ISSN 1063-780X, Plasma Physics Reports, 2008, Vol. 34, No. 8, pp. 678–687. © Pleiades Publishing, Ltd., 2008.
Published in Russian in Fizika Plazmy, 2008, Vol. 34, No. 8, pp. 736–745.

PLASMA INSTABILITY

High-Frequency Extensions of Magnetorotational Instability in Astrophysical Plasmas

A. B. Mikhailovskii[a], J. G. Lominadze[b], A. P. Churikov[c], V. D. Pustovitov[a], N. N. Erokhin[a], V. S. Tsypin[d †], and R. M. O. Galvão[d, e]

[a] *Institute of Nuclear Fusion, Russian Research Centre Kurchatov Institute, pl. Kurchatova 1, Moscow, 123182 Russia*
[b] *Kharadze Abastumani National Astrophysical Observatory, Kazbegi Ave. 2a, Tbilisi 0160, Georgia*
[c] *Syzran Branch of Samara Technical University, Sovetskaya ul. 45, Syzran, Samara oblast, 446001 Russia*
[d] *Brazilian Center for Physics Research, Rua Xavier Sigaud, 15, 22290-180, Rio de Janeiro, Brazil*
[e] *Physics Institute, University of São Paulo, Cidade Universitaria, 05508-900, São Paulo, Brazil*

Received October 12, 2007; in final form, January 16, 2008

Abstract—High-frequency extensions of magnetorotational instability driven by the Velikhov effect beyond the standard magnetohydrodynamic (MHD) regime are studied. The existence of the well-known Hall regime and a new electron inertia regime is demonstrated. The electron inertia regime is realized for a lesser plasma magnetization of rotating plasma than that in the Hall regime. It includes the subregime of nonmagnetized electrons. It is shown that, in contrast to the standard MHD regime and the Hall regime, magnetorotational instability in this subregime can be driven only at positive values of $d\ln\Omega/d\ln r$, where Ω is the plasma rotation frequency and r is the radial coordinate. The permittivity of rotating plasma beyond the standard MHD regime, including both the Hall regime and the electron inertia regime, is calculated.

PACS numbers: 52.35.Bj, 94.30.Cq

DOI: 10.1134/S1063780X08080060

1. INTRODUCTION

The Velikhov effect [1], which leads to magnetorotational instability (MRI) [2, 3], was predicted for the standard MHD regime, in which the ion cyclotron frequency ω_{Bi} exceeds substantially both the oscillation frequency ω and the plasma rotation frequency Ω,

$$\omega_{Bi} \gg (\omega, \Omega). \tag{1.1}$$

The importance of MRI was understood due to [3], in which this instability was applied to explain anomalously high viscosity in accretion disks [4]. As a whole, MRI and related instabilities in the MHD regime became a cumulating point of the astrophysical subtrend dealing with accretion disks around a compact object in a binary system. This subtrend was summarized in a review by Balbus and Hawley [5]. Meanwhile, in [6] and, then, in [7–15] and a series of other papers, a regime quite different from the standard MHD regime was considered. This regime was called the Hall regime. Note that, when resistive effects are neglected, this regime corresponds to the condition

$$\omega_{Bi} \ll (\Omega, \omega). \tag{1.2}$$

In accordance with [6–15], the Velikhov effect, i.e., the driving mechanism caused by a decreasing rotation frequency profile, survives in this regime. The importance of studying MRI in the Hall regime was motivated by the problem of the evolution of protostellar disks [6, 7, 14, 15] and the quiescent phase of dwarf nova disks [8, 9].

MRI in the Hall regime was also studied in [16], in which the case of hot-electron plasma was analyzed. The dispersion relation for small-scale perturbations, which simultaneously allows for rotation, finite electron pressure, and electron pressure anisotropy, was derived in [16]. It was pointed out that finite electron pressure should be taken into account in the Hall regime for $\beta \geq 1$, where β is the ratio of the electron pressure to the magnetic field pressure. According to [16], the effects of order β weaken or even suppress MRI. It was shown that, in the presence of electron pressure anisotropy, a hybrid of MRI and anisotropic instability appears, similar to that pointed out for the standard MHD regime in [17].

Physically, the Hall regime is the whistler regime [18–21] modified by the rotation effects. In this regime, the electron component behaves as a magnetized medium. In other words, it is required for the Hall regime that the condition

$$\omega_{Be} \gg (\omega, \Omega) \tag{1.3}$$

be satisfied, where ω_{Be} is the electron cyclotron frequency. Thus, in the Hall regime, one deals with the case of strong electron magnetization.

† Deceased.

In this connection, one can introduce the notion of the weak electron magnetization regime (opposite to the Hall regime), characterized by the condition

$$\omega_{Be} \lll (\omega, \Omega). \quad (1.4)$$

This regime includes the subregime of nonmagnetized electrons, which implies

$$\omega_{Be} \longrightarrow 0. \quad (1.5)$$

The main goals of the present paper are to describe the weak magnetization regime, to bridge the Hall regime to this regime (including the subregime of nonmagnetized electrons), and to elucidate whether the Velikhov effect survives in this subregime. Then, we should allow for electron inertia. In this context, the term "electron inertia regime" is a synonym for the weak magnetization regime.

Note that electron inertia was allowed for in [22]. However, the formulation of the problem and the results obtained in [22] differ from ours. More detailed comments on these aspects of this paper will be given below.

At the same time, [22] is valuable for its indications of astrophysical media that can be areas of applications of the notion of weak magnetization regime. According to [22], studying this regime, called in [22] the weak-field limit, may be of interest for understanding collective effects in accretion systems in primordial galaxies, in which seed fields may be very weak, and protostellar accretion flows, in which the charge resides primarily on massive grains.

Condition (1.1) does not enter explicitly into the one-fluid MHD theory of MRI. Instead of this, the theory is based on the assumption that the Ohm law can be written in the form

$$\mathbf{E} + \frac{1}{c}[\mathbf{V} \times \mathbf{B}] = 0. \quad (1.6)$$

Here, $\mathbf{V}$ is the plasma velocity (i.e., the velocity of the ion component of the plasma, $\mathbf{V} = \mathbf{V}_i$), $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, and c is the speed of light. Meanwhile, Eq. (1.6) is nothing but the equation of electron motion. Therefore, using Eq. (1.6), one assumes that the ion velocity is equal to the electron velocity $\mathbf{V}_e$,

$$\mathbf{V}_i = \mathbf{V}_e. \quad (1.7)$$

To study MRI beyond condition (1.1), one should allow for the difference between $\mathbf{V}_i$ and $\mathbf{V}_e$, i.e., take

$$\mathbf{V}_i \neq \mathbf{V}_e, \quad (1.8)$$

which corresponds to taking into account the Hall effect.

The approach of papers of the Hall trend consists in that one substitutes

$$\mathbf{V} \longrightarrow \mathbf{V}_e = \mathbf{V}_i - \mathbf{j}/en_0 \quad (1.9)$$

into Ohm's law (1.6), where $\mathbf{j}$ is the electric current density, e is the ion charge, and n_0 is the equilibrium plasma number density. Then, Eq. (1.6) transforms into

$$\mathbf{E} + \frac{1}{c}[\mathbf{V}_i \times \mathbf{B}] - \frac{1}{en_0 c}[\mathbf{j} \times \mathbf{B}] = 0. \quad (1.10)$$

Section 2 is addressed to the description of electrons with allowance for their inertia. The MRI dispersion relation covering both the Hall regime and the electron inertia regime is derived and analyzed in Section 3. We call the family of instabilities in both these regimes the high-frequency MRI.

In accordance with above said, in Sections 2 and 3, we are dealing with small-scale perturbations. It should be emphasized that we restrict ourselves to axisymmetric perturbations. Meanwhile, in accordance with [23], such perturbations can be described by means of a dispersion relation expressed in terms of the permittivity tensor. Thereby, the permittivity is one of the basic attributes of the theory of axisymmetric MRI. In [23], the permittivity tensor was calculated for the standard MHD regime. Similar calculations for the high-frequency regimes are given in Section 4.

Thus, our analysis in Sections 2 and 3 is restricted to the case of small-scale axisymmetric perturbations. Similarly to [3], we work in these sections with a dispersion relation derived in the scope of the local approach based on the assumption that the radial wavenumber of perturbations is larger than the inverse scale length of inhomogeneity of the rotation frequency. Then, this inhomogeneity, i.e., the Velikhov effect, is allowed for by means of the frozen-in condition (see Eqs. (2.5)–(2.7)).

An alternative to the local approach is the Frieman–Rotenberg (FR) approach [24–29]. It is important that, in the scope of this approach, one deals with equations for perturbed values whose characteristic scale lengths are arbitrary as compared to the scale length of inhomogeneity of the rotation frequency. An intrinsic peculiarity of MRI is that, in contrast to the electrostatic modes described in terms of a single variable (as a rule, the electrostatic potential [30]), the modes relevant to MRI are *multivariable*; i.e., they are described in terms of several basis variables. In the present paper, the role of these variables is played by the radial and azimuthal perturbed magnetic fields, $\tilde{B}_r$ and $\tilde{B}_\theta$. Instead of $\tilde{B}_\theta$, the authors of [24] proposed to use the sum of the perturbed plasma pressure and the perturbed magnetic field pressure—the so-called FR variable, denoted by p_*. Accordingly, the basic equations of the FR approach are a pair of equations for $\tilde{B}_r$ and p_*. Paying tribute to the authors of [25, 26], who originally formulated these equations, we call them the Hameiri–Bondeson–Iacono–Bhattacharjee (HBIB) equations.

Meanwhile, in deriving the mode equation, one excludes the FR variable and, thereby, arrives at a single equation for the variable $\tilde{B}_r$. Then, in addition to the term responsible for the Velikhov effect, additional terms can appear in the mode equation that contain the radial derivative of the rotation frequency. In the problem of standard MRI, the effect of these terms is opposite to the Velikhov effect. In other words, they produce the "*anti-Velikhov effect*." Having obtained the mode equation, one can pass to the local approximation. Then, one finds that, due to the anti-Velikhov effect, the resultant local dispersion relation differs from that derived by means of the standard local approach [3]. Therefore, in order to be sure that the results presented in Sections 2–4 are correct, we should justify that the anti-Velikhov effect does not appear in our problem. This procedure is the topic of Section 5.

The results obtained in this study are discussed in Section 6. The paper also contains the *Appendix*, aimed at explaining the appearance of the anti-Velikhov effect in the standard MHD regime.

2. DESCRIPTION OF ELECTRONS

We use the equation of electron motion in the form

$$\mathbf{E} + \frac{1}{c}[\mathbf{V}_e \times \mathbf{B}] - \frac{M_e}{e}\frac{d_e\mathbf{V}_e}{dt} = \frac{1}{e}\left(\frac{\nabla p_e}{n} - M_e\mathbf{g}\right). \quad (2.1)$$

Here, M_e and $-e$ are the mass and charge of an electron, n is the electron number density, $d_e/dt = \partial/\partial t + \mathbf{V}_e \cdot \nabla$, p_e is the electron pressure, and $\mathbf{g}$ is the gravity acceleration. We assume that, in the equilibrium state, the plasma has a cylindrical symmetry characterized by the coordinates (r, θ, z), where r and θ are the radius and azimuthal angle, respectively, and the z axis coincides with the cylinder axis. The electrons and ions in the equilibrium state are assumed to rotate with the same frequency Ω, so that their equilibrium velocities $\mathbf{V}_{e0}$ and $\mathbf{V}_{i0}$ are

$$\mathbf{V}_{e0} = \mathbf{V}_{i0} = r\Omega\mathbf{e}_\theta, \quad (2.2)$$

where $\mathbf{e}_\theta$ is the unit vector along θ. It then follows from the rth projection of the equilibrium part of Eq. (2.1) that there is a radial equilibrium electric field E_{0r} determined by

$$E_{0r} = -\frac{\Omega r B_0}{c} - \frac{1}{en_0}\frac{dp_{e0}}{dr} + M_e\frac{r\Omega^2 + g}{e}. \quad (2.3)$$

In addition, we use Maxwell's equation

$$\partial\mathbf{B}/\partial t = -c\nabla \times \mathbf{E}. \quad (2.4)$$

Acting on Eq. (2.1) by the operator $\nabla\times$ and using Eq. (2.4), we arrive at the frozen-in condition

$$\frac{\partial\mathbf{B}}{\partial t} - [\nabla \times (\mathbf{V}_e \times \mathbf{B})] - \frac{M_e c}{e}\left[\nabla \times \frac{d_e\mathbf{V}_e}{dt}\right] = 0. \quad (2.5)$$

The spatiotemporal dependence of the perturbed values is represented in the form $\exp(-i\omega t + ik_r r + ik_z z)F(r)$, where $F(r)$ is a slowly varying function, so that ω is the mode frequency and k_z and k_r are the longitudinal and radial projections of the wave vector, respectively. Such a representation corresponds to the local approach [3]. It follows from Eq. (2.5) that

$$-i\omega\tilde{B}_r - ik_zB_0\left[\tilde{V}_{er} + \frac{1}{\omega_{Be}}\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_\theta\right] = 0, \quad (2.6)$$

$$-i\omega\tilde{B}_\theta - \frac{d\Omega}{d\ln r}\tilde{B}_r - ik_zB_0 \times\left\{\tilde{V}_{e\theta} - \frac{1}{\omega_{Be}}\left[\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_r - \frac{k_r}{k_z}\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_z\right]\right\} = 0. \quad (2.7)$$

Here, $\omega_{Be} = -eB_0/M_ec$ is the electron cyclotron frequency, $\tilde{B}_r$ and $\tilde{B}_\theta$ are the r and θ components of the perturbed magnetic field, $\tilde{V}_{er}$ and $\tilde{V}_{e\theta}$ are the components of the perturbed electron velocity, and the tilde over (…) means the perturbed part of (…).

From Eq. (2.1), we obtain

$$\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_r = -i\omega\tilde{V}_{er} - 2\Omega\tilde{V}_{e\theta}, \quad (2.8)$$

$$\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_\theta = -i\omega\tilde{V}_{e\theta} + \frac{\kappa^2}{2\Omega}\tilde{V}_{er}, \quad (2.9)$$

$$\widetilde{\left(\frac{d_e\mathbf{V}_e}{dt}\right)}_z = -i\omega\tilde{V}_{ez}. \quad (2.10)$$

Here,

$$\kappa^2 = \frac{2\Omega}{r}\frac{d}{dr}(r^2\Omega) \quad (2.11)$$

and $\tilde{V}_{ez}$ is the perturbed longitudinal electron velocity. Using the formula

$$\tilde{V}_{ez} = -\tilde{j}_z/en_0, \quad (2.12)$$

where $\tilde{j}_z$ is the perturbed longitudinal electric current, and the z component of Ampère's law

$$(\nabla \times \tilde{\mathbf{B}})_z = 4\pi\tilde{j}_z/c, \quad (2.13)$$

we find

$$\tilde{V}_{ez} = -\frac{ik_rc}{4\pi en_0}\tilde{B}_\theta. \quad (2.14)$$

Then, we can see that allowing for the term with $\widetilde{(d_e\mathbf{V}_e/dt)}_z$ in Eq. (2.7) leads to the appearance of the coefficient $(1 + k_r^2c^2/\omega_{pe}^2)$ in front of $\tilde{B}_\theta$, where ω_{pe} is the electron plasma frequency, defined by $\omega_{pe}^2 = 4\pi e^2 n_0/M_e$.

As a results, Eqs. (2.6) and (2.7) are reduced to

$$-i\omega\tilde{B}_r - ik_r B_0\left(\mu_2\tilde{V}_{er} - \frac{i\omega}{\omega_{Be}}\tilde{V}_{e\theta}\right) = 0, \qquad (2.15)$$

$$-i\omega\left(1 + \frac{k_r^2c^2}{\omega_{pe}^2}\right)\tilde{B}_\theta - \frac{d\Omega}{d\ln r}\tilde{B}_r - ik_zB_0\left(\mu_1\tilde{V}_{e\theta} + \frac{i\omega}{\omega_{Be}}\tilde{V}_{er}\right) = 0, \qquad (2.16)$$

where

$$\mu_1 = 1 + 2\Omega/\omega_{Be}, \qquad (2.17)$$

$$\mu_2 = 1 + \kappa^2/(2\Omega\omega_{Be}). \qquad (2.18)$$

It, hence, follows that

$$\tilde{V}_{er} = -\frac{\omega}{D_ek_zB_0}\left[\mu_2\tilde{B}_r + \frac{i\omega}{\omega_{Be}}\left(1 + \frac{k_r^2c^2}{\omega_{pe}^2}\right)\tilde{B}_\theta\right], \qquad (2.19)$$

$$\tilde{V}_{e\theta} = -\frac{\omega}{D_ek_zB_0}\left[\mu_2\left(1 + \frac{k_r^2c^2}{\omega_{pe}^2}\right)\tilde{B}_\theta - i\left[\frac{\omega}{\omega_{Be}} + \frac{\mu_2}{\omega}\frac{d\Omega}{d\ln r}\right]\tilde{B}_r\right], \qquad (2.20)$$

where

$$D_e = \mu_1\mu_2 - \omega^2/\omega_{Be}^2. \qquad (2.21)$$

3. HIGH-FREQUENCY MRI IN ELECTRON–ION PLASMA

3.1. Dispersion Relation

Knowing the perturbed electron velocities $\tilde{V}_{er}$ and $\tilde{V}_{e\theta}$ and assuming the perturbations to be pure electronic, we find the perturbed currents $\tilde{j}_r$ and $\tilde{j}_\theta$,

$$\tilde{j}_r = -en_0\tilde{V}_{er}, \qquad (3.1)$$

$$\tilde{j}_\theta = -en_0\tilde{V}_{e\theta}. \qquad (3.2)$$

In addition, we allow for the r and θ components of Ampère's law,

$$(\nabla\times\tilde{\mathbf{B}})_r = 4\pi\tilde{j}_r/c, \qquad (3.3)$$

$$(\nabla\times\tilde{\mathbf{B}})_\theta = 4\pi\tilde{j}_\theta/c, \qquad (3.4)$$

and Maxwell's equation $\nabla\cdot\tilde{\mathbf{B}} = 0$, leading to

$$k_r\tilde{B}_r + k_z\tilde{B}_z = 0. \qquad (3.5)$$

Equations (3.3)–(3.5) yield

$$\tilde{B}_\theta = i4\pi\tilde{j}_r/(ck_z), \qquad (3.6)$$

$$\tilde{B}_r = -i4\pi k_z\tilde{j}_\theta/(ck^2), \qquad (3.7)$$

where $k^2 = k_r^2 + k_z^2$. It follows from Eqs. (3.1), (3.2), (3.6), and (3.7) that

$$\tilde{V}_{er} = ick_z\tilde{B}_\theta/(4\pi en_0), \qquad (3.8)$$

$$\tilde{V}_{e\theta} = -ick^2\tilde{B}_r/(4\pi en_0k_z). \qquad (3.9)$$

By means of Eqs. (2.17), (2.18), (3.8), and (3.9), we arrive at the following set of equations for $\tilde{B}_r$ and $\tilde{B}_\theta$:

$$\mu_2k_z^2c^2\tilde{B}_\theta - i\omega\omega_e\left(1 + \frac{c^2k^2}{\omega_{pe}^2}\right)\tilde{B}_r = 0, \qquad (3.10)$$

$$\left(\mu_1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln r}\right)\tilde{B}_r + i\frac{\omega\omega_e}{c^2k^2}\left(1 + \frac{c^2k^2}{\omega_{pe}^2}\right)\tilde{B}_\theta = 0, \qquad (3.11)$$

where

$$\omega_e = 4\pi en_0c/B_0. \qquad (3.12)$$

Hence, we obtain the dispersion relation

$$\mu_2k_z^2c^2\left(\mu_1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln r}\right) - \frac{\omega^2\omega_e^2}{c^2k^2}\left(1 + \frac{c^2k^2}{\omega_{pe}^2}\right)^2 = 0. \qquad (3.13)$$

3.2. Analysis of the Dispersion Relation

3.2.1. The Hall regime. With $(\Omega, \omega) \ll \omega_{Be}$ and $c^2k^2 \ll \omega_{pe}^2$, Eq. (3.13) is reduced to

$$\omega^2 = \omega_{wh}^2\left(1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln r}\right), \qquad (3.14)$$

where ω_{wh} is the whistler frequency [16–21], defined by

$$\omega_{wh}^2 = k_z^2k^2c^4\omega_{Be}^2/\omega_{pe}^4. \qquad (3.15)$$

Hence, one can see that, for

$$B_0\frac{d\Omega}{d\ln r} < -\frac{ck^2}{4\pi n_0e}, \qquad (3.16)$$

there is an instability related to the whistler oscillation branches. This instability was analyzed in studies on the Hall trend (see, e.g., [7]). In contrast to the standard MRI, it depends on the relationship between the signs of the gradients of the plasma rotation frequency and the magnetic field, i.e., in terms of [7], on the helicity. It is driven when these signs are mutually opposite. At the same time (as in the case of the standard MRI), for this instability to develop, it is necessary that the perturbation wavelength be sufficiently long.

It should be noted that, although dispersion relation (3.14) is independent of the ion mass, it refers to electron–ion plasma. An alternative to this case is electron–positron plasma, whose analysis, however, goes beyond the scope of the present study.

3.2.2. Subregime of nonmagnetized plasma. It follows from Eq. (3.13) that, for $\omega_{Be} \longrightarrow 0$, i.e., in the subregime of nonmagnetized plasma, there are oscillation branches with oscillation frequencies satisfying the dispersion relation

$$(\omega_{pe}^2 + c^2k_r^2)(\omega_{pe}^2 + c^2k^2)\omega^2 = \kappa^2 c^4 k_z^2 k^2\left(1 - \frac{\omega_{pe}^2}{2c^2k^2}\frac{d\ln\Omega}{d\ln r}\right). \quad (3.17)$$

The perturbations under study are unstable for

$$\frac{d\ln\Omega}{d\ln r} > \frac{2c^2k^2}{\omega_{pe}^2}. \quad (3.18)$$

One can see that, in this subregime (as in the Hall regime), the Velikhov effect also survives. Note also that, in the subregime of nonmagnetized plasma (similarly to the standard MHD regime and the Hall regime), it is necessary for the onset of instability that the perturbation wavelength be sufficiently long. At the same time, in contrast to the standard MHD regime, perturbations are unstable for a positive rotation frequency gradient, $d\ln\Omega/d\ln r > 0$.

4. PERMITTIVITY OF ROTATING PLASMA BEYOND THE STANDARD MHD REGIME

4.1. Dispersion Relation and Permittivity Tensor

Here, we are interested in axisymmetric MRI. According to [23], this type of MRI can be studied using the dispersion relation

$$\begin{vmatrix} \varepsilon_{11} - c^2k_z^2/\omega^2 & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} - c^2k^2/\omega^2 \end{vmatrix} = 0, \quad (4.1)$$

where $\varepsilon_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) are the components of the permittivity tensor.

It was shown in [23] that the permittivity tensor can be calculated if one knows the expressions for the perturbed currents $\tilde{j}_r$, $\tilde{j}_\theta$ in terms of the perturbed magnetic fields $\tilde{B}_r$ and $\tilde{B}_\theta$. Such calculations are performed using the formulas

$$\tilde{j}_r = \frac{\omega^2}{4\pi i k_z c}(\varepsilon_{11}\tilde{B}_\theta - \varepsilon_{12}\tilde{B}_r), \quad (4.2)$$

$$\tilde{j}_\theta = \frac{\omega^2}{4\pi i k_z c}(\varepsilon_{21}\tilde{B}_\theta - \varepsilon_{22}\tilde{B}_r). \quad (4.3)$$

4.2. Regimes Dependent on the Ion Contribution

4.2.1. Plasma equilibrium. We consider a rotating plasma characterized by the following equilibrium condition:

$$-\rho_0 r\Omega^2 = g\rho_0, \quad (4.4)$$

where ρ_0 is the equilibrium plasma mass density and p_0 is the plasma equilibrium pressure. It is important for our following analysis that the equilibrium can be realized for both $B_0 \neq 0$ and $B_0 = 0$. In other words, one can pass from the case of a magnetized plasma to that of a nonmagnetized plasma.

4.2.2. Ion perturbations. Considering plasma perturbations, we assume for simplicity that the effects of the plasma temperature are negligibly small. Then, the perturbed part of the equation of ion motion can be represented in the form

$$\rho_0\left(\frac{d_i\mathbf{V}_i}{dt}\right)^{\sim} = \frac{1}{c}(\mathbf{j}\times\mathbf{B})^{\sim}, \quad (4.5)$$

where $d_i/dt = \partial/\partial t + \mathbf{V}_i \cdot \nabla$. The left-hand side of Eq. (4.5) contains the term $-\tilde{\rho}\, r\Omega^2$, while the right-hand side, the term $\tilde{\rho}\, g$, where $\tilde{\rho}$ is the perturbed ion mass density. However, equilibrium condition (4.4) implies that these terms are mutually cancelled. From Eq. (4.5) we have

$$\tilde{j}_r = -\frac{c\rho_0}{B_0}\left(-i\omega\tilde{V}_{i\theta} + \frac{\kappa^2}{2\Omega}\tilde{V}_{ir}\right), \quad (4.6)$$

$$\tilde{j}_\theta = \frac{c\rho_0}{B_0}(-i\omega\tilde{V}_{ir} - 2\Omega\tilde{V}_{i\theta}). \quad (4.7)$$

Here, $\tilde{V}_{ir}$ and $\tilde{V}_{i\theta}$ are the r and θ components of the perturbed ion velocity. From Eqs. (4.6) and (4.7), we find

$$\tilde{V}_{ir} = \frac{B_0}{c\rho_0(\kappa^2 - \omega^2)}(-i\omega\tilde{j}_\theta - 2\Omega\tilde{j}_r), \quad (4.8)$$

$$\tilde{V}_{i\theta} = \frac{B_0}{c\rho_0(\kappa^2 - \omega^2)}\left(i\omega\tilde{j}_r - \frac{\kappa^2}{2\Omega}\tilde{j}_\theta\right). \quad (4.9)$$

4.2.3. Equations for calculating the perturbed currents as functions of the perturbed magnetic fields. The perturbed currents are related to the perturbed ion and electron velocities by

$$\tilde{j}_r = en_0(\tilde{V}_{ir} - \tilde{V}_{er}), \quad (4.10)$$

$$\tilde{j}_\theta = en_0(\tilde{V}_{i\theta} - \tilde{V}_{e\theta}). \quad (4.11)$$

According to Eqs. (4.8) and (4.9), the perturbed ion velocities $\tilde{V}_{ir}$ and $\tilde{V}_{i\theta}$ are expressed in terms of the perturbed currents, while, according to Eqs. (2.17) and (2.18), the perturbed electron velocities $\tilde{V}_{er}$ and $\tilde{V}_{e\theta}$ are functions of the perturbed fields. Therefore, formulas (4.11) and (4.12) allow one to calculate $\tilde{j}_r$ and $\tilde{j}_\theta$ as functions of $\tilde{B}_r$ and $\tilde{B}_\theta$ and, then, using Eqs. (4.2) and (4.3), to obtain the permittivity tensor.

4.2.4. Permittivity tensor for arbitrary ratios between ω_{Bi}, Ω, and ω. Taking $(\omega, \Omega) \ll \omega_{Be}$, we reduce Eqs. (2.17) and (2.18) to

$$\tilde{V}_{er} = -\omega\tilde{B}_r/(k_z B_0), \tag{4.12}$$

$$\tilde{V}_{e\theta} = -\frac{\omega}{k_z B_0}\left(\tilde{B}_\theta - \frac{i}{\omega}\frac{d\Omega}{d\ln r}\tilde{B}_r\right). \tag{4.13}$$

With expressions (4.8), (4.9), (4.12), and (4.13), Eqs. (4.10) and (4.11) yield

$$\begin{aligned}&\left(1+\frac{\kappa^2}{2\Omega\omega_{Bi}}\right)\tilde{j}_r - \frac{i\omega}{\omega_{Bi}}\tilde{j}_\theta \\ &= -\frac{\omega c\rho_0}{k_z B_0^2}(i\omega\tilde{B}_\theta + 2\Omega\tilde{B}_r),\end{aligned} \tag{4.14}$$

$$\begin{aligned}&\left(1+\frac{2\Omega}{\omega_{Bi}}\right)\tilde{j}_\theta + \frac{i\omega}{\omega_{Bi}}\tilde{j}_r \\ &= \frac{\omega c\rho_0}{k_z B_0^2}\left[-i\omega\left(1-\frac{1}{\omega^2}\frac{d\Omega^2}{d\ln r}\right)\tilde{B}_r + 2\Omega\tilde{B}_\theta\right].\end{aligned} \tag{4.15}$$

Hence, we find

$$\tilde{j}_r = -\frac{en_0\omega}{k_z B_0 D_0}[(\kappa^2-\omega^2+2\Omega\omega_{Bi})\tilde{B}_r - i\omega\omega_{Bi}\tilde{B}_\theta], \tag{4.16}$$

$$\begin{aligned}\tilde{j}_\theta = -\frac{en_0\omega}{k_z B_0 D_0}\Bigg\{&i\omega\omega_{Bi}\left[1-\frac{1}{\omega_{Bi}}\right. \\ &\left.\times\frac{d\Omega}{d\ln r}(\kappa^2-\omega^2+2\Omega\omega_{Bi})\right]\tilde{B}_r \\ &+(\kappa^2-\omega^2+2\Omega\omega_{Bi})\tilde{B}_\theta\Bigg\},\end{aligned} \tag{4.17}$$

where

$$D_0 = \kappa^2 - \omega^2 + [\kappa^2/(2\Omega) + 2\Omega]\omega_{Bi} + \omega_{Bi}^2. \tag{4.18}$$

Comparing Eqs. (4.16) and (4.17) with Eqs. (4.2) and (4.3), we conclude that, for arbitrary ratios between ω_{Bi}, Ω, and ω, the permittivity tensor is given by

$$\varepsilon_{11} = -\omega_{pi}^2/D_0, \tag{4.19}$$

$$\varepsilon_{12} = -\varepsilon_{21} = i\frac{4\pi en_0}{B_0\omega D_0}(\kappa^2-\omega^2+2\Omega\omega_{Bi}), \tag{4.20}$$

$$\varepsilon_{22} = -\frac{\omega_{pi}^2}{D_0}\left[1-\frac{1}{\omega_{Bi}}\frac{d\Omega}{d\ln r}(\kappa^2-\omega^2+2\Omega\omega_{Bi})\right], \tag{4.21}$$

where $\omega_{pi}^2 = 4\pi e^2 n_0/M_i$ is the squared ion plasma frequency.

Passing to the case $\omega_{Bi} \gg (\Omega, \omega)$, we obtain from Eqs. (4.19)–(4.21) the standard one-fluid MHD expressions for the permittivity tensor [23],

$$\varepsilon_{\alpha\beta} = \frac{c^2}{v_A^2}\begin{pmatrix} 1 & -\dfrac{2i\Omega}{\omega} \\ \dfrac{2i\Omega}{\omega} & 1-\dfrac{1}{\omega^2}\dfrac{d\Omega^2}{d\ln r}\end{pmatrix}, \quad (\alpha, \beta = 1, 2). \tag{4.22}$$

In the opposite limit, $\omega_{Bi} \ll (\Omega, \omega)$, it follows from Eqs. (4.19)–(4.21) that

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -\dfrac{i\omega_e}{\omega} \\ \dfrac{i\omega_e}{\omega} & -\dfrac{\omega_e}{\omega^2}\dfrac{d\Omega}{d\ln r}\end{pmatrix}. \tag{4.23}$$

4.3. Permittivity Tensor in the Electron Inertia Regime

In the electron inertia regime, the ion contribution to expressions (4.10) and (4.11) is insignificant, so the perturbed currents are given by formulas (3.1) and (3.2) with $\tilde{V}_{er}$, $\tilde{V}_{e\theta}$ of form (2.19) and (2.20). By the above manner, we find the perturbed currents and then compare the results obtained with formulas (4.2) and (4.3). Then, we arrive at the following expressions for the permittivity tensor:

$$\varepsilon_{11} = -\frac{\omega_e}{D_e\omega_{Be}}\left(1+\frac{c^2k_r^2}{2\omega_{pe}^2}\right), \tag{4.24}$$

$$\varepsilon_{12} = \frac{i\omega_e}{D_e\omega}\left(1+\frac{\kappa^2}{2\Omega\omega_{Be}}\right), \tag{4.25}$$

$$\varepsilon_{21} = -\frac{i\omega_e}{D_e\omega}\left(1+\frac{c^2k_r^2}{2\omega_{pe}^2}\right)\left(1+\frac{\kappa^2}{2\Omega\omega_{Be}}\right), \tag{4.26}$$

$$\varepsilon_{22} = \frac{\omega_e}{D_e\omega_{Be}}\left[1+\frac{\omega_{Be}}{\omega^2}\frac{d\Omega}{d\ln r}\left(1+\frac{\kappa^2}{2\Omega\omega_{Be}}\right)\right]. \tag{4.27}$$

In the limit $\omega_{Be} \longrightarrow 0$, Eqs. (4.24)–(4.27) are reduced to

$$\varepsilon_{11} = \frac{\omega_{pe}^2}{\kappa^2 - \omega^2}\left(1 + \frac{c^2 k_r^2}{2\omega_{pe}^2}\right), \tag{4.28}$$

$$\varepsilon_{12} = i\frac{\omega_{pe}^2}{\kappa^2 - \omega^2}\frac{\kappa^2}{2\Omega\omega}, \tag{4.29}$$

$$\varepsilon_{21} = -i\frac{\omega_{pe}^2}{\kappa^2 - \omega^2}\frac{\kappa^2}{2\Omega\omega}\left(1 + \frac{c^2 k_r^2}{2\omega_{pe}^2}\right), \tag{4.30}$$

$$\varepsilon_{22} = \frac{\omega_{pe}^2}{\kappa^2 - \omega^2}\left(1 + \frac{1}{\omega^2}\frac{\kappa^2}{2\Omega}\frac{d\Omega}{d\ln r}\right). \tag{4.31}$$

In the case of high-frequency oscillations (i.e., for $\omega^2 \gg \kappa^2$), it follows from Eqs. (4.28)–(4.31) that the Velikhov effect disappears. In the opposite case of strong plasma rotation ($\kappa^2 \gg \omega^2$), we obtain from Eqs. (4.28)–(4.31)

$$\varepsilon_{\alpha\beta} = \omega_{pe}^2 \begin{pmatrix} \frac{1}{\kappa^2}\left(1 + \frac{c^2 k_r^2}{2\omega_{pe}^2}\right) & \frac{i}{2\Omega\omega} \\ -\frac{i}{2\Omega\omega}\left(1 + \frac{c^2 k_r^2}{2\omega_{pe}^2}\right) & \frac{1}{2\omega^2}\frac{d\ln\Omega}{d\ln r} \end{pmatrix}. \tag{4.32}$$

5. FRIEMAN–ROTENBERG APPROACH FOR ELECTRONIC MODES

A specific feature of the FR approach [26–31] is the use of the FR variable p_*, which is the sum of the perturbed plasma pressure and the perturbed magnetic field pressure,

$$p_* = \tilde{p} + B_0\tilde{B}_z/(4\pi). \tag{5.1}$$

We are interested in perturbations with $\tilde{p}_e = 0$. Therefore, in our case, we have

$$p_* = B_0\tilde{B}_z/(4\pi). \tag{5.2}$$

Using Maxwell's equation (cf. Eq. (3.5))

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r) + ik_z\tilde{B}_z = 0, \tag{5.3}$$

we obtain

$$\tilde{B}_z = i\tau_B/k_z, \tag{5.4}$$

where

$$\tau_B = \frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r). \tag{5.5}$$

In accordance with [25], the first canonical HBIB equation has the form

$$D\tau_B = C_1\tilde{B}_r - i4\pi k_z B_0 C_2 p_*. \tag{5.6}$$

It follows from Eqs. (5.2), (5.4), and (5.5) that, in our case,

$$D = 1, \tag{5.7}$$

$$C_1 = 0, \tag{5.8}$$

$$C_2 = 1/B_0^2. \tag{5.9}$$

The r and θ components of linearized equations (2.1) are

$$-\frac{M_e}{e}(-i\omega\tilde{j}_r - 2\Omega\tilde{j}_\theta) = -\frac{\partial p_*}{\partial r} + i\frac{k_z B_0}{4\pi}\tilde{B}_r - en_0\tilde{E}_r - \frac{ien_0}{ck_z}r\Omega\tau_B, \tag{5.10}$$

$$-\frac{M_e}{e}\left(-i\omega\tilde{j}_\theta + \frac{\kappa^2}{2\Omega}\tilde{j}_r\right) = \frac{ik_z B_0}{4\pi}\tilde{B}_\theta - en_0\tilde{E}_\theta. \tag{5.11}$$

The current $\tilde{j}_r$ is expressed in terms of $\tilde{B}_\theta$ by using formula (3.6), while, using expression (5.4), similarly to formula (3.7), we find

$$\tilde{j}_\theta = \frac{ic}{4\pi}\left(k_z\tilde{B}_r - \frac{1}{k_z}\frac{\partial\tilde{B}_z}{\partial r}\right). \tag{5.12}$$

In addition, the perturbed electric fields $\tilde{E}_r$ and $\tilde{E}_\theta$ in Eqs. (5.10) and (5.11) should also be expressed in terms of the perturbed magnetic fields. Using the induction equation

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{5.13}$$

we find

$$\tilde{E}_r = \frac{\omega}{ck_z}\tilde{B}_\theta - \frac{i}{k_z}\frac{\partial\tilde{E}_z}{\partial r}, \tag{5.14}$$

$$\tilde{E}_\theta = -\omega\tilde{B}_r/(ck_z). \tag{5.15}$$

As a result, Eqs. (5.10) and (5.11) transform into (cf. Eqs. (3.10), (3.11))

$$\frac{M_e ck_z}{4\pi e}\left[\omega\tilde{B}_\theta + i2\Omega\left(\tilde{B}_r - \frac{1}{k_z^2}\frac{\partial\tau_B}{\partial r}\right)\right] = -\frac{\partial p_*}{\partial r} + \frac{ik_z B_0}{4\pi}\tilde{B}_r - \frac{en_0}{k_z}\left(\frac{\omega}{c}\tilde{B}_\theta - i\frac{\partial\tilde{E}_z}{\partial r}\right) - \frac{ien_0}{ck_z}r\Omega\tau_B, \tag{5.16}$$

$$\mu_2\tilde{B}_\theta + \frac{i\omega}{\omega_{Be}}\left[\left(1 + \frac{\omega_{pe}^2}{c^2k_z^2}\right)\tilde{B}_r - \frac{1}{k_z^2}\frac{\partial\tau_B}{\partial r}\right] = 0. \tag{5.17}$$

Using the parallel component of Eqs. (2.1) and (2.12), we obtain

$$\tilde{E}_z = \frac{r\Omega}{c}\tilde{B}_r + \frac{r\Omega}{c}\tau_B + \frac{i\omega M_e}{e^2 n_0}\tilde{j}_z. \quad (5.18)$$

According to Eq. (2.13), we have

$$\tilde{j}_z = \frac{c}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_\theta). \quad (5.19)$$

It follows from Eqs. (5.18) and (5.19) that

$$\frac{\partial \tilde{E}_z}{\partial r} = \frac{\tilde{B}_r}{c}\frac{d\Omega}{d\ln r} + \frac{i\omega c}{\omega_{pe}^2}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_\theta)\right]. \quad (5.20)$$

In accordance with [23], the term with $d\Omega/d\ln r$ in Eq. (5.20) is responsible for the Velikhov effect.

Substituting Eq. (5.20) into Eq. (5.16) and taking into account that, according to Eqs. (5.6)–(5.9),

$$\tau_B = -i4\pi k_z p_*/B_0, \quad (5.21)$$

we find

$$\mu_1\frac{\partial p_*}{\partial r} = \frac{ik_z B_0}{4\pi}\left(\mu_1 + \frac{\omega_e}{k_z^2 c^2}\frac{d\Omega}{d\ln r}\right)\tilde{B}_r$$
$$-\frac{en_0\omega}{ck_z}\left(1 + \frac{c^2k_z^2}{\omega_{pe}^2}\right)\tilde{B}_\theta - \frac{M_e c\omega}{4\pi e k_z}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_\theta)\right]. \quad (5.22)$$

By means of Eq. (5.17), we find an expression for $\tilde{B}_\theta$ in terms of $\tilde{B}_r$. Substituting this expression into Eq. (5.22), we arrive at the generalized second canonical HBIB equation,

$$\bar{D}_e\frac{\partial p_*}{\partial r} = \frac{ik_z B_0}{4\pi}\hat{D}_e[\tilde{B}_r - \hat{H}_e(\tilde{B}_r)]. \quad (5.23)$$

Here,

$$\bar{D}_e = \mu_1 - \frac{\omega^2}{\omega_{Be}^2\mu_2}\left(1 + \frac{\omega_{pe}^2}{c^2k_z^2}\right), \quad (5.24)$$

$$\hat{D}_e = \mu_1 + \frac{\omega_e}{k_z^2c^2}\frac{d\Omega}{d\ln r} - \frac{\omega^2}{\omega_{Be}^2\mu_2}\left(1 + \frac{\omega_{pe}^2}{c^2k_z^2}\right)^2, \quad (5.25)$$

$$\hat{H}_e(\tilde{B}_r) = \frac{\omega^2}{k_z^2\omega_{Be}^2}$$
$$\times\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\frac{1}{\mu_2}\left[\left(1 + \frac{\omega_{pe}^2}{c^2k_z^2}\right)\tilde{B}_r - \frac{1}{k_z^2}\frac{\partial\tau_B}{\partial r}\right]\right\}. \quad (5.26)$$

Thus, we obtained a pair of equations for $\tilde{B}_r$ and p_*, Eqs. (5.6) and (5.23). In order to understand whether the anti-Velikhov effect appears in our problem, let us compare the structure of these equations with that of the canonical equations for these variables relevant to the standard MHD regime and explained in the Appendix. Then, we can see that, according to Eq. (5.8), the parameter C_1 in our case vanishes, $C_1 = 0$. Therefore, the parameter b, defined by Eq. (A.8) and responsible for the anti-Velikhov effect, also vanishes. This justifies our local approach used in Sections 2 and 3.

Due to the vanishing of the coefficient C_1 in our problem, the procedure of obtaining the mode equation is reduced to replacing p_* with τ_B in Eq. (5.23) by using formula (5.21). Then, we arrive at the mode equation

$$\hat{D}_e\tilde{B}_r - \frac{1}{k_z^2}\bar{D}_e\frac{\partial\tau_B}{\partial r} - \hat{H}_e(\tilde{B}_r) = 0. \quad (5.27)$$

Recalling formula (5.21), we can see that Eq. (5.27) is the fourth-order differential equation, in contrast to the standard MHD regime, in which one deals with a second-order differential equation (see Eq. (A.5)).

Assuming that the radial dependence of $\tilde{B}_r$ is close to $\exp(ik_r r)$ (see the comment after Eq. (2.5)), we obtain from Eq. (5.27) dispersion relation (3.13).

6. DISCUSSION

Our analysis has demonstrated the existence of three physically different regimes of MRI: the standard MHD regime, the Hall regime, and the electron inertia regime. MRI in the Hall regime is described by local dispersion relation (3.14). The condition for the onset of this instability is given by Eq. (3.16) and depends on the helicity. The electron inertia regime includes the subregime of a nonmagnetized plasma. In this subregime, MRI is described by dispersion relation (3.17).

As in the standard MHD regime, in the subregime of a nonmagnetized plasma, only perturbations with sufficiently long wavelengths can be unstable. At the same time, in contrast to the standard MHD regime, it is necessary for the onset of MRI that the value of $d\ln\Omega/d\ln r$ be positive. Characteristic wavenumbers relevant to this instability prove to be on the order of $k \simeq \omega_{pe}/c$, which is typical for Weibel instability [18, 31, 32]. It seems that surviving the Velikhov effect for $\omega_{Be} \longrightarrow 0$, i.e., the existence of MRI in the absence of a magnetic field, is of primary importance in connection with astrophysical problems like those discussed in [6–15, 22].

Turning to electron motion equation (2.1), we can see that, in the electron inertia regime, both the perturbed electric and magnetic fields are important. In [22], however, the effect of the perturbed magnetic field on the electron motion was ignored. Therefore, it seems that the analysis of [22] is incomplete.

We have calculated the permittivity tensor of rotating plasma beyond the standard MHD regime. For arbitrary ratios between ω_{Bi}, Ω and ω and ignoring electron inertia, this tensor is given by Eqs. (4.19)–(4.21). For $\omega_{Bi} \gg (\Omega, \omega)$, it is of form (4.22), corresponding to the standard MHD regime, while for $\omega_{Bi} \ll (\Omega, \omega)$, it is given by Eq. (4.23).

In turn, in the electron inertia regime, the permittivity tensor is defined by Eqs. (4.24)–(4.27), which transform into Eqs (4.28)–(4.31) in the limit $\omega_{Be} \longrightarrow 0$ and into Eq. (4.32) for $\omega^2 \gg \kappa^2$. In considering the FR approach for the regimes under study, we have derived the pair of canonical equations (5.6) and (5.27). Then, we have demonstrated that this approach leads to the same dispersion relation as the standard local approach, in which variations in the perturbation amplitudes are ignored.

We hope that the results of the present paper can be useful for subsequent astrophysical applications to the problems considered in [3, 5–15, 22].

ACKNOWLEDGMENTS

We are grateful to E.A. Pashitskii, A.I. Smolyakov, and S.V. Vladimirov for useful comments. This work was supported by the Russian Foundation for Basic Research (project no. 06-02-16767), the RF Program for State Support of Leading Scientific Schools (grant no. 9878.2006.2), the International Science and Technology Center (grant no. G-1217), the Science and Technology Center in Ukraine (grant no. 3473), the Research Foundation of the State of São Paulo (FAPESP), the National Council of Scientific and Technological Development (CNPq), and the Ministry of Science and Technology of the Federative Republic of Brazil.

APPENDIX

Explanation of the Appearance of the Anti-Velikhov Effect in the Standard MHD Regime

In the scope of the one-fluid approach in the ideal plasma, one arrives at a pair of canonical equations of the FR type, one of which is of form (5.6), while second one is as follows [28, 29]:

$$i4\pi k_z B_0 D p'_* = -i4\pi k_z B_0 C_1 p_* + C_3 \tilde{B}_r, \quad (A.1)$$

where the prime stands for the radial derivative and C_3 is a function of the radial coordinate (similar to D, C_1, and C_2). An important feature of Eqs. (5.6) and (A.1) is the antisymmetry of the cross coefficients: the coefficient C_1 enters into both these equations. In order to obtain the mode equation, we should exclude the variable p_* from the above set of equations. Then, from Eq. (5.6) we find

$$p_* = (C_1 \tilde{B}_r - D\tau_B)/(i4\pi k_z B_0 C_2). \quad (A.2)$$

Substituting (A.2) into (A.1), we obtain

$$D\left(\frac{D\tau_B}{C_2}\right)' - D\left(\frac{C_1\tilde{B}_r}{C_2}\right)' = -\frac{DC_1\tau_B}{C_2} + \left(\frac{C_1^2}{C_2} - C_3\right)\tilde{B}_r \quad (A.3)$$

and, then, transform the second term on the right-hand side of Eq. (A.3),

$$\left(\frac{C_1\tilde{B}_r}{C_2}\right)' = \frac{C_1\tau_B}{C_2} + r\left(\frac{C_1}{rC_2}\right)'\tilde{B}_r. \quad (A.4)$$

The terms with τ_B on the left- and right-hand sides of Eq. (A.3) are mutually cancelled, so that we arrive at the mode equation [28, 29]

$$D(D\tau_B/C_2)' + \Lambda\tilde{B}_r = 0, \quad (A.5)$$

where

$$\Lambda = a + b, \quad (A.6)$$

$$a = C_3 - C_1^2/C_2, \quad (A.7)$$

$$b = -Dr[(C_1/rC_2)]'. \quad (A.8)$$

For small-scale modes, Eq. (A.5) yields the dispersion relation [28, 29]

$$-k_r^2 D^2/C_2 + \Lambda = 0. \quad (A.9)$$

Following the local approach and escaping the mode equation, it is impossible to obtain the term b in Eq. (A.6). However, it is this term that is responsible for the anti-Velikhov effect. Note that, for small-scale perturbations, the term b (i.e., the second term on the right-hand side of Eq. (A.4) is small as compared with the contribution from τ_B to this equation. However, in accordance with Eq. (A.4), mode equation (A.3) contains *two terms* with τ_B, which are mutually cancelled. If one deals with a pair of FR-type equations, whose structure differs substantially from that of Eqs. (5.6) and (A.1), one can ignore the anti-Velikhov effect. Such a situation is considered in Section 5.

REFERENCES

1. E. P. Velikhov, Zh. Éksp. Teor. Fiz. **36**, 1398 (1959) [Sov. Phys. JETP **9**, 995 (1959)].
2. S. Chandresekhar, Proc. Nat. Acad. Sci. U.S.A. **46**, 253 (1960).
3. S. A. Balbus and J. F. Hawley, Astrophys. J. **376**, 214 (1991).
4. N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. **24**, 337 (1973).
5. S. A. Balbus and J. F. Hawley, Rev. Mod. Phys. **70**, 1 (1998).
6. M. Wardle, Mon. Not. Roy. Astron. Soc. **307**, 849 (1999).
7. S. A. Balbus and C. Terquem, Astrophys. J. **552**, 235 (2001).
8. T. Sano and J. M. Stone, Astrophys. J. **570**, 314 (2002).
9. T. Sano and J. M. Stone, Astrophys. J. **577**, 534 (2002).
10. P. D. Mininni, D. O. Gomez, and S. M. Mahajan, Astrophys. J. **584**, 1120 (2003).
11. S. J. Desch, Astrophys. J. **608**, 509 (2004).

12. L. L. Kitchatinov and G. Rüdiger, Astron. Astrophys. **608**, 509 (2004).
13. G. Rüdiger and D. Shalybkov, Phys. Rev. E **69**, 016303 (2004).
14. G. Rüdiger and L. L. Kitchatinov, Astron. Astrophys. **434**, 629 (2005).
15. V. Urpin and G. Rüdiger, Astron. Astrophys. **437**, 23 (2005).
16. A. B. Mikhailovskii, J. G. Lominadze, R. M. O. Galvão, et al., Phys. Plasmas **14**, 112 108 (2007).
17. A. B. Mikhailovskii, J. G. Lominadze, A. P. Churikov, et al., Zh. Éksp. Teor. Fiz. **133**, 429 (2008) [JETP **106**, 371 (2008)].
18. A. B. Mikhailovskii, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Atomizdat, Moscow, 1972; Consultants Bureau, New York, 1975), Vol. 6.
19. A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, et al., *Plasma Electrodynamics* (Nauka, Moscow, 1974; Pergamon, Oxford, 1975).
20. V. D. Shafranov, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Gosatomizdat, Moscow, 1963; Consultants Bureau, New York, 1967), Vol. 3.
21. N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (Academic, New York, 1973; Mir, Moscow, 1975).
22. J. H. Krolik and E. G. Zweibel, Astrophys. J. **644**, 651 (2006).
23. A. B. Mikhailovskii, J. G. Lominadze, A. P. Churikov, et al., Zh. Éksp. Teor. Fiz. **133**, 183 (2008) [JETP **106**, 154 (2008)].
24. E. Frieman and N. Rotenberg, Rev. Mod. Phys. **32**, 898 (1960).
25. E. Hameiri, J. Math. Phys. **22**, 2080 (1981).
26. A. Bondeson, R. Iacono, and A. Bhattacharjee, Phys. Fluids **30**, 2167 (1987).
27. R. Keppens, F. Casse, and J. P. Goedbloed, Astrophys. J. **569**, L121 (2002).
28. A. B. Mikhailovskii, J. G. Lominadze, A. P. Churikov, et al., Fiz. Plazmy **34** (2008) (in press).
29. A. V. Mikhailovskii, J. G. Lominadze, N. N. Erokhin, et al., Phys. Plasmas **15**, 052103 (2008).
30. A. B. Mikhailovskii, *Theory of Plasma Instabilities* (Atomizdat, Moscow, 1971; Consultants Bureau, New York, 1974).
31. E. S. Weibel, Phys. Rev. Lett. **2**, 83 (1959).
32. N. Shukla and P. K. Shukla, Phys. Lett. A **362**, 221 (2007).

Translated by the authots