Nonlocal magnetorotational instability

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An analytical theory of the nonlocal magnetorotational instability (MRI) is developed for the simplest astrophysical plasma model. It is assumed that the rotation frequency profile has a steplike character, so that there are two regions in which it has constant different values, separated by a narrow transition layer. The surface wave approach is employed to investigate the MRI in this configuration. It is shown that the main regularities of the nonlocal MRI are similar to those of the local instability and that driving the nonaxisymmetric MRI is less effective than the axisymmetric one, also for the case of the nonlocal instability. The existence of nonlocal instabilities in nonmagnetized plasma is predicted. © *2008 American Institute of Physics*.
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I. INTRODUCTION

The magnetorotational instability[1,2] (MRI) was treated in Ref. 3 within the local approximation. Actually, the local approach applied both for the MRI and for many types of instabilities in an inhomogeneous plasma[4–6] is valuable, in particular for understanding the physics of the phenomenon considered, since it leads to rather simple analytical results. Therefore, it is reasonable that the local approach to the MRI started in Ref. 3 has been adopted in numerous later works (see, e.g., Refs. 7–15). Furthermore, according to the general theory of plasma instabilities, there is a class of problems in which nonlocal perturbations can be investigated analytically without making the local assumption. We have in mind the scenario of steplike profiles of equilibrium parameters solved by the so-called method of surface waves. This approach goes back, in particular, to the paper by Kruskal and Schwartzschild[16] addressed to the flute (interchange) instability in a plasma with a sharp boundary in the gravitation field (see also Ref. 4 generalizing the results of Ref. 16 by allowing for the ion drift effects). In Sec. 1.7 of Ref. 17, this approach has been used for studying the electrostatic instability in an electron flow with steplike velocity profile (a variety of the electrostatic Kelvin-Helmholtz instability). By the same approach, in Sec. 10.5 of Ref. 6 the classical Kelvin-Helmholtz instability in a magnetized plasma has been considered. Therefore, it seems reasonable to consider a steplike profile of plasma rotation frequency and to study its stability using surface waves. This is the scheme adopted in the present paper to investigate the nonlocal MRI.

Evidently, the above-noted program for analyzing nonlocal perturbations should be preceded by a derivation of appropriate equations for such perturbations. We recall that recently, a general scheme for describing them has been elaborated in Ref. 15, based upon the Frieman–Rotenberg approach.[18–21] In order to facilitate understanding the essence of these equations, in the present paper we derive them from first principles, starting with the simplest situation and moving on to more complex ones in the sequel.

In Sec. II the theory of nonlocal axisymmetric MRI in an incompressible plasma is developed having in mind a high-β plasma, where β is the ratio of the plasma pressure to the magnetic field pressure. In contrast to this, Sec. III is addressed to nonlocal axisymmetric MRI in a cold plasma; i.e., in a low-β plasma. Thereby, in these sections we arrive at the simplest results of the above theory. A more complicated version of the theory of axisymmetric MRI is given in Sec. IV, where an arbitrary-β plasma is analyzed. In contrast to Secs. II–IV, in Secs. V and VI the nonlocal nonaxisymmetric MRI is treated. Section V addresses the description of such a MRI, while its analysis is performed in Sec. VI. Discussions of the results are given in Sec. VII. In addition, the paper contains Appendix A explaining regularities of the local nonaxisymmetric MRI and Appendix B treating the nonlocal modes in nonmagnetized plasma.

II. AXISYMMETRIC MRI IN AN INCOMPRESSIBLE PLASMA

A. The problem statement and basic equations

We consider a cylindrical plasma placed in a magnetic field $\mathbf{B}_0$ and rotating in azimuthal direction with the angular frequency $\Omega=\Omega(r)$, where r is the radial coordinate. Both the equilibrium magnetic field $\mathbf{B}_0$ and gravitation acceleration $\mathbf{g}$ are assumed uniform, with the former in the longitudinal and the latter in the radial directions, i.e., $\mathbf{B}_0=B_0\hat{z}$ and $\mathbf{g}=g\hat{r}$. The equilibrium equation is then given by

$$r\rho_0\Omega^2 = p_0' - \rho_0 g, \tag{1}$$

where p_0 and ρ_0 are the equilibrium plasma pressure and mass density, respectively, and the prime stands for the radial derivative. Hence, we can see that the plasma rotation can be balanced by gravitation, provided that $g<0$, or, if $g=0$, by a pressure gradient $p_0' \neq 0$. Therefore, two limiting cases can be distinguished; the one that we refer to as the *simplest astrophysical equilibrium*, described by the equation

$$r\rho_0\Omega^2 + \rho_0 g = 0, \tag{2}$$

and the *laboratory plasma equilibrium*, given by

$$r\rho_0\Omega^2 - p_0' = 0. \tag{3}$$

In the sequel, we consider only the simplest astrophysical equilibrium case. The perturbations are taken to be dependent on time t and longitudinal coordinate z as $\exp(-i\omega t + ik_z z)$ and axisymmetric; i.e., independent of the azimuthal coordinate θ. The plasma is supposed to be incompressible, so that the (z,r) components of the perturbed plasma velocity, i.e., $\tilde{V}_z$ and $\tilde{V}_r$, are interrelated by

$$ik_z\tilde{V}_z + \frac{1}{r}\frac{\partial}{\partial r}(r\tilde{V}_r) = 0. \tag{4}$$

Similarly, for the Maxwell equation $\nabla\cdot\tilde{\mathbf{B}}=0$, where $\tilde{\mathbf{B}}$ is the perturbed magnetic field, its parallel $(\tilde{B}_z)$ and radial $(\tilde{B}_r)$ components are related by

$$ik_z\tilde{B}_z + \frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r) = 0. \tag{5}$$

The perturbed plasma motion equation has the following (r,θ,z) components:

$$-i\omega\tilde{V}_r - 2\Omega\tilde{V}_\theta - \frac{iv_A^2 k_z}{B_0}\tilde{B}_r + \frac{1}{\rho_0}\frac{\partial\tilde{p}}{\partial r} + \frac{v_A^2}{B_0}\frac{\partial\tilde{B}_z}{\partial r} = 0, \tag{6}$$

$$-i\omega\tilde{V}_\theta + \frac{\kappa^2}{2\Omega}\tilde{V}_r - \frac{iv_A^2 k_z}{B_0}\tilde{B}_\theta = 0, \tag{7}$$

$$-i\omega\tilde{V}_z + \frac{1}{\rho_0}ik_z\tilde{p} = 0. \tag{8}$$

Here, $v_A^2 = B_0^2/(4\pi\rho_0)$ is the squared Alfvén velocity, ρ_0 is the equilibrium plasma mass density assumed to be uniform, $\kappa^2 = (2\Omega/r)d(r^2\Omega)/dr$, $\tilde{V}_\theta$ and $\tilde{B}_\theta$ are the azimuthal components of the perturbed plasma velocity and the perturbed magnetic field, respectively, and $\tilde{p}$ is the perturbed plasma pressure.

We note that there appear in Eq. (6) two other terms with the perturbed density, one of which is proportional to the gravitation force and the second to the Coriolis force. However, these terms cancel each other.

The standard freezing condition (the induction equation)[1,12] yields

$$-i\omega\tilde{B}_r - ik_z B_0\tilde{V}_r = 0, \tag{9}$$

$$-i\omega\tilde{B}_\theta - \frac{d\Omega}{d\ln r}\tilde{B}_r - ik_z B_0\tilde{V}_\theta = 0. \tag{10}$$

Using Eqs. (4), (5), and (7)–(10), all the variables are expressed in terms of $\tilde{B}_r$:

$$\tilde{V}_r = -\omega\tilde{B}_r/(k_z B_0), \tag{11}$$

$$\tilde{V}_z = -\frac{i\omega}{k_z B_0}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r), \tag{12}$$

$$\tilde{B}_z = \frac{i}{k_z}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r), \tag{13}$$

$$\tilde{p} = -i\frac{\rho_0\omega^2}{k_z^3 B_0}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r), \tag{14}$$

$$\tilde{B}_\theta = -i\frac{2\Omega\omega}{\omega^2 - k_z^2 v_A^2}\tilde{B}_r, \tag{15}$$

$$\tilde{V}_\theta = i\frac{\omega^2\kappa^2/(2\Omega) - k_z^2 v_A^2 d\Omega/d\ln r}{k_z B_0(\omega^2 - k_z^2 v_A^2)}\tilde{B}_r. \tag{16}$$

Substituting Eqs. (11)–(16) into Eq. (6) leads to the following second-order differential equation:

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)\right] - \mu(r)\tilde{B}_r = 0. \tag{17}$$

Here,

$$\mu(r) = k_z^2[(\gamma^2 + k_z^2 v_A^2)^2 + \gamma^2\kappa^2 + k_z^2 v_A^2 d\Omega^2/d\ln r]/(\gamma^2 + k_z^2 v_A^2)^2 \tag{18}$$

and $\gamma^2 \equiv -\omega^2$.

Let us digress briefly to discuss the validity of the ideal model that we are considering. It is well known that resistivity and viscosity establish two different dissipative scales in magnetohydrodynamic (MHD) turbulence, so that if the value of their ratio, the magnetic Prandtl number P_m, is much larger than 1, dynamic stresses are relevant in the dissipation process of the magnetic field at the small resistive scale, as in the case of laboratory experiments with liquid metals.[22] In the astrophysical case, the value of P_m is usually small in standard models of accretion disks. However, in a recent paper it has been shown the this value can be larger than 1 at distances smaller than approximately 50 Schwarzschild radii of the central object.[23] Therefore, the question of the relevance of the two dissipation mechanisms is still under investigation. Nonetheless, these dissipative effects are not relevant for describing the MRI in its linear phase, which is the subject of this work, as the instability is not driven by singular dissipation. Therefore, the ideal MHD assumption is justified within the scope of this work.

B. Solution of differential equation and derivation of dispersion relation

Let us assume that the rotation frequency profile is such that $\Omega=\Omega_1=\text{const}$, in the inside region "1," corresponding to $r<r_0$, and that $\Omega=\Omega_2=\text{const}$, in the outside region "2," $r>r_0$, the width of the transition layer between these regions being $a\ll r_0$. In addition, we take for simplicity $k_z r_0\gg 1$. Equation (17) then reduces to the quasislab form

$$\frac{\partial^2\tilde{B}_r}{\partial r^2}-\mu(r)\tilde{B}_r=0. \tag{19}$$

Far from the transition layer, the function $\mu(r)$ can by approximated by a constant. The solution of Eq. (19) decreasing with increasing $|r-r_0|$ is then given by

$$(\tilde{B}_r)_{1,2}=\tilde{B}_r^{(0)}\exp(-\sigma_{1,2}|r-r_0|), \tag{20}$$

where $\tilde{B}_r^{(0)}$ is a constant,

$$\sigma_{1,2}=\mu_{1,2}^{1/2}=|k_z|[1+4\Omega_{1,2}^2\gamma^2/(\gamma^2+k_z^2v_A^2)^2]^{1/2}. \tag{21}$$

Now we integrate Eq. (19) over a region of width of order δ, including the transition layer assuming $a\ll\delta\ll 1/\sigma_{1,2}$ and take that $\tilde{B}_r$ is continuous. We then obtain

$$\left(\frac{\partial\tilde{B}_r}{\partial r}\right)_{r_0-\delta}^{r_0+\delta}-\tilde{B}_r^{(0)}I=0, \tag{22}$$

where

$$I=\int_{r_0-\delta}^{r_0+\delta}\mu(r)dr. \tag{23}$$

Substituting here $\tilde{B}_r$ from Eq. (20), we arrive at the dispersion relation

$$\sigma_2+\sigma_1+I=0. \tag{24}$$

For $\mu(r)$ given by Eq. (18), the integral (23) yields

$$I=\frac{k_z^2 r_0}{\gamma^2+k_z^2v_A^2}(\Omega_2^2-\Omega_1^2). \tag{25}$$

Substituting Eqs. (21) and (25) into Eq. (24), we obtain

$$\left[\left(1+\frac{\gamma^2}{k_z^2v_A^2}\right)^2+\frac{4\Omega_2^2\gamma^2}{k_z^4v_A^4}\right]^{1/2}+\left[\left(1+\frac{\gamma^2}{k_z^2v_A^2}\right)^2+\frac{4\Omega_1^2\gamma^2}{k_z^4v_A^4}\right]^{1/2}+\frac{r_0(\Omega_2^2-\Omega_1^2)}{|k_z|v_A^2}=0. \tag{26}$$

This is the explicit form of the dispersion relation for axisymmetric perturbations in incompressible plasma; i.e., in the case of high-β plasma. We note that the local theory of axisymmetric MRI in incompressible plasma, which takes the radial dependence of the perturbation proportional to $\exp(ik_z r)$, leads to the dispersion relation

$$\gamma^4+\gamma^2\left(2k_z^2v_A^2+\frac{k_z^2}{k^2}\kappa^2\right)+k_z^4v_A^4\left(1+\frac{1}{k^2v_A^2}\frac{d\Omega^2}{d\ln r}\right)=0, \tag{27}$$

where $k^2=k_r^2+k_z^2$, k_r is the radial projection of the wave vector (see details in Refs. 3 and 12).

C. Analysis of the dispersion relation

One can see that, since the two first terms of the right-hand side of Eq. (26) are positive, this equation can be satisfied only if

$$\Omega_1^2>\Omega_2^2. \tag{28}$$

This condition agrees with the general result that the MRI is possible only for a decreasing profile of the rotation frequency, i.e., for $d\Omega^2/d\ln r<0$, as obtained in Refs. 1–3 and verifiable from Eq. (27).

One can consider two limiting cases: $\gamma<|k_z|v_A$ (the case of magnetized plasma) and $\gamma>|k_z|v_A$ (the case of nonmagnetized plasma). Here we deal with the first case, while the nonmagnetized plasma is studied in Appendix B.

In order to obtain the instability boundary, we take $\gamma=0$ in Eq. (26). We then find that the perturbations are marginally stable for $k_z=k_{z0}$, satisfying the condition

$$|k_{z0}|=r_0(\Omega_1^2-\Omega_2^2)/(2v_A^2). \tag{29}$$

Turning to Eq. (27), one can see that, instead of Eq. (29), the local theory yields

$$(k_z^2+k_r^2)^{\text{loc}}=-(d\Omega^2/d\ln r)/v_A^2, \tag{30}$$

indicating that the two conditions are indeed in qualitative agreement.

Near the instability boundary it is reasonable to take $\gamma^2\ll k_z^2v_A^2$. Equation (26) then reduces to

$$\gamma^2=\frac{1}{2}|k_{z0}|r_0\frac{\Omega_1^2-\Omega_2^2}{1+(\Omega_1^2+\Omega_2^2)/(k_{z0}v_A)^2}\left(1-\frac{|k_z|}{|k_{z0}|}\right). \tag{31}$$

At the same time, the expression for the squared growth rate near the instability boundary following from the local dispersion relation (27) is

$$(\gamma^2)^{\text{loc}}=-k_z^2v_A^2\frac{k^2v_A^2+d\Omega^2/d\ln r}{2k^2v_A^2+\kappa^2}, \tag{32}$$

again indicating qualitative agreement between the two approaches.

The instability condition following from Eq. (31) takes the form

$$|k_z|<|k_{z0}|, \tag{33}$$

or, in explicit form,

$$\Omega_1^2-\Omega_2^2>2v_A^2|k_z|/r_0. \tag{34}$$

One can see that, as in the case of local perturbations,[3] it is necessary for the nonlocal MRI that the wave number should be sufficiently small. This is in correspondence with the general results of Refs. 2 and 3.

III. AXISYMMETRIC MRI IN A COLD PLASMA

In the case of cold plasma, Eq. (6) is substituted by

$$-i\omega\tilde{V}_r - 2\Omega\tilde{V}_\theta - \frac{iv_A^2k_z}{B_0}\tilde{B}_r + \frac{v_A^2}{B_0}\frac{\partial\tilde{B}_z}{\partial r} = 0. \quad (35)$$

The expressions for $\tilde{V}_r$, $\tilde{B}_z$, and $\tilde{V}_\theta$ in terms of $\tilde{B}_r$ represented in Sec. II remain in force and are given by Eqs. (11), (13), and (16). Substituting them into Eq. (35), we arrive at Eq. (17) with the function $\mu(r)$ given by

$$\mu(r) = k_z^2 + \frac{1}{v_A^2}\left[\gamma^2\left(1 + \frac{4\Omega^2}{\gamma^2 + k_z^2v_A^2}\right) + \frac{d\Omega^2}{d\ln r}\right]. \quad (36)$$

As in Sec. II, we approximate Eq. (17), with $\mu(r)$ of form (36), by the quasislab Eq. (19). Repeating the procedure of Sec. II, we then arrive at the dispersion relation given by Eq. (24) with

$$\sigma_{1,2} = |k_z|\left[1 + \frac{\gamma^2}{k_z^2v_A^2}\left(1 + \frac{4\Omega_{1,2}^2}{\gamma^2 + k_z^2v_A^2}\right)\right]^{1/2}, \quad (37)$$

$$I = r_0(\Omega_2^2 - \Omega_1^2)/v_A^2. \quad (38)$$

Substituting Eqs. (37) and (38) into Eq. (24) yields

$$\left[1 + \frac{\gamma^2}{k_z^2v_A^2}\left(1 + \frac{4\Omega_2^2}{\gamma^2 + k_z^2v_A^2}\right)\right]^{1/2} + \left[1 + \frac{\gamma^2}{k_z^2v_A^2}\left(1 + \frac{4\Omega_1^2}{\gamma^2 + k_z^2v_A^2}\right)\right]^{1/2} + \frac{r_0(\Omega_2^2 - \Omega_1^2)}{|k_z|v_A^2} = 0. \quad (39)$$

We note that, in accordance with Refs. 7 and 12, the local theory yields the dispersion relation

$$(\gamma^2 + k_z^2v_A^2)(\gamma^2 + k^2v_A^2) + \gamma^2\kappa^2 + k_z^2v_A^2 d\Omega^2/d\ln r = 0. \quad (40)$$

By means of Eqs. (39) and (40), one can compare the results of the nonlocal and local theories of the axisymmetric MRI in the low-β plasma. For $\gamma^2=0$, Eq. (39) reduces to Eq. (29). Thereby, the necessary condition for axisymmetric MRI obtained for $\beta\to\infty$ remains in force for $\beta\to 0$. At the same time, for $\gamma^2=0$, the local dispersion relation (40) coincides with the respective version of Eq. (27). In other words, Eq. (30) is valid not only for $\beta\to\infty$ but also for $\beta\to 0$. Therefore, interrelation between the nonlocal and local instability boundaries for $\beta\to 0$ is the same as for $\beta\to\infty$.

Near the instability boundary the growth rate of nonlocal perturbations in a low-β plasma proves to be the following:

$$\gamma^2 = \frac{r_0|k_z|(\Omega_1^2 - \Omega_2^2)}{1 + 2(\Omega_2^2 + \Omega_1^2)/(k_{z0}v_A)^2}\left(1 - \frac{|k_z|}{|k_{z0}|}\right). \quad (41)$$

Similarly, it follows from Eq. (40) that the squared growth rate of the local perturbations near the instability boundary is given by

$$(\gamma^2)^{\text{loc}} = -k_z^2v_A^2\frac{k^2v_A^2 + d\Omega^2/d\ln r}{(k^2 + k_z^2)v_A^2 + \kappa^2}. \quad (42)$$

A comparison of Eqs. (41) and (42) with Eqs. (31) and (32) shows that there is a difference between the growth rates in high-β and low-β cases, both for the nonlocal and local perturbations.

IV. AXISYMMETRIC MRI IN A FINITE-β PLASMA

In the case of finite-β plasma, Eq. (6) remains in force, but the contribution of the perturbed pressure $\tilde{p}$ into this equation should be calculated by a manner different from that of Sec. II. We then use the adiabatic condition

$$\frac{d}{dt}\left(\frac{p}{\rho^\Gamma}\right) = 0, \quad (43)$$

where Γ is the adiabatic exponent, p and ρ are, respectively, the total plasma pressure and plasma mass density determined by $p=p_0+\tilde{p}$, $\rho=\rho_0+\tilde{\rho}$, where p_0 and ρ_0 and the equilibrium parts of these functions, and $\tilde{\rho}$ is the perturbed mass density. For calculation of $\tilde{p}$, we turn to the plasma continuity equation

$$\frac{\partial}{\partial t}\tilde{\rho} + \rho_0\nabla\cdot\tilde{\mathbf{V}} = 0. \quad (44)$$

It then follows from Eqs. (43) and (44) that

$$\tilde{p} = \frac{c_s^2\rho_0}{\omega}\left[-\frac{i}{r}\frac{\partial}{\partial r}(r\tilde{V}_r) + k_z\tilde{V}_z\right], \quad (45)$$

where $c_s^2=\Gamma p_0/\rho_0$ is the squared sound velocity.

The perturbed parallel velocity $\tilde{V}_z$ is related to $\tilde{p}$ by Eq. (8). Substituting Eq. (45) into Eq. (8) and using Eq. (11), we find

$$\tilde{V}_z = \frac{ic_s^2}{\alpha_sB_0\omega}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r), \quad (46)$$

where $\alpha_s=1-k_z^2c_s^2/\omega^2$. By means of Eqs. (46) and (11), Eq. (45) takes the form

$$\tilde{p} = \frac{ic_s^2\rho_0}{\alpha_sk_zB_0}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r). \quad (47)$$

Expressions for $\tilde{B}_z$ and $\tilde{V}_\theta$ are the same as in Sec. II and given by Eqs. (13) and (16). Substituting Eqs. (11), (13), (16), and (47) into Eq. (6), we arrive at Eq. (17) with $\mu(r)$ given by

$$\mu(r) = \frac{(\gamma^2 + k_z^2v_A^2)^2 + \gamma^2\kappa^2 + k_z^2v_A^2 d\Omega^2/d\ln r}{v_A^2(\gamma^2 + k_z^2v_A^2)(1 + \beta/\alpha_s)}, \quad (48)$$

where $\beta=c_s^2/v_A^2$.

One can see that the problem considered leads to the dispersion relation (24) with

$$\sigma_{1,2}=\left[\frac{(\gamma^2+k_z^2v_A^2)^2+4\Omega_{1,2}^2\gamma^2}{v_A^2(\gamma^2+k_z^2v_A^2)(1+\beta/\alpha_s)}\right]^{1/2}, \tag{49}$$

$$I=\frac{r_0(\Omega_2^2-\Omega_1^2)}{v_A^2(1+\beta/\alpha_s)}. \tag{50}$$

Thus, according to Eqs. (24), (49), and (50), the explicit form of the dispersion relation of the nonlocal axisymmetric MRI for arbitrary β is the following:

$$[(\gamma^2+k_z^2v_A^2)^2+4\Omega_2^2\gamma^2]^{1/2}+[(\gamma^2+k_z^2v_A^2)^2+4\Omega_1^2\gamma^2]^{1/2} + (\gamma^2+k_z^2v_A^2)^{1/2}\frac{r_0(\Omega_2^2-\Omega_1^2)}{v_A(1+\beta/\alpha_s)^{1/2}}=0. \tag{51}$$

The respective local dispersion relation is given by[7,12]

$$(\gamma^2+k_z^2v_A^2)[\gamma^4+\gamma^2k^2(v_A^2+c_s^2)+k_z^2k^2c_s^2v_A^2] + (\gamma^2+k_z^2c_s^2)(\gamma^2\kappa^2+k_z^2v_A^2 d\Omega^2/d\ln r)=0. \tag{52}$$

It follows from Eq. (51) that the necessary instability condition (28) remains in force. For $\gamma^2=0$, Eq. (51) reduces to Eq. (29), so that the instability boundary is the same as in Sec. II. The instability boundary of the local perturbations, according to Eq. (52), is also the same as in Sec. II. The expression for the growth rate of the local perturbations near the instability boundary in the case of not too small β proves to be the same as in Sec. II and is given by Eq. (31). The same concerns also the local perturbations. As for the case of low-β plasma, one should transit in Eqs. (51) and (52) to the region

$$k_z^2v_A^2\gg\gamma^2\simeq k_z^2c_s^2. \tag{53}$$

Equation (51) is then transformed to

$$1+\frac{r_0}{2|k_z|v_A^2}(\Omega_2^2-\Omega_1^2)+\frac{\gamma^2}{k_z^2v_A^2}\left[1+\frac{\Omega_2^2+\Omega_1^2}{k_z^2v_A^2} + \frac{r_0(\Omega_2^2-\Omega_1^2)}{4|k_z|v_A^2}\left(1-\frac{\beta}{2}\frac{\gamma^2}{\gamma^2+k_z^2c_s^2}\right)\right]=0. \tag{54}$$

Respectively, one has from Eq. (52)

$$\gamma^2\left[k_z^2v_A^2\left(1+\frac{1}{k^2v_A^2}\frac{d\Omega^2}{d\ln r}\right)+\gamma^2\left(1+\frac{k_z^2}{k^2}+\frac{\kappa^2}{k^2v_A^2}\right)\right] + k_z^2c_s^2\left[k_z^2v_A^2\left(1+\frac{1}{k^2v_A^2}\frac{d\Omega^2}{d\ln r}\right)+\gamma^2\left(2+\frac{\kappa^2}{k^2v_A^2}\right)\right]=0. \tag{55}$$

Keeping in these equations the terms of order $\beta\gamma^2/(\gamma^2+k_z^2c_s^2)$ allows one to obtain results relevant to arbitrary-β plasma.

V. DESCRIPTION OF NONAXISYMMETRIC MRI

A. Derivation of the mode equation

In contrast to Secs. II–IV, now we assume that the perturbations depend not only r and z but also on θ. Their θ dependence is taken in the form $\exp(im\theta)$.

Appealing to Ref. 15, dealing with the local nonaxisymmetric perturbations, we arrive at the following generalization of the plasma equations of motion (6)–(8) for the case of nonlocal modes

$$-i\tilde{\omega}\tilde{V}_r-2\Omega\tilde{V}_\theta-\frac{iv_A^2k_z}{B_0}\tilde{B}_r+\frac{1}{\rho_0}\frac{\partial\tilde{p}}{\partial r}+\frac{v_A^2}{B_0}\frac{\partial\tilde{B}_z}{\partial r}=0, \tag{56}$$

$$-i\tilde{\omega}\tilde{V}_\theta+\frac{\kappa^2}{2\Omega}\tilde{V}_r-\frac{iv_A^2k_z}{B_0}\tilde{B}_\theta+\frac{ik_y}{\rho_0}\tilde{p}+\frac{iv_A^2}{B_0}k_y\tilde{B}_z=0, \tag{57}$$

$$-i\tilde{\omega}\tilde{V}_z=-ik_z\tilde{p}/\rho_0. \tag{58}$$

Here, $k_y=m/r$, and $\tilde{\omega}$ is the Doppler-shifted oscillation frequency defined by

$$\tilde{\omega}=\omega-m\Omega. \tag{59}$$

Physically, the presence of the Doppler shift of the oscillation frequency means that we deal with the drifting modes.

The freezing conditions (9) and (10) are modified by the redefinition $\omega\to\tilde{\omega}$, so that now we have

$$\tilde{V}_r=-\frac{\tilde{\omega}}{k_zB_0}\tilde{B}_r, \tag{60}$$

$$\tilde{V}_\theta=-\frac{1}{k_zB_0}\left(\tilde{\omega}\tilde{B}_\theta-i\frac{d\Omega}{d\ln r}\tilde{B}_r\right). \tag{61}$$

The adiabatic condition (43) yields in the case $m\neq 0$:

$$-i\tilde{\omega}\tilde{p}+c_s^2\rho_0\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{V}_r)+ik_y\tilde{V}_\theta+ik_z\tilde{V}_z\right]=0. \tag{62}$$

Substituting Eq. (58) into Eq. (62), we find

$$\tilde{p}=-\frac{ic_s^2}{\tilde{\omega}\tilde{\alpha}_s}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{V}_r)+ik_y\tilde{V}_\theta\right], \tag{63}$$

where

$$\tilde{\alpha}_s=1-k_z^2c_s^2/\tilde{\omega}^2. \tag{64}$$

Using Eqs. (60) and (61), Eq. (60) is transformed to [cf. Eq. (47)]

$$\tilde{p}=\frac{ic_s^2\rho_0}{k_zB_0\tilde{\alpha}_s}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)+ik_y\tilde{B}_\theta\right]. \tag{65}$$

Equation (5) is also modified for $m\neq 0$, leading to [cf. Eq. (13)]

$$\tilde{B}_z=\frac{i}{k_z}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)-\frac{k_y}{k_z}\tilde{B}_\theta. \tag{66}$$

Substituting Eqs. (60), (61), (65), and (66) into Eqs. (56) and (57), we arrive at the following equation system for $\tilde{B}_r$ and $\tilde{B}_\theta$:

$$\left(\tilde{\omega}^2 - k_z^2 v_A^2 - \frac{d\Omega^2}{d\ln r}\right)\tilde{B}_r - i2\Omega\tilde{\omega}\tilde{B}_\theta + \frac{\partial}{\partial r}\left\{\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r) + ik_y\tilde{B}_\theta\right]\right\} = 0, \quad (67)$$

$$D_\theta\tilde{B}_\theta + i2\Omega\tilde{\omega}\tilde{B}_r + ik_y\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r) = 0, \quad (68)$$

where

$$D_\theta = \tilde{\omega}^2 - (k_z^2 + k_y^2)v_A^2 - k_y^2 c_s^2/\tilde{\alpha}_s. \quad (69)$$

It follows from Eq. (68) that

$$\tilde{B}_\theta = -\frac{i}{D_\theta}\left[2\Omega\tilde{\omega}\tilde{B}_r + k_y\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)\right]. \quad (70)$$

Substituting Eqs. (70) into Eq. (68) yields

$$\frac{\partial}{\partial r}\left[G\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)\right] - H\tilde{B}_r = 0, \quad (71)$$

where

$$G = \left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\frac{\tilde{\omega}^2 - k_z^2 v_A^2}{D_\theta}, \quad (72)$$

$$H = -\tilde{\omega}^2\left(1 - \frac{4\Omega^2}{D_\theta}\right) + k_z^2 v_A^2 + \frac{d\Omega^2}{d\ln r} - 2\frac{d}{d\ln r}\left[\frac{\Omega\tilde{\omega}}{D_\theta}\frac{k_y}{r}\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\right]. \quad (73)$$

One can see that, since the Doppler-shifted oscillation frequency $\tilde{\omega}$ depends on the radial coordinate r [see Eq. (59)], the structure of Eq. (71) differs from that of the mode equations considered in Secs. II–IV.

B. Derivation of dispersion relation

Similarly to Eq. (19), we turn to the quasislab version of Eq. (71):

$$\frac{\partial}{\partial r}\left(G\frac{\partial\tilde{B}_r}{\partial r}\right) - H\tilde{B}_r = 0. \quad (74)$$

Considering this equation far from the transition layer, we arrive at the solutions of the form of Eq. (20), where

$$\sigma_{1,2} = (H_{1,2}/G_{1,2})^{1/2}. \quad (75)$$

Now we integrate Eq. (71) over the transition layer, obtaining the dispersion relation

$$G_2\sigma_2 + G_1\sigma_1 + I = 0, \quad (76)$$

where

$$G_{2,1} = \left\{\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\frac{\tilde{\omega}^2 - k_z^2 v_A^2}{D_\theta}\right\}_{2,1}, \quad (77)$$

$$I = r_0\left\{\Omega_2^2 - \Omega_1^2 - 2\frac{m}{r_0^2}\left[\frac{\Omega\tilde{\omega}}{D_\theta}\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\right]_1^2\right\}. \quad (78)$$

Substituting Eq. (75) into Eq. (76) leads to

$$(GH)_2^{1/2} + (GH)_1^{1/2} + I = 0. \quad (79)$$

where

$$(GH)_{2,1}^{1/2} = \left\{\left(v_A^2 + \frac{c_s^2}{\tilde{\alpha}_s}\right)\frac{\tilde{\omega}^2 - k_z^2 v_A^2}{D_\theta}\left[k_z^2 v_A^2 - \tilde{\omega}^2\left(1 - \frac{4\Omega^2}{D_\theta}\right)\right]\right\}_{2,1}^{1/2}. \quad (80)$$

VI. ANALYSIS OF NONAXISYMMETRIC MRI

A. Nonaxisymmetric MRI in incompressible plasma

For $c_s^2 \to \infty$, Eqs. (78) and (80) yield

$$I = \frac{r_0}{k_y^2 + k_z^2}\left[(\Omega_2^2 - \Omega_1^2)(k_z^2 - k_y^2) + \frac{2m}{r_0^2}\omega(\Omega_2 - \Omega_1)\right], \quad (81)$$

$$(GH)_{1,2}^{1/2} = \frac{1}{(k_z^2 + k_y^2)^{1/2}}\left[(k_z^2 v_A^2 - \tilde{\omega}^2)^2 - 4\Omega^2\tilde{\omega}^2\frac{k_z^2}{k_z^2 + k_y^2}\right]_{1,2}^{1/2}. \quad (82)$$

We restrict ourselves to the case of weak rotation frequency jump, so that

$$\Omega_1^2 - \Omega_2^2 \ll \Omega_1^2. \quad (83)$$

One can then introduce the auxiliary Doppler-shifted oscillation frequency $\hat{\omega}$, defined by

$$\hat{\omega} = \omega - m\bar{\Omega}, \quad (84)$$

where

$$\bar{\Omega} = (\Omega_1 + \Omega_2)/2. \quad (85)$$

In terms of $\hat{\omega}$, Eq. (81) takes the form

$$I = \frac{r_0}{k_y^2 + k_z^2}\left[k_z^2(\Omega_2^2 - \Omega_1^2) + \frac{2m}{r_0^2}\hat{\omega}(\Omega_2 - \Omega_1)\right]. \quad (86)$$

We assume $\hat{\omega}$ to be a small parameter and find it by the method of successive approximations. At the same time, in Eqs. (82) we take for simplicity that $\hat{\omega} \gg m|\Omega_1 - \Omega_2|/2$. Equations (82) and (86) then yield

$$(GH)_2^{1/2} = (GH)_1^{1/2} = \frac{k_z^2 v_A^2}{(k_z + k_y^2)^{1/2}}\left\{1 - \frac{\hat{\omega}^2}{k_z^2 v_A^2}\left[1 + \frac{2\bar{\Omega}^2}{v_A^2(k_z^2 + k_y^2)}\right]\right\}, \quad (87)$$

$$I = -\frac{2r_0\Delta_\Omega}{k_y^2 + k_z^2}\left(k_z^2\bar{\Omega} + \frac{m\hat{\omega}}{r_0^2}\right), \quad (88)$$

where $\Delta_\Omega = \Omega_1 - \Omega_2$.

Using Eqs. (87) and (88), dispersion relation (79) reduces to

$$\hat{\omega}^2\left[1 + \frac{2\bar{\Omega}^2}{v_A^2(k_z^2 + k_y^2)}\right] + \frac{m}{r_0(k_z^2 + k_y^2)^{1/2}}\Delta_\Omega\hat{\omega} + f = 0, \quad (89)$$

where

$$f = k_z^2\left[\frac{r_0(\Omega_1^2-\Omega_2^2)}{2(k_z^2+k_y^2)^{1/2}} - v_{\mathrm{A}}^2\right]. \tag{90}$$

The term with $\hat{\omega}$ describes the overstable effect (cf. Appendix A). Neglecting this term, we find from Eq. (89) that the perturbations are unstable for

$$f > 0. \tag{91}$$

For $k_y=0$, this instability condition reduces to Eq. (34). Comparing Eq. (91) with Eq. (34), one can see that driving the nonaxisymmetric modes is hampered compared with that of the axisymmetric modes (cf. Appendix A). Allowing for the overstable effect, Eq. (89) leads to the instability condition

$$f - \frac{m^2\Delta_\Omega^2}{4r_0^2(k_z^2+k_y^2+2\Omega^2/v_{\mathrm{A}}^2)} > 0. \tag{92}$$

Comparing Eq. (91) with Eq. (A12), we see that, as in case of local modes, the overstable effect results in additional stabilization of nonlocal nonaxisymmetric modes.

B. Nonaxisymmetric MRI in low-β plasma

For $c_s^2 \to 0$, we obtain from Eqs. (78) and (80), instead of Eqs. (81) and (82):

$$I = r_0\left[\Omega_2^2 - \Omega_1^2 - 2\frac{mv_{\mathrm{A}}^2}{r_0^2}\left[\frac{\Omega\tilde{\omega}}{\tilde{\omega}^2-(k_z^2+k_y^2)v_{\mathrm{A}}^2}\right]\Bigg|_1^2\right\}, \tag{93}$$

$$(GH)_{2,1}^{1/2} = v_{\mathrm{A}}\left\{\frac{k_z^2v_{\mathrm{A}}^2-\tilde{\omega}^2}{(k_z^2+k_y^2)v_{\mathrm{A}}^2-\tilde{\omega}^2}\times\left[k_z^2v_{\mathrm{A}}^2-\tilde{\omega}^2-\frac{4\Omega^2\tilde{\omega}^2}{(k_z^2+k_y^2)v_{\mathrm{A}}^2-\tilde{\omega}^2}\right]\right\}_{2,1}^{1/2}. \tag{94}$$

As in subsection A, we consider the weak rotation frequency jump assuming $\hat{\omega}$ to be a small parameter. Equations (93) and (94) then reduce to Eqs. (87) and (88), respectively. As a result, dispersion relation (89) remains in force. Therefore, the instability conditions (91) and (92) are valid also for $\beta \to 0$.

VII. DISCUSSION

We have shown that the nonlocal axisymmetric perturbations of an incompressible (a high-β) rotating plasma are described by the mode equation (17). In the case of steplike rotation frequency profile, this mode equation leads to the dispersion relation (26). This dispersion relation shows that the nonlocal axisymmetric MRI, as the local one, is possible only for decreasing rotation frequency profile [see Eq. (28)].

The instability boundary is given by Eq. (29), revealing that, as in the local case, only the perturbations with sufficiently small k_z can be unstable. The growth rate of the nonlocal MRI near the instability boundary is given by Eq. (31).

A comparison of the results of nonlocal and local theories shows that both varieties of the axisymmetric MRI have the same qualitative behavior. Our analysis of the nonlocal axisymmetric MRI in the opposite case of low-β plasma has shown that this variety of MRI behaves similarly to the case $\beta \to \infty$. The same concerns also the case of arbitrary-β plasma (see in detail Secs. III and IV). In addition, in Appendix B we show existence of nonlocal instabilities in a nonmagnetized plasma.

A more complicated picture is revealed in analyzing the nonlocal nonaxisymmetric MRI. The main circumstance leading to such a complication is the radial dependence of the Doppler-shifted oscillation frequency $\tilde{\omega}$.

The dispersion relation of nonlocal nonaxisymmetric MRI is given by Eq. (79), complemented by Eqs. (77), (78), and (69). Its instability condition is given by Eqs. (91) and (92). According to these equations, driving the nonlocal nonaxisymmetric MRI is more difficult than that of the axisymmetric one. There are two effects resulting in weakening of this drive: enhancement of the magnetoacoustic stabilization, expressed in terms of the substitution $k_z^2 \to k_z^2 + k_y^2$, and overstable effect.

Numerical calculations of nonlocal nonaxisymmetric MRI have been performed in Refs. 24–28. One can suggest that the analytical results obtained is the present paper will be useful for understanding these calculations and for performing additional numerical calculations on both the nonaxisymmetric and axisymmetric MRI aiming at elucidating physical regularities of these instabilities.

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APPENDIX A: LOCAL NONAXISYMMETRIC MRI

For comparing the consequences of Eq. (79) with those of the local theory of nonaxisymmetric MRI, let us represent the local dispersion relation following from the mode equation (74). Taking $\tilde{B}_r \propto \exp(ik_r r)$, we obtain from (74)

$$k_r^2 G + H = 0. \tag{A1}$$

1. High-β plasma

For $\beta \to \infty$ and small $\tilde{\omega}$, one has from Eqs. (72) and (73),

$$G = -\frac{\tilde{\omega}^2 - k_z^2 v_A^2}{k_z^2 + k_y^2}, \tag{A2}$$

$$H = -\tilde{\omega}^2 \Delta_1^{NA} + k_z^2 v_A^2 + \frac{k_z^2}{k_z^2 + k_y^2} \frac{d\Omega^2}{d \ln r} + \frac{k_y^2}{m(k_z^2 + k_y^2)} \tilde{\omega} \frac{d\Omega}{d \ln r}, \tag{A3}$$

where

$$\Delta_1^{NA} = 1 + \frac{4\Omega^2}{v_A^2 (k_z^2 + k_\perp^2)}. \tag{A4}$$

Equation (A1) then reduces to

$$\tilde{\omega}^2 \Delta_1^{NA} - \frac{2\tilde{\omega}}{m} \frac{d\Omega}{d \ln r} \frac{k_y^2}{k_z^2 + k_\perp^2} + k_z^2 v_A^2 \Delta^{NA} = 0, \tag{A5}$$

where

$$\Delta^{NA} = -\left[1 + \frac{1}{(k_z^2 + k_\perp^2) v_A^2} \frac{d\Omega^2}{d \ln r}\right]. \tag{A6}$$

a. *The approximation $\tilde{\omega}/m \to 0$*

Without the term with $\tilde{\omega}/m$, Eq. (A5) reduces to

$$\gamma^2 = k_z^2 v_A^2 \Delta^{NA} / \Delta_1^{NA}, \tag{A7}$$

where $\gamma \equiv \mathrm{Im}\, \tilde{\omega}$. The instability boundary is then given by

$$\Delta^{NA} = 0, \tag{A8}$$

while the nonaxisymmetric instability region corresponds to

$$\Delta^{NA} > 0. \tag{A9}$$

Comparing Eq. (A7) with Eq. (32), we conclude that the nonaxisymmetric perturbations are less dangerous than the axisymmetric ones.

b. *Nonaxisymmetric overstabilities*

Equation (A5), with $\tilde{\omega}/m$, shows that the nonaxisymmetry leads to transition of the aperiodic instabilities to the overstabilities (oscillatory instabilities), $\mathrm{Re}\, \tilde{\omega} \neq 0$, $\mathrm{Im}\, \tilde{\omega} = 0$. In this case, one has

$$\tilde{\omega} = \hat{\omega} + i\gamma, \tag{A10}$$

where

$$\hat{\omega} \equiv \tilde{\omega} + \frac{1}{m \Delta_1^{NA}} \frac{d\Omega}{d \ln r} \frac{k_y^2}{k_z^2 + k_\perp^2}, \tag{A11}$$

$$\gamma^2 = k_z^2 v_A^2 \frac{\Delta^{NA}}{\Delta_1^{NA}} - \left(\frac{1}{m \Delta_1^{NA}} \frac{d\Omega}{d \ln r} \frac{k_y^2}{k_z^2 + k_\perp^2}\right)^2. \tag{A12}$$

One can see that the overstable effect leads to additional stabilization of the nonaxisymmetric modes.

2. Low-β plasma

It follows from Eqs. (72) and (73) for $\beta \to 0$ and small $\tilde{\omega}$ that Eqs. (A2) and (A3) for G and H remain in force. One then again arrives at Eqs. (A11) and (A12).

APPENDIX B: NONLOCAL AXISYMMETRIC INSTABILITIES IN A NONMAGNETIZED PLASMA

Taking $v_A^2 \to 0$ in Eq. (26), we arrive at the nonlocal dispersion relation for the nonmagnetized plasma

$$\gamma[(\gamma^2 + 4\Omega_2^2)^{1/2} + (\gamma^2 + 4\Omega_1^2)^{1/2}] + r_0 |k_z| (\Omega_2^2 - \Omega_1^2) = 0. \tag{B1}$$

For the condition (28), we then have

$$\gamma = -\frac{r_0 |k_z|}{2(|\Omega_2| + |\Omega_1|)} (\Omega_2^2 - \Omega_1^2). \tag{B2}$$

This dispersion relation describes nonlocal axisymmetric instabilities in a nonmagnetized plasma.

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