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CHIRAL BOSONS: BOSONIC DESCRIPTION AND QUANTIZATION\*

by

Prem P. SRIVASTAVA

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq  
Rua Dr. Xavier Sigaud, 150  
22290 - Rio de Janeiro, RJ - Brasil

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**ABSTRACT**

The bosonic formulations of chiral bosons are reviewed. It is shown that the formulation based on the conventional Lagrange multiplier method poses no problem and it offers itself as a self-consistent bosonic formulation.

**Key-words:** Chiral bosons; Gauge theory quantization.

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## 1. INTRODUCTION

Chiral bosons are relevant in the construction of many string theory models<sup>1</sup>. It is advantageous to keep their bosonic description, instead of fermionizing them, for example, for the loop corrections. Also much attention has recently been drawn to chiral bosonization in connection with the left-right asymmetric fermionic theories, particularly for anomalous gauge theories, in two dimensions.

There are two bosonic formulations currently in use to describe chiral bosons. The Siegel's action<sup>2</sup>  $\mathcal{L} = \partial_- \phi \partial_+ \phi + \lambda (\partial_- \phi)^2$  describes chiral bosons through an auxiliary gauge field  $\lambda$ . The gauge symmetry becomes anomalous at the quantum level and the action must be modified<sup>3</sup> by adding a local counter term fine tuned to cancel the anomaly. This then makes its consistent coupling to gravity rather difficult. Alternatively, additional fields<sup>4</sup> and further constraints must be added to remove the anomaly. The other action,  $\mathcal{L} = \phi'(\dot{\phi} - \phi')$  proposed by Floreanini and Jackiw<sup>5</sup> does not require the auxiliary field at the price of not having the manifest Lorentz invariance. Moreover, the chiral boundary conditions are needed to be imposed, since, it is the space derivative of  $\phi$  which satisfies the self-duality condition and not the field itself. On quantizing the theory we find the violation of microscopic causality. In spite of this the theory has been coupled to gravity<sup>6</sup> and supergravity<sup>7</sup>.

## 2. CONSISTENT BOSONIC FORMULATION

We propose here to reconsider the conventional action and show that it poses no problem. A self-consistent bosonic formulation<sup>8</sup> of the chiral boson is obtained with no violation of the microcausality. The Lagrangian is  $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + B_\mu (\eta^{\mu\nu} + \epsilon^{\mu\nu}) \partial_\nu \phi$  where  $B_\mu$  is a vector multiplier field. The eqn. of motion  $(\eta^{\mu\nu} + \epsilon^{\mu\nu}) \partial_\nu \phi = 0$  indicates that  $\phi$  is self-dual and leads to  $\partial_\mu \partial^\mu \phi = 0$  but the other one

$(\eta^{\mu\nu} + \epsilon^{\mu\nu}) \partial_\nu B_\mu = 0$  does not lead to  $\partial_\mu \partial^\mu B_\nu = 0$  for all the components of the multiplier field. We may not declare, a priori,  $B_\mu$  to be a dynamical field. It is necessary to follow, say, Dirac's procedure<sup>9</sup> (or Faddeev and Jackiw's method<sup>10</sup>) to implement all the constraints and then count in the reduced phase space the dynamical degrees of freedom. The situation is analogous to that found in the Yang-Mills theory where the Lagrange multiplier field  $A_0$  is required to enforce the Gauss' law constraint and the Lagrange eqns. do contain time derivative of this field.

On choosing a convenient metric and indicating by  $p_\lambda$  and  $\Pi \equiv \Pi_0 = \dot{\phi} + \lambda$  (where  $\Pi^\mu = \partial \mathcal{L} / \partial (\partial_\mu \phi)$ ) the canonical momenta corresponding to  $\lambda = B_0 + B_1$  and  $\phi$  respectively, the constraints in the theory are derived to be  $T \equiv \Pi - \phi' - \lambda \cong 0$ ,  $p_\lambda \cong 0$ . They are clearly second class<sup>9</sup> and the Dirac bracket<sup>8</sup> with respect to them is given by

$$\begin{aligned} \langle f(x,t), g(y,t) \rangle^* &= \langle f, g \rangle + \\ & 2 \iint dz du \partial_z \delta(z-u) \langle f, p_\lambda(z,t) \rangle \langle p_\lambda(u,t), g \rangle + \int dz \\ & [ \langle f, p_\lambda(z,t) \rangle \langle T(z,t), g \rangle - \langle T \leftrightarrow p_\lambda \rangle ] . \end{aligned}$$

The constraints now may be set as strong relations which remove  $p_\lambda$ , express  $\lambda$  as a dependent variable and the chiral (anti-chiral) boson is characterized by  $\Pi_- = \Pi_0 - \Pi_1 = 0$  ( $\Pi_+ = 0$ ). The Dirac brackets of  $\phi$  and  $\Pi$  coincide with the standard Poisson brackets showing no violation of microcausality and the Lagrangian is found to be  $\Pi(\dot{\phi} - \phi')$ .

It is also possible to build a gauge theory<sup>11</sup> of chiral boson with only the first class constraints if we add<sup>12</sup> to the theory above an auxiliary Wess-Zumino field  $\theta$  with the action  $\mathcal{L} = -\frac{1}{2} \partial_\mu \theta \partial^\mu \theta + \theta (\eta^{\mu\nu} + \epsilon^{\mu\nu}) \partial_\nu B_\mu$  in order to enforce  $\Pi_- = 0$ . The two versions lead to the same effective action on quantization<sup>11</sup>. The coupling to gravitation is straight-forward.

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