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On the Behaviour of Spatial Wilson Loops in the High Temperature Phase of L.G.T.

by

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Abstract

The behaviour of the space-like string tension in the high temperature phase is studied. Data obtained in the Z_2 gauge model in (2+1) dimensions are compared with predictions of a simple model of a fluctuating flux tube with finite thickness. It is shown that in the high temperature phase contributions coming from the fluctuations of the flux tube vanish. As a consequence we also show that in (2+1) dimensional gauge theories the thickness of the flux tube coincides with the inverse of the deconfinement temperature.

Key-words: Lattice gauge theory; String theory.

1 Introduction

It is well known that in finite temperature Lattice Gauge Theories (LGT) the "spatial" string tension (namely that extracted from Wilson loops orthogonal to the compactified imaginary time direction) is no longer the order parameter of confinement and is, in general, different from zero even in the deconfined phase. Recently, in a set of interesting papers [1, 2, 3] the behaviour of this spatial string tension was studied for SU(2) and SU(3) gauge theories in (2+1) and (3+1) dimensions, leading to new ideas and to a better understanding of the high temperature behaviour of LGT's. At the same time, it was observed in [1, 4] in the case of SU(2) in (2+1) dimensions (and it will be confirmed in the present paper, in the case of the gauge Ising model), that the inverse of the deconfinement temperature $1/T^c$ almost coincides with the thickness L_c of the flux tube joining the quark-antiquark pair in the confined phase. This implies that looking to space-like Wilson loops in the deconfined phase is almost equivalent to probing the interior of the flux tube and gives us a powerful tool to test the effective flux tube models of confinement. Moreover, the fact that $1/T^c$ and L_c have similar values is a quite interesting phenomenon in itself and deserves further investigation.

In this paper we want to further pursue this analysis, comparing some predictions of the effective flux tube picture of LGT's with a set of high statistics Monte Carlo data on the 3d Ising gauge model. The main results of our analysis are the following:

a] In the deconfined region ($T > T^c$) the space-like string tension increases as the temperature increases. This trend is rather impressive: almost two orders of magnitude are gained moving from the deconfinement point to the highest temperature we can measure . Moreover in the deconfined phase two distinct regimes can be identified: a first, rather smooth, crossover region ($T^c < T < 2T^c$) and a second, high temperature regime ($T > 2T^c$), where the string tension scales with the temperature, which is the only remaining dimensional scale in this regime. In particular, this second region ($T > 2T^c$) is the one in which dimensional reduction has been shown to apply in the case of the (3+1) dimensional SU(2) gauge theory [3]. All these results are in complete agreement with the scenarios described in ref.s [1, 2, 3], and can be understood within the framework of a "compressed" flux tube model.

b] In the range $T^c < T < 2T^c$ the simple picture of a compressed flux tube of uniform flux density for the space-like Wilson loops is quite accurate. It predicts a string tension rising linearly with the temperature and allows a rather precise estimate of the thickness L_c of the flux tube.

c] Under the assumption that in the deconfined region ($T > T^c$) the space-like flux tube is "frozen" and the contribution of its transverse quantum fluctuations is zero, it is possible to show that L_c and $1/T^c$ must coincide. Suitable ratios of Wilson loops can be introduced to test this assumption, which turns out to be in good agreement with our Monte Carlo simulations. These ratios can be used as order parameters for this phase, being zero in the high temperature region and different from zero in the confining phase.

d] There is an impressive agreement between some dimensionless ratios of physical observables that we obtain in the Z_2 case and those obtained by Teper [1, 4] for SU(2). This fact suggests that the behaviour of the theory near the deconfinement point is dominated by the macroscopic properties of the flux tube, which are largely independent of the precise ultraviolet details of the models and only relies on the infrared effective string action. As a consequence some "super-universal" behaviours, like those that we have observed, are expected for dimensionless ratios of physical observables. Assuming this point of view, we expect that the effective models and ideas that we shall discuss in the next section should be valid for any gauge theory in 2+1 dimension.

Let us conclude this section by noticing that in (3+1) dimensions, there is another interesting situation (which we will not discuss in this paper) where finite size effects of the type described in this paper can be observed. It is the cylindric geometry described in ref.s [5, 6] in which two spatial directions are chosen to be approximately of the same size as the inverse deconfinement temperature.

This paper is organized as follows: in sect.2 we give a short discussion of the flux tube model, in sect.3 we describe the details of the Monte Carlo simulation and in sect.4 we compare theoretical predictions and numerical results.

2 The fluctuating flux tube model

The flux tube model is based on the idea that in the confined phase of a gauge theory the quark-antiquark pair should be joined together by a thin, fluctuating flux tube [7]. The simplest version of the model (which should be a good description when heavy quark-antiquark pairs are studied at large interquark separation R) assumes that the chromoelectric flux is confined inside a tube of small but nonzero thickness L_c, that L_c is constant along the tube (neglecting boundary effects near the quarks) and independent of the interquark distance.

An immediate consequence of this picture is linear confinement: the interquark potential $V(R)$ rises linearly according to the law $V(R) = \sigma R$.

A second, important consequence is that the string tension σ and the effective cross-section of the flux tube A_t are related by the law

$$\sigma = \frac{c_t}{A_t} \quad , \tag{1}$$

where A_t is defined as

$$A_t \equiv \frac{\left(\int dA\ E\right)^2}{\int dA\ E^2} \quad . \tag{2}$$

In some cases the constant c_t and, in particular, its dependence on the quark representation, can also be determined (see ref. [8] for a discussion on this point and for further references). If the chromoelectric flux E is constant across the tube, then the simple dimensional relation $A_t \propto L_c^{(d-2)}$ holds, where $(d-2)$ is the number of transverse dimensions.

Eq. (1) can be tested, for instance, by measuring the string tension on asymmetric lattices, with one space direction, say L_s, smaller than the others. If $L_s > L_c$ no effect on the string tension is expected. On the contrary, when $L_s < L_c$ the flux tube is squeezed and the string tension increases. In particular, if we assume that the flux density is uniform inside the flux tube, then eq. (1) suggests that the rising of the string tension is linear and obeys the law

$$\sigma(L_s) = \frac{L_c}{L_s}\sigma(\infty) \tag{3}$$

where $\sigma(\infty)$ denotes the string tension in the uncompressed situation, namely for $L_s >> L_c$ (in the following we will denote $\sigma(\infty)$ with σ for brevity).

This is the simplest possible assumption on the behaviour of the chromoelectric flux E across the tube and as a consequence the linear behaviour of eq.(3) is not at all mandatory and will have in general to be corrected when regions deep inside the flux tube are probed. Nevertheless, it has has been neatly observed recently in the case of the (2+1) dimensional SU(2) model [1]. The interesting aspect of such a linear behaviour implied by this uniform flux distribution is that it allows a rather precise determination of the flux tube thickness L_c, which turns out to be in good agreement with other independent evaluations. For instance, in the above mentioned case of the (2+1) dimensional SU(2) gauge model, the value quoted by Teper [1] agrees with the corresponding value obtained by Trottier and Woloshyn [8] with a different method.

The fact that the flux tube has a finite non-zero thickness must be carefully taken into account to avoid systematic errors in the evaluation of the string tension from numerical simulations. It is in fact clear from the above discussion that only Wilson loops of size greater than L_c must be used to extract the string tension in order to avoid the appearance of unphysical effects, in particular an artificial enhancement of the string tension. This is particularly important since small size loops, due to their small relative errors, dominate the fits and can strongly influence the determination of σ. An important cross-check of these systematic errors is given by looking at the variation of $\chi^2/d.o.f.$ (from now on denoted by χ^2_r) as a function of the minimal size L_{min} of the Wilson loops considered in the fits. Only when L_{min} is of the order of L_c good χ^2_r are obtained and stable values of the string tension are found . Notice also that these systematic deviations from the area law at small distances cannot be considered as lattice artifacts, because of their good scaling behaviour [9]. They could in principle be used as an independent way to estimate L_c.

Let us finally remark that the enhancement of the string tension at short distances is a general phenomenon: it can be precisely seen also in the case of 4d SU(2) and SU(3) gauge theories (see for instance fig.2 of ref. [10]) and also when Polyakov line correlations are studied (see for instance fig.1 and fig.4 of ref. [11]).

A third, important, consequence emerges if one tries to take into account the quantum fluctuations of the flux tube. Below the roughening transition the Wilson loop follows the area law, which is the lattice counterpart of the "classical" linearly rising potential, but the continuum limit, hence the connection with the physical flux tube, can be achieved only after the roughening transition, where the quantum fluctuations of the surface bordered by the loop play a crucial role. It is by now generally accepted that these quantum fluctuations can be effectively described by a massless two-dimensional free field theory. This idea traces back to the seminal work by Lüscher, Symanzik and Weisz [12] and has been discussed from then in several papers both in the context of lattice gauge theories (see for instance ref. [13] and references therein) and in the dual context of interfaces in 3d spin systems (see for instance ref. [14] and references therein). Let us briefly review this approach and fix notations and conventions.

2.1 Quantum fluctuations of the flux tube

The starting point is the assumption that the fluctuations of the surface bordered by the Wilson loop are described by an effective Hamiltonian proportional to the change they produce in the area of the surface itself

$$H_{eff} = \sigma \int_0^{L_1} dx_1 \int_0^{L_2} dx_2 \left[\sqrt{1 + \left(\frac{\partial h}{\partial x_1}\right)^2 + \left(\frac{\partial h}{\partial x_2}\right)^2} - 1\right] \quad , \tag{4}$$

where the field $h(x_1, x_2)$ describes the surface displacement from the equilibrium position as a function of the longitudinal coordinates x_1 and x_2, L_1 (L_2) is the size of the Wilson loop in the x_1 (x_2) direction and σ is the string tension [1].

The contribution to the Wilson loop expectation value due to surface fluctuations is then given by

$$Z_{eff} = tr\, e^{-H_{eff}} \quad . \tag{5}$$

The Hamiltonian of eq. (4) is too difficult to be handled exactly. However a crucial observation is that this theory can be expanded in the adimensional parameter $(\sigma L_1 L_2)^{-1}$ and the leading order term is the gaussian model, which will be a good approximation when large enough Wilson loops are studied. Then we replace eq. (4) with the $\sigma L_1 L_2 \to \infty$ limit $H \to H_G$

$$H_G = \frac{\sigma}{2} \int_0^{L_1} dx_1 \int_0^{L_2} dx_2 \left[\left(\frac{\partial h}{\partial x_1}\right)^2 + \left(\frac{\partial h}{\partial x_2}\right)^2\right] \quad . \tag{6}$$

Within this approximation, the integration over h implied in eq. (5) can be done exactly. The integral is divergent but can be regularized using, for instance, a suitable generalization of the Riemann ζ-function regularization (see *e.g.* [15]). The result depends only on the geometrical properties of the boundary. In particular, for a Wilson loop with fixed ("Dirichlet") boundary conditions along the loop, the gaussian contribution turns out to be

$$Z_G(L_1, L_2) = \frac{c}{\sqrt{\eta(\tau)\sqrt{L_2}}} \quad , \quad \tau = i\frac{L_1}{L_2} \quad , \tag{7}$$

where η denotes the Dedekind eta function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q = e^{2\pi i \tau} \quad , \tag{8}$$

[1]Note, as a side remark, that eq. (4) coincides with the Nambu-Goto string action in a special frame where only a subset of surface configurations are allowed. However the results of this section are also valid in a more general frame where all the possible configurations are taken into account.

c is an undetermined constant and we have assumed $L_1 \geq L_2$, without loss of generality.

Thus, taking into account the area term $\sigma L_1 L_2$ (coming from the classical or zeroth order contribution of the saddle point) and the undetermined perimeter contribution $p(L_1 + L_2)$ (which depends on the lattice regularization), the Wilson loop expectation value $W(L_1, L_2)$ may be written in the form

$$W(L_1, L_2) = e^{-\sigma L_1 L_2 + p(L_1+L_2)} Z_G \quad , \tag{9}$$

where the contribution Z_G of the quantum fluctuations can be expanded as follows

$$Z_G(L_1, L_2) = \frac{c}{\sqrt[4]{L_2}} q^{-\frac{1}{24}} \sqrt{1 + q + 2q^2 + 3q^3 + 5q^4 + \ldots} \tag{10}$$

with $q = exp(-2\pi \frac{L_1}{L_2})$. Eq. (10) is symmetric under the exchange $L_1 \leftrightarrow L_2$, as one can check by applying the modular transformation $\tau \to -\tau^{-1}$ at eq. (7) (see *e.g.* [15]).

As a consequence, a simple area-perimeter-constant law cannot fit the Wilson loop expectation values, and this shows up in high χ_r^2's and unacceptable confidence levels[2]. On the contrary, if the contribution of the fluctuations is taken into account, impressive reductions of the χ_r^2 are found, and eventually acceptable confidence levels are reached (see for instance [9]). This is up to now one of the strongest evidences for the correctness of this description of the fluctuations of the flux tube.

One of the consequences of the above description in terms of a free bosonic field is that the mean width of the flux tube is expected to grow logarithmically as a function of the interquark distance [12], while there are reasons to believe that such a width should be constant (see for instance Ref.[16]). Actually there is a slight modification of this theory which accounts for this fixed thickness, based on the observation that the free boson can be seen as a limit of a one-parameter family of conformal field theories, where this parameter can be simply related to the width L_0 of the flux tube.

In Ref.[16] it has been argued that the value of this parameter can be determined by matching the boundary conditions of the conformal theory with those of the underlying gauge field theory, combined with the obvious assumption that the flux tube cannot self-overlap freely. The latter assumption can be formulated more precisely in the Ising gauge theory by saying that the flux tube must sweep in its time evolution self-avoiding surfaces (see Ref. [13] for a discussion of this point).

In this way one finds [16] $L_0 = \sqrt{\pi/4\sigma}$ for a flux tube *at zero physical temperature*. In the following we shall not use this result, because we want to evaluate such a width by applying a completely independent argument to a different physical situation (the flux tube at the critical temperature T^c). We anticipate that the resulting value of L_0 at T^c turns out to be very near the one quoted above.

Another consequence of this mentioned modification of the free boson theory is that the contribution due to the quantum fluctuations of the flux tube has now a different form:

[2]Notice, for completeness, that in the case of gauge theories with continuous gauge groups, perturbative contributions (for instance one gluon exchange terms) are also expected for small interquark separations. However these are completely independent from the above discussed infrared contributions and should be separately taken into account in fitting Wilson loop expectation values [10]. Notice also that the above discussion requires non-trivial modifications if "fuzzy" or "blocked" versions of the Wilson loop are studied.

$$Z_{SA}(L_1, L_2) = \tilde{c} q^{-\frac{1}{192}} \sqrt[4]{1 + q^{\frac{1}{2}} + q^{\frac{3}{2}} + q^2 + \dots} \tag{11}$$

where $\tilde{c}$ is an undetermined constant and $q = exp(-2\pi \frac{L_1}{L_2})$.

Notice however that, while this modification has important consequences in the low temperature, confining regime, it is actually irrelevant in the high temperature phase we are discussing in this paper. Hence we will only refer to this choice when comparing our results with data taken in the low temperature phase.

2.2 Finite Temperature

The fact that the flux tube has a non-zero thickness becomes particularly relevant when the Wilson loop is studied at a finite temperature T, because this introduces into the game a new scale *i.e.* the length of the lattice in the time direction $L_t = 1/T$. For high enough temperatures, this length eventually becomes comparable with the finite thickness of the flux tube and the free field picture described above breaks down.

Notice however that, since we are studying Wilson loops orthogonal to the time direction, this "temperature" interpretation is not mandatory. A space-like Wilson loop at high temperature is completely equivalent to an ordinary Wilson loop in a zero temperature environment, with the lattice size in one of the remaining space-like directions smaller than the other. This is exactly the situation described at the beginning of this section and studied by Teper in [1]: all the results listed there still hold with the simple exchange of L_s with L_t. Exactly as before, choosing higher temperatures, namely looking at smaller and smaller values of L_t, we are actually probing the interior of the flux tube.

In finite temperature LGT's the interquark potential can be extracted only by looking at the correlations of Polyakov loops. The surface bordered by two Polyakov loops undergoes a roughening transition exactly as in the Wilson loop case and the same arguments and techniques described above for the quantum fluctuations of Wilson loops apply also in this case [17, 18]. Using these results Olesen was able to predict the ratio between the deconfinement temperature and the square root of the zero-temperature (namely $L_t = \infty$) string tension σ [18]

$$\frac{T^c}{\sqrt{\sigma}} = \sqrt{\frac{3}{\pi(d-2)}} \tag{12}$$

where $(d-2)$ is the number of transverse dimensions of the flux tube. This prediction turns out to be in good agreement with the results obtained with Monte Carlo simulations in (3+1) dimensions, both for the SU(2) and SU(3) gauge theories. The comparison with the numerical data for models in (2+1) dimensions is less good but the disagreement is anyway contained within 20%. For a review of these data see tab.III and ref. [13].

2.3 Vanishing of the quantum fluctuations

Let us make at this point our main assumption. We assume that the *quantum corrections Z_G should disappear* when the flux tube fills the whole lattice, namely when $T > T^c$ or $L_s < L_c$, depending on the geometry we are interested in. Intuitively this is equivalent to assume some sort of self-avoiding behaviour of the flux tube, since in this case , when the flux tube fills the whole lattice there is no more space left for it to fluctuate. We will show below that this assumption is satisfied in the case of the (2+1) Z_2 model. This vanishing of quantum fluctuations can be described in a more rigorous way by noticing that the compactification in one lattice direction (say, L_s) naturally induces a compactification of the field $h(x_1, x_2)$ on a circle of radius $R = \frac{L_s}{2\pi}$. The quantum field theory of a two-dimensional bosonic field compactified on a circle is by now rather well understood. In particular the spectrum of states and the partition function are exactly known (at least for the so called "rational models" for which R^2 is a rational number). It is thus possible to follow the behaviour of the quantum corrections as a function of R: $Z_G = Z_G(R)$. What is interesting is that there are precisely four values of R (but only two of them are independent, the others being related by duality) for which the contribution of such quantum corrections vanishes (they correspond to the so called "topological field theories", see ref. [13] for notations and bibliography on this subject). According to ref. [13, 20], we can then predict that the value (let us call it $L_0 = 2\pi R_0$) of the lattice size which corresponds to a zero-contribution point must be related to the (zero temperature) string tension as follows

$$L_0 = \sqrt{\frac{\pi}{3\sigma}} \quad . \tag{13}$$

Following our assumption we can thus say that *moving toward higher temperatures corresponds in the flux tube effective model to a flow toward one of these zero-contribution points* and that *L_0 must coincide with L_c*, the flux tube thickness. By comparing eq.s (13) and (12) we then see that in (2+1) dimension the flux tube thickness L_c coincides with the inverse deconfinement temperature.

Eq.(13), and the fact that $L_c = 1/T^c$ are in rather good agreement (within at most a 20% of deviation depending on the model) with the result of the simulations of the (2+1) dimensional SU(2) model [1, 4] and with our simulations of the (2+1) Z_2 gauge model (see tab.III). According to the subsection [d] of the introduction we suggest that this similarity should hold for any gauge theory in (2+1) dimensions.

This similarity allows us to identify the deconfined regime with the compressed flux tube regime and to interpret *the increasing of the space-like string tension in the deconfined phase as a signature of the compression of the flux tube.*

3 (2+1) Z_2 gauge Monte Carlo simulation

We compared our predictions with a set of high statistics simulations of the Z_2 gauge model in (2+1) dimensions. Let us briefly list some of the reasons for this choice (a more detailed discussion can be found for instance in ref.s [9, 11]).

a] Due to the fact that this model is the dual of the 3d Ising model, very precise values of all bulk quantities (critical couplings and indices) are known. Moreover, in the region that we study an excellent agreement with the scaling laws has been found.

Choosing the standard normalization for the action

$$S = -\beta \sum_p U_p \quad , \tag{14}$$

where U_p denotes the product of Z_2 elements associated to the links belonging to the plaquette p, we have a roughening transition at $\beta_R \simeq 0.4964$, a (zero temperature) deconfinement transition at $\beta_c \simeq 0.7614$ and a critical index for the correlation function $\nu \simeq 0.63$. Precise information on the finite temperature deconfinement transition can be found, for instance, in [21]. The scaling region of this transition starts approximatively at $L_t = 4$. Data obtained with the Monte Carlo renormalization group approach for $L_t = 4$ and $L_t = 8$ give: $\beta_c(L_t = 4) = 0.7315(5)$ and $\beta_c(L_t = 8) = 0.7516(5)$ [21] (where the notation $\beta_c(L_t)$ denotes the value of β at which deconfinement occurs in a lattice with temporal size L_t). Defining the critical temperature T^c as

$$\frac{1}{L_t(\beta)} = T_0^c \; (\beta_c - \beta(L_t))^\nu \quad , \tag{15}$$

we have $T_0^c = 2.3$ (1).

b] The model is simple enough to allow high statistics simulations, but shares the same infrared behaviour (in the $\beta < \beta_c$ region) with non abelian lattice gauge theories. In particular, near the finite temperature deconfinement point, the Svetitsky-Yaffe conjecture tells us that Z_2 and SU(2) models should be in the same universality class.

c] Recently the interface behaviour in 3d statistical models has attracted a lot of interest. In d=3 the physics of interfaces is exactly equivalent, through duality, to the physics of Wilson loops and, in the particular case of the Ising model, not only theoretical tools but also numerical results can be borrowed from one context to the other (see ref. [14] for a discussion of this problem). In particular, high precision estimates of the string tension σ (equivalent to the interface tension) exist. In the scaling region the string tension behaves as

$$\sigma(\beta) = \sigma_0 \; (\beta_c - \beta)^{2\nu} \; . \tag{16}$$

It turns out that this law is very well fitted by the existing data [14] with $\sigma_0 =$ 3.70 (4). By inserting the value for the critical temperature obtained from eq. (15), we can construct the adimensional ratio: $T_0^c/\sqrt{\sigma_0} = 1.19$ (6).

3.1 The simulation

The simulation was performed on a $48^2 \times L_t$ lattice (L_t = lattice spacing in the time direction) with periodic boundary conditions in all space-time directions. A standard heat-bath algorithm was used to update links. Four values of β, with various choices of L_t, were studied. They are listed in tab.I .

β	L_t	$L_c(\beta)$
0.7420	13	5.2 (2)
0.7500	10, 12	7.3 (3)
0.7525	2 − 8	8.5 (4)
0.7585	2 − 16	17.2 (7)

Tab.I. *Set of measured data. In the third column, the corresponding inverse critical temperatures (in unities of the lattice spacing) are presented.*

The reason for these choices is the following. The data at $\beta = 0.7585$ have been measured to make quantitative estimates of the temperature dependence of $\sigma(L_t)$. Since in this case the finite temperature deconfinement point is at $L_t \sim 17$, a wide range of values of L_t can be used to check the predicted behaviour of the string tension. The data at $\beta = 0.7525$ have been taken to check the scaling behaviour and the overall reliability of our results. In both cases all deconfined values of L_t were studied, namely $2 \leq L_t \leq 8$ for $\beta = 0.7525$ and $2 \leq L_t \leq 16$ for $\beta = 0.7585$.

Finally, to make a comparison with the above data, we studied three samples in the confining phase at intermediate finite temperatures (from $T/T^c \sim 0.4$ to $T/T^c \sim 0.73$): these have been chosen in order to have low enough temperature to guarantee an uncompressed behaviour of the flux tube (in particular it can fluctuate freely). The values of β ($\beta = 0.742$ and $\beta = 0.750$) have been tuned so as to have values of the string tension comparable with those of the compressed case. In particular, $\sigma(\beta = 0.742) \sim \sigma(\beta = 0.7525, L_t = 4) \sim \sigma(\beta = 0.7575, L_t = 4)$ and $\sigma(\beta = 0.7575, L_t = 6) < \sigma(\beta = 0.750) \sim \sigma(\beta = 0.7525, L_t = 6) < \sigma(\beta = 0.7575, L_t = 5)$, (see tab.II*a*,*b*,*c*).

All Wilson loops orthogonal to the time direction $W(L_1, L_2)$ in the range $2 \leq L_1, L_2 \leq 20$ were measured. Also this choice requires some explanation. It is well known that improved versions of the Wilson loop operator [22] give more precise results, but notice that only in the case of ordinary Wilson loops the gaussian determinants described in the previous section can be evaluated exactly. Since our goal is to show the fate of the quantum fluctuations of the flux tube across the deconfinement transition, more than to have precise evaluations of, say, L_c, we decided to concentrate on ordinary, not improved, Wilson loop expectation values.

3.2 The cross-correlation problem

The major problems one has to face for these values of β are the critical slowing down and the huge cross-correlations among Wilson loops. The first problem was kept under control separating each measurement on the lattice with 32 sweeps. The second was taken into account, as usual, by weighting the data in the fitting procedure with the inverted cross-correlation matrix . In the most severe cases, namely those near the critical point ($L_t = 6 - 8$ for $\beta = 0.7525$, and $L_t = 13 - 16$ for $\beta = 0.7585$) a scattering procedure was also implemented, measuring in each iteration only one single Wilson loop of fixed size and scattering the measure of the others in the Monte Carlo time[3]. Let us stress that all these steps were absolutely crucial to obtain reliable confidence levels in the fits. Moreover in these last cases further runs with lattice size $96^2 \times L_t$ were performed, measuring loops up to 40×40, to check the reliability of our results. Notice finally, as a side remark, that improved Wilson loop estimators experience even worse cross-correlations.

All the quoted errors were obtained with a standard jacknife procedure.

[3] In these cases the cross-correlation matrix was so flat, within the statistical errors, that the reliability of the whole inversion procedure and cross-correlated fit calculation, without a proper scattering procedure, was rather doubtful.

4 Results and conclusions

For each value of β and L_t we fitted the Wilson loop expectation values with a pure area law

$$W(L_1, L_2) = exp\{-\sigma L_1 L_2 \ + \ p(L_1 + L_2) \ + \ c\} \tag{17}$$

(we shall call this choice "type 1" fit in the following) and with an area law corrected in order to take into account the quantum fluctuations of the Wilson loop surface

$$W(L_1, L_2) = Z_G(L_1, L_2) \ exp\{-\sigma L_1 L_2 \ + \ p(L_1 + L_2) \ + \ c\} \tag{18}$$

("type 2" fit in the following). For the three samples in the confined phase we fitted the data also with the modified version of the gaussian contribution proposed in eq. (11)

$$W(L_1, L_2) = Z_{SA}(L_1, L_2) \ exp\{-\sigma L_1 L_2 \ + \ p(L_1 + L_2) \ + \ c\} \tag{19}$$

("type 3" fit in the following).

We performed the fits setting a lower threshold R_t in the size of the Wilson loops. To be precise we constructed the following subsets of our Wilson loops samples

$$S(R_t) = \{W(L_1, L_2), \ L_1 \geq L_2 \geq R_t\} \tag{20}$$

choosing $R_t = L_t$ in the deconfined phase. In the confined region we chose instead $R_t = L_c(\beta)$ (namely the flux tube thickness, extrapolated through scaling at those values of β). This means $R_t = 4$ for $\beta = 0.742$ and $R_t = 6$ for $\beta = 0.750$. Notice that, as a consequence of this cutoff procedure, severe problems of precision exist near the critical temperature. The results of the fits are collected in tab.II*a*,*b*,*c* and fig.1,2.

Let us make some comments on these data.

χ_r^2 *behaviour*

In the deconfined phase type 1 fits have in general better confidence levels than type 2 fits (see tab.II*a* and tab.II*b*). This is quite evident in the high temperature regime $T > 2T^c$, while near the critical point the two χ_r^2's almost coincide, indicating that the quantum fluctuations corrections in these region are smaller than the precision of our data and could not be detected in any case even if they are present. This behaviour is even more distinct if compared with analogous fits for the two samples of data at low temperature, in the confined region (see tab.IIc), where definitely better confidence levels are obtained with fits of type 2 and 3. This is a first signature of the vanishing of quantum fluctuations.

$T^c < T < 2T^c$

Let us concentrate on the data at $\beta = 0.7585$ (tab.II*a*). We fitted the values of $\sigma(L_t)$ in the range $9 \leq L_t \leq 16$ according to the linear law of eq. (3). As it can be seen in fig.1, the linear law is in good agreement with the data, and in fact the fit shows a good confidence level: C.L.=98%. As a result we obtain $L_c = 18.5$ (9), which implies: $L_c = 1.07(5)(5)T_c^{-1}$ and $L_c\sqrt{\sigma_0} = 0.90(5)(2)$, where the second error in these relations is due respectively to the uncertainty in the critical temperature and the string tension.

L_t	T/T^c	σ	σ_0	χ_1^2	χ_2^2
2	8.6 (4)	0.120 (3)	189. (5)	1.3	7.1
3	5.7 (3)	0.0480 (2)	75.6 (3)	0.71	8.1
4	4.3 (2)	0.0244 (3)	38.8 (5)	0.85	5.1
5	3.45 (17)	0.0150 (4)	23.6 (6)	1.2	1.8
6	2.87 (14)	0.0100 (3)	15.8 (5)	0.91	1.8
7	2.46 (12)	0.0083 (5)	13.1 (8)	1.3	1.3
8	2.15 (11)	0.0068 (4)	10.7 (6)	0.78	1.3
9	1.92 (10)	0.0049 (4)	7.7 (6)	0.90	1.08
10	1.73 (9)	0.0044 (4)	6.9 (6)	1.1	1.4
11	1.57 (8)	0.0037 (4)	5.8 (6)	0.72	0.80
12	1.44 (7)	0.0038 (4)	6.0 (6)	1.01	1.06
13	1.33 (7)	0.0033 (5)	5.2 (8)	0.90	0.90
14	1.23 (6)	0.0033 (5)	5.2 (8)	0.52	0.74
15	1.15 (6)	0.0025 (8)	3.9 (1.3)	0.46	0.48
16	1.08 (5)	0.0022 (9)	3.5 (1.4)	0.85	0.85

a

L_t	T/T^c	σ	σ_0	χ_1^2	χ_2^2
2	4.2 (2)	0.128 (2)	49.1 (8)	0.79	2.73
3	2.84 (14)	0.0520 (4)	19.9 (2)	0.81	1.68
4	2.13 (10)	0.0270 (10)	10.0 (4)	1.08	1.44
5	1.70 (8)	0.0172 (5)	6.6 (2)	0.96	1.00
6	1.42 (7)	0.0137 (7)	5.3 (3)	0.70	0.81
7	1.22 (6)	0.0112 (7)	4.3 (3)	0.80	0.80
8	1.06 (5)	0.0085 (9)	3.3 (3)	0.70	0.70

b

β	L_t	T/T^c	σ	σ_0	χ_1^2	χ_2^2	χ_3^2
0.742	13	0.40 (2)	0.027 (2)	3.88 (20)	1.7	0.85	0.80
0.750	12	0.61 (3)	0.0132 (10)	3.71 (29)	1.4	1.1	1.02
0.750	10	0.73 (3)	0.0127 (11)	3.56 (32)	1.4	1.1	0.89

c

Tab.II. (a,b,c) *Values of the string tension extracted from Wilson loops at $\beta = 0.7585$ (a); $\beta = 0.7525$ (b); $\beta = 0.7420,\ 0.7500$ (c). In the first two columns the lattice size in the time direction and the corresponding temperature (in units of the critical temperature) are reported. The last columns contain the reduced χ^2 of the fits of type 1 and 2 (in tab.IIc also fits of type 3 are considered). The third column contains the string tension $\sigma(L_t)$, extracted from fits of type 1 in the cases a,b and fits of type 3 in the case c. In the fourth column are reported the corresponding scaling values $\sigma_0(L_t)$, according to eq. (16).*

A slightly lower confidence level (35%) is obtained if the same fit is performed on the $\beta = 0.7525$ data (tab.II*b*), with $L_c = 8.7(5)$, which corresponds to $L_c = 1.02(5)(5)T_c^{-1}$ and $L_c\sqrt{\sigma_0} = 0.86(5)(2)$.

These results are summarized in tab.III, where they are also compared with the corresponding values for the SU(2) model [1, 4] and with our predictions eq.s(12) and (13).

model	$L_c\sqrt{\sigma}$	$T^c/\sqrt{\sigma}$	L_cT^c
$SU(2)$	~ 1.09	1.12(1)	~ 1.22
Z_2	0.90(7)	1.19(6)	1.07(10)
FTM	1.024	0.97	1

Tab.III. *Comparison between Z_2 and $SU(2)$ gauge theories in (2+1) dimensions. In the last row are reported the flux tube model (FTM) predictions eq.s(12) and (13).*

$T > 2T^c$

In this region the linear behaviour of eq. (3) is completely lost. This indicates the appearance of some non-trivial structure deep inside the flux tube. The data show a good scaling behaviour as a function of T and suggest that the only remaining physical scale in this regime is the temperature.

This can be seen by looking at fig.2. Since the errors in this region are very small, the χ_r^2 becomes a very efficient tool to select among various possible behaviours. Indeed, as can be seen in fig.2, both the data at $\beta = 0.7525$ and those at $\beta = 0.7585$ have a very precise power law behaviour

$$\sigma(L_t) = a\ L_t^{\alpha} \tag{21}$$

and the two values of α and a at the two β's are almost compatible within the errors.

The output of the fits is

$\beta = 0.7525$, range: $4 \geq L_t \geq 2$, C.L.= 66%, $a = 0.58(2)$, $\alpha = -2.19\ (4)$,

$\beta = 0.7585$, range: $6 \geq L_t \geq 2$, C.L.=75%, $a = 0.59(2)$, $\alpha = -2.28\ (4)$.

4.1 Vanishing of quantum fluctuations

In order to have an independent check of the vanishing of quantum fluctuations in the deconfined phase we constructed the ratios

$$C(L,n) = \frac{W(L,L)}{W(L+n,L-n)} \quad . \tag{22}$$

If the Wilson loops are described by a pure area law then $C(L,n)$ does not depend on L

$$C(L,n) = e^{-\sigma n^2} \tag{23}$$

while, if a contribution coming from the flux tube fluctuations is present, a decreasing function of L is expected

$$C(L,n)=\frac{Z_G(L,L)}{Z_G(L+n,L-n)}e^{-\sigma n^2} \tag{24}$$

or, following eq. (11),

$$C(L,n)=\frac{Z_{SA}(L,L)}{Z_{SA}(L+n,L-n)}e^{-\sigma n^2} \quad . \tag{25}$$

Note that, being only two Wilson loops involved in these ratios, one has much smaller errors than, for instance, in the case of Creutz ratios. We decided to compare samples of data with (almost) the same string tension and the same expected flux tube thickness in order to minimize systematic errors (due, for instance, to different lower thresholds in fitting the data). Hence we compared the Wilson loop expectation values at $\beta = 0.7585$, $L_t = 4$ and $\beta = 0.7525$, $L_t = 4$ with those at $\beta = 0.7420$, $L_t = 13$. It can be neatly seen by looking at fig.3 that the two samples in the deconfined phase (compressed flux tube) have almost coinciding ratios and show no contribution coming from flux tube fluctuations, while the ratios of the sample in the confining region lie in between the slopes predicted by eq. (24) and eq. (25). The same picture is confirmed by fig.4, where the samples at $\beta = 0.7585$, $L_t = 5, 6$ and $\beta = 0.7525$, $L_t = 6$ are compared with those at $\beta = 0.7500$, $L_t = 10, 12$. Notice that in this case the lower threshold is $R_t = 6$, nevertheless also the data at lower values of R have been included in the figure, in order to show the enhancement of the string tension below the threshold, as discussed in sect.2. Note also that the proposal of eq. (11) and eq. (25) for the flux tube fluctuations seems to describe better the data exactly in that region in which the self-avoiding constraint plays a major role.

Thus we can conclude that the information extracted from $C(L,n)$ is in complete agreement with those coming from the χ_r^2 of the fits and support the picture of the vanishing of quantum flux tube fluctuations in the high temperature phase.

4.2 Comparison with the SU(2) gauge theory

A remarkable feature of our results is their similarity with those obtained by Teper in the case of SU(2) in d=2+1 dimensions (see tab.III). Indeed, this also is a consequence of the flux tube picture, which, at least in a first order approximation, does not depend on the ultraviolet details of the model (not even on the fact that the gauge group is discrete or continuous, abelian or non-abelian), but only on its infrared behaviour (hence essentially on the existence of a confining phase beyond the roughening transition and on the space time dimensions). Let us stress that the comparison made in tab.III is only a partial check of this supposed universality, since both the SU(2) and the Ising model have the same center. It would be quite interesting to check it also in other cases, and in particular for the SU(3) model.

Acknowledgements

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Figure Captions

Fig.1. *The string tension in scaled units ($\sigma_0(L_t)$ in tab.IIa ,b,c) is plotted as a function of T/T^c. Data corresponding to tab.IIa (triangles), tab.IIb (squares), tab.IIc (crosses), are reported. The horizontal line is the uncompressed value of the string tension* $\sigma_0 = 3.70$. *The other line is the best fit to eq. (3) for the sample of data at* $\beta = 0.7585$ *in the range* $9 \leq L_t \leq 16$.

Fig.2. *The string tension ($\sigma(L_t)$ in tab.IIa ,b) is plotted as a function of T ($1/L_t$ in tab.IIa,b). Data corresponding to tab.IIa (triangles) and tab.IIb (squares), are reported. Both $\sigma(L_t)$ and T are in log. scale. The continuum line is the best fit to eq. (21) for the sample of data at* $\beta = 0.7585$ *in the range* $2 \leq L_t \leq 6$.

Fig.3. *The logarithm of the Wilson loop ratios of eq. (22), with n=1, are plotted for* $\beta = 0.7420, L_t = 13$ *(triangles),* $\beta = 0.7525, L_t = 4$ *(squares) and* $\beta = 0.7585, L_t = 4$ *(crosses). The corresponding value of the string tension (as it is listed in tab.II) is subtracted for each sample. The continuous line is the expected result if a pure gaussian correction is taken eq. (24), the dashed line is the self-avoiding case eq. (25).*

Fig.4. *Same as fig.3. The ratios are evaluated at step n=2. The various samples plotted are:* $\beta = 0.7500, L_t = 10$ *(diamonds),* $\beta = 0.7500, L_t = 12$ *(circles),* $\beta = 0.7525, L_t = 6$ *(squares),* $\beta = 0.7585, L_t = 5$ *(crosses),* $\beta = 0.7585, L_t = 6$ *(triangles).*

fig.1.

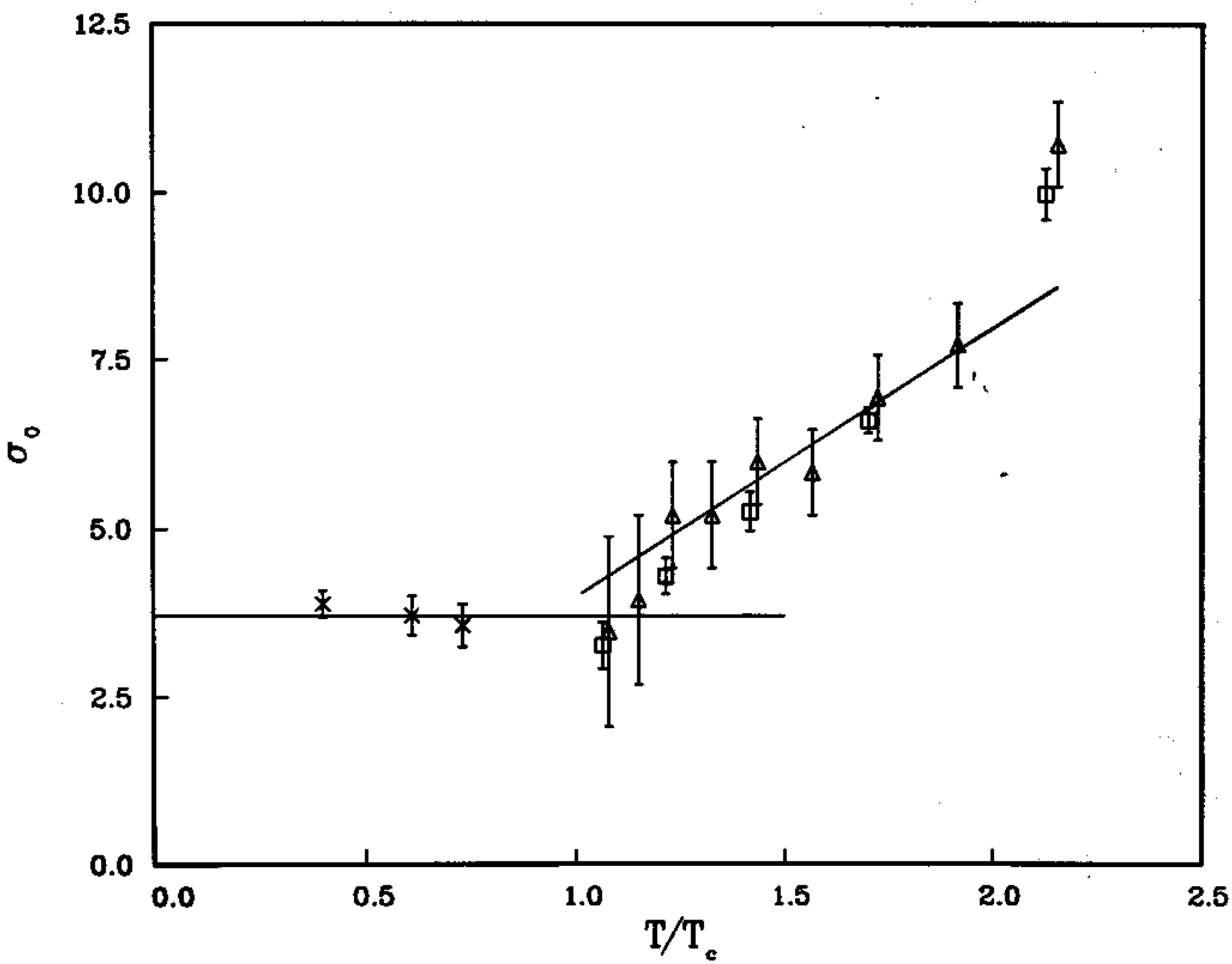


fig.2.

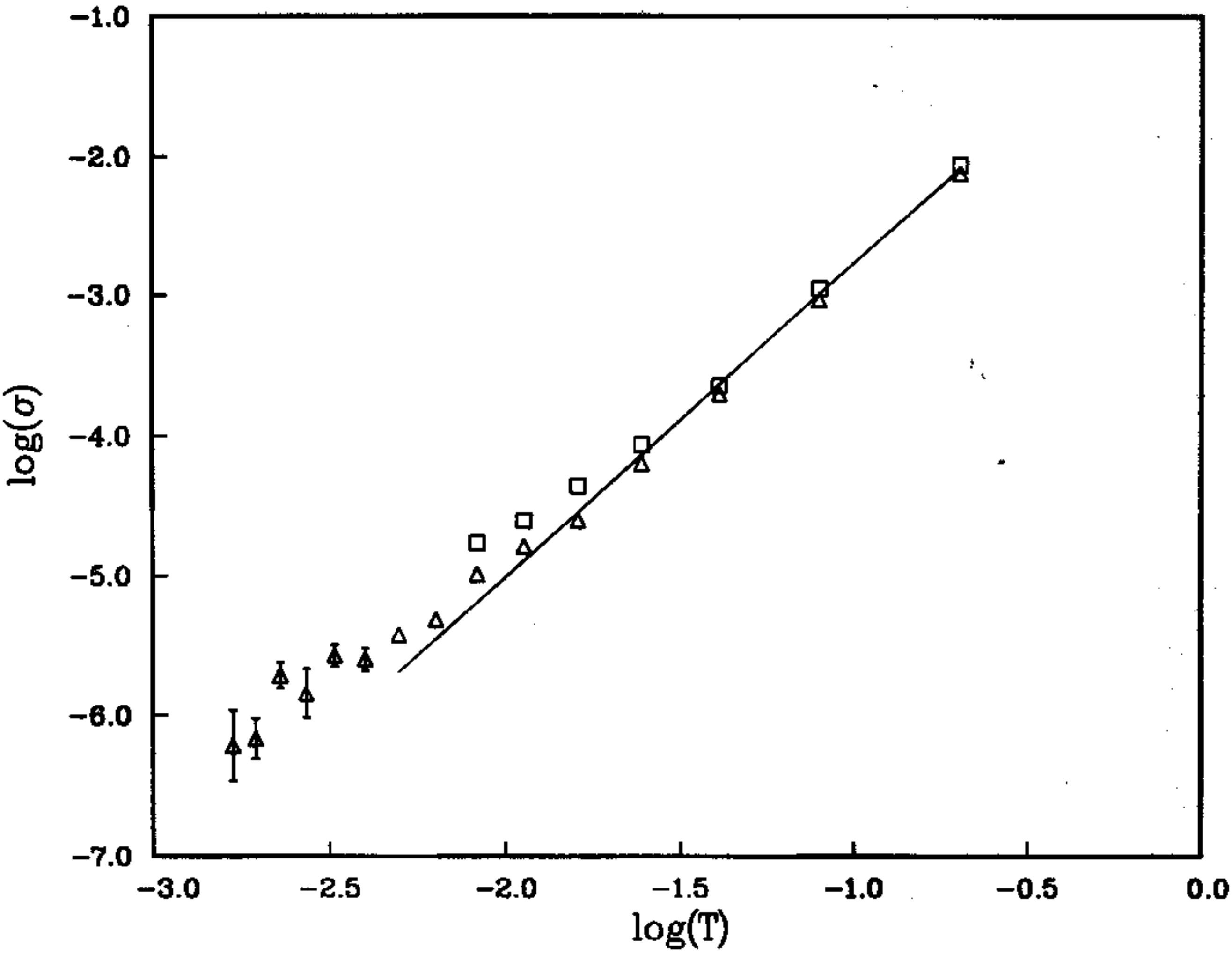


fig.3.

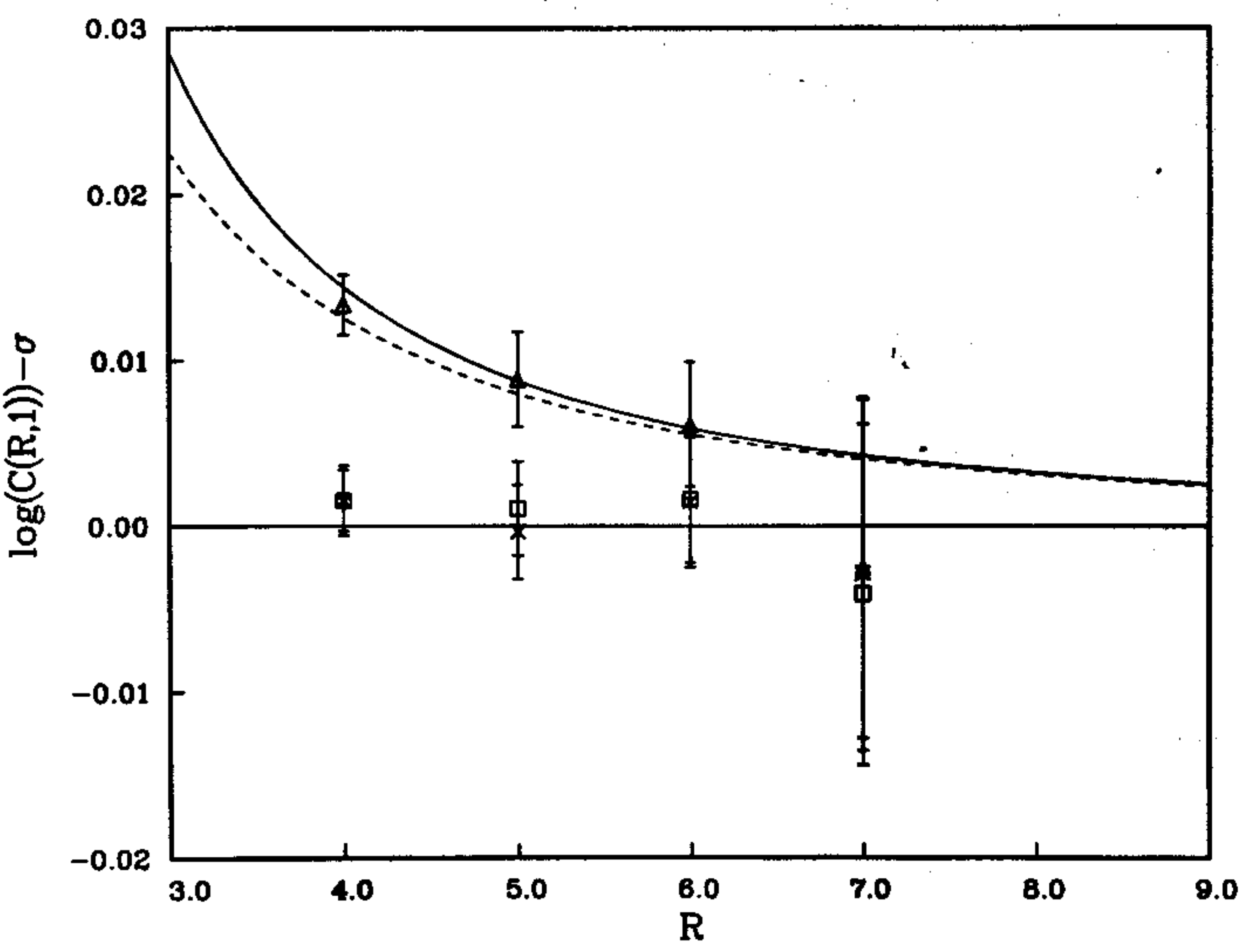
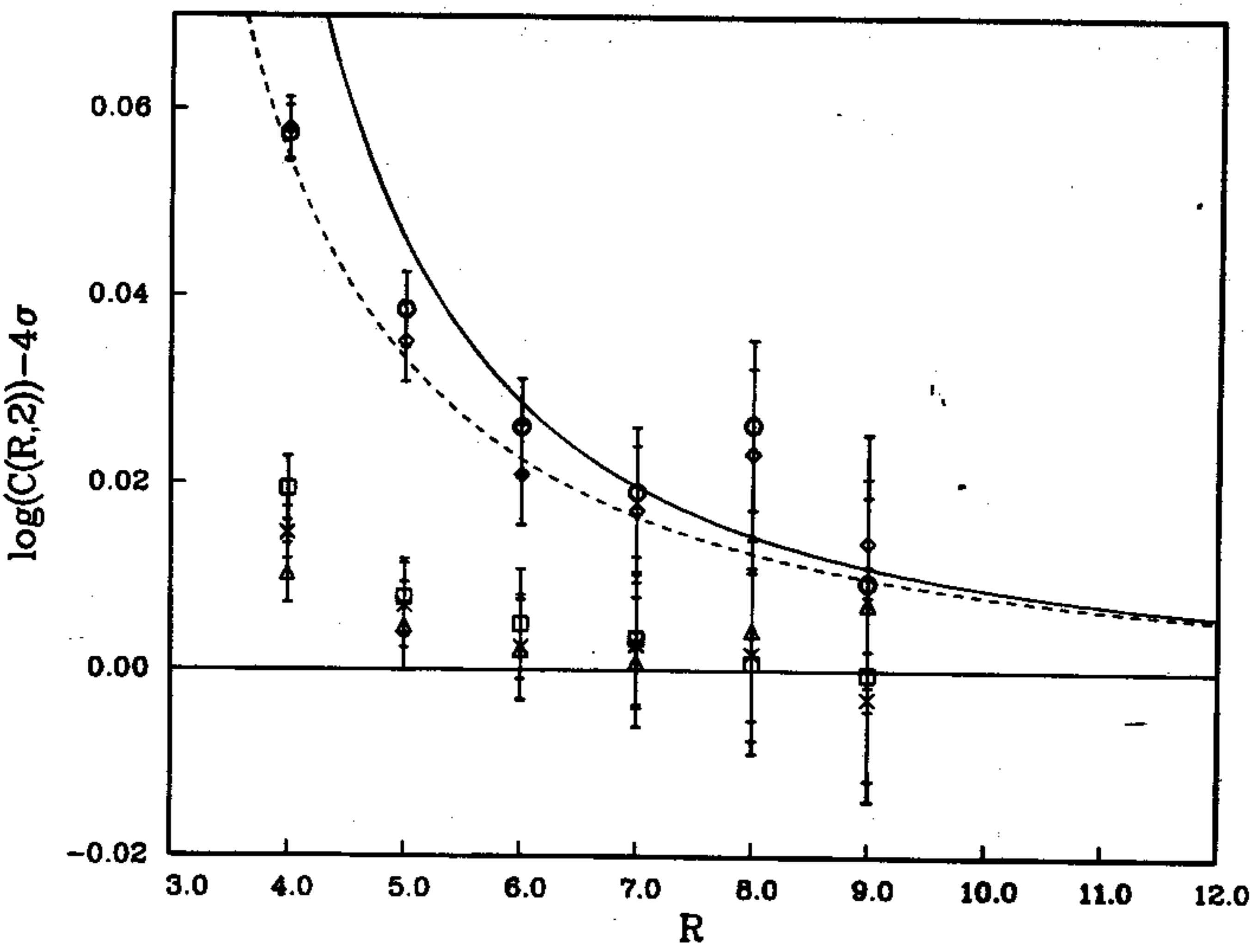


fig.4.



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