. 8. Asymptotic proportion of cooperators (ρ_∞) as a function of initial proportion of cooperators— ρ_0 —and connectivity— z —for $T=1$. Players adopt DES. Specifically, (a) ρ_∞ for even z values, with $z=[2; 30]$ (without self-interaction); (b) ρ_∞ for odd z values, with $z=[3; 29]$ (with self-interaction); (c) fluctuation of ρ_∞ displayed in (a); (d) the same for (b).

increases the cooperative phase, once a cooperator has at least one positive payoff and a defector has a null payoff. In this way, self-interaction is advantageous to the cooperator and they can be replicated more easily (greater payoff), so cooperation emerges in the system.

In Fig. 8a, ρ_∞ is larger or lower than the “expected” value $\rho_\infty = \rho_0$. For $\rho_0 < 0.4$, $\rho_\infty < \rho_0$ and $SD \sim 0.5$. On one hand, if the proportion of cooperators is small ($\rho_0 \sim 0$), these cooperators receive several null payoffs (from interactions with defectors) and their payoffs decrease, while the defector payoffs increase. Consequently, cooperators switch their states and cooperation does not emerge. On the other hand, when $\rho_0 > 0.4$, the system has more cooperators in the beginning of the dynamics. Cooperators playing against themselves receive a positive payoff. The total payoff of cooperators become greater than the one of defectors that confronted other defectors, so these cooperators do not switch their states. In Fig. 8b, the cooperation emerges for $\rho_0 \geq 0.1$ due to self-interaction, which increases the cooperators payoff, as explained previously.

Some defectors, despite exploiting neighboring cooperators, may also confront other defectors, which lead to a decrease in their total payoff. So they switch their states by copying the cooperative neighbors, with a greater payoff, driving the system to the cooperative phase. Meanwhile, for $z=2$, there is one exception: the cooperation does not emerge because cooperator j , which plays against cooperator i and defector k , has a payoff $G_j=1$. If defector k interacts with another cooperator, he/she has a payoff $G_k=2$, so the cooperator j copies the player k state. In this way, defection becomes the main behavior of the players, raising the defective phase. This allows one to conclude that greater connectivity favors cooperation, since it increases the chance of a cooperator to interact with other cooperators.

Figs. 8c and d show the fluctuations of ρ_∞ for even and odd z values, respectively. If $SD \sim 0.5$, the system is in the chaotic phase for that region on the parameter space. Chaotic phase is present only for even z values (Fig. 8c) and occurs between the cooperative and defective phases as ρ_0 decreases (see the cliff in Fig. 8a). For self-interacting players, cooperation increases and the chaotic phase does not appear at all (see Fig. 8d).

4.2.2. Players adopting Pavlovian evolutionary strategy

For PES, when two defectors are playing against themselves, their payoff is negative ($P = -T = -1$). This is enough for both to switch their states, but they can still interact with other cooperators of their neighborhoods. If even by exploiting the neighbors, their payoffs do not become positive, they switch their states and cooperation emerges in the system. The $\rho_\infty(T=1; \rho_0; z)$ plots are depicted in Fig. 9.

Fig. 9a without self-interaction (even z values) and Fig. 9b with self-interaction (odd z values) are very different from those observed for DES (Figs. 8a and b). Nevertheless, they also confirm that proportion of cooperators is not trivial for $T=1$. Here, the majority of the players cooperate in all the parameter space. The exceptions occur for $z=2$ and 4 (without self-interaction, see Fig. 9a), where the *quasi*-regular phase emerges with $\rho_\infty \sim 0.5$.

Notice that ρ_∞ decreases as z increases, with a more pronounced stiffness in the presence of self-interaction, because defectors always receive a null payoff due to his/her self-interaction. Besides, a ρ_∞ symmetry occurs regarding the $\rho_0 = 1/2$, because for $\rho_0 = \beta$, rL players receive a positive payoff and $(1-r)L$ player receives a negative one, where as, for $\rho_0 = 1-\beta$, $(1-r)L$ players receive a positive payoff and rL players receive a negative one, where r is an arbitrary proportion of players that depends on the distribution of the players in each time step.

In all the parameter space SD values are small, what confirms the non-existence of the chaotic phase. The plots demonstrate that the cooperative phase is dominant. The *quasi*-regular phase occurs only for $z=2$ and 4. The proportion of cooperators is lower than the “expected” only when the *quasi*-regular phase occurs.

4.3. Spatio-temporal patterns

As the system evolves, some cooperative/defective clusters emerge and their patterns are generated from local interactions among players. When the players adopt DES, the differences between the border player payoffs are fundamental to determine the system dynamics (Pereira et al., 2008b). In PES, these border payoff differences are not as crucial as in DES. For a more detailed

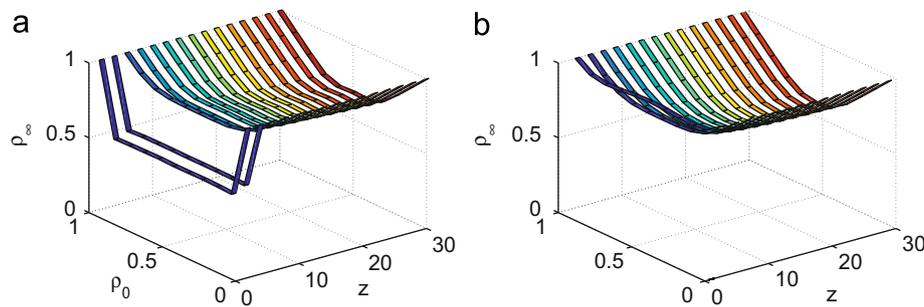


Fig. 9. Asymptotic proportion of cooperators (ρ_∞) as a function of initial proportion of cooperators— ρ_0 —and connectivity— z —for $T=1$. Players adopt PES. Specifically, (a) ρ_∞ for even z values, with $z=[2; 30]$ (without self-interaction); (b) ρ_∞ for odd z values, with $z=[3; 29]$ (with self-interaction).

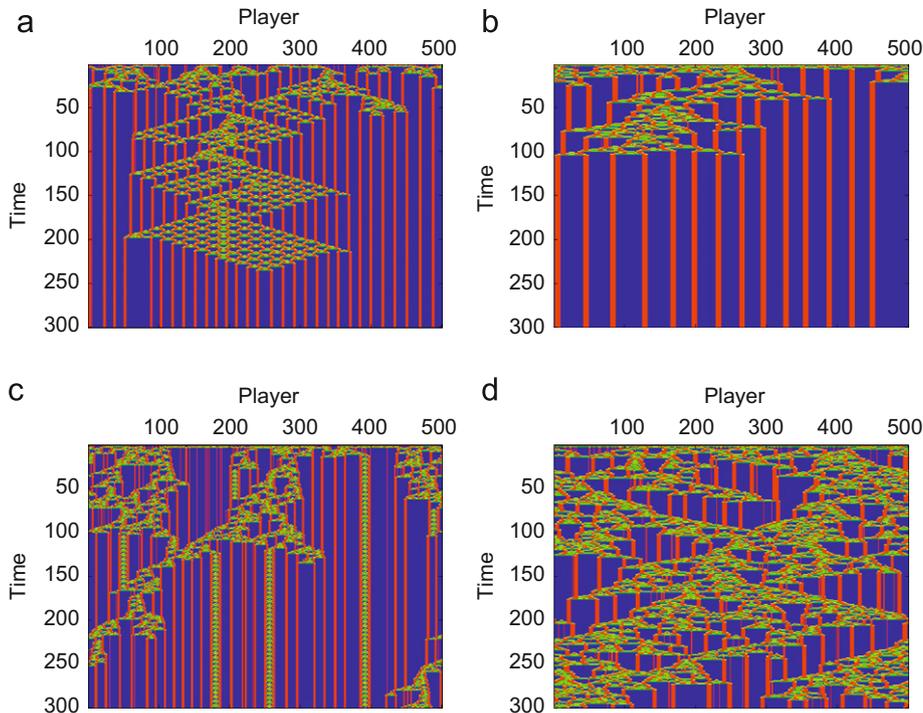


Fig. 10. Illustration of intersections of *glider* with *fingers* and other *gliders*. The parameters of these simulations are: $L=500$, $t=300$, all without self-interaction, and (a) $T=1.40$, $\rho_0=0.7$, $z=14$; (b) $T=1.70$, $\rho_0=0.3$, $z=26$; (c) $T=1.40$, $\rho_0=0.5$, $z=12$; and (d) $T=1.40$, $\rho_0=0.3$, $z=24$.

explanation of pattern formation, see Appendix A. In the pattern images, the cell color scheme is the following: blue is a cooperator; red is a defector; green is a cooperator that was defector in a previous round; yellow is a defector that was cooperator in a previous round. The spatio-temporal representation shows us that these clusters can form *fingers* and *gliders*, and they can interact with each other.

A *finger* is a cluster that extends along a straight line during dynamics. It can be simple (flat) or complex (composed by regular oscillations, like a saw-tooth, for example). The *finger* interior can consist in cooperators/defectors or intricate combinations of cooperators and defectors. It may present symmetry with respect to the central player of the pattern and periodicity in the player states. A *glider* is a cluster that extends itself diagonally, and it has the same features as the *fingers*.

One may notice that clusters composed exclusively by cooperators sustain themselves by maintaining cooperation among them. However, cooperation remains only when the size of the cooperative cluster is large enough to maintain a positive payoff for its members. The members placed in the borders are exploited by defectors, but they do not switch their states because their payoffs are positive, even though their payoffs are

lower than the payoffs of exploiters and inner players of the cooperative cluster. Remembering the migratory animals, which cooperative interface members can be exploited by defective ones, but these cooperators do not need to retaliate, because they can get the necessary for their well being (resource/help to feed) from the interactions within their own cooperative group. The same for the hunters that cooperate within their group and even with rivals (another group or lonely animals). This is possible when the group has a minimum number of cooperative skilled hunters, because they can “provide” enough resources to the group survival, besides the rival exploitation/concurrence.

Nevertheless, if several skilled selfish hunters get together, the group may split due to the severe competition that they impose to themselves. If a defective cluster is large enough to produce negative payoffs of their members, it will not be stable. Thus, players with negative payoffs will switch their states immediately. Therefore, Tragedy of Commons⁴ does not occur because the

⁴ The Tragedy of Commons occurs when multiple individuals act independently, aiming only his own interest. When this action is done by multiple individuals simultaneously, it can destroy the advantage desired by all of them, for example, finishing the desired resources in the environment.

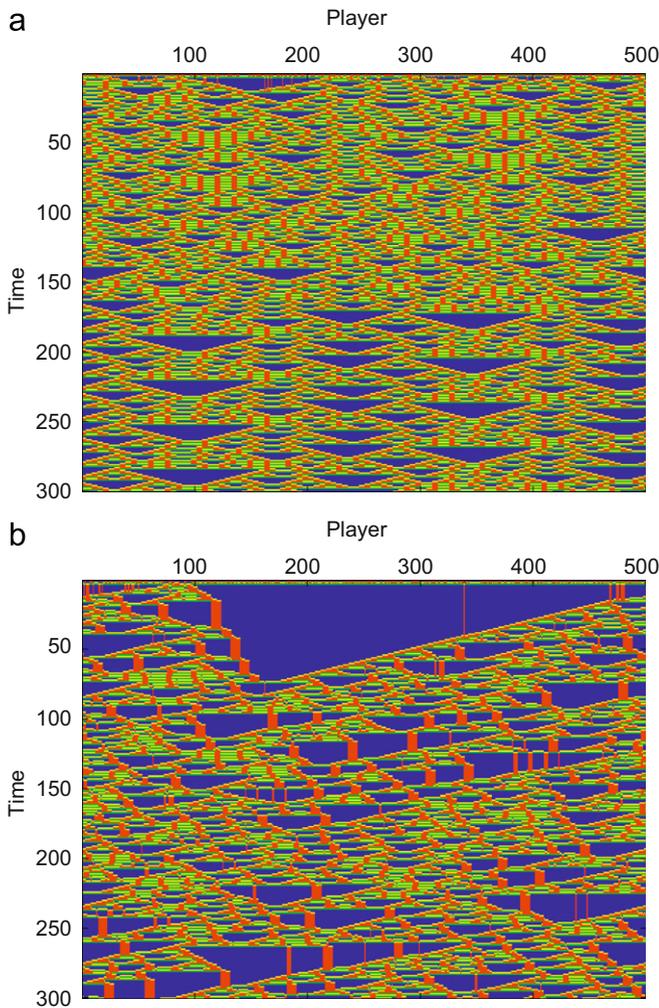


Fig. 11. Formation and evolution of the *quasi-regular* system. The parameters of these simulations are: $L=500$, $t=300$ and (a) $T=1.90$, $\rho_0=0.3$, and $z=26$ (without self-interaction) and (b) $T=1.90$, $\rho_0=0.5$, and $z=29$ (with self-interaction). Note the presence of a triangle-like at time $t=50$ at players 200–250.

negative payoff of the players does not persist for more than one round, as in the case of players adopting DES. In this way, the mean payoff of the population is greater when PES is adopted rather than DES, where the Tragedy of Commons takes longer to vanish (if it vanishes) (Pereira et al., 2008a, b). In PES, a player uses his/her own payoff to decide whether he/she will switch his/her state or not. It is an individual decision based on the personal aspiration level. Therefore, collective features may not occur.

Different neighborhood configurations may generate positive or negative payoffs for the players. It is simple to calculate the maximum defective and the minimum cooperative cluster size, which can remain together during system evolution (stable clusters). In the case of the cooperative cluster, player i does not switch his/her state if there are at least c_{\min} cooperators in his/her neighborhood. This guarantees a positive payoff $G_i^{c_{\min}}(\theta_i = 1) > 0$. Thus, from Eq. (3) one has

$$c_{\min} > z/(1+R/T). \tag{8}$$

The situation is reversed in the case of a defective cluster, where the player i must have a maximum of d_{\max} defectors in his/her neighborhood, so that $G_i^{d_{\max}}(\theta_i = 0) > 0$, and

$$d_{\max} < z/(1+R/T). \tag{9}$$

For example, defective *fingers* may be composed of, at most, d_{\max} defectors, and cooperative ones of, at least, c_{\min} cooperators.

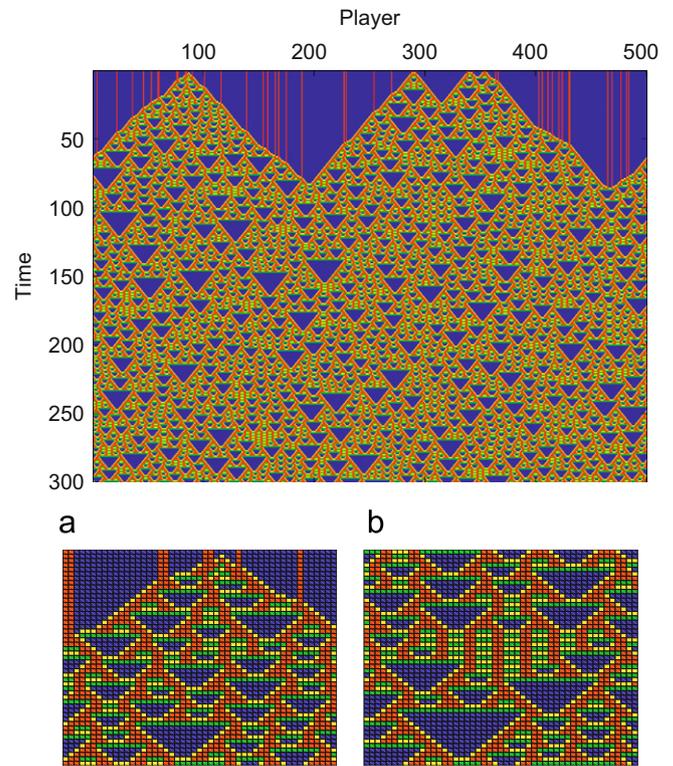


Fig. 12. Formation and evolution of the *quasi-regular* system. The parameters of this simulation are: $L=500$, $t=300$, $T=2.00$, $\rho_0=0.9$ and $z=6$ (without self-interaction). (a) Magnification of the region around to $t=1$ for the players close to player 90; (b) magnification of the region: $t=260$ around player 160.

For instance, for $z=2$, *fingers* formed by up to two players are always smooth and continuous (see simple and complex *fingers* in Appendix A). In general, the stable clusters are the cooperative ones (with at least c_{\min} cooperators) and small defective ones. Defective clusters that are greater than d_{\max} destabilize themselves rapidly in a few rounds.

The transient regime is the time necessary for the pattern interactions to cease or stabilize, and it varies depending on the employed set parameters (see Fig. 10). Other possibility is the emergence of the *quasi-regular* phase, which is stationary, but there is a very large number of players who switch their states but do not emerge *fingers* or *gliders*, and $\rho_{\infty} \sim 0.5$.

The intersection among cluster patterns generates very interesting structures. For example, Fig. 10 illustrates the presence of *gliders*⁵ that interact among themselves and with *fingers*. These interactions can generate either simple (Figs. 10a–d) or complex (Fig. 10c) *fingers*.

One can understand the *quasi-regular* phase by observing the behavior of cooperative/defective clusters. If defectors of a particular cluster receive a negative payoff at moment t , these players switch their status to cooperators at $t+1$. If this action is synchronized among different clusters, cooperation may emerge. Otherwise, if they are not synchronized and one cluster switches at the instant t and its neighbors at the instant $t+1$, these clusters alternate between cooperation and defection consequently, there is a balance among cooperators, who switch their states to defectors and *vice versa*, keeping the proportion of cooperators almost constant, with small oscillations due to the different size of clusters.

⁵ In systems that adopt DES, the inclination of *glider* is determined by the direction of upgrade of player states. If the system adopts PES, the *glider* can propagate both from left to right or *vice versa*.

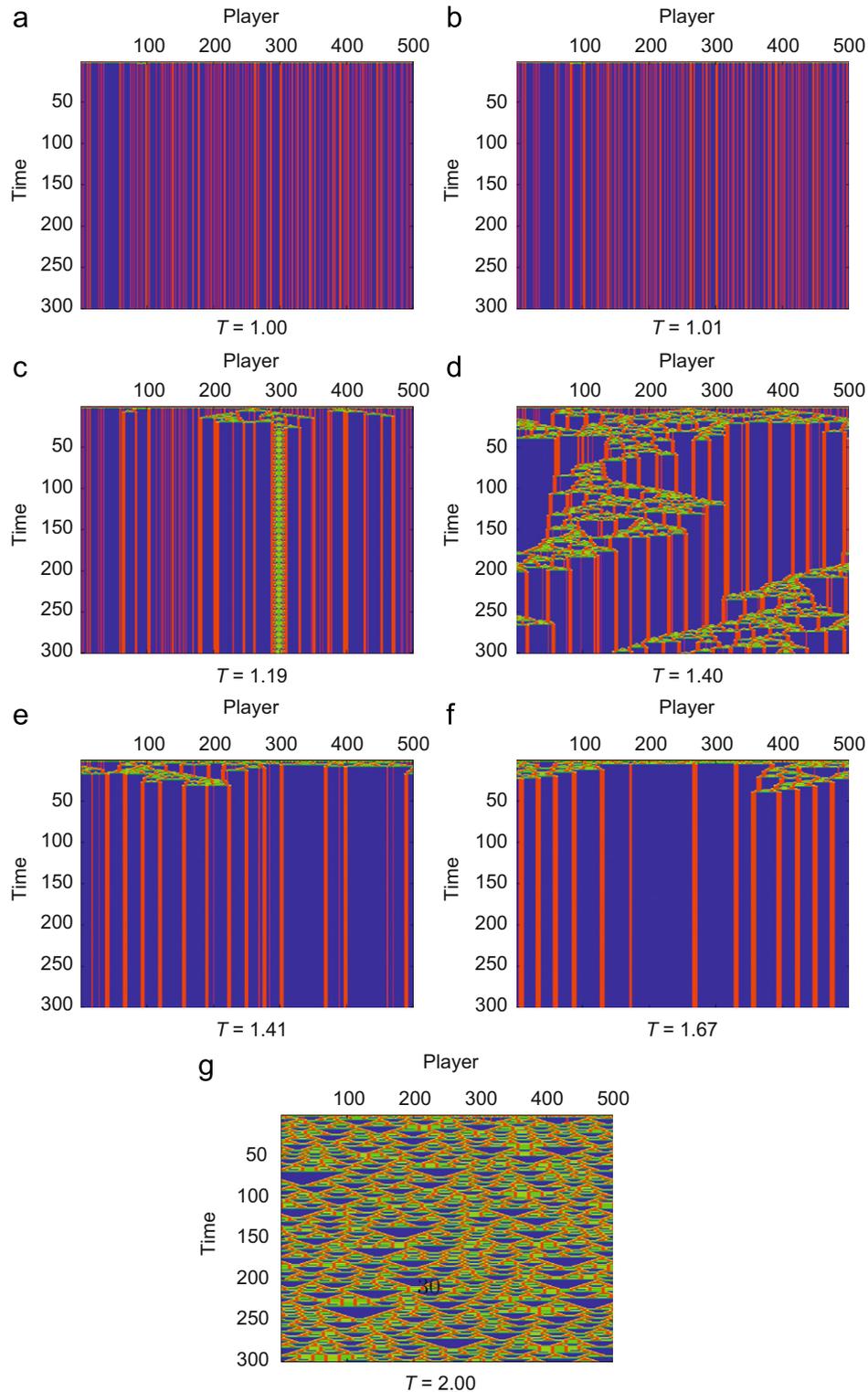


Fig. 13. Sequence of numerical simulations that show how the variation of the temptation— T —alters the cooperative/defective cluster patterns. In the interval between the presented T values there are no changes in patterns. The parameters of these simulations are: $L=500$, $t=300$, $\rho_0=0.3$ and $z=24$ (without self-interaction). To: (a) $T=1.00$; (b) $T=1.01$; (c) $T=1.19$; (d) $T=1.40$; (e) $T=1.41$; (f) $T=1.67$; and (g) $T=2.00$.

Figs. 11 and 12 show some examples where the synchronization among clusters has not occurred, $\rho_\infty \sim 0.5$, and many players switch their states at each round, giving rise to the *quasi-regular* phase. The triangles appear when adjacent defectors switch their states to cooperation at the same time. Fig. 11a shows a transient

followed by the *quasi-regular* phase with periodicity. It also shows another triangular pattern, but its interior is not exclusively composed by cooperators or defectors, but by complex cooperative and defective *fingers*. This pattern is a triangle with not well defined sides, called triangle-like, see Fig. 11a. In Fig. 11b,

one can notice that the size of cooperative clusters is large for a system with greater connectivity, but the system phase remains *quasi-regular*.

In Fig. 12, few defective clusters are enough to drive the system to *quasi-regular* phase instead of a cooperative one, despite the high initial proportion of cooperators. This occurs because $T=2$. In Fig. 12a, one can see a magnification of the defective clusters. Fig. 12b illustrates a so-called triangle-like that emerges at $t=260$ around player 160.

There are T intervals where increases or decreases in its value do not alter the system dynamics and ρ_∞ for the same system (identical initial configuration of cooperators in the lattice and z). In Fig. 13, for $z=24$, when the system passes through the critical temptation values $T_c=\{1; 13/11; 7/5; 15/11; 8/5; 17/9\}$, transient changes and patterns increase in quantity and variety, characterizing the phase transitions. For instance, from $T=13/11$ (see Fig. 13c), some *gliders* appear in the initial steps and a complex *finger* emerges and propagates during all system evolution. From $T=7/5$ (see Fig. 13d) the initial *gliders* are increased and propagate during the dynamics, and the *finger* does not emerge as before. Furthermore, for $1 \leq T < 2$, the system presents the cooperative phase in the steady regime, and for $T=2$ (see Fig. 13g), the system enters in the *quasi-regular* phase.

5. Conclusion

In this work we have adopted the Pavlovian Evolutionary Strategy to explore the one-dimensional cellular automaton, where each cell is a player who plays the Prisoner Dilemma with z neighbors.

Our results for PES were as follows: (i) phase transitions occur in well defined values of temptation T_c , which have been analytically calculated; (ii) there exist a cooperative and a new phase: the *quasi-regular*, which depend on the temptation value to defect; (iii) defective and chaotic phases are absent; (iv) the Tragedy of the Commons does not take place; (v) the cooperation is more remarkable, compared with systems with DES. The stationary state of the system was also analyzed, and the patterns of the clusters due to the local interactions of players were explained. The symmetry of ρ_∞ in the phase diagram is explained by arguments of equivalence of events occurrence.

Results of $T=1$ demonstrate that the system is not static and trivial when players play the IPD with more than one neighbor ($z > 1$), as previously supposed. On the one hand, if players adopt DES, cooperative, defective, and chaotic phases may occur. The chaotic phase appears for even z values only, without self-interaction. The more remarkable result is the presence of a cooperative phase ($\rho_\infty \sim 1$) for $\rho_0 > 0.5$, without self-interaction and for $\rho_0 > 0$ with it. On the other hand, adopting PES, as z increases, $\rho_\infty(1; \rho; z \gg 1)$ decreases with the exception $z=\{2; 4\}$, with a more pronounced decrease when the self-interaction is present. For $z=\{2; 4\}$, the system presents the *quasi-regular* phase. The increase in connectivity favors cooperation for DES, but it decreases cooperation for PES. Cooperation emerges even when cooperators and defectors have the same payoff in the IPD.

For systems adopting PES, the dynamics depends strongly on the composition of the cluster neighborhood. However, the location of players in the neighborhood (configuration) is irrelevant for determination of the total payoff. We have also noticed that the transient and steady regimes depend on system parameters. When T varies, the duration of the transient regime changes. In the steady regime, there are changes in the ρ_∞ value when T passes through the T_c values. A system can present the cooperative or *quasi-regular* phases. In the *quasi-regular* phase,

the system can yield a transient and latter, it achieves a periodic ρ_∞ oscillation.

The mean payoff of players is greater when the players are concerned with their own payoff only. If the players copy the action of the neighbor who received the largest payoff, as in DES, they may worsen the outcome of the whole system. Thus, the comparison with the greed for the greatest payoff can cause the ruin of all players. In the situation where there is no way to coordinate the moves of players, the best action would be that everyone seeks to have a positive payoff, even if this positive payoff is not the maximum possible. Thus, they could maximize the payoff of the population as a whole.

Acknowledgements

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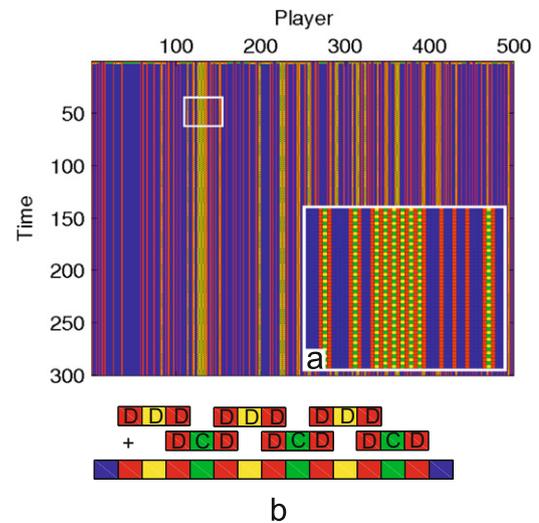


Fig. 14. Formation and evolution of smooth and complex *fingers*. Parameters of this simulation are: $L=500$, $t=300$, $T=2.00$, $\rho_0=0.3$ and $z=3$ (with self-interaction). (a) Magnification of the marked area. (b) Formation of a pattern composed by elementary patterns.

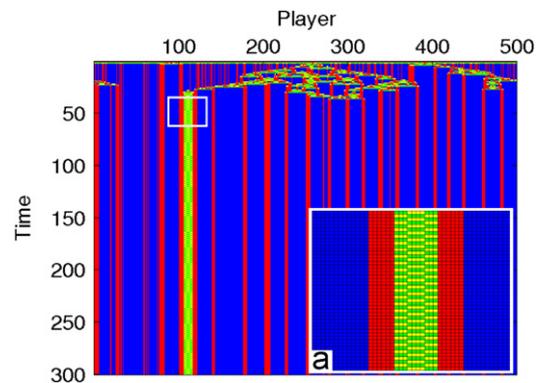


Fig. 15. Formation and evolution of smooth and complex *fingers*. The parameters of this simulation are: $L=500$, $t=300$, $T=1.30$, $\rho_0=0.3$, and $z=22$ (without self-interaction). (a) Magnification of the marked area.

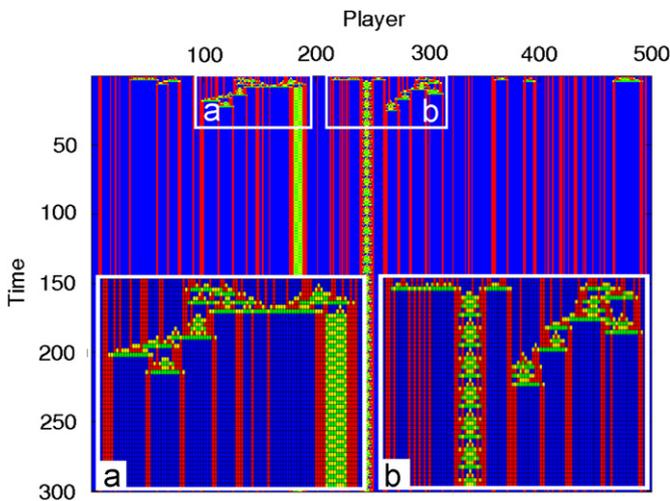


Fig. 16. Formation and evolution of smooth and complex *fingers*. The parameters of this simulation are: $L=500$, $t=300$, $T=1.10$, $\rho_0=0.7$, and $z=12$ (without self-interaction). (a) and (b) Magnification of the marked areas.

Appendix A. Patterns formation

In the following we analyze particular cases to explain the evolution dynamics. The magnification in the images of Fig. 14 shows the complex *finger* with three players with the pattern⁶: $\{D \mathbf{D} D \rightarrow D \mathbf{C} D\}$ and the pattern with 13 players is $\{D \mathbf{D} D \mathbf{C} D D D C D D D C D D D C D D D\}$. The pattern of 13 players is a composition formed by the alternation of the patterns of three players, with overlap (see Fig. 14b). Namely, the patterns $\{D \mathbf{D} D\}$ and $\{D \mathbf{C} D\}$ combine themselves so that the third player of one pattern is the first of the following one. Other combinations formed by the addition of patterns with or without overlapping of edges can be observed, as well.

In the magnification of Fig. 15 there are simple *fingers* with 12 defectors, at most, and also a complex one with the pattern: $\{6D \mathbf{3C} \mathbf{4D} \mathbf{3C} 6D \rightarrow 6D \mathbf{3D} \mathbf{4C} \mathbf{3D} 6D\}$. In the magnifications of Fig. 16, emerging *fingers* have the pattern: Fig. 16a: $\{4D \mathbf{C} \mathbf{3D} \mathbf{3C} D \mathbf{4D} \rightarrow 4D \mathbf{D} \mathbf{3C} \mathbf{3D} C \mathbf{4D}\}$ and Fig. 16b: $\{C D \mathbf{2C} \mathbf{D} \mathbf{2C} D C \rightarrow C D C \mathbf{3D} C D C \rightarrow 3D \mathbf{3C} \mathbf{3D} \rightarrow 2D C \mathbf{3D} C \mathbf{2D} \rightarrow D C D \mathbf{3C} D C D \rightarrow D C \mathbf{5D} C D \rightarrow C D \mathbf{5C} D C\}$. Note the periodicity present in these patterns.

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⁶ C: cooperator player, D: defector one. The player at center of the pattern is printed in boldface.