

$N=1$ Super- τ_3 QED from Atiyah-Ward Space-Time

M. A. De Andrade* and O. M. Del Cima**

Centro Brasileiro de Pesquisas Físicas - CBPF
Departamento de Teoria de Campos e Partículas (DCP)
Rua Dr. Xavier Sigaud, 150
22290-180 Rio de Janeiro/RJ, Brasil

*Internet e-mail: marco@cbpfsul.cat.cbpf.br

**Internet e-mail: oswaldo@cbpfsul.cat.cbpf.br

ABSTRACT

In this letter, we present the action for the massive super-QED $_{2+2}$. A pair of chiral and a pair of anti-chiral superfields with opposite $U(1)$ -charges are required. We also carry out a dimensional reduction *à la* Scherk from $(2+2)$ to $(1+2)$ dimensions, and we show that, after suitable truncations are performed, the supersymmetric extension of the τ_3 QED $_{1+2}$ naturally comes out.

Key-words: Supersymmetry; Atiyah-Ward space-time; Dimensional reduction.

The idea of considering space-times with several time directions and indefinite signature has deserved a great deal of attention since a self-dual Yang-Mills theory in (2+2) dimensions [1] has been related to the Atiyah-Ward conjecture [2]: this theory might be the source for various integrable models in lower dimensions, after appropriate dimensional reductions are carried out.

More recently, Gates, Ketov and Nishino [3] have pointed out the existence of Majorana-Weyl spinors in the Atiyah-Ward space-time, and $N=1$ self-dual supersymmetric Yang-Mills theories and self-dual supergravity models have been formulated for the first time in this particular space. Afterwards, $N=2$ self-dual super-Yang-Mills and $N=2$ and $N=4$ self-dual supergravities have been formulated and these results have been useful for the conjecture that $N=2$ superstrings have no possible counter-terms at quantum level to all orders in string loops [4].

Since over the past years 3-dimensional field theories [5] have been shown to play a central rôle in connection with the behaviour of 4-dimensional theories at finite temperature [6], as well as in the description of a number of problems in Condensed Matter Physics [7, 8, 9], it seems reasonable to concentrate efforts in trying to understand some peculiar features of gauge-field dynamics in 3 dimensions. Also, the recent result on the Landau gauge finiteness of Chern-Simons theories is a remarkable property that makes 3-dimensional gauge theories so attractive [10]. Very recently, this line of investigation has been well-motivated in view of the possibilities of providing a gauge-theoretical foundation for the description of Condensed Matter phenomena, such as high- T_c superconductivity [8], where the QED₃ and τ_3 QED₃ [8, 9] are some of the theoretical approaches that been forwarded as an attempt to understand more deeply about high- T_c materials.

Our purpose in the present letter is to show the relationship between massive Abelian $N=1$ super-QED₂₊₂ [11] and $N=1$ super- τ_3 QED, after a dimensional reduction *à la* Scherk is carried out and suitable supersymmetry-preserving truncations are made in order to suppress non-physical propagating modes in three dimensions.

The supersymmetric extension of the massive QED in $D=1+3$ requires two chiral superfields carrying opposite $U(1)$ -charges [12]. On the other hand, to introduce mass in the matter sector in $D=2+2$, without breaking gauge-symmetry, we have to introduce four scalar superfields: a pair of chiral and a pair of anti-chiral superfields; the supermultiplets of each pair exhibit opposite $U(1)$ -charges.

The massive Abelian $N=1$ super-QED₂₊₂ is described by the action : ¹

$$S_{\text{inv}}^{\text{AW}} = -\frac{1}{8} \left(\int ds W^c W + \int d\tilde{s} \tilde{W}^c \tilde{W} \right) + \int dv \left(\Psi_+^\dagger e^{4qv} \tilde{X}_+ + \Psi_-^\dagger e^{-4qv} \tilde{X}_- \right) + \\ + i m \left(\int ds \Psi_+ \Psi_- - \int d\tilde{s} \tilde{X}_+ \tilde{X}_- \right) + \text{h.c.} \quad , \quad (1)$$

where q is a dimensionless coupling constant and m is a parameter with dimension of mass. The + and - subscripts in the matter superfields refer to their respective $U(1)$ -charges. To build up the interaction terms, we have used a mixing between the chiral and anti-chiral superfields

¹In this letter, we are adopting $\eta_{\mu\nu}=(+, -, -, +)$ for the A-W space-time metric, $ds \equiv d^4 x d^2 \theta$, $d\tilde{s} \equiv d^4 x d^2 \tilde{\theta}$ and $dv \equiv d^4 x d^2 \theta d^2 \tilde{\theta}$, where θ and $\tilde{\theta}$ are Majorana-Weyl spinors. Notice that we are using the following convention for charge conjugation of all Weyl spinors: $\psi^c \equiv i\sigma_y \psi^*$ and $\tilde{\chi}^c \equiv i\sigma_y \tilde{\chi}^*$. Our conventions for the supersymmetry covariant derivatives are: $D_\alpha = \partial_\alpha - i\tilde{\rho}_{\alpha\dot{\alpha}} \tilde{\theta}^{\dot{\alpha}}$ and $\tilde{D}_{\dot{\alpha}} = \tilde{\partial}_{\dot{\alpha}} - i\tilde{\rho}_{\dot{\alpha}\alpha} \theta^\alpha$. The spinor indices are raised and lowered with the help of the following antisymmetric tensors: $\epsilon_{\alpha\beta} = -\epsilon^{\alpha\beta} = i\sigma_y$ and $\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} = -\tilde{\epsilon}^{\dot{\alpha}\dot{\beta}} = -i\sigma_y$. We use the abbreviations $\theta\sigma^\mu\tilde{\theta} \equiv \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \tilde{\theta}^{\dot{\alpha}}$, $\theta\psi \equiv \theta^\alpha \psi_\alpha$ and $\tilde{\theta}\tilde{\chi} \equiv \tilde{\theta}^{\dot{\alpha}} \tilde{\chi}_{\dot{\alpha}}$. For more details about notation and conventions in $D=2+2$, see ref.[11].

(in order to justify such a procedure, we refer to the works of Gates, Ketov and Nishino [4]). This mixed interaction term establishes that the vector superfield be *complex*.

In the action of $N=1$ super-QED₂₊₂, given by eq.(1), the chiral superfields Ψ_+ and Ψ_- ($\tilde{D}_{\dot{\alpha}}\Psi_{\pm}=0$), are defined as follows :

$$\Psi_{\pm}(x, \theta, \tilde{\theta}) = e^{i\tilde{\theta}\tilde{\theta}\theta} [A_{\pm}(x) + i\theta\psi_{\pm}(x) + i\theta^2 F_{\pm}(x)] \quad , \quad (2)$$

$$\tilde{\phi}_{\dot{\alpha}\alpha} \equiv \tilde{\epsilon}_{\dot{\alpha}\beta}\tilde{\sigma}^{\mu\beta}_{\alpha}\partial_{\mu} \quad \text{and} \quad \tilde{\sigma}^{\mu\beta}_{\alpha} = (i\sigma_x, -\sigma_y, \sigma_z, \mathbf{1}_2)^{\beta}_{\alpha} \quad , \quad (3)$$

where A_+ and A_- are complex scalar fields, ψ_+ and ψ_- are Weyl spinors, and F_+ and F_- are complex auxiliary scalar fields. Moreover, the anti-chiral superfields, \tilde{X}_+ and \tilde{X}_- ($D_{\alpha}\tilde{X}_{\pm}=0$), are defined by :

$$\tilde{X}_{\pm}(x, \theta, \tilde{\theta}) = e^{i\theta\tilde{\theta}\tilde{\theta}} [B_{\pm}(x) + i\tilde{\theta}\tilde{\chi}_{\pm}(x) + i\tilde{\theta}^2 G_{\pm}(x)] \quad , \quad (4)$$

$$\tilde{\phi}_{\alpha\dot{\alpha}} \equiv \epsilon_{\alpha\beta}\sigma^{\mu\beta}_{\dot{\alpha}}\partial_{\mu} \quad \text{and} \quad \sigma^{\mu\beta}_{\dot{\alpha}} = (-i\sigma_x, \sigma_y, -\sigma_z, \mathbf{1}_2)^{\beta}_{\dot{\alpha}} \quad , \quad (5)$$

where B_+ and B_- are complex scalar fields, $\tilde{\chi}_+$ and $\tilde{\chi}_-$ are Weyl spinors, and G_+ and G_- are complex auxiliary scalar fields.

In the Wess-Zumino gauge [12], a complex *vector* superfield, V , is written as

$$V(x, \theta, \tilde{\theta}) = \frac{1}{2}i\theta\sigma^{\mu}\tilde{\theta}B_{\mu}(x) - \frac{1}{2}\tilde{\theta}^2\theta\lambda(x) - \frac{1}{2}\theta^2\tilde{\theta}\tilde{\rho}(x) - \frac{1}{4}\theta^2\tilde{\theta}^2D(x) \quad , \quad (6)$$

where D is a complex auxiliary scalar field, λ and $\tilde{\rho}$ are Weyl spinors and B_{μ} is a *complex* vector field.

The field-strength superfields, W_{α} and $\tilde{W}_{\dot{\alpha}}$, defined by

$$W_{\alpha} = \frac{1}{2}\tilde{D}^2 D_{\alpha} V \quad \text{and} \quad \tilde{W}_{\dot{\alpha}} = \frac{1}{2}D^2 \tilde{D}_{\dot{\alpha}} V \quad , \quad (7)$$

respectively, satisfy the chiral and anti-chiral conditions, $\tilde{D}_{\dot{\beta}}W_{\alpha}=0$ and $D_{\beta}\tilde{W}_{\dot{\alpha}}=0$; they read as below :

$$\left\{ \begin{array}{l} W_{\alpha} = e^{i\tilde{\theta}\tilde{\theta}\theta} [\lambda_{\alpha} + \theta^{\beta} (\epsilon_{\alpha\beta}D - \sigma_{\alpha\beta}^{\mu\nu}G_{\mu\nu}) + i\theta^2\sigma_{\alpha\dot{\alpha}}^{\mu}\partial_{\mu}\tilde{\rho}^{\dot{\alpha}}] \\ \tilde{W}_{\dot{\alpha}} = e^{i\theta\theta\tilde{\theta}} [\tilde{\rho}_{\dot{\alpha}} + \tilde{\theta}^{\dot{\beta}} (\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}D - \tilde{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu}G_{\mu\nu}) + i\tilde{\theta}^2\tilde{\sigma}_{\dot{\alpha}\alpha}^{\mu}\partial_{\mu}\lambda^{\alpha}] \end{array} \right. \quad , \quad (8)$$

$$\sigma^{\mu\nu\alpha}_{\beta} = \frac{1}{4}(\sigma^{[\mu}\tilde{\sigma}^{\nu]})^{\alpha}_{\beta} \quad \text{and} \quad \tilde{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}} = \frac{1}{4}(\tilde{\sigma}^{[\mu}\sigma^{\nu]})^{\dot{\alpha}}_{\dot{\beta}} \quad , \quad (9)$$

where $G_{\mu\nu}=\partial_{\mu}B_{\nu}-\partial_{\nu}B_{\mu}$ is the field-strength.

By considering the superfields defined above, the following component-field action stems from the superspace action of eq.(1) :

$$\begin{aligned} S_{\text{inv}}^{\text{AW}} = & \int d^4x \left\{ -\frac{1}{4}i \left(\lambda^c \tilde{\phi} \tilde{\rho} + \tilde{\rho}^c \tilde{\phi} \lambda \right) - \frac{1}{8} G_{\mu\nu}^* G^{\mu\nu} - \frac{1}{4} D^* D + \right. \\ & - F_+^* G_+ - A_+^* \square B_+ - \frac{1}{2} i \psi_+^c \tilde{\phi} \tilde{\chi}_+ - q B_{\mu} \left(\frac{1}{2} i \psi_+^c \sigma^{\mu} \tilde{\chi}_+ + A_+^* \partial^{\mu} B_+ - B_+ \partial^{\mu} A_+^* \right) + \\ & \left. + i q \left(A_+^* \tilde{\chi}_+ \tilde{\rho} + B_+ \psi_+^c \lambda \right) - (qD + q^2 B_{\mu} B^{\mu}) A_+^* B_+ + \right. \end{aligned}$$

$$\begin{aligned}
 & - F_-^* G_- - A_-^* \square B_- - \frac{1}{2} i \psi_-^c \not{\partial} \tilde{\chi}_- + q B_\mu \left(\frac{1}{2} i \psi_-^c \sigma^\mu \tilde{\chi}_- + A_-^* \partial^\mu B_- - B_- \partial^\mu A_-^* \right) + \\
 & - i q \left(A_-^* \tilde{\chi}_- \tilde{\rho} + B_- \psi_-^c \lambda \right) + (q D - q^2 B_\mu B^\mu) A_-^* B_- + \\
 & + m \left(\frac{1}{2} i \psi_+ \psi_- - \frac{1}{2} i \tilde{\chi}_+ \tilde{\chi}_- - A_+ F_- - A_- F_+ + B_+ G_- + B_- G_+ \right) \Big\} + \text{h.c.} \quad (10)
 \end{aligned}$$

Due to the fact that in massive super-QED₂₊₂ one must have two opposite $U(1)$ -charges to introduce mass at tree-level, and a complex vector superfield in order to build up the gauge invariant interactions, we can read directly from the action (1), the following set of local $U(1)_\alpha \times U(1)_\gamma$ transformations :

$$\left\{ \begin{array}{l} \delta_g A_\pm^* = \pm i q \beta(x) A_\pm^* \\ \delta_g \psi_\pm^c = \pm i q \beta(x) \psi_\pm^c \\ \delta_g F_\pm^* = \pm i q \beta(x) F_\pm^* \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \delta_g B_\pm = \mp i q \beta(x) B_\pm \\ \delta_g \tilde{\chi}_\pm = \mp i q \beta(x) \tilde{\chi}_\pm \\ \delta_g G_\pm = \mp i q \beta(x) G_\pm \end{array} \right. , \quad (11)$$

where $\beta \equiv \alpha - i\gamma$ is an arbitrary infinitesimal complex function. Notice that the complexified gauge transformations (11) read as above because one has previously fixed to work in the Wess-Zumino gauge. As for the gauge superfield components surviving the Wess-Zumino gauge, we have :

$$\left\{ \begin{array}{l} \delta_g \lambda = \delta_g \tilde{\rho} = 0 \quad , \\ \delta_g D = 0 \quad \text{and} \\ \delta_g B_\mu = i \partial_\mu \beta \quad . \end{array} \right. \quad (12)$$

Therefore, in the Wess-Zumino gauge, the real part of B_μ gauges the $U(1)_\gamma$ -symmetry with real gauge function γ , whereas its imaginary part gauges the $U(1)_\alpha$ -symmetry with real gauge function α . The latter is an ordinary phase symmetry, and we associate it with the electric charge. Indeed, as we will see later on, the imaginary component of B_μ will be taken as the photon field. The parameter γ generates a local Weyl-like invariance [13]. However, the vector field that gauges such a symmetry, namely the real part of B_μ , will be suppressed in the process of dimensional reduction, so that such an invariance will not leave track in $D=1+2$.

It should be emphasized that the mass bilinears in the action given by eq.(10) preserve the local $U(1)_\alpha \times U(1)_\gamma$ -symmetry, since their component matter fields (fermion and scalar) carry opposite charges. Therefore, the opposite values of the $U(1)$ -charges play the central rôle of introducing mass to matter fields without breakdown of the gauge-symmetry, similarly to what happens in $D=1+3$.

$D=2+2$	$D=1+2$
$G_{\mu\nu}^* G^{\mu\nu}$	$G_{mn}^* G^{mn} + 2\partial_m \phi^* \partial^m \phi$
$\psi^c \partial \tilde{\chi}$	$\bar{\psi} \gamma^m \partial_m \chi$
$\tilde{\chi}^c \tilde{\phi} \psi$	$\bar{\chi} \gamma^m \partial_m \psi$
$iB_\mu \psi^c \sigma^\mu \tilde{\chi}$	$iB_m \bar{\psi} \gamma^m \chi - \phi \bar{\psi} \chi$
$B_\mu B \partial^\mu A^*$	$B_m B \partial^m A^*$
$iA^* \tilde{\chi} \tilde{\rho}$	$A^* \bar{\chi}^c \rho$
$iB \psi^c \lambda$	$-B \bar{\psi} \lambda$
$B_\mu B^\mu A^* B$	$B_m B^m A^* B + \phi^2 A^* B$
$i\psi_+ \psi_-$	$-\bar{\psi}_+^c \psi_-$
$i\tilde{\chi}_+ \tilde{\chi}_-$	$\bar{\chi}_+^c \chi_-$

Table 1: Dimensional reduction rules from $D=2+2$ to $D=1+2$.

It is well-known that outstanding supersymmetric models with extended supersymmetry are closely related to simple supersymmetries in higher dimensions [14, 15]. As we are interested in simple supersymmetric models in $D=1+2$, since those should be more relevant for applications in Condensed Matter Physics [7, 8, 9], we concentrate our efforts to investigate which sort of model comes out after a suitable compactification from Atiyah-Ward space-time to 3 space-time dimensions is accomplished. Therefore, it will be interesting to carry out a dimensional reduction² (*à la* Scherk [14]) of the massive $N=1$ super-QED₂₊₂. Bearing in mind that this procedure should extend the supersymmetry [14, 15] to $N>1$, truncations will be needed in order to remain with a simple supersymmetry and to suppress unphysical modes, *i.e.* spurious degrees of freedom coming from $D=2+2$ dimensions.

To perform the dimensional reduction of the massive $N=1$ super-QED₂₊₂ action (10) to $D=1+2$, use has been made of the rules presented in the Table 1. As a result, it can be directly found the following 3-dimensional action :³

$$\begin{aligned}
S_{\text{inv}}^{D=3} = & \int d^3 \hat{x} \left\{ -\frac{1}{4} i \left(\bar{\lambda} \gamma^m \partial_m \rho + \bar{\rho} \gamma^m \partial_m \lambda \right) - \frac{1}{8} \left(G_{mn}^* G^{mn} + 2\partial_m \phi^* \partial^m \phi \right) - \frac{1}{4} D^* D + \right. \\
& - F_+^* G_+ - A_+^* \square B_+ - \frac{1}{2} i \bar{\psi}_+ \gamma^m \partial_m \chi_+ - q B_m \left(\frac{1}{2} i \bar{\psi}_+ \gamma^m \chi_+ + A_+^* \partial^m B_+ - B_+ \partial^m A_+^* \right) + \\
& + \frac{1}{2} q \phi \bar{\psi}_+ \chi_+ + q \left(A_+^* \bar{\chi}_+^c \rho - B_+ \bar{\psi}_+ \lambda \right) - (qD + q^2 B_m B^m + q^2 \phi^2) A_+^* B_+ + \\
& - F_-^* G_- - A_-^* \square B_- - \frac{1}{2} i \bar{\psi}_- \gamma^m \partial_m \chi_- + q B_m \left(\frac{1}{2} i \bar{\psi}_- \gamma^m \chi_- + A_-^* \partial^m B_- - B_- \partial^m A_-^* \right) + \\
& - \frac{1}{2} q \phi \bar{\psi}_- \chi_- - q \left(A_-^* \bar{\chi}_-^c \rho - B_- \bar{\psi}_- \lambda \right) + (qD - q^2 B_m B^m - q^2 \phi^2) A_-^* B_- + \\
& \left. - m \left(\frac{1}{2} \bar{\psi}_+^c \psi_- + \frac{1}{2} \bar{\chi}_+^c \chi_- + A_+ F_- + A_- F_+ - B_+ G_- - B_- G_+ \right) \right\} + \text{h.c.} \quad , \quad (13)
\end{aligned}$$

²One uses the trivial dimensional reduction where the time-derivative, ∂_3 , of all component fields vanishes, $\partial_3 \mathcal{F} = 0$. Also, it was assumed that B_μ is reduced in the following manner: $B^\mu = (B^m, \phi)$, where ϕ is a complex scalar field and the 3-dimensional metric becomes $\eta_{mn} = (+, -, -)$. Note that, λ , ρ , ψ and χ are now Dirac spinors in $D=1+2$.

³For notations and conventions adopted in this letter for $D=1+2$, see ref.[11].

where, after dimensional reduction, the coupling constant g acquires dimension of $(\text{mass})^{\frac{1}{2}}$.

Analysing the 3-dimensional action given by eq.(13), it can be easily shown that the spectrum will unavoidably be spoiled by the presence of ghost fields, since the free sector of the action is totally off-diagonal. Therefore, truncations are needed in order to remove the spurious degrees of freedom, as well as to give rise to a simple supersymmetric action in $D=1+2$. First of all, to make the truncations possible, we need to diagonalize the whole free sector, in order that the ghost fields be identified.

The diagonalization is achieved by looking for suitable linear combinations of the fields which yield a diagonal free action (13). After extremely tedious algebraic manipulations, we find the following transformations which diagonalize the action $S_{\text{inv}}^{D=3}$:

1. gauge sector :

$$\lambda = \frac{1}{\sqrt{2}} (\hat{\rho} + \hat{\lambda}) \quad \text{and} \quad \rho = \frac{1}{\sqrt{2}} (\hat{\rho} - \hat{\lambda}) \quad ; \quad (14)$$

2. fermionic matter sector :

$$\psi_+ = \frac{1}{\sqrt{2}} (\hat{\psi}_+ - \hat{\psi}_- + \hat{\chi}_+ + \hat{\chi}_-) \quad \text{and} \quad \psi_- = \frac{1}{\sqrt{2}} (\hat{\psi}_- + \hat{\psi}_+ + \hat{\chi}_- - \hat{\chi}_+) \quad ; \quad (15)$$

$$\chi_+ = \frac{1}{\sqrt{2}} (\hat{\chi}_+ + \hat{\chi}_- - \hat{\psi}_+ + \hat{\psi}_-) \quad \text{and} \quad \chi_- = \frac{1}{\sqrt{2}} (\hat{\chi}_- - \hat{\chi}_+ - \hat{\psi}_- + \hat{\psi}_+) \quad ; \quad (16)$$

3. bosonic matter sector :

$$A_+ = \frac{1}{\sqrt{2}} (\hat{A}_+ - \hat{B}_+) \quad \text{and} \quad A_- = \frac{1}{\sqrt{2}} (\hat{A}_- - \hat{B}_-) \quad ; \quad (17)$$

$$B_+ = \frac{1}{\sqrt{2}} (\hat{A}_+ + \hat{B}_+) \quad \text{and} \quad B_- = \frac{1}{\sqrt{2}} (\hat{A}_- + \hat{B}_-) \quad ; \quad (18)$$

$$F_+ = \frac{1}{\sqrt{2}} (\hat{F}_+ + \hat{G}_+) \quad \text{and} \quad F_- = \frac{1}{\sqrt{2}} (\hat{F}_- + \hat{G}_-) \quad ; \quad (19)$$

$$G_+ = \frac{1}{\sqrt{2}} (\hat{G}_+ - \hat{F}_+) \quad \text{and} \quad G_- = \frac{1}{\sqrt{2}} (\hat{G}_- - \hat{F}_-) \quad . \quad (20)$$

On the other hand, to simplify the Yukawa-interaction terms (gaugino-matter couplings), we find that following field redefinitions for the bosonic matter sector are convenient :

$$\hat{A}_+ = \frac{1}{\sqrt{2}} (\check{A}_+ - \check{A}_-) \quad \text{and} \quad \hat{A}_- = \frac{1}{\sqrt{2}} (\check{A}_+ + \check{A}_-) \quad ; \quad (21)$$

$$\hat{F}_+ = \frac{1}{\sqrt{2}} (\check{F}_+ - \check{F}_-) \quad \text{and} \quad \hat{F}_- = \frac{1}{\sqrt{2}} (\check{F}_+ + \check{F}_-) \quad . \quad (22)$$

By replacing these field redefinitions into the action (13), one ends up with a diagonalized action, where the fields, ϕ , $\hat{\rho}$, $\hat{\chi}_+$, $\hat{\chi}_-$, \hat{B}_+ and \hat{B}_- appear like ghosts in the framework of an $N=2$ -supersymmetric model. Therefore, in order to suppress these unphysical modes, truncations must be performed. Bearing in mind that we are looking for an $N=1$ supersymmetric 3-dimensional model (in the Wess-Zumino gauge), truncations have to be imposed on the ghost fields, ϕ , $\hat{\rho}$, $\hat{\chi}_+$, $\hat{\chi}_-$, \hat{B}_+ and \hat{B}_- . To keep $N=1$ supersymmetry in the Wess-Zumino gauge, we

must simultaneously truncate the component fields, \hat{G}_+ , \hat{G}_- , D , a_m and τ^4 . The truncation of τ is dictated by the suppression of a_m . Now, the choice of truncating a_m , instead of A_m , is based on the analysis of the couplings to the matter sector: A_m couples to both scalar and fermionic matter and we interpret it as the photon field in 3 dimensions.

After performing these truncations, and omitting the (\wedge) and (\vee) symbols, we find the following action in $D=1+2$:

$$\begin{aligned}
 S_{N=1}^{\tau_3\text{QED}} = \int d^3\hat{x} \left\{ \frac{1}{2}i\bar{\lambda}\gamma^m\partial_m\lambda - \frac{1}{4}F_{mn}F^{mn} + \right. \\
 - A_+^*\square A_+ - A_-^*\square A_- + i\bar{\psi}_+\gamma^m\partial_m\psi_+ + i\bar{\psi}_-\gamma^m\partial_m\psi_- + F_+^*F_+ + F_-^*F_- + \\
 - qA_m\left(\bar{\psi}_+\gamma^m\psi_+ - \bar{\psi}_-\gamma^m\psi_- + iA_+^*\partial^m A_+ - iA_-^*\partial^m A_- - iA_+\partial^m A_+^* + iA_-\partial^m A_-^*\right) + \\
 - iq\left(A_+\bar{\psi}_+\lambda - A_-\bar{\psi}_-\lambda - A_+^*\bar{\lambda}\psi_+ + A_-^*\bar{\lambda}\psi_-\right) + q^2A_mA^m(A_+^*A_+ + A_-^*A_-) + \\
 \left. - m\left(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_- + A_+^*F_+ - A_-^*F_- + A_+F_+^* - A_-F_-^*\right) \right\} , \quad (23)
 \end{aligned}$$

where it can be easily concluded that this is a supersymmetric extension of a parity-preserving action, namely, $\tau_3\text{QED}$ [9]. However, to render our claim more explicit, we are going next to rewrite (23) in terms of the superfields of $N=1$ supersymmetry in 3 dimensions.

In order to formulate the $N=1$ super- $\tau_3\text{QED}$ action (23) in terms of superfields, we refer to the work by Salam and Strathdee [16], where the superspace and superfields in $D=1+3$ were formulated for the first time. Extending their ideas to our case in $D=1+2$, the elements of superspace are labeled by (x^m, θ) , where x^m are the space-time coordinates and the fermionic coordinates, θ , are Majorana spinors, $\theta^c = \theta$.⁵

Now, we are ready to introduce the formulation of $N=1$ super- $\tau_3\text{QED}$ in terms of superfields. To begin with, we define the complex scalar superfields with opposite $U(1)$ -charges, Φ_+ and Φ_- , as

$$\Phi_{\pm} = A_{\pm} + \bar{\theta}\psi_{\pm} - \frac{1}{2}\bar{\theta}\theta F_{\pm} \quad \text{and} \quad \Phi_{\pm}^{\dagger} = A_{\pm}^* + \bar{\psi}_{\pm}\theta - \frac{1}{2}\bar{\theta}\theta F_{\pm}^* , \quad (24)$$

where A_+ and A_- are complex scalar fields, ψ_+ and ψ_- are Dirac spinors and F_+ and F_- are complex auxiliary scalar fields.

In the Wess-Zumino gauge, the gauge superconnection, Γ_a , is written as

$$\Gamma_a = i(\gamma^m\theta)_a A_m + \bar{\theta}\theta\lambda_a \quad \text{and} \quad \bar{\Gamma}_a = -i(\bar{\theta}\gamma^m)_a A_m + \bar{\theta}\theta\bar{\lambda}_a , \quad (25)$$

where A_m is the gauge-field and λ_a is the gaugino (Majorana spinor).

Defining the field-strength superfield, W_a , according to :

$$W_a = \frac{1}{2}\bar{D}_b D_a \Gamma_b , \quad (26)$$

with superderivatives given by

$$D_a = \bar{\partial}_a - i(\gamma^m\theta)_a\partial_m \quad \text{and} \quad \bar{D}_a = \partial_a - i(\bar{\theta}\gamma^m)_a\partial_m , \quad (27)$$

⁴The a_m field is the real part of B_m , since we are assuming $B_m = a_m + iA_m$. Also, as $\hat{\lambda}$ is a Dirac spinor, it can be written in terms of two Majorana spinors in the following manner: $\hat{\lambda} = \tau + i\check{\lambda}$.

⁵The adjoint and charge-conjugated spinors are defined by $\bar{\psi} = \psi^1\gamma^0$ and $\psi^c = -C\bar{\psi}^T$, respectively, where $C = \sigma_y$. The γ -matrices we are using arised from the dimensional reduction to $D=1+2$ are: $\gamma^m = (\sigma_x, i\sigma_y, -i\sigma_z)$. Note that for any spinorial objects, ψ and χ , the product $\bar{\psi}\chi$ denotes $\bar{\psi}_a\chi_a$.

it can be found that

$$W_a = \lambda_a + \Sigma^{mn}{}_{ab} \theta_b F_{mn} - \frac{i}{2} \bar{\theta} \theta \gamma^m{}_{ab} (\partial_m \lambda_b) \quad (28)$$

and

$$\bar{W}_a = \bar{\lambda}_a - \bar{\theta}_b \Sigma^{mn}{}_{ba} F_{mn} + \frac{i}{2} \bar{\theta} \theta (\partial_m \bar{\lambda}_b) \gamma^m{}_{ba} \quad , \quad (29)$$

where $\Sigma^{mn} = \frac{1}{4}[\gamma^m, \gamma^n]$ are the generators of the Lorentz group in $D=1+2$.

The gauge covariant derivatives we are defining for the matter superfields with opposite $U(1)$ -charges, Φ_+ and Φ_- , are given by

$$\nabla_a \Phi_{\pm} = (D_a \mp iq \Gamma_a) \Phi_{\pm} \quad \text{and} \quad \bar{\nabla}_a \Phi_{\pm}^{\dagger} = (\bar{D}_a \pm iq \bar{\Gamma}_a) \Phi_{\pm}^{\dagger} \quad , \quad (30)$$

where q is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$.

By using the previous definitions of the superfields, (24), (25), (28) and (29), and the gauge covariant derivatives, (30), we found how to build up the $N=1$ super- τ_3 QED action, given by eq.(23), in superspace; it reads :

$$S_{N=1}^{\tau_3\text{QED}} = - \int d\hat{v} \left\{ \frac{1}{2} \bar{W} W + (\bar{\nabla} \Phi_+^{\dagger})(\nabla \Phi_+) + (\bar{\nabla} \Phi_-^{\dagger})(\nabla \Phi_-) - m(\Phi_+^{\dagger} \Phi_+ - \Phi_-^{\dagger} \Phi_-) \right\} \quad , \quad (31)$$

where the superspace measure we are adopted is $d\hat{v} \equiv d^3 \hat{x} d^2 \theta$ and the Berezin integral is taken as $\int d^2 \theta = -\frac{1}{4} \bar{\partial} \partial$. Therefore, we finally show, by using the superspace formulation (31), that the action (23) we have found after a dimensional reduction *à la* Scherk, and some suitable truncations of the massive $N=1$ super-QED $_{2+2}$, is certainly the simple supersymmetric version of τ_3 QED.

Our final conclusion is that the massive Abelian $N=1$ super-QED $_{2+2}$ proposed in ref.[11] shows interesting features when an appropriate dimensional reduction is performed. The dimensional reduction *à la* Scherk we have applied to our problem becomes very attractive, since, after doing some truncations to avoid unphysical modes, the $N=1$ super- τ_3 QED is obtained as a final result. In fact, the Atiyah-Ward space-time shows to be very fascinating as a starting point to formulate models to be studied in lower dimensions.

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