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BRST-BFV QUANTIZATION OF CHIRAL SCHWINGER MODEL*

by

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ABSTRACT

The BRST-BFV procedure of quantization is applied to establish, in a gauge independent manner, the equivalence of the gauge noninvariant and gauge invariant formulations of the Chiral Schwinger model.

Key-words: Anomaly; Schwinger model; Gauge theory quantization.

1. INTRODUCTION

The quantization of anomalous gauge theories involving chiral fermions has drawn much interest recently. For the case of Chiral Schwinger model (CSM) in two dimensions Jackiw and Rajaraman [1] showed that inspite of it being a potentially anomalous theory it is possible to quantize it consistently. Faddeev and Shatashvili [2] have suggested the modification of the canonical quantization by addition of new degrees of freedom through a Wess-Zumino action. At the present two formulations of CSM are available: the gauge noninvariant one [1] and the gauge invariant one [3,2].

Recently also the well known procedure of quantizing a gauge theory making use of BRST [4] symmetry has been further extended [5] and has been generalized into an Osp(1,1|2) symmetry over an extended phase space [6,5]. This ensures the cancellation of the unphysical degrees of freedom by the Parisi-Sourlas mechanism [7] so that the final result is the reduced phase space theory. In the present paper we will use this procedure to establish the equivalence of the two versions of the CSM mentioned above. We will show that they lead to the same effective action after we have functionally integrated over the variables in the unphsyical sector. The formulation does not require the computation of the Dirac brackets and the result is independent of the choice of the gauge-fixing fermion introduced in the theory [5,8]. The gauge independance of the result is thus automatically implemented. The result is in agreement with the one derived using canonical quantization [3] and the discussion parallels the one made recently in the context of a consistent bosonic formulation of chiral boson [9].

2. Chiral Schwinger Model

The Chiral Schwinger model is defined by the classical action

$$\int d^2x \left[- \frac{1}{2} F_{\mu\nu}F^{\mu\nu} + \bar{\psi} \gamma^{\mu} \left\{ i \partial_{\mu} + e \sqrt{\pi} \ A_{\mu}(1 - \gamma_5) \right\} \psi \right. \tag{1}$$

which is manifestly gauge invariant under the transformations $\delta A_{\mu} = \partial_{\mu}\alpha$, $\delta \psi = i \sqrt{\pi} \ e \ \alpha \ (1-\gamma_5) \ \psi$. The functional integral over fermions may be done explicitly and it introduces in the resulting effective action a free parameter 'a' due to the ambiguity of the regularization of the fermionic determinant. The gauge invariance is lost and anomalies are present [1]. We may ,however, obtain a gauge invariant action if we add [11] in the theory a Wess-Zumino [10] field such that its presence in the action makes the absence of genuine anomalies in the theory transparent. The effective Lagrangians corresponding to the two versions of CSM are written as

$$\mathcal{L}^{N} = - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} \partial_{\mu}\phi \ \partial^{\mu}\phi + e \ A_{\nu} \ (\eta^{\mu\nu} - \varepsilon^{\mu\nu}) \ \partial_{\mu}\phi + \frac{e}{2} \ a \ A_{\mu} \ A^{\mu} \tag{2}$$

and

$$\mathcal{L}^{I} = \mathcal{L}^{N} + \mathcal{L}^{WZ} \tag{3}$$

where

$$\mathcal{L}^{WZ} = \frac{1}{2} \ (a-1) \ \partial_{\mu}\theta \ \partial^{\mu}\theta - e \ A_{\nu} \ [(a-1) \ \eta^{\mu\nu} + \varepsilon^{\mu\nu}] \ \partial_{\mu}\theta \tag{4}$$

They have been re-expressed in local form by introducing an auxiliary field ϕ to remove the nonlocal terms and N stands for gauge noninvariant while I for gauge invariant. An additional Wess-Zumino field θ appears in $\mathcal{L}^{I}$ and the action (3) is invariant under the gauge transformation: $\delta A_{\mu} = -\frac{1}{e}\partial_{\mu}\omega$, $\delta \phi = \omega$, $\delta \theta = - \omega$.

The theory is shown [1] to be unitary and Lorentz invariant for a>1 with a massive ($m^2 = e^2a^2/(a-1)$) and a massless degree of freedom and for a=1 we obtain a free field theory.

3. BRST-BFV Quantization

(a) *The Case $a > 1$*

Consider first the gauge invariant formulation (3). The canonical momenta corresponding to A_0, A_1, ϕ and θ are indicated by Π_0, $E = F_{01} = \dot{A}_1 - A'_0$, $\Pi = \dot{\phi} + e\,(A_0 - A_1)$ and $\Pi_\theta = (a-1)\,\dot{\theta} - e\,[(a-1)\,A_0 + A_1]$ respectively. Here an overdot indicates the time derivative while a prime the space derivative and we adopt $\eta_{\mu\nu} = \mathrm{diag}(1,-1)$, $\varepsilon^{01} = 1$. The primary constraint is $\Pi_0 \approx 0$ and the canonical Hamiltonian is found to be

$$\mathcal{H}_c = \frac{1}{2}\left[\Pi - e\,(A_0 - A_1)\right]^2 + \frac{1}{2(a-1)}\left[\Pi_\theta + e\left\{(a-1)\,A_0 + A_1\right\}\right]^2$$

$$+ \frac{1}{2}\left[E^2 + \phi'^2 - e^2\,a\,(A_0^{\,2} - A_1^{\,2}) + (a-1)\,\theta'^2\right] + E\,A'_0$$

$$- e\,\phi'\,(A_0 - A_1) - e\,\theta'\left[A_0 + (a-1)\,A_1\right] \tag{5}$$

Following the Dirac's procedure [12] and requiring the persistency in time of the primary constraint we derive one additional secondary constraint $T \equiv E' + e\,(\Pi - \Pi_\theta + \theta' + \phi')$ and the two constraints are first class [3]. For the sake of simplicity, though without any loss of generality, we will fix the gauge $A_0 \approx 0$ (and $\dot{A}_0 \approx 0$) to take care of the (trivial) first class constraint $\Pi_0 \approx 0$. The Dirac brackets of the remaining variables with respect to this set of constraints evidently coincide with the standard Poisson brackets. The gauge invariant theory is then described by $\mathcal{H}_o = \mathcal{H}_c|_{A_o=0}$ together with a single first class constraint $T \approx 0$.

Following the BFV procedure [5] the Lagrange multiplier field λ required to enforce this constraint along with the corresponding canonical momentum Π_λ are now treated as dynamical fields over an extended phase space to which we add also the fermionic dynamical ghost fields η, $\bar{\eta}$ along with the correspnding canonical momenta P, $\bar{P}$. The non vanishing equal time graded Poisson brackets of the extended phase space variables are defined by

$$\{P , \bar{\eta}\} = \{\bar{P} , \eta\} = \{\Pi_\lambda , \lambda\} = \{\Pi , \phi\} = \{\Pi_\theta , \theta\} = -1 \tag{6}$$

The BRST charge which is nilpotent and conserved is found to be $\Omega = P\,\Pi_\lambda + \eta\, T$ while the anti-BRST charge is $\bar{\Omega} = -\bar{P}\,\Pi_\lambda + \bar{\eta}\, T$.

We now construct the following effective action [5]

$$S_{eff} = \int d^2x \left[\Pi\,\dot{\phi} + \Pi_\theta\,\dot{\theta} + \Pi_\lambda\,\dot{\lambda} + E\,\dot{A}_1 + \dot{\eta}\,\bar{P} + \dot{\bar{\eta}}\,P - \mathcal{H}_o + \{\Omega , \Psi\} \right] \tag{7}$$

where Ψ is an arbitrary gauge-fixing fermionic operator. The quantized theory is then obtained from the following functional integral

$$Z = \int [d\,\mu]\ \exp\,(i\,S_{eff}) \tag{8}$$

where $[d\mu]$ is the Liouville measure of the extended phase space, $[d\mu] = [d\phi]\,[d\Pi]\,[d\theta]\,[d\Pi_\theta]\,[d\lambda]\,[d\Pi_\lambda]\,[dA_1]\,[dE]\,[d\eta]\,[d\bar{P}]\,[d\bar{\eta}]\,[dP]$, and a normalization factor in (8) is understood. A convenient choice for Ψ is found to be $\Psi = \frac{1}{\beta}\theta\,\bar{\eta} + \bar{P}\,\lambda$ where β is a parameter and we find

$$\left\{\Omega,\ \Psi\right\} = -\frac{1}{\beta}\,\theta\,\Pi_\lambda - \lambda\,T - P\,\bar{P} + \frac{e}{\beta}\,\eta\,\bar{\eta} \tag{9}$$

The contribution of the ghosts to the functional integral is a field independent factor. A functional integration over Π_λ brings down the delta functional $\delta\,(\dot{\lambda} - \frac{1}{\beta}\,\theta)$. Integrating over θ, (a factor β originating from the delta functional is cancelled by $1/\beta$ arising from integral over the ghosts), and making $\beta \to 0$ leaves us with the following effective action

$$S_{eff} = \int d^2x\,\Big[\,\Pi\,\dot{\phi} + E\,\dot{A}_1 - \frac{1}{2}\,(\Pi + e\,A_1)^2 - \frac{1}{2(a-1)}\,(\Pi_\theta + e\,A_1)^2 - \frac{1}{2}\,(\,E^2 + \phi'^{\,2} + e^2\,a\,A_1^{\,2}) - e\,\phi'\,A_1 - \lambda\,\Big\{\,E' + e\,(\Pi - \Pi_\theta + \phi')\,\Big\}\,\Big] \tag{10}$$

Performing a shift transformation $\Pi_\theta \to \left\{\,\Pi_\theta - e\,A_1\,/\,\sqrt{(a-1)} + e\,\lambda\,\sqrt{(a-1)}\right\}$ we obtain a Gaussian integral over Π_θ which gives rise to a numerical factor which is absorbed in the normalization. The effective action after a rescaling of $\lambda \to \lambda/[e\,\sqrt{(a-1)}]$ reduces to

$$S_{eff} = \int d^2x\,\Big[\,\Pi\,\dot{\phi} + E\,\dot{A}_1 - \frac{1}{2}\,(\Pi + e\,A_1)^2 - \frac{1}{2}\,(\,E^2 + \phi'^{\,2} + e^2\,a\,A_1^{\,2}) - e\,\phi'\,A_1$$

$$+ \frac{1}{2} \lambda^2 - \lambda \left\{ \frac{1}{e} E' + \Pi + \phi' + e A_1 \right\} / \sqrt{(a-1)} \Bigg] \tag{11}$$

A shift transformation on λ followed by a Gaussian functional integration finally leads to the following effective Hamiltonian over the remaining variables

$$\mathcal{H}_{eff} = \frac{1}{2} (\Pi + e A_1)^2 + \frac{1}{2} (E^2 + \phi'^2 + e^2 a A_1^2) + e \phi' A_1 + \frac{1}{2} e^2 (a-1) (A_0)^2 \tag{12}$$

where $A_0 \equiv -\frac{1}{e (a-1)} \left[\frac{1}{e} E' + \Pi + \phi' + e A_1 \right]$ and the operator ordering ambiguities must be taken care of. The result is independent of the choice of the gauge-fixing fermionic function Ψ as follows from the Fradkin-Vilkovisky theorem [5,8] and agrees with the result following from the canonical quantization [3].

The gauge noninvariant case may be similarly treated The corresponding canonical Hamiltonian is obtained by dropping the second term in (5) and setting $\theta=0$. The two constraints are now second class [13]. It is easily seen that we may conveniently rewrite them as

$$G \equiv E' + e [\Pi + \phi' + e (a-1) A_0 + e A_1] \approx 0,$$

$$F \equiv - \frac{1}{e^2 (a-1)} \Pi_0 \approx 0 \tag{13}$$

so that

$$\{G, F\} = -1, \qquad \{G, G\} = \{F, F\} = 0 \tag{14}$$

The auxiliary fields λ, Π_λ in the present case are identified with the Lagrange multipliers required to enforce this pair of constraints of the reduce phase space theory [14]. The local transformations generated by G lead to $\delta\ \phi =$ eu etc. which result in $\delta\ \mathcal{H}_c \neq 0$.

For the nilpotent BRST operator we take the symmetric form [14]

$$\Omega = \frac{1}{\sqrt{2}} \left[\eta\ (G + \lambda) + \mathcal{P}\ (F + \Pi_\lambda) \right] \tag{15}$$

The extended phase space contains now ϕ, A_0, A_1, λ, $\bar{\eta}$, η, Π, Π_0, E, Π_λ, $\mathcal{P}$, $\bar{\mathcal{P}}$. They satisfy an algebra analogous to that in (6).

The effective action now takes the form

$$S_{eff} = \int d^2x \left[\Pi\ \dot{\phi} + \Pi_0 \dot{A}_0 + E\ \dot{A}_1 + \Pi_\lambda\ \dot{\lambda} + \dot{\eta}\ \bar{\mathcal{P}} + \dot{\bar{\eta}}\ \mathcal{P} - \mathcal{H}_0 + \{\Omega\ ,\Psi\} \right] \tag{16}$$

and the path integral (8) is defined over the phase space under consideration here.

A convenient choice for Ψ now is $\Psi = \sqrt{2} (\frac{1}{\beta}\ F\ \bar{\eta} + \bar{\mathcal{P}}\ \lambda)$ where β is an arbitrary parameter. We find

$$\{\Omega,\ \Psi\} = -\lambda\ (G + \lambda) - \frac{1}{\beta}\ F\ (F + \Pi_\lambda) + \mathcal{P}\ \bar{\mathcal{P}} - \frac{1}{\beta}\ \eta\ \bar{\eta} \tag{17}$$

The ghost integration contributes a numerical factor while the integration over Π_λ brings down a delta functional $\delta(\frac{1}{\beta}$ $F - \dot{\lambda})$ which allows us to integrate functionally over Π_0 and in the limit $\beta \to 0$ we obtain

$$S_{eff} = \int d^2x \left[\Pi\, \dot{\phi} + E\, \dot{A}_1 + \frac{1}{2}\, e^2(a-1)\, A_o^2 + \left\{ K - \lambda\, e^2(a-1) \right\} A_o - \lambda\, K - \lambda^2 - \mathscr{H}_o \; |_{A^o=0} \right] \tag{18}$$

where $K \equiv G \; |_{a=1}$. Integration over A_o is done by making a shift transformation as usual and we obtain

$$S = \int d^2x \left[\Pi\, \dot{\phi} + E\, \dot{A}_1 - \frac{K^2}{2\, e^2\, (a-1)} - \frac{1}{2}\, \lambda^2 \left\{ 2 + e^2(a-1) \right\} - \mathscr{H}_o \; |_{A^o = 0} \right] \tag{19}$$

which leads again to the same result (12) on integrating over λ.

(b) *The Case a = 1*

For a = 1 we easily derive from the equations of motion following from (2) that $\epsilon^{\mu\nu}F_{\mu\nu} \equiv 2\, E = 0$ and that A_μ is no more independent being given by

$$A_\mu = - \frac{1}{e}\, (\eta^{\mu\nu} - \epsilon^{\mu\nu})\, \partial_\nu \phi = \epsilon_{\mu\nu}\, A^\nu \tag{20}$$

It follows from (20) that $e\, \partial_\mu A^\mu = -\, \partial_\mu \partial^\mu\, \phi = 0$ implying that ϕ is a free field. The result may also be derived by making use of the functional integral. We briefly mention the salient points. Following the Dirac's method we obtain one (primary) first class constraint $\Pi_o \approx 0$,which generates the time independent gauge transformations of the action and two (secondary) second class constraints $K \equiv G|_{a=1} \approx 0$ and

$E \approx 0$. It is worth pointing out that requiring the persistency in time of these constraints does not determine the Lagrange multiplier u required to enforce $\Pi_o \approx 0$ in the extended Hamiltonian arising in the context of the Dirac's method. We may then follow the procedure described above for the first and second class constraints to write the functional integral. Alternatively, we may add to the set a gauge-fixing condition $A_o \approx 0$ so that we are left with only the set of second class constraints. The requirement of the persistency in time of $A_o \approx 0$ determines u to vanish and thereby reduces the extended Hamiltonian to $\mathcal{H}_o = \mathcal{H}_c|_{A^o=0}$. The functional integration over the extended phase space are done as above and the effective action is easily shown to be that of the free action for the field ϕ.

For the gauge invariant version we obtain two first class $\Pi_o \approx 0$, $E' + e\,(\,\Pi - \Pi_\theta + \theta' + \phi'\,) \approx 0$ and two second class $E \approx 0$, $\Pi_\theta + e\,A_1 \approx 0$ constraints. We may, if we wish, simplify the calculation as in the previous paragraph by adding the gauge condition $A_o \approx 0$ (and $\dot{A}_o \approx 0$). It is evident, *without making the actual computation*, that the Dirac brackets with respect to the set Π_o, A_o, E, $\Pi_\theta + e\,A_1$ of second class constraints of the remaining variables ϕ, Π, θ, Π_θ coincide with their Poisson brackets. We are then left with a gauge theory with the (nontrivial) first class constraint $(\,\Pi - \Pi_\theta + \theta' + \phi'\,) \approx 0$ and $\mathcal{H}_o = \frac{1}{2}\,(\Pi - \Pi_\theta)^2 + \frac{1}{2}\,\phi'^{\,2} + \frac{1}{2}\,\Pi_\theta^{\,2} - \Pi_\theta\,\phi'$ when we set the second class constraints equal to zero as strong relations which in its turn removes all the dependence on the gauge field A_μ. Recalling the arbitrary nature of the gauge fermion we may take it to be $\Psi = \frac{1}{\beta}\,\theta\,\bar{\eta} + \bar{P}\,\lambda$. Proceeding as in the earlier case it is easily shown that the final result after the functional integrations over θ and Π_{θ} is again the free action for the field ϕ.

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