ISSN 0029-3865

CBPF-NF-055/89

TCP (TRUNCATED COMPOUND POISSON) PROCESS FOR MULTIPLICITY DISTRIBUTIONS IN HIGH ENERGY COLLISIONS*

by

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*Extended version of ICTP, Trieste preprint IC/88/127. To appear in Int. Jl. Mod. Phys. A (1989).

ABSTRACT:

On using the Poisson distribution truncated at zero for intermediate cluster decay in a compound Poisson process we obtain TCP distribution which describes quite well the multiplicity distributions in high energy collisions. A detailed comparison is made between TCP and NB for UA5 data. The reduced moments up to the fifth agree very well with the observed ones. The TCP curves are narrower than NB at high multiplicity tail, look narrower at very high energy and develop shoulders and oscillations which become increasingly pronounced as the energy grows. At lower energies the curves are very close to the NB ones. We also compare the parameterizations by these two distributions of the data for fixed intervals of rapidity for UA5 data and for the data (at low energy) for e^+e^- annihilation and pion-proton, proton-proton and muon-proton scattering. A discussion of compound Poisson distribution ,expressions of reduced moments and Poisson transforms are also given. The TCP curves and curves of the reduced moments for different values of the parameters are also presented.

Key-words: Multiplicity; High energy collisions.

I. INTRODUCTION:

Recently a fair amount of experimental data has been accumulated on multiplicity distribution (md.) of charged particles in high energy collisions in the energy range of 10 GeV up to around 900 GeV. The main experiments are, the UA5 Collaboration[1] at SPS $p\bar{p}$ collider ($E_{c.m}$=200,546,900 GeV), NA22 Collaboration[2] (pp and πp collisions at $E_{c.m}$=22 GeV), HRS collaboration[3] at PEP (e^+e^- annihilation at $E_{c.m.}$ = 29 GeV), EM Collaboration[4] (deep inelastic muon-proton scattering for $E_{c.m}$= 4-6 GeV to 18-20 GeV) among others.

The violation of KNO scaling[5] in the mds. at energies above 200 GeV was detected by UA5 group who found that the distributions grew broader with energy. They used the negative binomial (NB) distribution to fit the data in full phase space with remarkable success over a wide range of c.m. energies up to 900 GeV. Also the reduced moments calculated using NB parameterization of the multiplicity data for fixed, symmetric or asymmetric, rapidity intervals are found to agree with experiments[1-4].

In a recent publication it was pointed out that the above mentioned mds. may as well be described very well by a Truncated Compound Poisson distribution (TCP)[6] obtained by compounding two Poisson distributions in contrast to the case of NB where a logarithmic distribution appears compounded with a Poisson one. The agreement for full phase space UA5 data of the calculated reduced moments using TCP with the experimental values is found to be as good as in the case of NB and the distribution curves of TCP and NB almost coincide at lower energies. At higher energies ,however, some significant differences begin to appear in the form of shoulders and oscillations in the case of TCP while they are absent in NB and several other currently proposed[7] hadroproduction distributions. For the same values of the two parameters (Sec.II) which characterize TCP or NB the former is always narrower than the latter at higher multiplicity points and it may be possible to test these distributions more closely by a careful analysis of the tails of the mds., say, at 900 GeV and at Tevatron energies. An evidence of marked oscillations in the mds. at SSC, SLC and LEP would be in favor of TCP or a situation in between the two distributions. We remark that in the interpretation of a compound Poisson process as a two step[8] process in which intermediate clusters (or clans[9]) are formed the average number of clusters is found to decrease with energy from 6 to 3, reminding us of cosmic rays data, for the UA5 data if we use TCP while it is found to increase and saturate around 8 for NB[9]. We discuss the properties of TCP distribution in Sec.II and the expressions for the various moments are given along with its Poisson transform. In Sec.III we compare the TCP predictions with the UA5 data in detail and compare also the TCP and NB parameterizations of the data from experiments mentioned above for fixed rapidity intervals by comparing the predicted values for the reduced moments in Tabs.3-7.

II. COMPOUND POISSON PROCESS. TCP AND NB DISTRIBUTIONS:

Both TCP and NB belong to the general class of the so called (discrete) compound Poisson distributions which are infinitely divisible. A compound Poisson process may conveniently be described by the following generating function

$$G(t) = G(g(t);\langle N\rangle) = \exp(\langle N\rangle[g(t)-1]) = \sum t^n P_n \qquad (1)$$

which corresponds to the multiplicity distribution

$$P_n = \sum_N e^{-\langle N\rangle}(\langle N\rangle^N/N!)\,(1/n!)\,(d^n/dt^n)\,(g(t))^N\Big|_{t=0} \qquad (2)$$

In our context it corresponds to a two step process. At the first stage N independent Poisson distributed clusters or clans are produced and each of which then subsequently decays according to the probability distribution corresponding to the generating function g(t) giving rise to a total number of n particles as observed experimentally. The TCP distribution[6] is obtained by choosing g(t)=

$(e^{Bt}-1)/(e^{B}-1)$,where B is a constant ,corresponding to a Poisson distribution truncated at zero in order to allow for at least one particle inside the decaying cluster (see also Sec.III). The NB is obtained from $g(t)=\ln(1-qt)/\ln(1-q)$. We have $P(0)=e^{-\langle N\rangle}$ where $\langle N\rangle$ is the average number of clusters formed. For the average number of particles produced we obtain $\langle n\rangle=\langle N\rangle\langle n_c\rangle$ where $\langle n_c\rangle=g'(1)$ gives the average multiplicity of the cluster decay. Defining the shape parameter k from $(\langle n^2\rangle-\langle n\rangle^2)/\langle n\rangle^2=1/\langle n\rangle+1/k$ we find $\langle N\rangle=k\, g''(1)/(g'(1)^2)$ and $\langle n\rangle/k=g''(1)/g'(1)=B$ for TCP while it equals $q/(1-q)$ for NB while $\langle N\rangle/k=1-e^{-\langle n\rangle/k}$ for TCP and $\ln(1+\langle n\rangle/k)$ for NB. The distributions under consideration are completely characterized in terms of two parameters k and $B\equiv\langle n\rangle/k$ which are determined from the fit to the experimental data. An important property of compound Poisson distribution is infinite divisibility[8] in that it can be represented as m-fold convolution of a probability distribution with itself (see (1)and ref.8). We also verify that

$$G[g_1(t);\langle N_1\rangle]\ G[g_2(t);\langle N_2\rangle]$$

$$= G[(\langle N_1\rangle g_1+\langle N_2\rangle g_2)/(\langle N_1\rangle+\langle N_2\rangle);\langle N_1\rangle+\langle N_2\rangle] \qquad (3)$$

Another property worth mentioning is that we may rewrite the generating function $G(g(t);\langle N\rangle)$ above in an infinite product form

$$e^{-a(1-t)}e^{-b(1-t^2)}e^{-c(1-t^3)} \qquad (4)$$

where the numbers of singlets, doublets, triplets etc. of particles involved have independent Poisson distributions with means a,b,c etc.

A discrete probability distribution may be associated to a continuous distribution through the following Poisson transform[10] f(x) defined by

$$P_n = \int_0^\infty dx\, f(x)\, [x^n e^{-x}/n!] \qquad (5)$$

$$\int_0^\infty dx\ f(x) = 1\ , \qquad \int_0^\infty dx\ \ x\, f(x) = \langle n\rangle \tag{6}$$

It follows that the generating function may be written as the Laplace transform of the corresponding Poisson transform. In fact, setting $s = 1-t$, $Q(s) = G(1-s)$,

$$Q(s) = \sum (1-s)^n\, P_n = \int_0^\infty dx\, f(x)\, e^{-sx} \tag{7}$$

The Poisson transform may itself be regarded as a probability distribution defined on $0 \le x \le \infty$ and ,for example, the k-th moment $\langle x^k\rangle$ gives the k-th factorial moment of the probability distributon P_n. Expanding $f(x)$ in terms of Laguerre functions $L_m(x) = \sum_{n=0}^{m} \binom{m}{n} (-x)^n/n!$ we easily derive

$$f(x) = \sum_{m=0}^{\infty} \sum_{n=0}^{m} (-1)^n \binom{m}{n} L_m(x)\ P_n \tag{8}$$

and $Q(s)$ follows on using the Laplace transform $(s-1)^m/s^{m+1}$ of L_m.

For TCP case we may derive the following probability distribution

$$P_n = e^{-\langle N\rangle}\, (B^n/n!)\ A_n(k\ e^{-B}),$$

$$\langle N\rangle = k\ (1-e^{-B}), \qquad \langle n_c\rangle = B/(1-e^{-B}) \tag{9}$$

where $B = \langle n\rangle/k$ and $A_n(x)$ are polynomials defined by the recurrence relation $A_{n+1}(x) = x\,[A_n(x) + A_n'(x)]$ satisfying $A_0(x) = 1$, $A_1(x) = x$ etc. In deriving P_n we used the expansion $\exp[x\,(e^t - 1)] = \sum A_n(x)\ t^n/n!$. The numerical coefficients in these polynomials are Stirling numbers of second kind and they grow very large with increasing n. Computer program, however , can be set up easily to handle the numerical computation required for obtaining the mds. We also note the relation $e^x\, A_n(x) = \sum (j^n/j!)\ x^j$ which was quite useful for computation at high multiplicity points. From (8) we obtain the corresponding Poisson transform

$$f(x) = \sum_{j=0}^{\infty} (e^{-k}\, k^j/j!)\ \delta\,(x-j\,B) \qquad (10)$$

This result could otherwise be obtained by expanding the generating function of TCP rewritten in the form $Q(s)= \exp(k(\exp(-B\,s)-1))$. For Poisson distribution with $Q(s)= e^{-\langle n\rangle s}$ we find from (7) that $f(x) = \delta(x-\langle n\rangle)$ while for NB it is[9] Gamma distribution $[B^k/\Gamma(k)]x^{k-1}e^{-Bx}$ corresponding to $P_n = [\Gamma(n+k)/(\Gamma(n+1)\Gamma(k))]\ \ B^n/(1+B)^{n+k}$. As expected the non-scaling behavior in the KNO limit of TCP distribution is like that of Poisson distribution. In any case the NB also does not scale strictly since k does depend on energy as the observations show.

The generating function $G(t)$ allows us to calculate efficiently the various moments[11] characteristic of TCP and NB distributions. For example, the raw moments about origin $\langle n^r\rangle = (d^r/dt^r)\ G(e^t)|_{t=0}$, the factorial cumulants or inclusive correlation integrals $f_r = (d^r/dt^r)\ln G(t)|_{t=1}$, the cumulant moments $\varkappa_r = (d^r/dt^r)\ln G(e^t)|_{t=0}$. The reduced moments C_r (or $C(r)$) are defined by $\langle n^r\rangle/\langle n\rangle^r$. We list some of these moments for comparison, $B = \langle n\rangle/k$,

TCP distribution:

$$C_2 = (Bk+B+1)/\langle n\rangle$$
$$C_3 = (B^2k^2+3B^2k+B^2+3Bk+3B+1)/\langle n\rangle^2$$
$$C_4 = (B^3k^3+6B^3k^2+7B^3k+B^3+6B^2k^2+18B^2k+6B^2+7Bk+7B+1)/\langle n\rangle^3$$
$$f_r = B^r k \qquad (11)$$

NB distribution:

$$C_2 = (Bk+B+1)/\langle n\rangle$$
$$C_3 = (B^2k^2+3B^2k+2B^2+3Bk+3B+1)/\langle n\rangle^2$$
$$C_4 = (B^3k^3+6B^3k^2+11B^3k+6B^3+6B^2k^2+18B^2k+12B^2+7Bk+7B+1)/\langle n\rangle^3$$
$$f_r = (r-1)!\ B^r k \qquad (12)$$

Poisson distribution:

$C_2 = (\langle n\rangle+1)/\langle n\rangle$

$C_3 = (\langle n\rangle^2+3\langle n\rangle+1)/\langle n\rangle^2$

$C_4 = (\langle n\rangle^3+6\langle n\rangle^2+7\langle n\rangle+1)/\langle n\rangle^3$

$f_r = 0$ for $r \geq 2$ (13)

The factorial cumulants f_r for TCP and NB furnish a measure of their deviations from Poisson distribution. It is clear that for reasonable values of $\langle n\rangle$ and not too large k both deviate from Poisson and for a given set of parameters TCP is narrower and closer to Poisson at he high multiplicity tail than NB. This is also evident from the presence of an extra factor $(r-1)!$ in the expression of f_r for NB. The reduced moments C_2 coincide for the two distributions. For $\langle n\rangle$ small and k large the TCP distribution tends towards a smooth Poisson like curve (see plots for various B and k). For the same set of parameters the TCP curves are narrower than those of NB at high multiplicity tail. We note also that $\langle n_c\rangle \geq 1$ tends to unity in the limit $\langle n\rangle/k \to 0$.

III. COMPARISON WITH EXPERIMENTAL DATA

The UA5 data for the full phase space as well as for fixed rapidity gaps can be very well parameterized using[1] NB (or TCP). A recent 'mature' analysis[12] of the old data shows the experimental curves to be narrower than the predictions of NB. In fact TCP curves as seen in the figures are narrower than NB in the region of high multiplicity tail. The charge conservation constraint must be taken into account for fitting the full phase space all charged distributions. From a distribution defined for n= 0,1,2,3..we may by ignoring odd multiplicities derive a modified probability distribution which allows for only the even multiplicities. The corresponding generating function may be written as

$$G_e(t) = \sum t^n P_{en} = \alpha \sum [1+(-1)^n] P_n t^n = \alpha [G(t)+G(-t)] \quad (14)$$

On demanding that $G_e(1) = 1$ we find $\alpha = 1/[1+G(-1)]$. The corresponding moment generating function is then given by $G_e(e^t) = \alpha [G(e^t) + G(e^{t+i\pi})]$. We may check by exact computation or use simple arguments to show that $\langle n^r \rangle_e = \alpha \langle n^r \rangle$ to very great accuracy both for TCP and NB. The inverse renormalization factor for TCP is $\alpha^{-1} = [1+ e^{-k[1-\exp(-2B)]}]$ while for NB it is given by $[1 + e^{-k\ln(1+2B)}]$. It can become quite appreciable if both B and k are small. In Tab.1 we compare the reduced moments C_r for full phase space data of UA5 with the predictions of TCP distribution. The agreement over the energy range from 10 GeV to 900 GeV is quite good. One verifies also that while fitting the data relatively small flexibility is allowed in the parameters B and k. The average number of clusters $\langle N \rangle$ is found to decrease with energy from about 6 to 3 contrary to the case of NB where it increases to around 8 at 900 GeV. With increasing energy TCP predicts less numerous and wider clusters with more energy content reminding us of the cosmic ray energy data.

The reduced moments C_2, C_3, C_4, C_5 computed from TCP show a smooth variation with parameters. For a given value of k they tend to flatten for large values of B while for smaller values of B they show a steep increase. Increasing the value of k lowers the reduced moments and the corresponding TCP distributions for $k \gg \langle n \rangle$ tend to become smooth Poisson like. Several KNO plots with varying parameters are presented here for a fixed k as well as for a fixed B. Drawn are also TCP curves at several energies comparing them with the best NB fits(1) as found in UA5 data. The predictions at 2, 20 and 40 Tev are also included by extrapolating the energy dependence(1) of $\langle n \rangle$ and k. The marked feature of TCP curves at higher energies is that they become narrower than NB ones at higher multiplicity points even though for lower multiplicity values they are slightly broader (up to 900 GeV) with shoulders and oscillations which become increasingly pronounced with energies as indicated by the predictions at 20 and 40 TeV. A careful analysis of the tails of the experimental distributions and on the presence of shoulders and oscillations, say, in 900 GeV and the forthcoming FERMILAB data at 1800 GeV may be warranted. As regards the (preliminary) UA5 data[1] with fixed rapidity gaps at 200, 546 and 900 GeV we compare in Tab.2 the reduced moments obtained from (11) and (12) corresponding to the quoted best fit NB parameters B,k (central values). The agreement with the quoted experimental data is satisfactory for both the distributions.

At lower energies where the e^+e^-[3], pion-proton[2], proton-proton[2], muon-proton[4] data has also been analysed in fixed symmetric as well as asymmetric rapidity gaps the TCP distribution is as suitable as NB for parameterizing the experimental data. We compare in Tabs.3-7 the reduced moments computed from (11) for TCP and from (12) for NB corresponding to the best fit NB parameters (central values) quoted in the cited references. For e^+e^- annihilation the two curves practically coincide, TCP staying closer to Poisson than NB (see remarks about f_r above). The same remark holds for the full phase space two jets and single jet data as well. Similar conclusions though to slightly lesser degree hold also for pp, pi-p and mu-p data. For none of the quoted best fit parameters do we find oscillations or shoulders in the TCP curves in the lower multiplicity region and which are predicted to be present for UA5 data at high energies discussed above. A detailed analysis of the forthcoming data at 1.8 TeV from FERMILAB and e^+e^- data at 50 GeV from KEK may be able to distinguish the two parameterizations under consideration.

IV. CONCLUSIONS:

TCP distribution seems to be a good alternative candidate for representing the mds. observed over the energy range from 10 to 900 GeV in hadroproduction data as well as e^+e^- annihilation and lepton-hadron collision data. It employs only Poisson distributions in its construction. It allows for shoulders and oscillations which become more and more pronounced in hadronic collisions as the energy grows ,assuming the extrapolation of the parameters(1) to be valid, making the distributions look 'narrower' compared to those of NB. For large values of k and <n> small TCP stays closer to Poisson than NB. A careful study of the high multiplicity tails and the presence of shoulders at lower multiplicity points in experimental data at higher energies may be useful to clarify if one or the other or something in between is favored.

ACKNOWLEDGEMENTS:

The author is grateful to Professor Abdus Salam, IAEA and UNESCO for the hospitality at ICTP under CNPq-ICTP agreement where this work was completed. Thanks are also due to Dr Renato P. Santos for his help in handling of the computer programs.

APPENDIX:

STIRLING NUMBERS OF SECOND KIND AND POLYNOMIALS $A_n(x)$.

Stirling numbers of the first kind S^m_n and the second kind ζ^m_n are defined(13) by

$$(x)_n = \sum_{m=o}^{n} S^m_n x^m \quad , \qquad x^n = \sum_{m=o}^{n} \zeta^m_n (x)_m$$

where $(x)_n = x(x-1)(x-2)\ldots\ (x-n+1)$. Those of the second kind are positive integers and satisfy the following properties

$$\zeta^m_{(n+1)} = \zeta^{(m-1)}_n + m\,\zeta^m_n$$

$\zeta^m_n = 0$ for $m > n$, $\zeta^0_n = \delta^0_n$, $\zeta^1_n = 1$, $\zeta^n_n = 1$, $\zeta^2_n = 2^{(n-1)} - 1$ etc.

and we may write $\zeta^m_n = \sum_{j=0}^{m} (-1)^{j-m}\, j^n / [(m-j)!\, j!]$.

The (Bell) polynomials $A_n(x)$ are defined by

$$A_n(x) = \sum_{m=0}^{n} \zeta^m_n x^m$$

We find $A_0(x)=1$, $A_1(x)= x$ etc. and derive the following recurrence relation $A_{(n+1)}(x) = x\,[A_n(x)+A'_n(x)]$ which may be used to generate the polynomials on the computer using REDUCE. Writing $A_n(x) = e^x u_n(x)$ we find $u_{(n+1)}(x) = x\, u_n'(x)$ which is seen to be satisfied by $u_n(x,L) = \sum_{j=0}^{L} (j^n/j!)x^j$. For large L,depending on the value of x, $u_n(x)$ may be computed quite accurately from $u_n(x,L)$. For L tending to infinity it goes over to $u_n(x)$. Clearly $u_n(x) = \sum (x^j/j!)(d^n/dt^n)e^{jt}|_{t=0} = (d^n/dt^n)\; e^{xe^t}|_{t=0}$ showing that $\mathrm{Exp}(xe^t)$ is the generating function of $u_n(x)$ while $\mathrm{Exp}(x[e^t-1])$ generates the polynomials $A_n(x)$.

FIGURE and TABLE CAPTIONS:

Figs. 1.1,1.2,1.3 Comparison of the mds. arising from TCP and NB distributions at 19.7,44.5,200,546,900,2000 and 40000 GeV for full phase space UA5 data. The extrapolation of the parameters is done using the energy dependence given in ref.1 and $z=n/\langle n\rangle$, $B=\langle n\rangle/k$. The Fig. 1.2 shows the best fit curves for NB (ref.1) and TCP at 546 and 900 GeV.

Fig. 1.4 Comparison of TCP distributions at 546,900 and 2000 GeV and at 20 and 40 TeV.

Fig. 2.1 Comparison of TCP curves for k=1.5,3.5,5.5 with a fixed value of B=2.0,5.0,8.0.

Fig. 2.2 Comparison of TCP curves for k=1.5,3.5,5.5 with fixed B=12.0and TCP distribution for B=32.0, k=2.0.

Fig. 2.3 Comparison of TCP curves for k=10,30 and 50 with fixed B=0.3.

Fig. 2.4 Comparison of TCP curves for B=2.0,12.0 with fixed k=1.5,3.5,5.5.

Figs. 3.1,3.2 Variations of reduced moments C_2,C_3,C_4,C_5 against B for different values of k as given by (11). For k=1.5 the C_4 curve must be multiplied by a factor of 3 while C_5 one by 12 while for k=3.5 the C_5 curve must be multiplied by a factor of 3.

Tab.1 Comparison of reduced moments $C_{2,3,4,5}$ obtained from TCP with UA5 data.

Tab.2-7 Comparison of reduced moments obtained from TCP and NB for various fixed rapidity intervals for experimentally quoted best fit NB parameters.

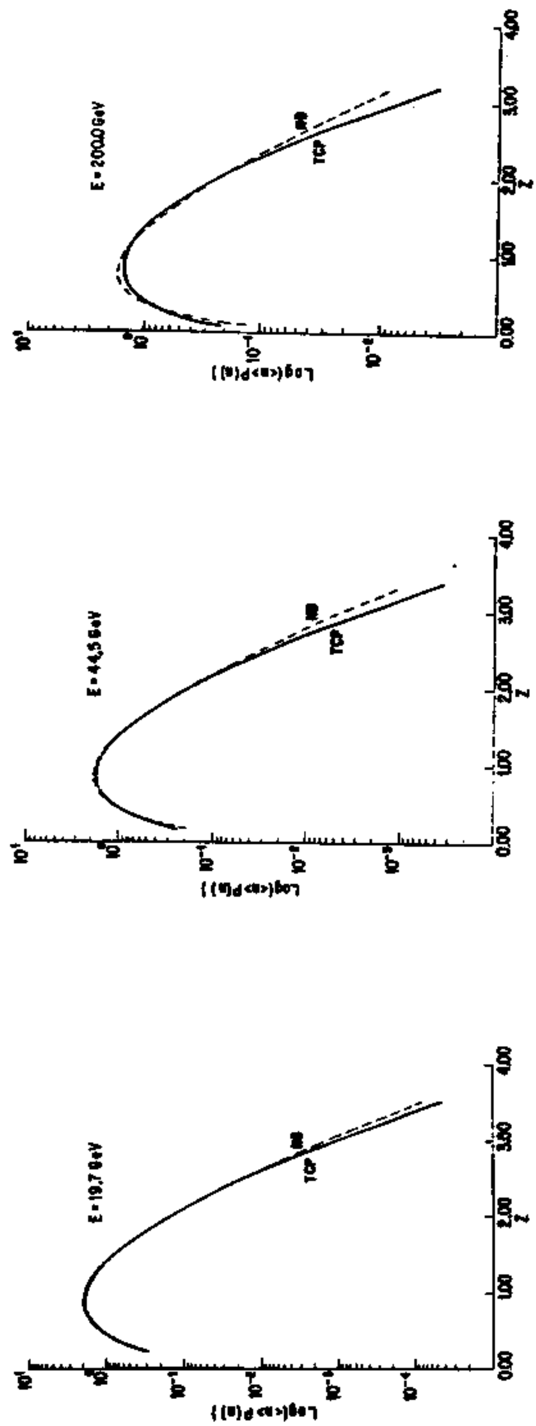


Fig. 1.1

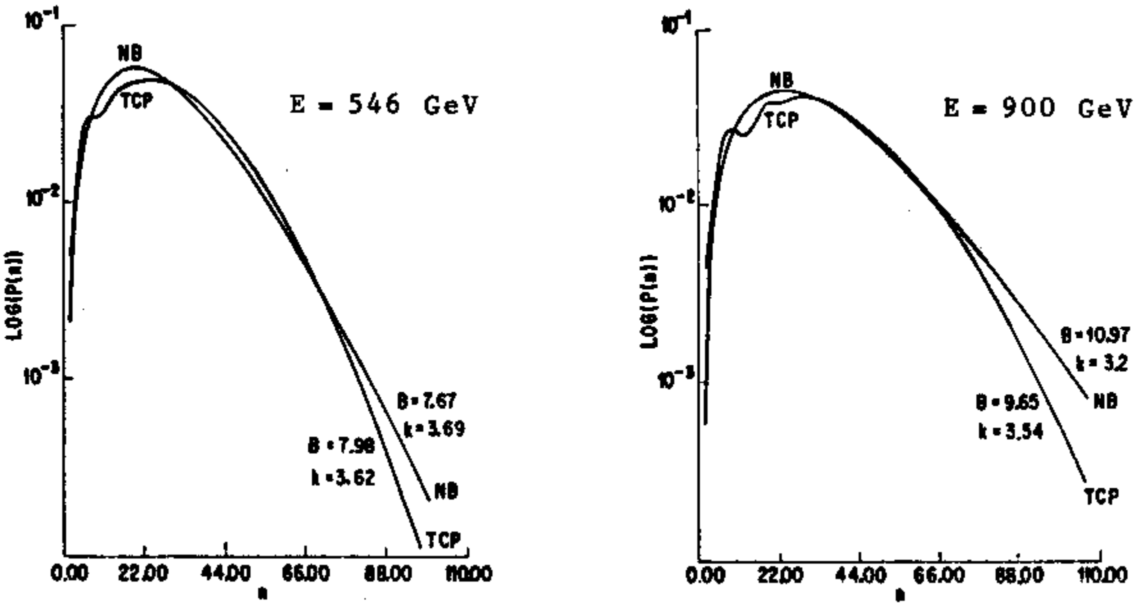

10⁻¹
NB
TCP
E = 546 GeV
10⁻²
LOG(P(n))
10⁻³
B = 7.67
k = 3.69
B = 7.98
k = 3.62
NB
TCP
0.00
22.00
44.00
66.00
88.00
110.00
n
10⁻¹
NB
TCP
E = 900 GeV
10⁻²
LOG(P(n))
10⁻³
B = 10.97
k = 3.2
NB
B = 9.65
k = 3.54
TCP
0.00
22.00
44.00
66.00
88.00
110.00
n

Fig. 1.2

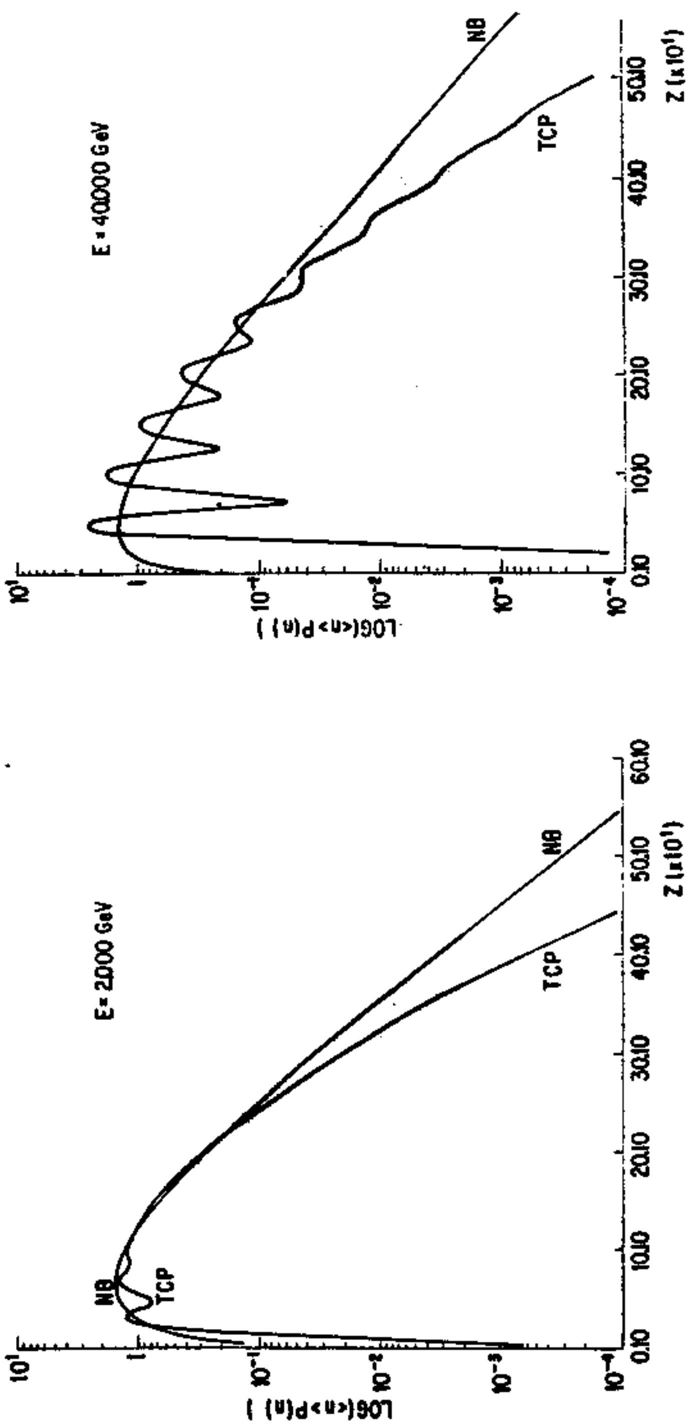
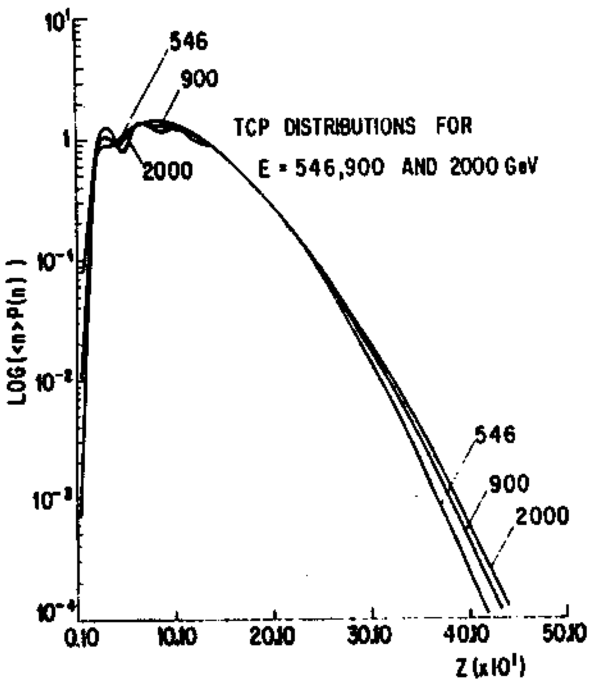


Fig. 1.3


TCP DISTRIBUTIONS FOR
E = 546,900 AND 2000 GeV
546
900
2000
LOG(<n>P(n))
Z (x10¹)

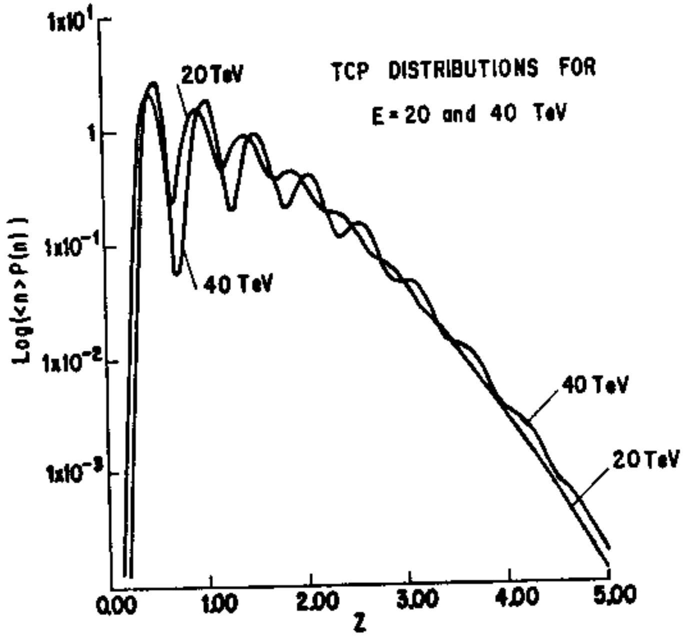

TCP DISTRIBUTIONS FOR
E = 20 and 40 TeV
20 TeV
40 TeV
Log(<n>P(n))
Z

Fig. 1.4

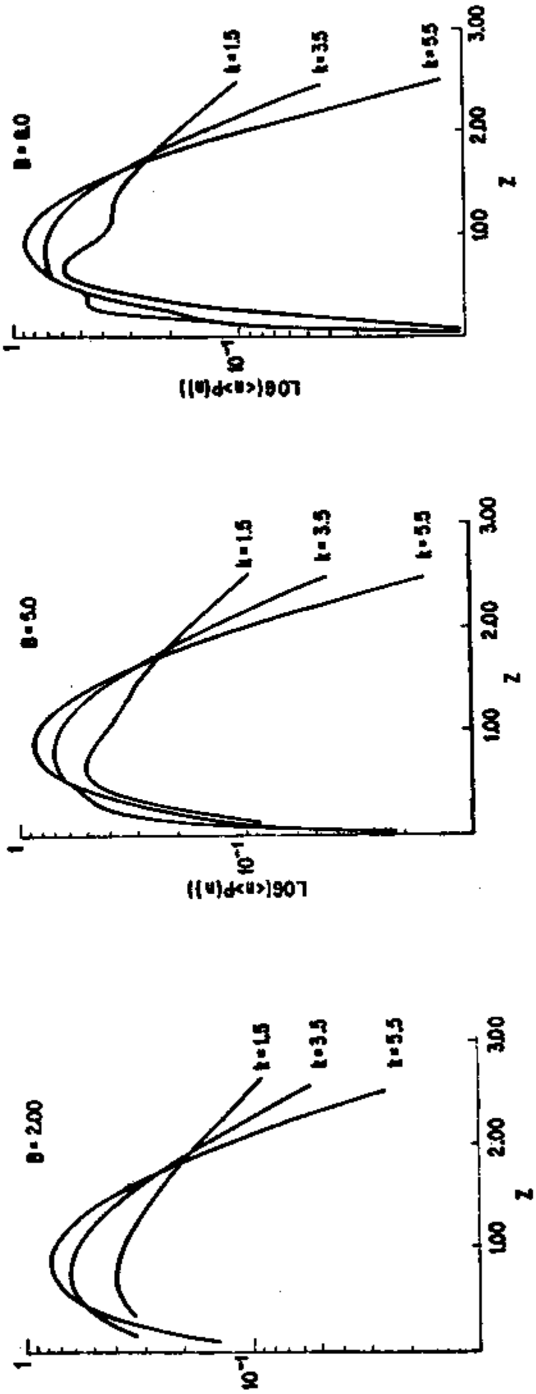
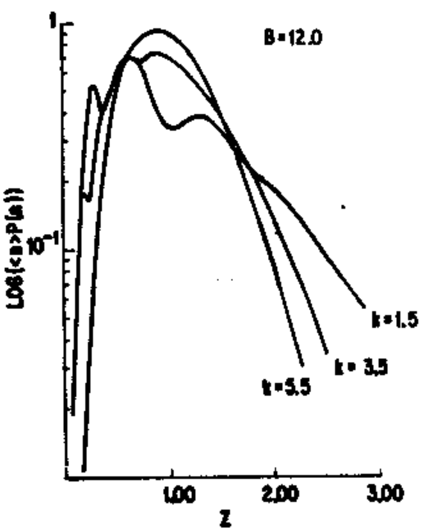


Fig. 2.1


B=12.0
1
10^{-1}
LOG(<n>P(n))
k=1.5
k=3.5
k=5.5
1.00
2.00
3.00
Z

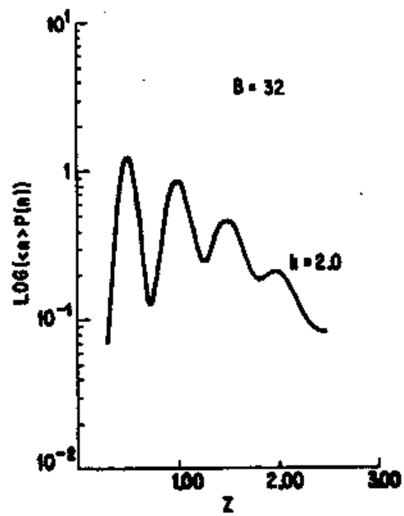

10^{1}
B = 32
1
LOG(<n>P(n))
k=2.0
10^{-1}
10^{-2}
1.00
2.00
3.00
Z

Fig. 2.2

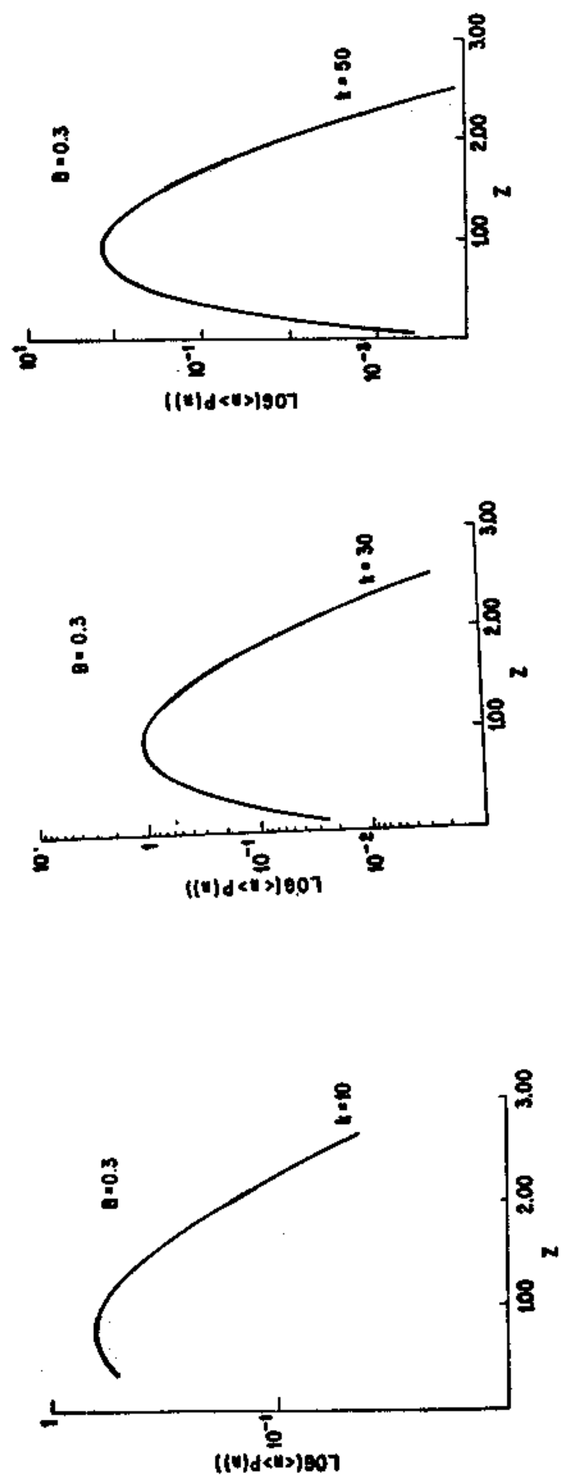


Fig. 2.3

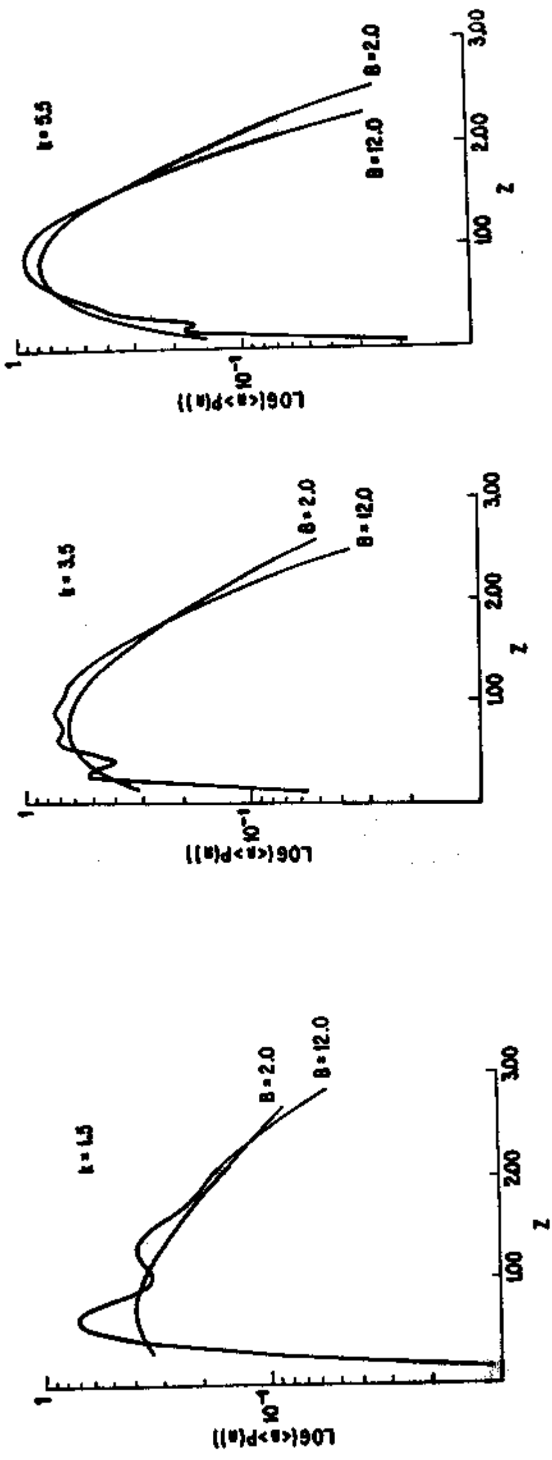
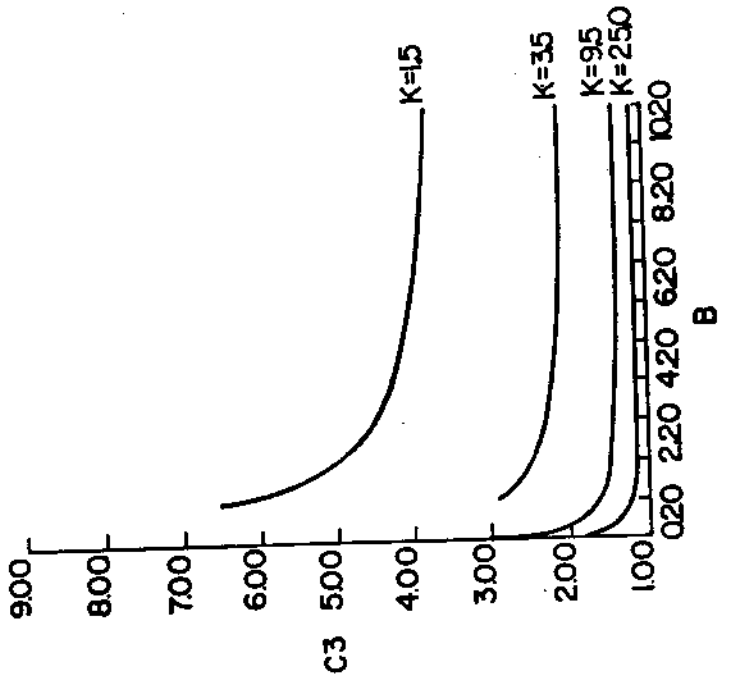

k = 5.5
B = 2.0
B = 12.0
LOG(<n>P(n))
1
10^{-1}
1.00
2.00
3.00
Z
k = 3.5
B = 2.0
B = 12.0
LOG(<n>P(n))
1
10^{-1}
1.00
2.00
3.00
Z
k = 1.5
B = 2.0
B = 12.0
LOG(<n>P(n))
1
10^{-1}
1.00
2.00
3.00
Z

Fig. 2.4


C3
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.20
2.20
4.20
6.20
8.20
10.20
B
K=1.5
K=3.5
K=9.5
K=25.0

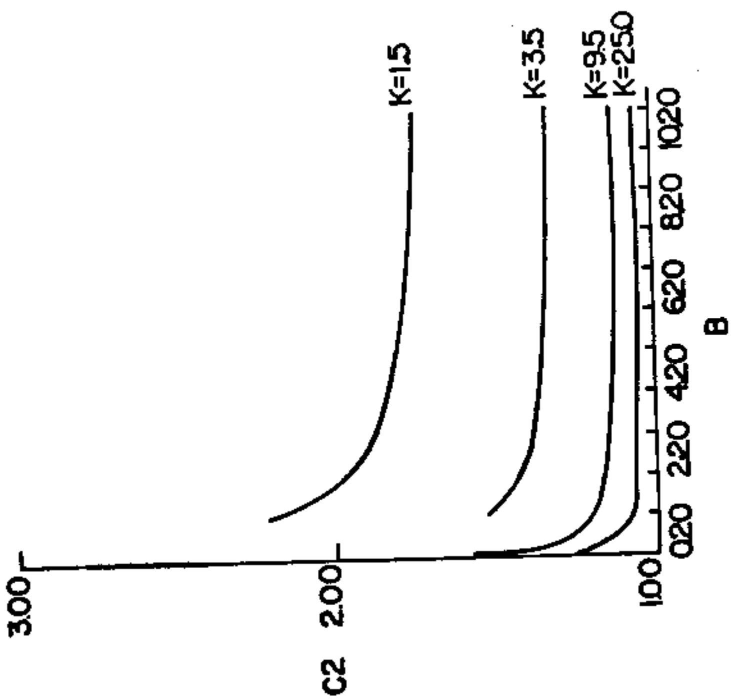

C2
3.00
2.00
1.00
0.20
2.20
4.20
6.20
8.20
10.20
B
K=1.5
K=3.5
K=9.5
K=25.0

Fig. 3.1

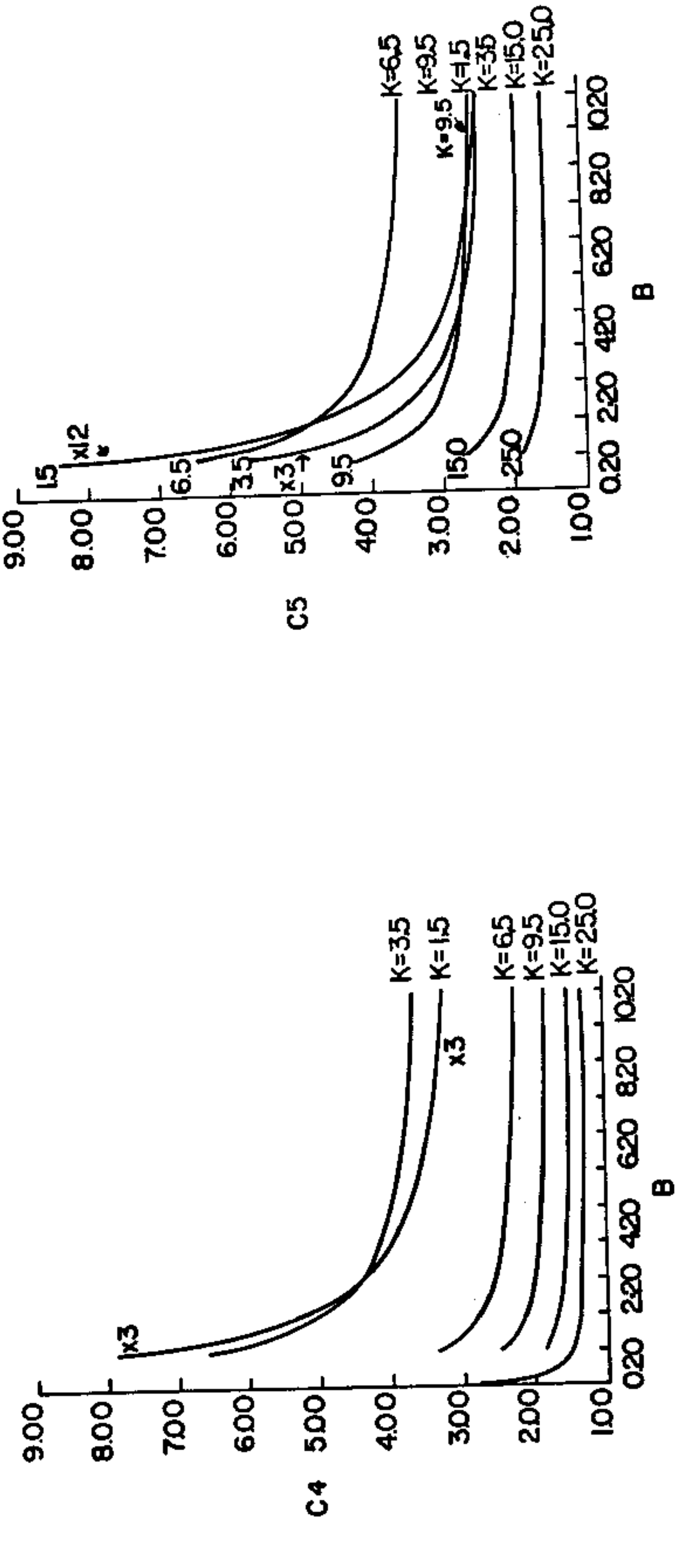


Fig. 3.2

$E_{c.m.}$ GeV	$\bar{n}$ (input) →	$B = \bar{n}/k$	$\bar{N}$	⟨n⟩	C_2	C_3	C_4	C_5
900	34.6	10.76	3.21	33.26 34.6±1.2	1.39 1.34±0.033	2.32 2.22±0.13	4.43 4.30±0.40	9.59 9.3±1.1
546	29.1	8.02	3.63	28.35 29.1±0.9	1.34 1.31±0.03	2.15 2.12±0.11	3.91 4.05±0.32	7.92 8.8±1.0
200	21.4	4.56	4.64	21.20 21.4±0.8	1.27 1.26±0.03	1.89 1.91±0.12	3.19 3.30±0.30	5.94 6.6±0.9
62.6	13.63	1.73	6.49	13.62 13.63±0.16	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.60±0.08	4.28 4.4±0.2
52.6	12.76	1.68	6.18	12.75 12.76±0.14	1.21 1.21±0.01	1.69 1.70±0.03	2.63 2.70±0.09	4.52 4.8±0.3
44.5	12.08	1.42	6.46	12.08 12.08±0.13	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.63±0.10	4.27 4.6±0.3
27.6	9.77	1.05	6.04	9.77 9.77±0.16	1.21 1.21±0.01	1.69 1.72±0.05	2.63 2.76±0.13	4.52 5.0±0.4
19.7	8.56	0.489	6.77	8.56 8.56±0.11	1.17 1.174±0.010	1.56 1.57±0.03	2.29 2.34±0.08	3.68 3.8±0.2
11.5	6.35	0.219	5.70	6.35 6.35±0.08	1.19 1.192±0.009	1.62 1.63±0.03	2.44 2.49±0.08	4.04 4.2±0.2

TABLE: 1

Comparison of moments C_q obtained from TCP distribution with experimental data[1] indicated below each computed value. $\bar{N}$ indicates the average number of clusters for each energy.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=2.48	k=1.7
1.991	1.991	
5.195	5.541	0.346
16.55	19.79	3.239
yo=1.0	<n>=5.1	k=2.0
1.696	1.696	
3.671	3.921	0.25
9.522	11.44	1.919
yo=1.5	<n>=7.8	k=2.3
1.563	1.563	
3.062	3.251	0.189
7.099	8.412	1.312
yo=3.0	<n>=15.4	k=3.3
1.368	1.368	
2.259	2.351	0.09183
4.307	4.849	0.5422
yo=5.0	<n>=20.5	k=4.3
1.281	1.281	
1.935	1.989	0.05409
3.32	3.615	0.2951

Tab.2.1: Comparison of reduced moments CT(I),eq.(11) and CN(I), eq.(12),I=2,3,4, for UA5 data for various rapidity intervals -yo<y<yo at 200 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=3.01	k=1.68
1.927	1.927	
4.84	5.195	0.3543
14.79	17.97	3.178
yo=1.0	<n>=6.17	k=1.81
1.715	1.715	
3.744	4.049	0.3052
9.791	12.15	2.361
yo=1.5	<n>=9.49	k=1.95
1.618	1.618	
3.291	3.554	0.263
7.943	9.835	1.893
yo=3.0	<n>=18.9	k=2.422
1.466	1.466	
2.636	2.807	0.1705
5.534	6.622	1.088
yo=5.0	<n>=26.4	k=3.166
1.354	1.354	
2.198	2.298	0.09976
4.104	4.683	0.5793

Tab.2.2: Same as Tab.2.1 but for UA5 data at various rapidity intervals -yo<y<yo at 546 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=3.51	k=1.56
1.926	1.926	
4.818	5.229	0.4109
14.64	18.3	3.663
yo=1.0	<n>=7.2	k=1.68
1.734	1.734	
3.824	4.178	0.3543
10.1	12.86	2.767
yo=1.5	<n>=11.0	k=1.84
1.634	1.634	
3.355	3.65	0.2954
8.175	10.32	2.145
yo=3.0	<n>=22.2	k=2.26
1.488	1.488	
2.72	2.916	0.1958
5.815	7.084	1.269
yo=5.0	<n>=32.4	k=3.1
1.353	1.353	
2.195	2.299	0.1041
4.09	4.693	0.6034

Tab.2.3:Same as Tab.2.1 but for UA5 data for various rapidity intervals -yo<y<yo at 900 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.1	<n>=0.403	k=4.85
3.688	3.688	
16.8	16.84	0.0425
94.54	95.39	0.8469
yo=0.5	<n>=2.12	k=7.32
1.608	1.608	
3.259	3.278	0.01865
7.871	8.011	0.1402
yo=1.0	<n>=4.27	k=10.56
1.329	1.329	
2.117	2.126	0.00897
3.882	3.935	0.05272
yo=1.5	<n>=6.65	k=15.95
1.213	1.213	
1.694	1.698	0.00393
2.651	2.672	0.0205
yo=2.0	<n>=8.97	k=26.18
1.15	1.15	
1.476	1.477	0.00147
2.078	2.085	0.0071
yo=2.5	<n>=10.83	k=57.18
1.11	1.11	
1.343	1.343	0.0003
1.752	1.753	0.00141

Tab.3: Same as Tab.2.1 but for e+e- annihilation for various rapidity intervals -yo<y<yo at 29 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
SINGLE JET <n>=6.43	k=7143.0	
1.156	1.156	
1.491	1.491	
2.107	2.107	
3.22	3.22	

Tab.3a: Same as Tab.3 but for the full phase space and single jet.

CT(I)	CN(I)	CN(I)-CT(I)
WHOLE EVENT <n>=12.87	k=256.4	
1.082	1.082	
1.252	1.252	0.00002
1.538	1.538	0.00006
1.994	1.995	0.0002

Tab.3b: Same as Tab.3 but for the full phase space and whole event.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=1.9	k=2.564
1.916	1.916	
4.794	4.946	0.1521
14.64	16.02	1.385
yo=1.0	<n>=3.72	k=3.125
1.589	1.589	
3.199	3.302	0.1024
7.683	8.422	0.7386
yo=1.5	<n>=5.42	k=4.0
1.435	1.435	
2.538	2.601	0.0625
5.264	5.661	0.3973
yo=3.0	<n>=9.06	k=12.99
1.187	1.187	
1.606	1.612	0.00592
2.416	2.446	0.02994
yo=3.0 ODD	<n>=7.19	k=17.24
1.197	1.197	
1.638	1.642	0.00336
2.5	2.517	0.01723

Tab.4: Same as Tab.2.1 but for pp data for various rapidity intervals $-yo<y<yo$ at 22 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=1.87	k=2.778
1.895	1.895	
4.677	4.807	0.1296
14.08	15.25	1.168
yo=1.0	<n>=3.67	k=3.571
1.552	1.552	
3.039	3.117	0.07841
7.073	7.624	0.5515
yo=1.5	<n>=5.42	k=4.274
1.419	1.419	
2.474	2.529	0.05476
5.045	5.389	0.3437
yo=3.0	<n>=9.53	k=16.13
1.167	1.167	
1.535	1.539	0.00384
2.231	2.25	0.019
yo=3.0 ODD	<n>=7.27	k=15.38
1.203	1.203	
1.658	1.662	0.00423
2.553	2.575	0.02176

Tab.5: Same as Tab.2.1 but for Pi-p data for various rapidity intervals -yo<y<yo at 22 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=1.57	k=4.717
1.849	1.849	
4.403	4.448	0.04491
12.72	13.12	0.3992
yo=1.0	<n>=3.12	k=6.135
1.484	1.484	
2.737	2.763	0.02657
5.952	6.131	0.179
yo=1.5	<n>=4.56	k=7.874
1.346	1.346	
2.187	2.203	0.01613
4.105	4.201	0.09598
yo=2.0	<n>=5.75	k=9.615
1.278	1.278	
1.929	1.94	0.01082
3.32	3.38	0.06017
yo=3.0	<n>=7.51	k=40.0
1.158	1.158	
1.503	1.503	0.00062
2.143	2.146	0.00308
yo=3.5	<n>=7.48	k=58.82
1.151	1.151	
1.477	1.477	0.00029
2.077	2.078	0.00141

Tab.6.1:Same as Tab.2.1 but for mu-p data for various rapidity intervals -yo<y<yo for energy interval 18-20 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=0.79	k=4.651
2.481	2.481	
7.907	7.954	0.04623
31.12	31.71	0.5857
yo=1.0	<n>=1.61	k=5.848
1.792	1.792	
4.11	4.139	0.02924
11.39	11.64	0.2509
yo=1.5	<n>=2.43	k=8.85
1.525	1.525	
2.895	2.908	0.01277
6.496	6.585	0.08983
yo=2.0	<n>=3.12	k=17.54
1.378	1.378	
2.293	2.297	0.00325
4.416	4.436	0.02015
yo=3.0	<n>=3.85	k=-166.7
1.254	1.254	
1.824	1.824	0.00003
2.982	2.982	0.00019
yo=3.5	<n>=3.96	k=-45.45
1.231	1.231	
1.739	1.74	0.00048
2.74	2.743	0.00263

Tab.6.2:Same as Tab.2.1 but for mu-p data for various rapidity intervals 0<y<yo for energy interval 18-20 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=0.77	k=4.785
2.508	2.508	
8.068	8.111	0.04369
32.05	32.61	0.5608
yo=1.0	<n>=1.49	k=4.115
1.914	1.914	
4.741	4.8	0.05905
14.31	14.85	0.5457
yo=1.5	<n>=2.13	k=3.731
1.737	1.737	
3.882	3.954	0.07183
10.47	11.06	0.5858
yo=2.0	<n>=2.66	k=4.098
1.62	1.62	
3.336	3.395	0.05955
8.22	8.665	0.4451
yo=3.0	<n>=3.43	k=14.29
1.362	1.362	
2.236	2.241	0.00491
4.241	4.271	0.02988
yo=3.5	<n>=3.49	k=16.67
1.347	1.347	
2.177	2.18	0.00361
4.053	4.075	0.02167

Tab.6.3:Same as Tab.2.1 but for mu-p data for various rapidity intervals -yo<y<0 for energy interval 18-20 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=1.47	k=14.71
1.748	1.748	
3.851	3.856	0.00463
10.15	10.19	0.0389
yo=1.0	<n>=2.85	k=18.18
1.406	1.406	
2.402	2.405	0.00302
4.763	4.782	0.01931
yo=1.5	<n>=4.12	k=34.48
1.272	1.272	
1.896	1.897	0.00085
3.203	3.207	0.00471
yo=2.0	<n>=4.84	k=-37.17
1.18	1.18	
1.566	1.567	0.00072
2.284	2.287	0.00369
yo=2.5	<n>=4.89	k=-29.41
1.171	1.171	
1.534	1.535	0.00116
2.199	2.204	0.00583

Tab.7.1:Same as Tab.6.1 but for the energy interval 6-8 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=0.74	k=10.99
2.442	2.442	
7.53	7.539	0.00829
28.41	28.51	0.1041
yo=1.0	<n>=1.48	k=21.28
1.723	1.723	
3.722	3.724	0.00221
9.587	9.605	0.01832
yo=1.5	<n>=2.1	k=-76.92
1.463	1.463	
2.598	2.598	0.00018
5.344	5.345	0.00113
yo=2.0	<n>=2.45	k=-19.61
1.357	1.357	
2.178	2.181	0.00261
3.967	3.984	0.01611
yo=2.5	<n>=2.6	k=-12.35
1.304	1.304	
1.972	1.978	0.00657
3.33	3.369	0.03874

Tab.7.2:Same as Tab.6.2 but for the energy interval 6-8 GeV.

CT(I)	CN(I)	CN(I)-CT(I)
yo=0.5	<n>=0.73	k=18.52
2.424	2.424	
7.373	7.376	0.00291
27.33	27.37	0.0365
yo=1.0	<n>=1.36	k=9.524
1.84	1.84	
4.304	4.315	0.01103
12.14	12.24	0.0985
yo=1.5	<n>=1.81	k=29.41
1.586	1.586	
3.122	3.123	0.00116
7.247	7.256	0.00865
yo=2.0	<n>=2.09	k=-22.22
1.433	1.433	
2.467	2.469	0.00201
4.873	4.887	0.01345
yo=2.5	<n>=2.14	k=-19.23
1.415	1.415	
2.394	2.397	0.00268
4.632	4.649	0.01769

Tab.7.3: Same as Tab.6.3 but for the energy interval 6-8 GEV.

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